NBER WORKING PAPER SERIES

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Working Paper 31852
http://www.nber.org/papers/w31852

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
November 2023

We thank Luigi Boccola, Antonio Coppola, Jeff Frieden, Brent Neiman, Stephen Redding, and Chenzi Xu for helpful comments. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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November 2023
JEL No. F02,F10,F5

ABSTRACT

Governments use their countries' economic strength from existing financial and trade relationships to achieve geopolitical and economic goals. We refer to this practice as geoeconomics. We build a framework based on three core ingredients: input output linkages, limited contract enforceability, and externalities. Geoeconomic power arises from the ability to jointly exercise threats arising from separate economic activities. Being able to retaliate against a deviating country across multiple arenas, often involving indirect threats from third parties also being pressured, increases the off equilibrium threats and, thus, helps in equilibrium to increase enforceability. A world hegemon, like the United States, exerts its power on firms and governments in its economic network by asking these entities to take costly actions that benefit the hegemon. We characterize the optimal actions and show that they take the form of mark-ups on goods or higher rates on lending, but also import restrictions and tariffs. The input-output amplification makes controlling some sectors more valuable for the hegemon since changes in the allocation of these strategic sectors have a larger influence on the world economy. This formalizes the idea of economic coercion as a combination of strategic pressure and costly actions. We apply the framework to two leading examples: national security externalities and the Belt and Road Initiative.

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1 Introduction

Governments use their countries' economic strength from existing financial and trade relationships to achieve geopolitical and economic goals. We refer to this topic as geoeconomics. We build a framework to understand the role of geoeconomics in shaping global real and financial activity. In this era of competition between US and China, we aim to provide a model to conceptualize how the great powers use their financial and economic strength to extract economic and political surplus from countries around the world.

Geoeconomic power is a form of soft indirect power. It is not as blunt as the direct threat to go to war, as it operates through commercial channels like the interruption of the supply or purchase of goods, the sharing of technology, or financial relationships and services. At the other extreme, this power operates in areas in which complete contracts are not feasible either because of very limited enforceability or because for political and legal reasons formal contracts are unpalatable. For example, government to government relationships take this nature due to the limited presence of courts with the power to adjudicate disputes.

We consider a collection of countries and productive sectors with an input-output network structure. The model features both production and consumer externailities and limited enforceability of contracts. Geoeconomic power arises from the ability of a country to consolidate disparate threats across multiple economic relationships, often with some of the threats carried out by third party entities also being pressured, to induce a target to take a desired action. We refer to countries that exerts such power as hegemons. We characterize when these threats are valuable and how the hegemon extracts the value from the target countries.

We model threats as trigger strategies that firms and governments can employ to punish other entities for deviating from contracts. For example, a supplier of a good might commit to not supply the good again to a customer who did not pay for an earlier shipment. A lender might withhold future financing from a borrower who defaulted on a loan. Joint threats are trigger strategies in which the trigger can be based on multiple economic relationships. For example, a hegemon can threaten to withhold future financing if the recipient country either defaults on a loan or breaches the contract for importing intermediates. A hegemon is characterized by its ability to coordinate many such threats both via its national entities and via their economic network abroad.

Many threats are either not feasible or not valuable in equilibrium. A threat may not be feasible in the sense that a particular hegemon does not control the economic relationship either directly or indirectly. Even if the threat can be made, it might not be valuable. First, the entity making the threat might be offering an input that can easily be sourced elsewhere. Second, many economic relationships have sufficiently high enforceability that they occur at their unconstrained scale to begin with. Joint threats on such activities generate no value. We show that the value arises, instead, by combining activities that are differentially constrained. In this case, joint threats use the economic value of each activity as an endogenous cost of default on the other activity. For example, sovereign lending might be completely unenforceable on its own, but might be viable in equilibrium if occurring jointly with manufacturing exports or military supplies, even if the latter are subject to expropriation risk.

We embed in the input-output structure both production and consumer externalities. Production externalities occur because an individual sector's productivity can depend of what other sectors are producing both within and across countries. These can capture both traditional economic forces such as economies of scale and endogenous technology innovation, and also externalities that are directly connected to geoeconomics such as national security. Production externalities can capture forces like network effects: if the adoption of a firm's information technology infrastructure by one country makes it more appealing for other countries to use these products. Consumer externalities impact directly the consumer utility function and can capture traditional economic forces such as pollution, but also political affinity to other countries. In particular, we can use consumer externalities to capture the idea that the size of various industries around the world may make citizens of one country feel less secure. For instance, the development of a cutting edge semiconductor or AI sector for military use in a country's geopolitical rival may show up as a negative externality above and beyond any effect on the profits of the country's own firms.

We show that the input-output network propagates the production externalities. For example, changes in the exogenous productivity of a sector or the price of its inputs propagate through the network. One sector producing more, might make another more productive, that sector producing more affects the productivity of another sector, and so on. We show that this propagation can be summarized by a Leontief inverse matrix based on the externalities. As we detail below the propagation is an important element in which sectors are strategic, in the sense of which sectors the hegemon aims to influence the most.

Hegemons that control valuable threats exert the resulting power by asking entities in their network to take costly actions. We consider a general set of such actions that comprises both side payments and wedges in the input-output relationships. These general theoretical tools can be specialized to cover many observed instruments in practice. For example, side payments cover both monetary transfer, mark ups on goods, higher interest rates on loans, but also the cost of lobbying efforts firms might be asked to undertake for a desired political concession. Wedges in the input-output relationship can be specialized to be import quantity restrictions that are good and destination specific or tariffs and price caps. These are common tools in the implementation of sanctions.

We derive a theory-based measure of which countries, sectors, and activities within a sector are *friends* or *enemies* from the perspective of the hegemon. An activity, like sourcing from a particular supplier, is friendly if an increase in that activity has a positive spillover into the hegemon (indirect) utility. This can happen in three ways: (i) directly because the hegemon benefits from that activity, (ii) indirectly because the activity benefits (harms) other activities that have a direct benefit (harm) for the hegemon, (iii) indirectly because it benefits firms in the hegemon network and increases the hegemon power over these firms. Firms, sector, and countries are collections of specific activities and therefore similar definitions apply. Of course, a firm or a country could feature both friendly and unfriendly activities at the same time.

We show that the hegemon treats friends and enemies differently. In all cases, since entities in the hegemon's network can revert to an outside option, the power of the hegemon is limited by the value of the threats that it can offer. Intuitively, the hegemon subsidizes (positive wedge) friendly activities and taxes unfriendly ones. The hegemon would also like to extract side payments from all entities it has power over, but faces a trade-off. A bigger side payment tightens the participation constraint of the target entity and, all else equal, shrinks the wedges that can be applied since those costly actions also tighten the participation constraint. We show that all surplus is always extracted by the hegemon from neutral or unfriendly entities, but friendly ones might keep part the surplus.

The input-output amplification makes controlling some sectors more valuable for the hegemon since changes in the allocation of these strategic sectors have a larger influence on the world economy. For example, financial services such as payments and clearing have a thick market externality. Everyone wants to use dollar clearing and payment systems such as SWIFT because they are widely used by everyone else. The same is true for some forms of telecommunications and information technologies. Controlling these sectors is more valuable for the hegemon and we show that the hegemon optimal wedges on these sectors, like curbing their availability to some hostile countries, load heavily on the indirect propagation throughout the world economy, including the parts of the network that the hegemon does not control. We formalize in this way the notion of which sectors are strategic.

We show that equilibria with a hegemon are constrained inefficient from a global perspective because, while threats are a positive-sum game from a global perspectives they increase enforcement and therefore economic activity, the actions that the hegemon extracts with its power can be a negative-sum game. First, the global planner and the hegemon can disagree on the notion of friends and enemies, that is they view the externalities differently. Second, monetary side payments are distortionary because they lower profits and, thereby, worsen incentives of the firms and governments that have to pay them. Indeed, the global planner would impose no side payments and in general a different set of wedges that the hegemon does.

After deriving the general model, we focus on two leading applications. In the first application, we show how production and national security externalities interact and generate demands from the hegemon to third party countries for restricting the use of inputs of an hostile country. An example is the US demand to European governments and firms that they stop using information technology (IT) infrastructure produced by China's Huawei. We think of a world composed on three regions: the US hegemon, third party countries, and China. China has a sector producing IT goods that the rest of the world firms use as an input. We assume that this IT infrastructure has positive production externalities so that more firms using that input makes a firm more productive in using that same input. This captures the market-depth externality common of communication systems whereby a technology is more attractive to an user the more other users are on the same technology.

We also assume that the US experiences a negative consumer externality, which we refer to as a national security externality, from the size of China's exports of the technology.

We show that in this application it is optimal for the US hegemon to demand governments and firms in third party countries that it can pressure to curb their imports of Chinese technology. The extent of the requested import restrictions is higher since the hegemon internalizes the amplification effect of the sanctions. As the firms in its network use this technology less, using the technology becomes less attractive also for firms that the hegemon cannot directly pressure. This Leontief-type of linkage also feeds back to the firms accepting the sanctions: knowing that other firms will respond by not using the technology either, makes accepting the sanctions easier on the margin. Of course, overall the foreign firms accept the import restriction voluntarily, but they understand it is a costly action that reduces profit since the Chinese technology was a profitable input. They agree to do so because the US provides valuable economic relationships and powerful threats, for example the threats to threaten financial institutions that rely on dollar clearing or the threat to withdraw intelligence sharing with the government.

Our second application focuses on the Belt and Road Initiative by China. We model it as a sovereign lending program that aims to join borrowing and trade decisions. We illustrate how sovereign debt can be represented in the form of a productive input in our framework, and show that a country's borrowing capacity increases when the hegemon lender, in this case China, is able to consolidate threats in the sovereign lending arena with activity in export markets. Even if sovereign lending is completely unenforceable on its own, so that as an isolated activity no lending would take place, we should that profitable trade relationships can act as an endogenous cost of default. The optimal contract extracts surplus for China in one of three forms: as a mark-up on the price of the exports, as a higher return on loans, or as a political concession. In practice, it seems the latter has been the dominant form of request by the Chinese hegemon. More generally, the application shows the futility of assessing the success of the Belt and Road Initiative lending or infrastructure investment in isolation. The sustainability of the debt and the return of the program are inextricably linked with other economic and political activities.

In the final section of the paper, we explore how hegemons compete with each other in the geoeconomic arena. Two hegemons compete by offering firms and governments to enter their sphere of influence (accept their contract). Firms and governments choose which hegemon contracts to accept, if any. When hegemons control no common threats (i.e., are not substitutable in the threats they can provide), there is no competition: each hegemon is able to offer the same contract as if it were the only hegemon. This is a multipolar but geographically fully segmented world. When hegemons provide some common threats, we show the existence of an equilibrium in which both hegemons offer maximal joint threats. A hegemon's ability to extract side payments is limited by the extent to which its threats overlap with those of the other hegemon. This provides a notion of substitutability between the two hegemons and the existence of an alternative hegemon impacts the equilibrium. If the two hegemons fully overlap, competition leads hegemons to extract no side payments, leaving firms with the full surplus of the relationship.

Literature Review. In two landmark contributions Hirschman (1945, 1958) relates the structure of international trade to international power dynamics and sets up forward and backward linkages in input-output structures as a foundation for structural economic development. Much of our model is inspired by this work and aims to provide a formal framework for the power dynamics. In doing so, we connect to four broad strands of literature.

First, the paper connects to the literature in political science on economic statecraft. The notion of economic statecraft, or the use of economic means for political ends, was explored in depth by Baldwin (1985) and the subsequent literature. A particular tool of economic statecraft, economic and financial sanctions, is a focus of this political science literature, including such contributions as Lindsay (1986), Kirshner (1997), Drezner (2003), and Mulder (2022). Blackwill and Harris (2016) explore the rise of geoeconomics, that is the use of economic power for geopolitical goals. Farrell and Newman (2019) and Drezner et al. (2021) introduce the idea of "weaponized interdependence" whereby governments can use the increasingly complex global economic network to influence and coerce other governments.[1]

Second, the paper relates to the literature on networks, industrial policy, and trade. There is a growing literature on networks in economics including Gabaix (2011), Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), Jones (2011), Bigio and La'o (2020), Baqaee and Farhi (2019), Baqaee and Farhi (2020, 2022), Liu (2019) and Elliott, Golub and Leduc (2022). Bachmann et al. (2022) and Moll et al. (2023) use this class of models to find limited impact for Germany of a stop of energy imports from Russia. Our notion of friends and enemies of the hegemon is related to the work of Kleinman, Liu and Redding (2020) who explore whether countries become more politically aligned as they trade more with each other. In trade we relate to the study of global value chains (Antràs and Staiger (2012); Caliendo and Parro (2015); Grossman et al. (2021); Antràs and Chor (2022)) as well as the study of optimal tariffs and trade agreements (Grossman and Helpman (1995); Ossa (2014)). Our supplier-client relationship also encompasses forms of trade credit (Schmidt-Eisenlohr (2013); Bocola and Bornstein (2023)). Antràs and Miquel (2011, 2023) explore how foreign influence affects tariff and capital taxation policy. Bartelme, Costinot, Donaldson and Rodriguez-Clare (2019) estimate sector-level economies of scale to quantify the expected gains from industrial policy.[2] At the intersection with political economy, Berger, Easterly, Nunn and Satyanath (2013) demonstrate that countries where the CIA intervened during the Cold War imported more from the United States. Kuziemko and Werker (2006) document that a country that rotates on the UN Security council experiences an increase in foreign aid. Juhász, Lane, Oehlsen and Pérez (2022) use textual analysis to measure industrial policy interventions around the world. Juhász, Lane and Rodrik (2023) surveys the recent literature on industrial policy.

[1] Mangini (2022) studies how states' attempts to use economic coercion interact with domestic political constraints.

[2] Camboni and Porcellacchia (2021) use a gravity framework to test for geopolitical competition.

Fourth, the paper uses several theory tools developed in economic theory and macroeconomics. We employ grim trigger strategies a la Abreu et al. (1986, 1990) as an enforcement mechanism.[3] Our notion of joint triggers relates to the literature on multitasking (Holmstrom and Milgrom (1991)) in principal-agent models in which the presence of multiple tasks can help to provide higher powered incentives. We introduce externalities a la Greenwald and Stiglitz 1986 and our study of the hegemon optimal usage of wedges and side payments is related to the analysis of inefficiency in the presence of externalities (Geanakoplos and Polemarchakis (1985)) and the macro-prudential tools that can be used to improve welfare (Farhi and Werning (2016)).

2 Model Setup

There are N countries in the world. Each country n is populated by a representative consumer and set of productive sectors $\mathcal{I}_n$. We define $\mathcal{I}$ to be the union of all productive sectors across all countries, $\mathcal{I} = \bigcup_{n=1}^{N} \mathcal{I}_n$. Each sector produces a differentiated good indexed by $i \in \mathcal{I}$ out of a local sector specific factor ℓ_i and intermediate inputs produced by other sectors. Each sector is populated by a continuum of identical firms. The good produced by sector i is sold on world markets at price p_i and we take the good produced by sector 1 as the numeraire so that $p_1 = 1$.

The representative consumer in each country has linear preferences over all goods and consumers are identical in all countries:

$$U_n = \sum_{i \in \mathcal{I}} \tilde{p}_i \; C_{ni} + u_n(z),$$

with $\tilde{p}_i > 0$ and $\tilde{p}_1 = 1$, and where z is a vector of aggregate variables which we use to capture externalities (e.g, Greenwald and Stiglitz 1986). Consumers take z as given. We assume that the representative consumer in each country owns all domestic firms and the endowments of the local factors. The representative consumer of country n faces a budget constraint given by:

$$\sum_{i \in \mathcal{I}} \tilde{p}_i \; C_{ni} \leq \sum_{i \in \mathcal{I}_n} \left[\Pi_i + w_i \bar{\ell}_i \right],$$

where Π_i are the profits made by firms in sector i and $w_i \bar{\ell}_i$ is the compensation earned by the local factor of production. Given linear preferences and provided that the consumption of each good by each consumer is strictly positive, the optimality conditions for the consumer problem imply that $p_i = \tilde{p}_i$ for all goods i. Hence, for the remainder of this article, we take goods prices p_i as exogenous.[4]

A firm in sector i produces output y_i using intermediate inputs x_{ij} of good j and local factor ℓ_i

[3] See also Thomas and Worrall (1994) on FDI, and Eaton and Gersovitz (1981) on sovereign debt.

[4] To ensure that consumption of each good by each consumer is strictly positive we assume that the endowment of the local factor of production is always sufficiently high.

in a separable production function given by:

$$y_i = \sum_{j \in \mathcal{I}} f_{ij}(x_{ij}, z) + \ell_i.$$

Firms take the externality vector z as given. Therefore for notational convenience in this section, we adopt the simpler notation $f_{ij}(x_{ij})$. We reintroduce the explicit dependency in Section 3. We assume $f_{ij}(0) = 0$, $f'_{ij} > 0$, and either $f''_{ij} < 0$ or $f''_{ij} = 0$, i.e. the production function is increasing and either linear or strictly concave. The function f_{ij} is good and input specific allowing us to capture technology, but also transport costs and relationship specific knowledge. The assumption of separable production is one of convenience since, as we make clear below, it makes the pattern of binding incentive compatibility constraints straightforward. We can extend the set-up to nonseparable submodular production functions, for example, to constant elasticity of substitution (CES) production function with weakly decreasing returns to scale (see Appendix B).[5]

Firm operating profits are given by $p_i y_i - \sum_{j \in \mathcal{I}} p_j x_{ij} - w_i \ell_i$. We assume that the local factor specific to sector i is purchased competitively by the unit mass of firms in that sector so that $w_i = p_i$, thus firms earn zero profits from the local factor. Profits only arise from production using intermediate inputs, with $\pi_{ij}(x_{ij}) = p_i f_{ij}(x_{ij}) - p_j x_{ij}$ being profits earned out of production using input j.

Input j is profitable if and only if $\pi'_{ij}(0) \geq 0$. Denote $\mathcal{J}_i = \{j \in \mathcal{I} | \pi'_{ij}(0) \geq 0\}$ to be the set of profitable inputs for sector i.[6] We denote $J_i = |\mathcal{J}_i|$ to be the cardinality, the number of elements, of set $\mathcal{J}_i$. We assume that sector i is only linked to suppliers in its profitable set $\mathcal{J}_i$. Hence the profitable sets define the network structure of the model. Total operating profits of firms in sector i are:

$$\Pi_i = \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}).$$

2.1 Incentives and Limited Contracting Problems

The timing of the model includes three subperiods: Beginning, Middle, and End. We describe here the game that unfolds between each individual firm in sector i and the continuum of suppliers in sector j, with a version of this game playing out in each of the relationships $j \in \mathcal{J}_i$ of a firm in sector i. We refer to the respective players as individual firm i and suppliers in j, highlighting that i is an individual firm within the sector while j is the continuum of firms in that sector, so that the game is between one firm in a downstream sector and many suppliers in its upstream sector.

[5]Consider a CES production function of the form $f_i(x_i) = A_i \left(\sum_{j \in \mathcal{J}_i} \alpha_{ij} x_{ij}^{\sigma} \right)^{\beta/\sigma}$. This function is submodular if $\beta \leq \sigma$. The return to scale parameter β, where $\beta < 1$ is decreasing returns to scale, is a lower bound for the degree of substitutability of the inputs, where $\sigma = 1$ is perfect substitutes. See also Bocola and Bornstein (2023).

[6]We assume that the set $\mathcal{J}_i$ is invariant to the externality vector z in solutions studied in this paper.

Appendix Figure 4 provides the extensive-form illustration of this game.

Beginning Subperiod. In the Beginning, an individual firm i places an order to suppliers in j for intermediate inputs x_{ij} and makes no payment. If firms accept the order, then we assume that the order is fulfilled equally by all firms in sector j. Firms in sector j deliver immediately only a fraction $\theta_{ij}(x_{ij}) \in [0, 1]$ of the order.

Middle Subperiod. In the Middle, individual firm i decides whether to *Pay* or *Steal* from its suppliers in j. We assume that individual firm i must treat all suppliers in j equally, but may make different choices of Pay or Steal across different supplier sectors (e.g., Pay suppliers in sector j and Steal from suppliers in sector k). If individual firm i chooses to Pay suppliers in j, then it makes a monetary payment of $p_j\theta_{ij}(x_{ij})x_{ij}$ to suppliers in j, split evenly among the suppliers. If individual firm i chooses Steal, it makes no payment to suppliers in j.

End Subperiod. In the End, regardless of the outcome of the Middle, it is possible for the remainder of the transaction to take place. Individual firm i and suppliers in sector j simultaneously choose whether to *Complete (C)* or *Not Complete (NC)* the remainder of the transaction. We assume that all suppliers in j play the same action.

If individual firm i and suppliers in j both choose Complete, the transaction occurs: individual firm i provides monetary payment $p_j(1 - \theta_{ij}(x_{ij}))x_{ij}$ to suppliers in j, while suppliers in j provide goods $(1 - \theta_{ij}(x_{ij}))x_{ij}$ to individual firm i. If one or both parties choose Not Complete, the transaction does not occur. If both sides choose Not Complete, individual firm i retains its money and distributes it to consumers in its country as part of profits, and suppliers in j retain their goods and sell them at market price on world markets. If individual firm i chooses Complete while suppliers in j chose Not Complete, suppliers in j retain and sell their goods, while individual firm i's money $p_j(1 - \theta_{ij}(x_{ij}))x_{ij}$ is lost (i.e. a deadweight loss). Similarly, if firm i chooses Not Complete while suppliers in j choose Complete, individual firm i retains its money whereas suppliers in j lose their goods $(1 - \theta_{ij}(x_{ij}))x_{ij}$ (i.e. deadweight loss).

There are two pure strategy Nash equilibria of the End subgame: (C,C) and (NC,NC).[7] Therefore, there are four candidate subgame perfect pure strategy outcomes of the full game between individual firm i and suppliers in j: Pay-(C,C), Steal-(C,C), Pay-(NC,NC), and Steal-(NC,NC). The total payoffs summed across subperiods for firm i and suppliers in j are summarized in Table 1, where by convention we net out the opportunity cost $-p_jx_{ij}$ of suppliers in j from the relationship.

In general, we allow for the off-path production following a failed transaction with suppliers in j to lead to profits $\pi^D_{ij}(x_{ij}) \le p_if_{ij}(\theta_{ij}(x_{ij})x_{ij})$ so that we can capture, for example, productivity

[7] We restrict attention to pure strategy equilibria, both of which exist in the relevant range of allocations. Although there are mixed strategy equilibria, their expected payoffs to both players lie between the payoffs of the pure strategy equilibria. Hence, (C, C) reflects the best outcome whereas (NC, NC) reflects the worst outcome.

Table 1: **Net Payoffs of Game for Firm i and Suppliers in Sector j**

Hisotry of Game	**Firm i payoff**	**Suppliers j payoff**
Pay-(C,C)	$\pi_{ij}(x_{ij})$	0
Steal-(NC,NC)	$p_i f_{ij}(\theta_{ij}(x_{ij})x_{ij})$	$-p_j\theta_{ij}(x_{ij})x_{ij}$
Steal-(C,C)	$\pi_{ij}(x_{ij}) + p_j\theta_{ij}(x_{ij})x_{ij}$	$-p_j\theta_{ij}(x_{ij})x_{ij}$
Pay-(NC,NC)	$\pi_{ij}(\theta_{ij}(x_{ij})x_{ij})$	0

Notes: Table provides End subperiod payoffs for the game between firm i and suppliers in sector j. The history of the game depends on whether firm i chose *Pay* or *Steal*, and the firms i and suppliers in sector j chose *Complete (C)* or *Not Complete (NC)*.

losses arising from severing the supplier relationships.[8] We assume that $\pi_{ij}^{D\prime}(0) < \pi_{ij}'(0)$. For technical reasons, we assume that $\exists\ \overline{x}_{ij} > 0$ such that $\pi_{ij}^{D\prime}(x_{ij}) > 0$ and $\pi_{ij}^{D\prime\prime}(x_{ij}) > 0$ for $x_{ij} \leq \overline{x}_{ij}$, and that $\pi_{ij}^{D}(\overline{x}_{ij}) > \pi_{ij}(\overline{x}_{ij})$.[9] We assume that suppliers in j are unwilling to sell more than $\overline{x}_{ij}$ to any individual firm, and that all solutions in the paper are interior, $0 < x_{ij} < \overline{x}_{ij}$.

Reduced-form of the Game. It is useful to summarize the crucial aspects of this game that matter for the rest of the paper. The set-up covers economic relationships with a repeated interaction between two players, the firm and the suppliers. There is an early transaction and a later one. If the firm Steals in the early transaction it has a short run gain, but the suppliers punish the firm in the second transaction (choose Not Complete). In this case the firm payoff is $\pi_{ij}^{D}(x_{ij})$. If the firm Pays in the early transaction, the second transaction also Completes, and the firm payoff is $\pi_{ij}(x_{ij})$. The suppliers' payoff is positive if the firm does not Steal in the initial transaction and negative otherwise. The suppliers by backward induction only accept orders from the firm that are incentive compatible with the firm not Stealing: $\pi_{ij}^{D}(x_{ij}) \leq \pi_{ij}(x_{ij})$.

This reduced-form game can capture many economic relationships that are based on repeated transactions and incomplete contracts. For example, it covers a lender/borrower relationship in finance, a supplier-customer relationship in the goods market, but service provider and customer relationship, and infrastructure building over multiple installments.

[8] For example, following an action involving stealing input j, we could restrict the firm to produce with an inefficient technology g_{ij} with $g_{ij} < f_{ij}$. In the three-subperiod game described above, this amounts to assuming that a failed transaction in the end subperiod also results in production using the inefficient technology.

[9] In the case in which the same technology is used when stealing, we can define θ_{ij} to equal a continuous function $t_{ij}(x_{ij})$ for $0 \leq x_{ij} \leq \overline{x}_{ij}$, with $t_{ij}(\overline{x}_{ij}) = 1$. We have $\pi_{ij}^{D}(\overline{x}_{ij}) > \pi_{ij}(\overline{x}_{ij})$, satisfying the final condition. π_{ij}^{D} is increasing on $[0, \overline{x}_{ij}]$ if $t_{ij}(x_{ij})x_{ij}$ is increasing, and π_{ij}^{D} is convex if $f_{ij}'(t_{ij}(x_{ij})x_{ij})\frac{d^2[t_{ij}(x_{ij})x_{ij}]}{dx_{ij}^2} + f_{ij}''(t_{ij}(x_{ij})x_{ij})\left(\frac{d[t_{ij}(x_{ij})x_{ij}]}{dx_{ij}}\right)^2 > 0$.

2.1.1 Strategies and Incentive Compatibility

In the Middle, firm i's action is a choice of the set $S \subset \mathcal{J}_i$ of inputs, if any, to steal. The action S denotes the choice to steal goods $j \in S$ and not to steal goods $j \notin S$. For example, $S = \{1, 2\}$ denotes the action of stealing goods 1 and 2 and not any others, and $S = \emptyset$ denotes no stealing. The set of all possible stealing actions of firm i is $P(\mathcal{J}_i)$, that is the set of all subsets of the firm's supplier relations $\mathcal{J}_i$.

We denote $\sigma_{ij}(S)$ to be the strategy of suppliers in j in the End following stealing action S by individual firm i in the Middle. We assume that suppliers in j follow a trigger strategy of Not Complete under either of two conditions, which are stated intuitively here and with details in the proof of Lemma 1. The first condition is an individual trigger: $\sigma_{ij}(S) = NC$ if individual firm i steals from suppliers in j, that is $j \in S$. The second condition is a joint trigger: $\sigma_{ij}(S) = NC$ if another supplier $k \in R_{ij}$ will choose NC in response to S, where $R_{ij} \subset \mathcal{J}_i$ is the joint trigger set of suppliers in j. If $R_{ij} = \emptyset$, then suppliers in j have no joint triggers for firm i. We assume all joint triggers are symmetric: $k \in R_{ij}$ if and only if $j \in R_{ik}$.

In the proof of Lemma 1, we show that an equilibrium exists in which the strategy of suppliers in j in the game with individual firm i can be described by a subset $K_{ij} \subset \mathcal{J}_i$ such that $\sigma_{ij}(S) = NC$ if and only if $\exists k \in K_{ij}$ such that $k \in S$, and in which the strategy of individual firm i is $\varsigma_{ij} = \sigma_{ij}$. An individual firm i that chooses stealing action $S \in P(\mathcal{J}_i)$ will therefore face equilibrium (NC,NC) in relationships with all suppliers in $\overline{K}_i(S) \equiv \bigcup_{j \in S} K_{ij}$,[10] and equilibrium (C,C) in other relationships. We say that the input purchase vector $x_i = \{x_{ij}\}_{j \in \mathcal{J}_i}$ by firm i is *incentive compatible* with respect to stealing action $S \in P(\mathcal{J}_i)$ if firm i prefers not to steal over stealing goods S, that is,

$$\sum_{j \in S} \pi_{ij}^D(x_{ij}) + \sum_{j \in \overline{K}_i(S) \setminus S} \left[\pi_{ij}^D(x_{ij}) - p_j \theta_{ij}(x_{ij}) x_{ij} \right] \leq \sum_{j \in \overline{K}_i(S)} \pi_{ij}(x_{ij}). \tag{1}$$

The first term on the left-hand side is goods that are stolen (i.e., outcome Steal-(NC,NC)). The second term is goods that are paid for but subject to a joint trigger (i.e., outcome Pay-(NC,NC)). All remaining supplier relationships have the outcome Pay-(C,C) and drop out from both sides.

Intuitively, equation (1) suggests that since individual firm i faces (NC, NC) by all firms in $\overline{K}_i(S)$ anyway, it should prefer to steal all goods in $\overline{K}_i(S)$ rather than stealing only S. This motivates providing a representation of incentive compatibility that focuses only on undominated stealing strategies of firms. We first adopt the following definition.

Definition 1 *A* ***(restricted) action set*** $\mathcal{S}_i$ *of firm* i *is a subset of stealing actions,* $\mathcal{S}_i \subset P(\mathcal{J}_i)$*, such that: (i)* $\emptyset \in \mathcal{S}_i$*; (ii)* $\bigcup_{S \in \mathcal{S}_i} S = \mathcal{J}_i$*; (iii)* $S \cap S' = \emptyset \ \ \forall S, S' \in \mathcal{S}_i \ (S \neq S')$.

The first property says that not stealing, $S = \emptyset$, is available. The second property says that every good $j \in \mathcal{J}_i$ can be stolen as part of some action. The third property says that for each good

[10] Given joint triggers are symmetric, $k \in K_{ij}$ if and only if $j \in K_{ik}$ (see the proof of Lemma 1).

$j \in \mathcal{J}_i$, there is only one action S in which that good is stolen. We now show that incentive compatibility of an allocation under any configuration of joint triggers can be re-represented as incentive compatibility under a (restricted) action set by eliminating dominated strategies.

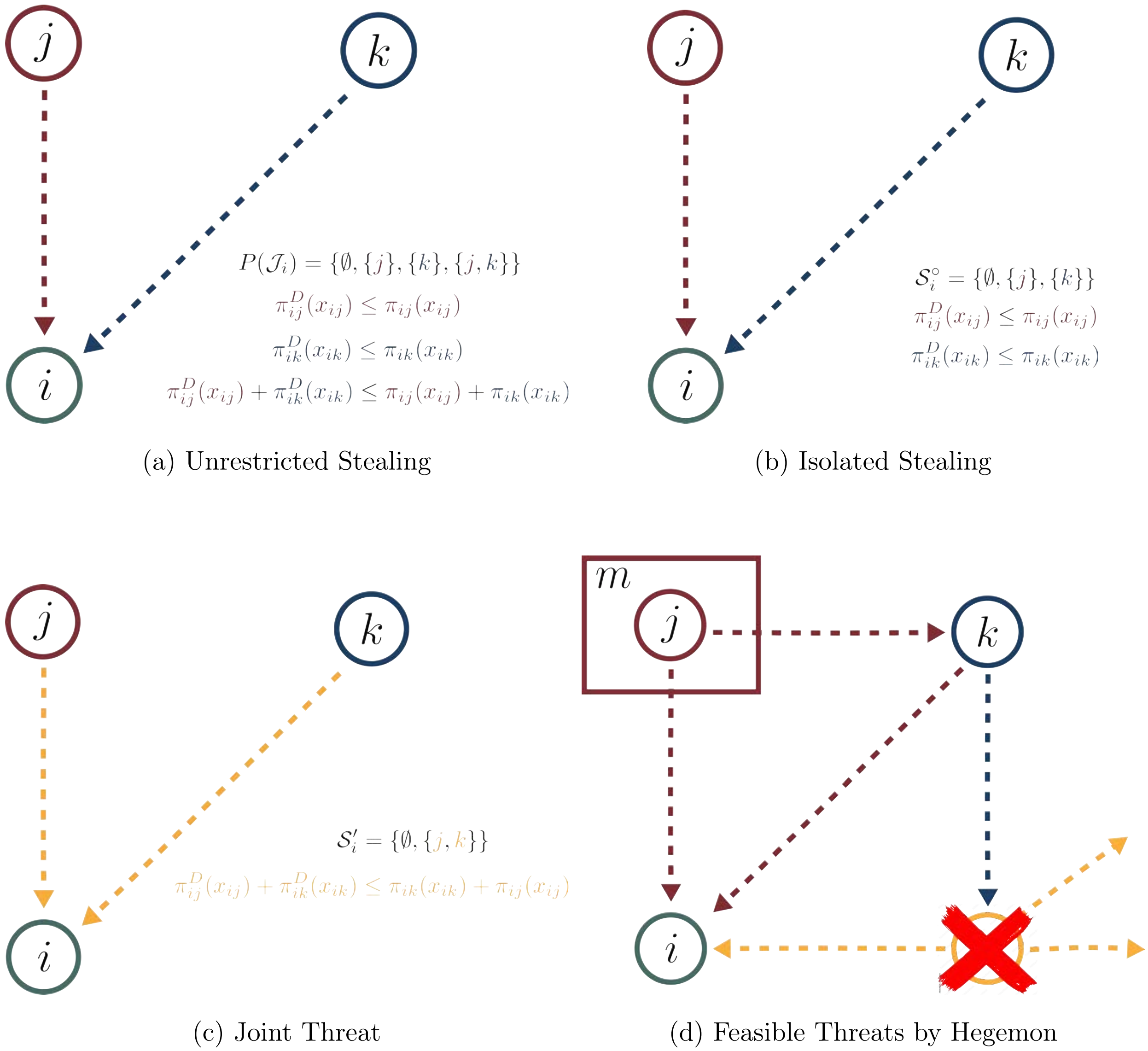
Lemma 1 *For any configuration of joint triggers, there exists a (restricted) action set $\mathcal{S}_i$ such that the allocation x_i is incentive compatible with respect to $P(\mathcal{J}_i)$ if and only if it is incentive compatible with respect to $\mathcal{S}_i$. The incentive compatibility constraint for $S \in \mathcal{S}_i$ is*

$$\sum_{j \in S} \pi_{ij}^D(x_{ij}) \leq \sum_{j \in S} \pi_{ij}(x_{ij}). \tag{2}$$

Lemma 1 allows us to represent arbitrary configurations of joint triggers in the much simpler form of a (restricted) action set, $\mathcal{S}_i$. This representation simplifies verifying incentive compatibility by restricting attention to a relatively small set of constraints. For expositional ease, henceforth we refer to $\mathcal{S}_i$ as simply the action set of firm i.

Figure 1 illustrates firm i action sets and related incentive compatibility constraints under different configurations. Consider a firm in sector i that sources inputs from suppliers in sectors j and k. Panel (a) considers a set-up in which suppliers in each of j and k have individual trigger strategies, that is they choose Not Complete if and only if firm i Steals from them. As a result, firm i entertains all possible stealing actions: no stealing, stealing only from j, stealing only from k, or stealing from both. This results in three IC constraints. Panel (b) uses Lemma 1 to restrict the action set: for the same trigger strategies as in panel (a), it is necessary and sufficient to check the individual deviations with respect to suppliers in either j and k, but not both. Under individual trigger strategies, a firm i allocation is incentive compatible to all possible stealing decisions if and only if it is incentive compatible with respect to pairwise stealing. Panel (c) considers joint triggers: suppliers in j commit to play Not Complete if and only if firm i steals from them or steals from suppliers k. Suppose firm i were stealing from suppliers k, firm i then knows that suppliers j will play Not Complete and that its best response is to chose Not Complete. Since firm i is facing a NC-NC outcome of the subgame with suppliers j, it then would choose to Steal from suppliers j. Hence, it is never optimal under the joint trigger for firm i to Steal from k but Pay j. Since we assume joint triggers to be symmetric, it is also never optimal under the joint trigger for firm i to Steal from j but Pay k. As a result, the joint trigger strategies can be tracked as a transformation in firm i action set that now only contains the empty set (no stealing at all) or stealing from both j and k. There is only one incentive constraint to keep track of, resulting from the joint stealing decision.

Figure 1: **Stealing, Action Sets, and Joint Threats**



(a) Unrestricted Stealing (b) Isolated Stealing

(c) Joint Threat (d) Feasible Threats by Hegemon

Notes: All panels focus on a firm in sector i with suppliers in sectors j and k. Panel (d) additionally considers the case of firms in sector k sourcing inputs from those in sector j and assumes that sector j is the only one located within the hegemon country m. The action sets and related incentive constraints are from the perspective of firm i under different configurations. Panel (a) illustrates unrestricted stealing. Panel (b) illustrates isolated stealing. Panel (c) illustrates a joint threat of j and k. Panel (d) consider the hegemon country m controlling the joint threat on firm i.

2.2 Optimal Firm Production

Letting $\mathcal{S}_i$ be the action set of firm i arising from Lemma 1, the optimization problem of firm i in the beginning subperiod is given by

$$V_i(\mathcal{S}_i) = \max_{x_i} \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}) \quad s.t. \quad \sum_{j \in S} \pi_{ij}^D(x_{ij}) \le \sum_{j \in S} \pi_{ij}(x_{ij}) \quad \forall S \in \mathcal{S}_i. \tag{3}$$

Since elements of an action set $\mathcal{S}_i$ are disjoint (Definition 1 and Lemma 1), this decision problem is separable across elements of $\mathcal{S}_i$. The decision problem associated with action $S \in \mathcal{S}_i$ is

$$v_i(S) = \max_{\{x_{ij}\}_{j\in S}} \sum_{j\in S} \pi_{ij}(x_{ij}) \quad s.t. \quad \sum_{j\in S} \pi_{ij}^D(x_{ij}) \leq \sum_{j\in S} \pi_{ij}(x_{ij}),$$

where $v_i(S_i)$ denotes the value function associated with the optimization problem involving action $S \in \mathcal{S}_i$. Observe, therefore, that $V_i(\mathcal{S}_i) = \sum_{S_i\in\mathcal{S}_i} v(s_i)$.

Suppose, first, that there are no joint triggers for suppliers in j with regard to firm i, that is $R_{ij} = \emptyset$. Then, firm i can steal good j without further reprecussions, that is $\{j\} \in \mathcal{S}_i$ is an available stealing action for firm i. The decision problem of firm i associated with $S = \{j\}$ is

$$\max_{x_{ij}} \ \pi_{ij}(x_{ij}) \quad s.t. \quad \pi_{ij}^D(x_{ij}) \leq \pi_{ij}(x_{ij}).$$

The solution to this problem is that either the constraint does not bind and we have an interior optimum x_{ij}^U satisfies $p_i f_{ij}'(x_{ij}^U) - p_j = 0$ or the constraint binds and the (nonzero) solution x_{ij}^D is given by $\pi_{ij}^D(x_{ij}^D) = \pi_{ij}(x_{ij}^D)$.[11] We denote λ_{ij} the Lagrange multiplier associated with this problem. We collect this result in the proposition below.

Proposition 1 *Let $\{j\} \in \mathcal{S}_i$. Then firm i's optimal input choice x_{ij}^* satisfies $x_{ij}^* = \min[x_{ij}^D, x_{ij}^U]$. The Lagrange multiplier is $\lambda_{ij} = 0$ if $x_{ij}^* = x_{ij}^U$ or $\lambda_{ij} = \frac{\sigma_{ij}}{\sigma_{ij}^D - \sigma_{ij}} > 0$ if $x_{ij}^* = x_{ij}^D$, where σ_{ij} and σ_{ij}^D are the elasticities of $\pi_{ij}(x_{ij})$ and $\pi_{ij}^D(x_{ij})$, respectively.*

Suppose, for example, that the production function is linear with productivity z_{ij}, so that $f_{ij}(x_{ij}) = z_{ij}x_{ij}$, that the fraction of the order that can be stolen is linear in the size of the order, $\theta_{ij}(x_{ij}) = \vartheta_{ij}x_{ij}$, and that conditional on stealing production occurs using f_{ij}. Then, we have $x_{ij}^* = \frac{1}{\vartheta_{ij}} \frac{p_i z_{ij} - p_j}{p_i}$. Intuitively, $p_i z_{ij} - p_j$ is the per-unit profit that the firm earns from production using input j, so that higher profitability relaxes the constraints and allows more production.

If all actions $S \in \mathcal{S}_i$ involve stealing only a single good, then Proposition 1 fully characterizes firm i's optimal production decision input-by-input. If instead there is an element $S \in \mathcal{S}_i$ with $|S| \geq 2$, reflecting the presence of joint triggers, then we obtain the following result.

Proposition 2 *For $|S| \geq 2$, if for all $j \in S$ we have $x_{ij} = x_{ij}^U$ satisfies the IC constraint, then it is the solution and $\lambda_{iS} = 0$. Otherwise, the IC constraint binds and*

$$\lambda_{iS} = \left(\sum_{j\in S} \frac{\sigma_{ij}^D - \sigma_{ij}}{\sigma_{ij}^D} \omega_{ij} \right)^{-1} - 1$$

where $\omega_{ij} = \frac{\pi_{ij}}{\sum_{j\in S} \pi_{ij}}$.

[11] If no such x_{ij}^U exists, we define $x_{ij}^U = \overline{x}_{ij}$.

Proposition 2 highlights an important property of the model: the Lagrange multiplier on a joint stealing action between good j and k is the weighted average of the same elasticities that characterize the Lagrange multipliers in the isolated stealing decisions for i and k separately. Intuitively, the action of $\{j,k\}$ shares the slack in the IC constraints of $\{j\}$ and $\{k\}$ and equalizes the elastisticities once weighted by how much each line of production is contributing to total profits.

2.3 Market Clearing and Equilibrium

Denote $\mathcal{D}_j = \{i \in \mathcal{I} | j \in \mathcal{J}_i\}$ the set of firms that source from supplier j, i.e. the firms immediately downstream from j. Market clearing for good j is given by

$$\sum_{n=1}^{N} C_{nj} + \sum_{i \in \mathcal{D}_j} x_{ij} = y_j,$$

while market clearing for factor i is $\ell_i = \bar{\ell}_i$.

An equilibrium of the model, given action sets $\{\mathcal{S}_i\}_{i \in \mathcal{I}}$ and externalities z, is prices for goods and factors p, w and allocations $\{x_{ij}, C_{ni}, y_i, \ell_i\}$ such that: (i) firms maximize profits, given prices; (ii) households maximize utility, given prices; (iii) markets clear.[12]

3 Hegemonic Power

Our main analysis focuses on when and how value can be created by generating joint threats in stealing decisions. We begin this section by defining and characterizing *pressure points* on firms, which heuristically denote a set of off the equilibrium threats on a firm that, when consolidated into a single joint threat, generate on the equilibrium path an increase in profits earned by that firm. We then introduce the problem of a hegemon that is able to join together threats, and ask when and how the hegemon can create and extract value by doing so.

3.1 Joint Threats and Pressure Points

A joint threat in our model is a coordination of trigger strategies across multiple inputs used by the same firm. We depict a simple example in Figure 1. Firms in sector j are supplying inputs to both sector i and k, and those in sector k are themselves also supplying to sector i. We refer to this configuration a triangle network. A joint threat, in this example, is the suppliers in k adopting a trigger strategy that commits not to deliver good k to firms in i in the end subperiod if those firms steals either good j or good k.

[12]Let $x_{ij}(p)$ and $y_i(p)$ denote, respectively, demand and production by firm i given prices p and factor prices $w = p$. For the equilibrium to feature prices $p = \tilde{p}$, we are assuming that $\sum_{i \in \mathcal{D}_j} x_{ij}(\tilde{p}) < y_j(\tilde{p})$ for all j. Observe that this can be guaranteed by assuming $\bar{\ell}_i$ is sufficiently large.

We now formally define joint threats as restrictions on the action set, which we then formalize as arising from trigger strategies in the underlying game.

Definition 2 *A simple* ***joint threat*** *is a transformation of action set $\mathcal{S}_i$ into a new action set $\mathcal{S}_i'$ that combines $n \geq 2$ elements $S_1, \ldots, S_n \in \mathcal{S}_i$ into a single action,*

$$\mathcal{S}_i' = \{\bigcup_{x=1}^{n} S_x\} \cup (\mathcal{S}_i \backslash \{S_1, \ldots, S_n\}).$$

A complex joint threat is a finite sequence of simple joint threats.

The action set $\mathcal{S}_i'$ formed from a simple joint threat embeds an (explicit or implicit) joint trigger on all inputs $j \in \bigcup_{x=1}^{n} S_x$, while leaving all other elements $S \in \mathcal{S}_i \backslash \{S_1, \ldots, S_n\}$ of the original action set $\mathcal{S}_i$ unchanged. A complex joint threat creates a new action set $\mathcal{S}_i^K$ via a path $\{\mathcal{S}_i^k\}_{k=0}^{K}$, where $\mathcal{S}_i^0 = \mathcal{S}_i$ and where $\mathcal{S}_i^{k+1}$ is a simple joint threat of $\mathcal{S}_i^k$. We refer to both simple and complex joint threats as joint threats for the remainder of the paper, although for the majority of the paper it suffices to consider simple joint threats. Going back to Figure 1, consider starting from the isolated stealing action set for firm i given by $\mathcal{S}_i = \{\{j\}, \{k\}, \emptyset\}$ (Panel (b)), then a (simple) joint threat action set is $\mathcal{S}_i = \{\{j, k\}, \emptyset\}$ (Panel (c)).

This set transformation is achieved in our model by a coordination of trigger strategies. In particular, we assume that supplier j commits to a trigger strategy that depends not only of the decision of firm i to steal good j but also on the decision of a subset of other suppliers to not deliver to firm i. If two or more suppliers are part of the same action S we assume that they all coordinate on not delivering if one of them does not deliver. Given these trigger strategies, firm i anticipates not to deliver to any suppliers that are part of the same joint threat and therefore steals from all of them as a best response. For this reason, we defined joint threats directly as restrictions to the firms action set.[13]

Joint threats generically generate value for the firm being threatened because they relax incentive constraints. This is natural in set-ups like ours in which trigger strategies can be used to threaten agents with punishments in order to induce good behavior. Formally, we have that for any joint threat action set $\mathcal{S}_i'$ formed from $\mathcal{S}_i$, we have:

$$V_i(\mathcal{S}_i') \geq V_i(\mathcal{S}_i).$$

Of course, in many cases the value creation is zero, for example when incentive constrains are all not binding, but our main interest is in the cases of strictly positive value. We define a pressure point for firm i as a joint threat that *strictly* increases the profits of firm i.[14]

[13] Indeed, pre-existing joint triggers, from the previous subsection, can also be thought of as arising from situations in which a legal contract formalizes the cross-default agreements among the suppliers, or situations in which this is a non binding equilibrium outcome.

[14] In identifying pressure points, it suffices to consider simple joint threats.

Definition 3 *A **pressure point** of firm i is a collection of stealing actions $S_1, \ldots, S_n \in \mathcal{S}_i$ that, when used to form a joint threat $\mathcal{S}_i'$, strictly increases profits, that is,*

$$V_i(\mathcal{S}_i') > V_i(\mathcal{S}_i).$$

We now prove a necessary and sufficient condition for identifying pressure points. As the preliminary to this condition, the optimization problem of firm i has a corresponding Lagrangian,

$$\mathcal{L}(x_i, \lambda|\mathcal{S}_i) \equiv \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}) - \sum_{S \in \mathcal{S}_i} \lambda_{iS} \sum_{j \in S} \left[\pi_{ij}^D(x_{ij}) - \pi_{ij}(x_{ij})\right],$$

where $\lambda_{iS} \geq 0$ is the Lagrange multiplier on the incentive compatibility constraint associated with $S \in \mathcal{S}_i$. We obtain the following result, which holds fixed the vector of aggregates z.

Proposition 3 *$S_1, \ldots, S_n \in \mathcal{S}_i$ is a pressure point of firm i if and only if $\lambda_{iS} \neq \lambda_{iS'}$ for some $S, S' \in \{S_1, \ldots, S_n\}$.*

Proposition 3 proves that a necessary and sufficient condition for a pressure point is that the Lagrange multipliers of the existing equilibrium differ among those input relationships that enter the joint threat. To build intuition, return to the triangle network in Figure 1. Consider the equilibrium under isolated stealing $\mathcal{S}_i = \{\emptyset, \{j\}, \{k\}\}$, then firms in sector i have a pressure point resulting from the joint threat actions $\{j\}, \{k\}$ if and only if $\lambda_{ij} \neq \lambda_{ik}$. Intuitively, if $\lambda_{ij} > \lambda_{ik}$, then the marginal value of slack in the incentive compatibility constraint for (stealing) good j is higher than for slack in the incentive compatibility constraint for good k. The joint threat creates value by consolidating the two constraints and altering relative production of the two goods, a means of redistributing slack. Heuristically, the joint threat facilitates a *decrease* in production using k in order to create slack that allows for an *increase* in production using j under the joint threat. By contrast if $\lambda_j = \lambda_k$, then slack is equally valuable across goods j and k, even when both multipliers are strictly positive and both constraints bind. In this case, no value is created by forming a joint threat: production under the joint threat is precisely the same as under isolated threats. The proof of Proposition 3 formalizes these intuitions for more general action sets $\mathcal{S}_i$.

This result is both intuitive and powerful. Intuitive, in the sense that combining disparate threats into a joint one, creates value by allowing profitable perturbations of the original allocation that now feasible under the joint threat. The ex-ante Langrange multipliers indicate whether adding slack to a particular input relationship is more valuable, and therefore guide the perturbation to increase that allocation and decrease the rest to preserve joint incentive compatibility. Powerful, in the sense that identifying pressure points only requires knowing the tightness of the constraints in the existing equilibrium.

3.2 Hegemon Problem

Consider a single country, m, comprising a collection $\mathcal{I}_m$ of sectors and consumer m. The country's government can pay a fixed cost $F_m \geq 0$ in order to become a hegemon, with the fixed cost paid for by the hegemon's representative consumer. For now, we think of all other country governments as facing arbitrarily large fixed costs, so that they do not become hegemons. If m becomes a hegemon, it gains the ability to coordinate its firms ("collusion"), including the ability to create joint threats. It can then propose take-it-or-leave-it offers to *all* downstream firms from $\mathcal{I}_m$, where contract terms will specify joint threats, side payments, and restrictions on inputs purchased.

3.2.1 Hegemon's Contract

Let $\mathcal{D}_m = \bigcup_{i\in\mathcal{I}_m} \mathcal{D}_i \backslash \mathcal{I}_m$ denote the set of foreign firms that source from at least one input from the firms in the hegemon's country. Let $\mathcal{C}_m \equiv \mathcal{I}_m \cup \mathcal{D}_m$ denote the set of firms the hegemon can contract with. Let $\mathcal{J}_{im} = \mathcal{I}_m \cap \mathcal{J}_i$ denote the set of inputs that firm i sources from (firms in) country m. Hegemon m proposes a take-it-or-leave-it offer to each firm $i \in \mathcal{C}_m$. The contract offered to firm i has three terms: (i) a joint threat action set $\mathcal{S}'_i$; (ii) nonnegative transfers (side payments) $\mathcal{T}_i \equiv \{T_{ij}\}_{j\in\mathcal{J}_{im}}$ from firm i to the hegemon's representative consumer (with $T_{ij} > 0$ representing a payment to the hegemon associated with stealing decision j of firm i); (iii) revenue-neutral taxes $\tau_i \equiv \{\tau_{ij}\}_{j\in\mathcal{J}_i}$ on purchases of x_{ij}, with equilibrium revenues $\tau_{ij}x^*_{ij}$ raised from sector i rebated lump sum to firms in sector i. Naturally, remitted revenues x^*_{ij} is a determined by the contract terms and of the externality vector z, as made clear below.

Wedges adjust the effective price the firm faces in its relationship to $p_j + \tau_{ij}$. Side payments and rebates occur contemporaneously in the Middle subperiod, i.e. concurrently with the Pay/Steal. Under the contract, if individual firm i Pays suppliers in j, then it pays $p_j\theta_{ij}(x_{ij})x_{ij}$ to suppliers in j and pays $\tau_{ij}\theta_{ij}(x_{ij})x_{ij} + T_{ij} - x^*_{ij}$ to the hegemon's consumer. If individual firm i Steals from suppliers in j, it makes no payments. In the End, an individual firm i that chooses Complete is commiting resources of $p_j(1-\theta_{ij}(x_{ij}))x_{ij}$ to paying suppliers in j and $\tau_{ij}(1-\theta_{ij}(x_{ij}))x_{ij}$ as payment to the hegemon's consumer.

Side payments can cover different interpretations: direct monetary payments, a firm-specific markup charged by the hegemon on sales of its goods, or the extraction of value in some other action the firm takes on behalf of the hegemon (see later discussion in this section).

The revenue-neutral taxes are a set of wedges in the problem of firm i that allow us to capture the ability of the hegemon to ask the firm to change its allocation of inputs. Wedges of this type are typical in the macro-prudential literature that focuses on pecuniary and demand externalities (Farhi and Werning (2016)). This can capture either quantity restrictions or taxes/subsidies.[15] Importantly, we allow these instruments to target bilateral readerships between two nodes. This

[15] For example, Clayton and Schaab (2022) show how different rebate rules can be implemented to cover quantity restrictions and ad valorem taxes.

covers, for example, restricting energy imports from Russia but not from other countries; or tariffs and quantity restrictions on inports of Chinese goods.

In principle, we could allow for bilateral taxes on sales by firm i sales in addition to input purchases. Since firm i has the outside option to sell to consumers, in equilibrium any sales taxes would be fully passed through to the buyer and, in this sense, they are captured by the input taxes that we consider. However, a difference is that the input taxes on firm i that arise from sales taxes on firm j would not in principle require firm i to agree to the contract. It is this latter situation that we do not consider in our choice to focus on input taxes alone.

Feasible Contracts. We restrict the joint threats that the hegemon can make to involve firms that are at most one node removed from the hegemon, that is involving either the hegemon's firms or its immediately downstream firms. We impose this restriction to prevent unrealistic situations in which the hegemon threatens a firm that it has no relationship with. Formally, we refer to the act of creating a joint threat from $S, S' \in \mathcal{S}_i$ as *consolidating* S and S', and define *direct transmission* of threats as follows.

Definition 4 *Hegemon m can consolidate $S \in \mathcal{S}_i$ under* direct transmission *if $\exists j \in S$ with either $j \in \mathcal{I}_m$ (direct control) or $j \in \mathcal{D}_m$ (indirect control). A joint threat is **feasible** if it can be achieved under direct transmission.*

Intuitively, Definition 4 says that the hegemon can create a joint threat using action $S \in \mathcal{S}_i$ if either the hegemon supplies a good $j \in S$ to firm i, or if the hegemon supplies a good to a firm $j \in \mathcal{D}_m$ that in turn is a supplier to firm i, that is $j \in S$. Indeed, the set of firms the hegemon can contract with, $\mathcal{C}_m$, is the union of firms over which it either has direct or indirect control. The former is a case of direct control: the hegemon coordinates a joint threat between two actions S and S' over which it has direct control by directly coordinating the trigger strategies of two or more firms, one in each action. The latter is a case of indirect control: the hegemon instead creates a joint threat via its downstream supplier, by requiring its downstream supplier, as part of its contract, to adopt the trigger strategy associated with the joint threat.

For each $i \in \mathcal{C}_m$, define the set of *direct transmission links* $\mathcal{S}_i^D \subset \mathcal{S}_i$ as the subset of elements $S \in \mathcal{S}_i$ that can be consolidated under direct transmission by the hegemon.[16] Observe that the ex-ante equilibrium can be implemented by a feasible contract, whereby the hegemon proposes the terms $\mathcal{S}_i' = \mathcal{S}_i$, $T_{ij} = 0$, and $\tau_{ij} = 0$ for all i, j.

[16] One can imagine threats being passed on over more than direct links, for example each firm passing on the threat to the next one over a chain. Further, one could imagine stipulating that the threats are agreed to be carried on with some probability less than one, so that at each link the threat becomes weaker in probability (decaying over the length of the chain). For now, we keep the length of the chain to be 1 and the threat to be carried out for sure.

3.2.2 Form of Externalities

We assume that externalites take the form $z = \{z_{ij}\}$, where in equilibrium $z^*_{ij} = x^*_{ij}$. That is externalities are based on the quantities of inputs in bilateral i and j relationships. This general formulation can be specialized to cover pure size externalities, in which it is the total output of a sector that matters, or export-import externalities, in which it is the fraction of output sold cross border that matters, but also thick market externalities, in which the extent to which an input is widely used by many sectors that matters.[17]

Unlike individual firms and consumers, the hegemon internalizes how the terms of its contract affect the vector of externalities.[18]

3.2.3 Firm Participation Constraints

Firm $i \in \mathcal{C}_m$ chooses whether or not to accept the take-it-or-leave it offer made by the hegemon. If individual firm i rejects the hegemon's contract and the vector of aggregates is z, it sources competitively and achieves the same value $V_i(\mathcal{S}_i, z)$ as arises in Section 2 when the externality vector is z. Individual firm i, being small, does not internalize the effect of its decision to accept or reject the contract on the prevailing externality vector z. If instead firm i accepts the offer, it chooses allocations to maximize profits given the contract terms.

As in Section 2, the decision problem of firm i is separable across actions $S \in \mathcal{S}_i$. Formally, we can write

$$V_i(\mathcal{S}'_i, \mathcal{T}_i, \tau_i, z) = \sum_{S \in \mathcal{S}} v_i(S, \mathcal{T}_i, \tau_i, z)$$

where

$$v_i(S, \mathcal{T}_i, \tau_i, z) = \max_{\{x_{ij}\}_{j \in S}} \sum_{j \in S} \Big[\pi_{ij}(x_{ij}, z) - T_{ij} - \tau_{ij}(x_{ij} - x^*_{ij}) \Big]$$

$$s.t. \quad \sum_{j \in S} \pi^D_{ij}(x_{ij}, z) \leq \sum_{j \in S} \Big[\pi_{ij}(x_{ij}, z) - T_{ij} - \tau_{ij}(x_{ij} - x^*_{ij}) \Big].$$

Recall that side payments and taxes occur in the Middle subperiod and are associated with the firm decision to Pay. Therefore, they enter the incentive constraint on the right hand side, but not the left hand side. Note that side payments T_{ij} tighten the incentive constraint, all else equal. At the level of the individual firm, taxes have two effects: (i) they affect the incentive constraint

[17] It is possible to remove the assumption of linear preferences for consumers, so that prices are not constant in equilibrium, and included prices in the vector of externalities z. This extension would feature pecuniary externalities and hegemons' incentives to manipulate terms of trade with tariffs. It would also allow for endogenous network amplification of shocks via the effect of the quantity produced/demanded on prices.

[18] It is without loss of generality to assume that firm-to-firm sales, y_{ij}, do not cause externalities, since $x_{ji} = y_{ij}$ already captures such sales on the buyer side. We can also capture aggregate production externalities through y_i, since production out of factors is fixed. Thus, we have only ruled out externalities arising from consumption of certain goods by consumers, above and beyond the externalities associated with production.

because they alter the perceived price of the input good; (ii) they affect the incentive constraint via loss of profits. In equilibrium, this latter effect washes out since taxes are rebated lump sum (i.e., $x_{ij} = x_{ij}^*$). The optimal allocation $x_{ij}^*(\mathcal{S}, \mathcal{T}_i, \tau_i, z)$, and hence remitted revenues, are defined implicitly as a function of contract terms and externalities by the above optimization problem.

For firm i to accept the contract, it must be better off under the contract than by rejecting it. This gives rise to the **participation constraint** of firm i,

$$V_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i, z) \geq V_i(\mathcal{S}_i, z). \tag{4}$$

The hegemon must propose contracts that respect the participation constraint (4) of firm i to avoid having that firm reject its contract.

3.2.4 Hegemon Objective Function and Maximization Problem

The hegemon's objective function is utility of its representative consumer, to whom all firm profits and side payments accrue. As derived above, the consumer's utility is total country wealth, that is,

$$U_m = \sum_{i \in \mathcal{I}_m} \left[V_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i, z) + w_i \bar{\ell}_i \right] + u_m(z) + \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_{im}} T_{ij},$$

where recall that T_{ij} are the contract side payments. This reduces to

$$U_m = \sum_{i \in \mathcal{I}_m} \left[\Pi_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i, z) + w_i \bar{\ell}_i \right] + u_m(z) + \sum_{i \in \mathcal{D}_m} \sum_{j \in \mathcal{J}_{im}} T_{ij}. \tag{5}$$

Because side payments from domestic firms to the hegemon's consumer net out from the consumer's perspective, the hegemon only values the operating profits $\Pi_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i, z) = V_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i, z) + \sum_j T_{ij}$ of its domestic firms. However, the hegemon values side payments from foreign firms, precisely because its consumer has no claim to their profits. Similarly, taxes on all firms are revenue neutral for the hegemon, and therefore only matter to the extent they affect either domestic firm profits or side payments from foreign firms.

Conditional on entering, the hegemon's maximization problem is choosing a contract (joint threats, side payments, and wedges, $\{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}_{i \in \mathcal{C}_m}$) to maximize its consumer utility (equation 5), subject to the participation constraints of firms (equation 4), the feasibility of joint threats (Definition 4), and the determination of externalities $z_{ij}^* = x_{ij}^*(\mathcal{S}_i', \mathcal{T}_i, \tau_i, z^*)$. Given its optimal contract conditional on entry and associated utility U_m^c, the hegemon enters if $U_m^c - F_m \geq \sum_{i \in \mathcal{I}_m} [V_i(\mathcal{S}_i, z) + w_i \bar{\ell}_i] + u_m(z)$ (for the equilibrium z that arises absent a hegemon).

3.3 Optimality of Maximal Joint Threats

We solve the hegemon's problem in two steps. First, we prove that the hegemon utilizes all threats it can make in a single "maximal" joint threat. Second, we characterize side payments and wedges under the optimal contract. We focus here on the first step.

For each $i \in \mathcal{C}_m$, define the maximal joint threat action set that is feasible under direct transmission as $\overline{\mathcal{S}}_i' = \{\cup_{S \in \mathcal{S}_i^D} S\} \cup (\mathcal{S}_i \backslash \mathcal{S}_i^D)$, which consolidates all $S \in \mathcal{S}_i^D$ into a single joint threat. We obtain the following result.

Lemma 2 *It is weakly optimal for the hegemon to offer a contract with maximal joint threats to every firm it contracts with, that is $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$ for all $i \in \mathcal{C}_m$.*

Intuitively, Lemma 2 follows from the observation that joint threats expand the set of feasible allocations, and so weakly increase firm profits. Formally, a hegemon that chose a contract that did not involve maximal joint threats could always implement the same side payments and allocations while offering a contract with maximal joint threats. Hence offering maximal joint threats can increase value to the hegemon but cannot decrease it.

Lemma 2 underlies our model's tractability in two dimensions. The first is that since the optimal joint threat is known to be the maximal joint threat $\overline{\mathcal{S}}_i'$, the decision problem of the hegemon becomes separable across firms $i \in \mathcal{C}_m$. The second is that since all hegemon firms that can enter a joint threat on firm i do so, then the side payments can be tracked at the joint threat (i.e. the IC constraint) level rather than the bilateral supplier level, that is $\overline{T}_i \equiv \sum_{j \in \mathcal{J}_{im}} T_{ij}$. We therefore abuse notation and write $\Pi_i(\overline{\mathcal{S}}_i', \overline{T}_i, \tau_i, z^*)$ and $V_i(\overline{\mathcal{S}}_i', \overline{T}_i, \tau_i, z^*)$ to stress that they depend on $\overline{T}_i$ rather than the full vector $\mathcal{T}_i$.

3.4 A First Pass: Optimal Contract and Efficiency

We begin by shutting off the externalities arising from the aggregate vector z, that is $u_n(z)$ and $f_{ij}(x_{ij}, z)$ are constant in z. In this environment, the proposition below characterizes the optimal contract offered by the hegemon, differentiating domestic and foreign firms.

Proposition 4 *Conditional on entry and in the absence of externalities from vector z, an optimal contract of the hegemon has the following terms:*

1. *All wedges are zero on all firms, $\tau_{ij}^* = 0$ for all $i \in \mathcal{C}_m$, $j \in \mathcal{J}_i$.*

2. *All side payments are zero for domestic firms, that is $\overline{T}_i^* = 0$ for all $i \in \mathcal{I}_m$.*

3. *Foreign firm i is charged a positive side payment $\overline{T}_i^* > 0$ if and only if the set of direct transmission links $\mathcal{S}_i^D$ is a pressure point on i. The side payment is then set so that the participation constraint binds, $V_i(\overline{\mathcal{S}}_i', \overline{T}_i^*, 0) = V_i(\mathcal{S}_i)$.*

We define a firm to be an *extraction point* for the hegemon if, under the optimal contract, it makes a strictly positive side payment. To understand the hegemon's optimal contract, we focus first on domestic firms. Since the hegemon's decision problem is separable across firms, the hegemon's optimization problem for a domestic firm is

$$\max_{\overline{T}_i, \tau_i} \Pi_i(\overline{S}_i', \overline{T}_i, \tau_i) \quad s.t. \quad V_i(\overline{S}_i', \overline{T}_i, \tau_i) \geq V_i(\mathcal{S}_i).$$

The hegemon sets $\tau_i = 0$ because revenue-neutral wedges, in absence of externalities z, can only decrease the firm's profits compare to its privately optimal decision, that is Π_i and V_i are maximized at $\tau_i = 0$.[19] Similarly, positive side payments directly tighten the firm's incentive constraint, and therefore reduce its profits. Since side payments from domestic firms are a wash for the hegemon's representative consumer, it is optimal to set them to zero.[20] Domestic firms, therefore, are never extraction points. If the hegemon's joint threat includes a pressure point on a domestic firm, the optimal contract features the threat, relaxes the firm's incentive constraint, and expands its profits. The firm participation constraint is slack, and the hegemon receives value of the increase in profits from the firm payout to consumers.

For a foreign firm, the hegemon's decision problem is different, since the objective is to extract side payments rather than maximize firm profits. Therefore, the hegemon solves

$$\max_{\overline{T}_i, \tau_i} \overline{T}_i \quad s.t. \quad V_i(\overline{S}_i', \overline{T}_i, \tau_i) \geq V_i(\mathcal{S}_i).$$

For the same reason as for domestic firms, the hegemon also sets $\tau_i = 0$ for foreign firms. In constrast, while side payments do reduce firm profits, similarly to the domestic firm case, the hegemon's consumer has no claim to these profits. The hegemon therefore would like to charge side payments to foreign firms. What limits the ability of the hegemon to do so is the participation constraint. If the joint threat that the hegemon offers does not include a pressure point, then the participation constraint binds even at no side payments. In this case, the hegemon has nothing of value to offer to the firm, and so cannot extract any side payment. If instead the hegemon's threat includes a pressure point, then the hegemon extracts the entire increase in firm value as a side payment.

We conclude that the hegemon has an extraction point if and only if it has a pressure point on that firm. This highlights the nature of geoeconomic power: it is not just the ability to threaten, it is that ability combined with the capacity to extract surplus from these economic relationships.

[19] Recall that we have assumed that factor endowments are always large enough that prices are constant due to consumers having linear preferences and being marginal in every market.

[20] Recall that we ruled out negative side payments. As usual in the macroprudential literature, the hegemon would want to use negative side payments (subsidies) to slacken incentive constraints of domestic firms. Consistent with the literature, we have ruled out these subsidies.

Entry by the Hegemon. We conclude the analysis of the optimal contract by pinning down the entry decision of the hegemon. Proposition 4 characterizes the increase in value obtained from becoming a hegemon as $\sum_{i \in \mathcal{C}_m} \Delta\Pi_i$. Thus given a fixed cost F_m, the country chooses to become a hegemon if $\sum_{i \in \mathcal{C}_m} \Delta\Pi_i \geq F_m$.

3.4.1 Efficient Allocations

We now contrast the hegemon outcome with an efficiency benchmark, focusing on the limiting case as the fixed cost of entry goes to zero, $F_m \to 0$. Since the fixed cost is zero, without loss the hegemon will enter.

An efficient allocation in this setting can be described as the solution to a global planning problem. The global planner chooses the entry decision and contract of the hegemon, but faces the same constraints as the hegemon. The global planner has a utilitarian objective, $\sum_{n=1}^{N} \sum_{i \in \mathcal{I}} \tilde{p}_i C_{ni}$, where as is standard we can think of lump sum transfers between consumers as being used to ensure Pareto efficiency. Thus from the same steps as above, the global planner's objective function is

$$\sum_{i \in \mathcal{I}} \left[\Pi_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i) + w_i \bar{\ell}_i \right]. \tag{6}$$

Contrasting equation 6 with 5, the global planner values the profits of all firms, and views all side payments as a wash. The hegemon's maximization problem is choosing a hegemon contract to maximize global welfare (equation 6), subject to the participation constraints of firms (equation 4) and the feasibility of joint threats (Definition 4). The following result characterizes the global planner's optimal contract.

Proposition 5 *An optimal contract of the hegemon from the global planner's perspective features maximal joint threats, zero side payments, and zero wedges, that is* $S_i' = \overline{S}_i'$, $\overline{T}_i = 0$, *and* $\tau_i = 0$ *for all* $i \in \mathcal{C}_m$.

The global planner's solution exactly coincides with the hegemon's solution for the hegemon's domestic firms, but now extends the same terms of the contract to foreign firms as well. Intuitively, both the global planner and hegemon perceive a benefit to expanding production and operating profits of foreign firms, and hence both wish to impose maximal joint threats, which expands operating profits the most. However, the global planner directly values the operating profits accruing to foreigners. In contrast, the hegemon's consumers do not own these foreign firms, and only benefit to the extent that the expanded profits are extracted as a side payment.

Propositions 4 and 5 highlight some crucial features of our model. The presence of geoeconomic power in our framework is not a zero-sum game. Geoeconomic power improves global outcomes, making everyone weakly better off, but the benefits accrue disproportionately to the hegemeon. The negative sum aspect arises from the side payments that destroy value at the global level, while transferring wealth from extraction points to the hegemon. The hegemon in extracting positive transfers is moving to the inside of the global Pareto frontier, but increasing the benefit to its own country.

3.5 General Analysis: Optimal Contract and Efficiency

We now reintroduce externalities arising from the vector of aggregates z. We first show that our economy has an input-output structure where amplification occurs via the externalities.

The equilibrium vector of externalities, z^*, must satisfy $x^*_{ij}(\overline{\mathcal{S}}'_i, \overline{T}_i, \tau_i, z^*) = z^*_{ij}$. We derive analysis focusing on maximal threats (Lemma 2), but the derivations also hold for equilibria under other possible (suboptimal) contracts. To clarify the ordering for matrix algebra, we have $z^*_i = \left(z^*_{1,\min \mathcal{J}_i}, \ \ldots, \ z^*_{1,\max \mathcal{J}_i} \right)^T$ which is a $|\mathcal{J}_i| \times 1$ vector, and $z^* = \left(z^{*T}_1, \ \ldots, \ z^{*T}_{|\mathcal{I}|} \right)^T$, which is a $\sum_{i\in\mathcal{I}} |\mathcal{J}_i| \times 1$ vector. For compactness, we use $|z^*| = \sum_{i\in\mathcal{I}} |\mathcal{J}_i|$. We stack x^* starting from elements x^*_{ij} in the same manner.

Consider a generic exogenous variable a. To understand the impact that a change in a has on the entire input-output system, we derive a Leontief inverse based on the endogenous response of z^*. That is, we are interested in computing the vector $\frac{dz^*}{da}$, which is a $|z^*| \times 1$ vector. We start by totally differentiating $x^*_{ij}(\overline{\mathcal{S}}'_i, \overline{T}_i, \tau_i, z^*) = z^*_{ij}$ in a,

$$\frac{\partial x^*_{ij}}{\partial a} + \frac{\partial x^*_{ij}}{\partial z^*}\frac{dz^*}{da} = \frac{dz^*_{ij}}{da},$$

where $\frac{\partial x^*_{ij}}{\partial z}$ is a $1 \times |z^*|$ vector. Stacking the system vertically, we write

$$\frac{\partial x^*}{\partial a} + \frac{\partial x^*}{\partial z^*}\frac{dz^*}{da} = \frac{dz^*}{da},$$

where $\frac{\partial x^*}{\partial a}$ is a $|z^*| \times 1$, and $\frac{\partial x^*}{\partial z^*}$ is a $|z^*| \times |z^*|$ matrix with each rows corresponding to the vector $\frac{\partial x^*_{ij}}{\partial z^*}$. We solve for $\frac{dz^*}{da}$ and obtain:

$$\frac{dz^*}{da} = \left(\mathbb{I} - \frac{\partial x^*}{\partial z^*}\right)^{-1} \frac{\partial x^*}{\partial a}.$$

We define $\Psi^z \equiv \left(\mathbb{I} - \frac{\partial x^*}{\partial z^*}\right)^{-1}$ and note that it is akin to a Leontief inverse matrix since it keeps track of all the successive amplification via the input-out structure of the original change in production. We collect the result in the Lemma below.

Proposition 6 *The aggregate response of z^* to a perturbation in exogenous variable a is $\frac{dz^*}{da} = \Psi^z \frac{\partial x^*}{\partial a}$, where $\Psi^z = \left(\mathbb{I} - \frac{\partial x^*}{\partial z^*}\right)^{-1}$.*

3.5.1 Optimal Contract

In characterizing the hegemon's optimal contract, we set up the following notation (see the proof of Proposition 7 for details). Letting $\mathcal{L}_m$ be the hegemon's Lagrangian, we denote $\eta_i \geq 0$ the Lagrange multiplier on the participation constraint of firm i, and $\Lambda_{iS} \geq 0$ the Lagrange multiplier on the incentive constraint of firm i for S. We define $\mathcal{E}_{ij} \equiv \frac{\partial \mathcal{L}_m}{\partial z^*_{ij}}$. For technical reasons, we assume that if $\mathcal{S}_i^D$ is not a pressure point on firm i at the optimal z^*, then it is also not a pressure point on i in a neighborhood of z^*. An optimal contract is characterized by the proposition below.[21]

Proposition 7 *Conditional on entry, an optimal contract of the hegemon has the following terms:*

1. *For domestic firms $i \in \mathcal{I}_m$, if $\mathcal{S}_i^D$ is a pressure point on i:*
   (a) *Wedges satisfy:* $(\Lambda_{iS} + \eta_i + 1)\tau^*_{ij} = -\mathcal{E}_{ij}$.
   (b) *Side payments are zero:* $\overline{T}^*_i = 0$.
2. *For foreign firms $i \in \mathcal{D}_m$, if $\mathcal{S}_i^D$ is a pressure point on i:*
   (a) *Wedges satisfy:* $(\Lambda_{iS} + \eta_i)\tau^*_{ij} = -\mathcal{E}_{ij}$.
   (b) *Side payments satisfy:* $\eta_i \geq 1 - \Lambda_{iS_i^D}$, *with equality if* $\overline{T}^*_i > 0$.
3. *If $\mathcal{S}_i^D$ is not a pressure point of firm i, then $\overline{T}_i = 0$ and $\tau_i = 0$.*

Intuitively, for a domestic firm and in the presence of externalities from z, the hegemon no longer finds it optimal to impose zero wedges because it uses wedges to correct externalities. Activities that generate positive externalities $\mathcal{E}_{ij} > 0$ are subsidized, while activities that generate negative externalities $\mathcal{E}_{ij} < 0$ are taxed. The wedges interact with both the incentive constraint and the participation constraint. If the constraints are tighter, i.e. higher Lagrange multipliers, the subsidies and taxes shrink towards zero. The hegemon trades off distorting private production decisions, which tightens the constraints, against the benefit of the distortion arising from externalities. Tighter constraints make this trade-off put more weight on private costs (for fixed externalities).

Familiar from Proposition 4, domestic firms are never charged side payments. However, this result is no longer immediate: in the presence of externalities, in principle the hegemon might want to use side payments to reduce firms' capacity to engage in negative-externality activities. However, in the presence of complete wedges, the hegemon can instead use wedges to achieve this goal, and so no side payments are charged.

Consider next a foreign firm. The hegemon's optimal wedge formula is almost identical to that for domestic firms, except that the magnitude of wedges (whether tax or subsidy) is higher.

[21] Proposition 7 provides necessary conditions for optimality. Formally, if for a foreign firm i we have $\eta_i = 0$ and $\Lambda_{iS} > 0$, it instead characterizes the limit of a sequence of wedges, each of which is part of a (different) optimal contract (see the proof for details).

Intuitively, this occurs because the hegemon does not (directly) value the profits of foreign firms, as it does for domestic firms. As a result, the hegemon is more willing to impose higher corrective wedges, even though they erode operating profits. While the hegemon still has an incentive to extract side payments from foreign firms, as in Proposition 4, in the presence of externalities there is a counterveiling force. Charging a higher side payment to a firm has the effect of tightening both the participation constraint and the incentive constraint, valued by the multipliers $\eta_i + \Lambda_{iS_i^D}$. At the same time, externality correction also has the effect of tightening these constraints. The hegemon therefore has to weigh using slack generated by its joint threat to extract side payments, or to manage externalities.

3.5.2 Classifying Friends and Enemies

Our framework provides a classification of "friends and enemies" of the hegemon based on externalities. This terminology and notion is related to Kleinman et al. (2020) who base it on a country real income response to a foreign country increase in productivity. Foreign firm i friendly, neutral, or unfriendly based on the value of the spillovers that that firm has from the hegemon's perspective.

Definition 5 *Under the hegemon's optimal contract, foreign firm i is:*

1. ***Unfriendly*** *to the hegemon if $\mathcal{E}_{ij} \leq 0$ for all $j \in \mathcal{J}_i$, with strict inequality for at least one j.*
2. ***Neutral*** *to the hegemon if $\mathcal{E}_{ij} = 0$ for all $j \in \mathcal{J}_i$.*
3. ***Friendly*** *to the hegemon if $\mathcal{E}_{ij} \geq 0$ for all $j \in \mathcal{J}_i$, with strict inequality for at least one j.*

Definition 5 delineates three types of relationships: friendly firms that have only (weakly) positive spillovers from the hegemon's perspective, neutral firms with no spillovers, and unfriendly firms with only (weakly) negative spillovers. Of course, firms can in general have some activities that generate positive spillovers and some activities that generate negative ones. We leave those firms unclassified in the definition above, as mixed firms.

The notion of friendship that we develop is both theoretically grounded and relevant for understanding how the hegemon interacts with these firms in its optimal contract. For example, a friendly firm i has its strictly positive-externality activities subsidized, while an unfriendly firm has its strictly negative-externality activities taxed. A neutral firm, in contrast, is neither taxed nor subsidized as long as at least one constraint binds ($\Lambda_{iS} + \eta_i > 0$), consistent with Proposition 4, in which all firms were neutral.

Friendship is also an important driver of which firms are held to their participation constraints and achive no surplus under the optimal contract. We obtain the following result.

Proposition 8 *Under the hegemon's optimal contract, the participation constraint of foreign firm i binds if $\mathcal{E}_{ij} \leq 0$ for all $j \in S_i^D$. Therefore, the participation constraint always binds for foreign unfriendly and neutral firms.*

Intuitively, if $\mathcal{E}_{ij} \leq 0$ for all $j \in S_i^D$, then the activities involved in the joint threat all entail weakly negative externalities. If hypothetically the participation constraint did not bind, the hegemon would be better off by curtailing some or all of these activities, and charging a higher side payment out of the slack generated.

Even friendly firms might have a binding participation constraint because, if all strictly-positive externality activities occur outside the joint threat, the hegemon finds it optimal to extract as much surplus as possible from the joint threats it supplies. On the other hand, we provide a simple example where not only is the constraint nonbinding, but in fact the hegemon imposes no side payments or wedges on the foreign friendly firm.

Example 1 (Only Consumer Externalities) *Consider an environment in which externalities z do not enter firm production, that is $f_{ij}(x_{ij}, z)$ is constant in z. Suppose further that consumer externalities are separable across firms, that is $u_n(z) = \sum_{i\in\mathcal{I}} u_n^i(z_i)$, and let $u_n^i(z_i) = \alpha_i \sum_{j\in\mathcal{J}_i} \pi_{ij}(z_{ij})$. For each firm i, we have $\mathcal{E}_{ij} = \frac{\partial u_m^i(z_i)}{\partial z_{ij}} = \alpha_i \frac{\partial \pi_{ij}(z_{ij})}{\partial z_{ij}}$. If $\alpha_i < 0$, then firm i is unfriendly to the hegemon. The hegemon will choose a combination of positive wedges and side payments to shrink this unfriendly firm's activities and make the participation constraint bind. On the contrary, if $\alpha_i > 0$, then firm i is friendly to the hegemon, and $\tau_{ij} = \overline{T}_i = 0$ is an optimal contract if and only if $\alpha_i \geq \lambda_{iS_i^D}^{-1}$.*

Example 1 highlights how the externalities in the consumer utility function can be used to capture the economics of cross-border ownership. For example, if $\alpha_i > 0$, it gives utility weight for the hegemon to the operating profits of firm i, much like an equity stake in the firm would do. We do not tackle endogenous cross-border ownership in this paper, but the framework can clearly be used to derive interesting implications from this potential extension.

3.5.3 Efficient Allocations

As in Section 3.4.1, we provide an efficiency benchmark by taking the perspective of a utilitarian global planner choosing a hegemon contract to maximize global welfare,

$$\sum_{i\in\mathcal{I}} \left[\Pi_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i, z) + w_i \bar{\ell}_i\right] + \sum_{n=1}^{N} u_n(z), \tag{7}$$

subject to the same constraints as the hegemon. We obtain the following result.

Proposition 9 *An optimal contract of the hegemon from the global planner's perspective features maximal joint threats $\mathcal{S}_i' = \overline{S}_i'$, zero side payments $\overline{T}_i = 0$, and wedges given by $(\Lambda_{iS} + \eta_i + 1)\tau_{ij}^* = -\mathcal{E}_{ij}^p$ for all firms $i \in \mathcal{C}_m$ on which the hegemon has a pressure point, where $\mathcal{E}_{ij}^p$ is defined in the proof. Wedges and side payments are zero if $\mathcal{S}_i^D$ is not a pressure point on i.*

The proposition above generalizes that the hegemon and the global planner agree threats are positive-sum game and that side-payments are negative sum game, but they have different objective functions. As in Proposition 5 the global planner imposes zero side payments. In this general case, however, the planner implements non-zero wedges, but these wedges are different from those imposed from the hegemon. In general, the global planner does not perceive friends and enemy the same way as the hegemon does. Intuitively, a sector might have a negative externality on the hegemon country but a positive one on others. Formally, this can be seen in the proposition above where $\mathcal{E}^p_{ij}$ tracks the impact of activity x_{ij} on the planner's Lagrangian rather than the hegemon's one.

3.5.4 Which Good are Strategic?

Our results provide guidance on how to think about what types of goods or industries are "strategic". In our model, strategic sectors are nodes through which the hegemon can extract high surplus if it controls them. A node is strategic if it allows the hegemon to form many valuable joint threats to pressure other nodes. In addition, it might be strategic because it has a high influence, in the Leontief sense, on the world economy and therefore controlling its choices via the wedges has a large indirect impact on the economy.

Control and pressure are central to our notion of strategic sectors. Sectors that have many linkages in the global economic network are good candidates for applying pressure. Goods with low transport costs, such as finance or information technology or natural resources like rare earths and oil, are more likely to be strategic goods than goods with high transport costs, such as concrete. Linkages are not sufficient, however, since they might provide no pressure points because threats have no bite if they have readily available substitutes or they have no added value.

Assuming a hegemon controls a certain sector, either directly in the domestic economy or abroad through pressure, the nature of that sector determines how strategic it is. Given the costly actions that the hegemon can demand of this node, how much value can it extract from the world economy? Direct value extraction from the node being pressured is only one part, and limited by the participation constraint. The indirect transmission is potentially much larger. By asking nodes it controls to take costly actions, such as curbing the usage of some inputs, the hegemon indirectly influences a larger part of the input-output network that it does not directly control. The propagation and amplification through the network structure (our externality based Leontief-inverse) is key to this effect. In an application below focusing on telecommunication infrastructure and national security, we show how the hegemon can extract value indirectly by using network amplification to contain an hostile country.

Our theory not only clarifies what is strategic, but also highlights shortcomings of existing measures. Our framework clarifies that that most threats, even if they can be made, are not valuable because of other means of enforceability, the presence of alternative providers, and the presence of close substitute inputs.[22] Overlooking this would lead to the mistaken perception of an industry as strategic. Other ideas of strategic industries may be incomplete in the sense that characteristics of a sector, for example whether it is downstream or upstream, are not necessarily the relevant metrics. Furthermore how strategic an industry is to a particular hegemon depends on what other sectors the hegemons controls.

4 Applications

We show how the model can be specialized to capture leading applications in geoeconomics. We focus on two applications. In the first, we show how the hegemon can combine lending and manufacturing activities to extract political concessions, which helps capture in the model programs such as China's Belt and Road Initiative. Dreher et al. (2022), Gelpern et al. (2022), Horn et al. (2021), Horn et al. (2023), and Liu (2023) document and analyze the rise of China as a global development and project finance lender.

In the second, we focus on a hegemon blocking third party countries from using a technology input provided by an unfriendly country. We assume the unfriendly technology is a national security threat for the hegemon, but a positive externality for production by firms in third party countries. This helps us capture bans on emerging technology such as semiconductors or the 5G telecommunication infrastructure provided by Huwaei.[23]

4.1 Official Lending, Infrastructure Projects, and Political Concessions

We specialize the model to the configuration in Figure 2. The hegemon country, in this application China, has two sectors. Sector k is a lender, while sector j is a manufacturer. For simplicity, both sectors produce only using their respective local factor. The target country, in this application an emerging economy, has a single sector i that uses both inputs from China to produce. We assume that there are no externalities in the production functions.

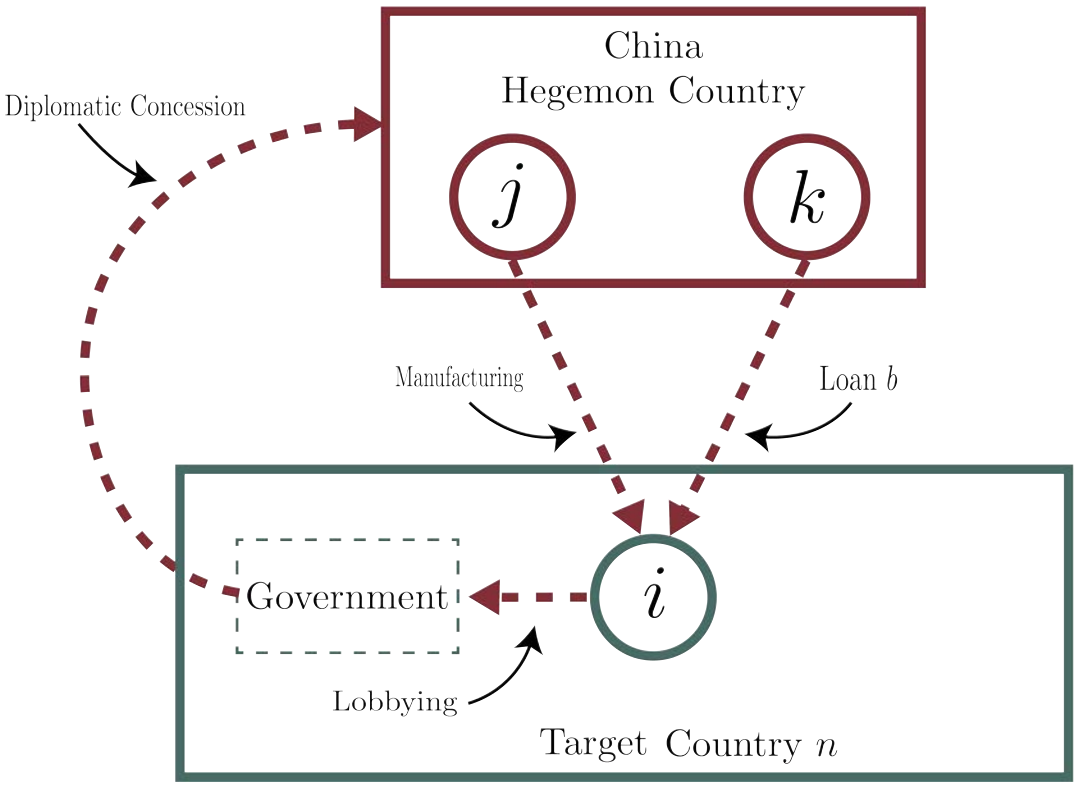
We think of the lending sector as providing a loan or buying a bond issued by sector i. The loan is for amount $x_{ik} = b$ and the gross interest rates is $p_k = R_k$. Much like in the sovereign default literature, we assume that the loan is not enforceable at all, so that $\theta_{ik}(b) = 1$. This sharpens the application because under isolated threats no lending can be sustained. Indeed, we have $\pi_{ik}(b) = p_i f_{ik}(b) - R_k b$ and $\pi^D_{ik}(b) = p_i f_{ik}(b)$, so the the incentive constraint under isolated threats is:

$$p_i f_{ik}(b) - R_k b \geq p_i f_{ik}(b) \quad \Rightarrow \quad b \leq 0.$$

[22]The literature in trade and finance are replete of examples of restrictions that have little equilibrium consequence precisely because of the endogenous response of economic actors that by-pass the restrictions or substitute away.

[23]See the discussions in Miller (2022) and Farrell and Newman (2023).

Figure 2: **Application: Belt and Road Initiative**



Notes: Figure depicts the model set-up for the application on the Belt and Road Initiative as described in Section 4.1.

Since the lending relationship is technologically viable, that is $p_i f'_{ik}(0) - R_k > 0$, the constraint binds under isolated threats with $b = 0$ and we the associated Lagrange multipliers is positive, $\lambda_{ik} > 0$.

To sharpen the application, we assume that, under isolated threats, the sourcing of input j occurs at the unconstrained level. That is $\pi'_{ij}(x^u_{ij}) = 0$ and $\lambda_{ij} = 0$. Therefore, without a hegemonic China, the equilibrium features no lending and a positive manufacturing relationship.

China can as a hegemon impose a joint threat that links together the provision of lending and manufacturing goods. If the target country defaults on either input, both are withdrawn in the subgame. Under the joint threat the incentive constraint of the target country sector i is:

$$b \leq \frac{\pi_{ij}(x_{ij}) - \pi^D_{ij}(x_{ij})}{R_k}.$$

At the ex-ante equilibrium $\pi_{ij}(x^u_{ij}) - \pi^D_{ij}(x^u_{ij}) > 0$ since input j was sourced at the unconstrained level. The value of this slack provides incentives to repay the debt in the joint threat, an endogenous cost of default on the loan. Indeed, the maximum amount that the target country can promise to repay $R_k b$ is bounded above by the value of the slackness in the manufacturing relationship.

By the envelope theorem, small variation in x_{ij} around x_{ij}^u only induces a second order profit loss form firms in sector i, while there is a first order gain to further borrowing. Under the joint threat, the equilibrium features strictly positive lending $b^* > 0$ and a smaller manufacturing relationship $x_{ij}^* \leq x_{ij}^u$. The surplus in extracted by China via positive side payment $T_i^* > 0$.

Our mechanism is related to that proposed in Bulow and Rogoff (1989), whereby lenders seize the exports of a country conditional on a default, thereby generating a cost of default. More recently, Mendoza and Yue (2012) consider a quantitative sovereign debt model where country's face an endogenous productivity cost of default that arises because a defaulting country loses access to trade finance, losing the ability to import intermediate goods, and is forced to switch to imperfect domestic substitutes for production. In our framework, joint threats offer a means for a country to voluntarily raise its cost of default through such a channel, thereby allowing it to borrow more. In particular, the more inputs are sourced from China, the more the borrowing constraint is relaxed.

One interpretation of the side payments are markups on the manufacturing goods being sold by China to the target country, or equivalently an interest rate on the loan above the market rate R_k. This application shows the futility of assessing China's lending programs in isolation: i.e. focusing only on the loans and their returns. Both the sustainability of the loans and the economic returns from the lending have to be assessed jointly with other activities, such as manufacturing exports, that are occurring jointly with the lending. The benefits to China might not even accrue in monetary form as we explore below.

Side Payments as Costly Actions and Political Concessions. The side payments that the hegemon extracts can cover costly actions that the hegemon asks the firm to undertake in exchange for the joint threat. In this case, the side payment T_i represents the private cost to the firm of the action. Here we focus on a leading example for geoeconomics in which China asks the firms to lobby their governments for a political concession.

We assume that a bilateral geopolitical concession can be made from country i to China. We let the concession, be the element z_n^c of aggregate vector z and assume that it enters positively in China's utility, $u_m(z_n^c) > 0$, and negatively in the target's country utility $u_n(z_n^c) < 0$. We assume that no utility is derived by either countries from all other elements of z. Governments care about consumer welfare and therefore internalize these utility costs and benefits. Governments also care about the profits of the firms in their country net of side payments. We assume that a hegemon asking a firm to make a positive side payment can alternatively ask that firm to transfer part or all of that side payment to the government in exchange for the government undertaking the geopolitical action, with any money not transferred being paid as usual to the hegemon. The geopolitical action is feasible to implement as long as country level side payments exceed the government utility cost of the concession.

These concessions can account, for example, for China asking countries who are part of the Belt and Road Initiative to not recognize Taiwan. This is consistent with the evidence in Dreher et al.

(2022) that recipients of Belt and Road lending are much less likely to recognize Taiwan.

4.2 National Security Externalities

There are three regions: the hegemon country m, a hostile foreign country h, and "rest of world" w (which may comprise multiple countries). Figure 3 illustrates the set-up of this application.

The hostile foreign country h has a single sector, which we denote $i = H$. We take the output of this sector to be the numeraire, $p_H = 1$. Sector H is not subject to externalities from z, that is $f_{Hj}(x_{Hj}, z)$ is constant in z. Firms in the hegemon country, $i \in \mathcal{I}_m$, are not subject to externalities from z, that is $f_{ij}(x_{ij}, z)$ is constant in z. For simplicity, we assume that firms in the hegemon country do not source from the hostile country's firms in H and vice-versa. We assume this to ensure that H cannot be used by the hegemon as part of a joint threat.

The main action in this application comes from rest-of-world firms, $i \in \mathcal{I}_w$. We assume that all rest-of-world firms source from H, and let $z^H \equiv \{z_{iH}\}_{i \in \mathcal{I}_w}$ be the vector of purchases by these firms of input H. Firm i's production out of H is given by

$$f_{iH}(x_{iH}, z) = A_{iH}(z^H) g_{iH}(x_{iH}). \tag{8}$$

We assume that $\frac{\partial A_{iH}}{\partial z_{jH}} > 0$ for all $i, j \in \mathcal{I}_w$, so that there are positive spillovers from greater usage of H. We further assume that $A_{iH}(z^H) g_{iH}(z_{iH})$ is concave in z^H. All production lines $f_{ij}(x_{ij}, z)$ of i apart from H are constant in z.

Finally, we assume all firms start from isolated threat action sets (i.e., no pre-existing joint threats).[24]

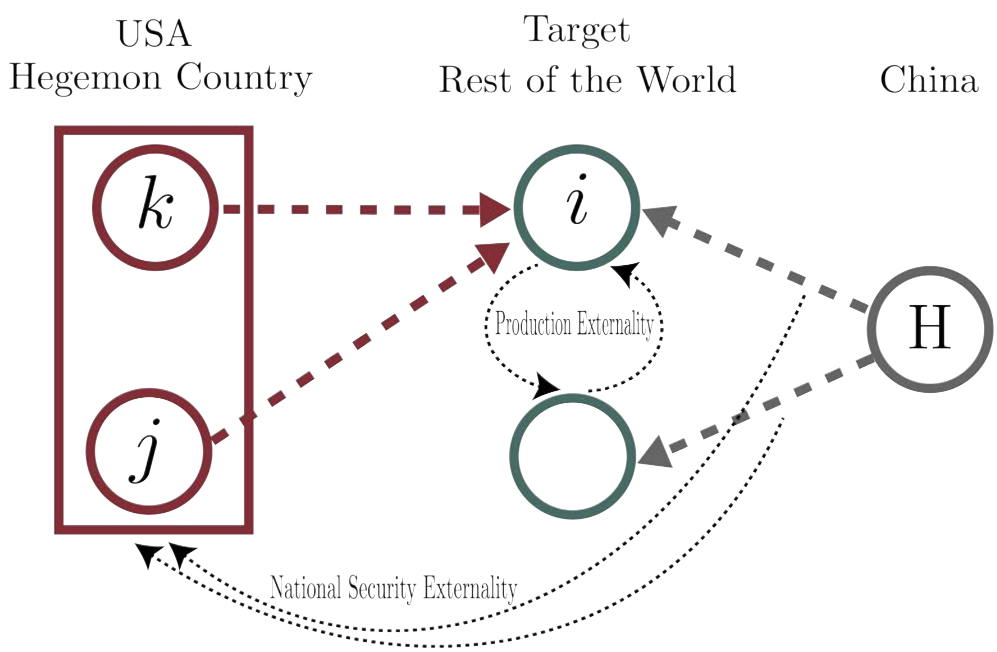
Hegemon Negative Externality from H. We assume that the hegemon's representative consumer's function $u_m(z) = u_m(z^H)$ has a negative externality from production using H, that is $\frac{\partial u_m}{\partial z_{iH}} < 0$ for all $i \in \mathcal{I}_w$. There are no other externalities on the consumer. From Lemma 2, maximal joint threats are optimal for all firms. Since there are no externalities associated with production by domestic firms, Proposition 7 tells us $\overline{T}_i = 0$ and $\tau_i = 0$ is an optimal contract for all domestic firms. Therefore, we focus on characterizing optimal contracts for foreign firms in the rest of world. The remaining part of the objective function of equation 5 related to these firms is

$$U_m = u_m(z^H) + \sum_{i \in \mathcal{D}_m} \overline{T}_i. \tag{9}$$

Participation Constraints. Consider a foreign firm $i \in \mathcal{D}_m$. Since the hegemon sells no inputs to H, H cannot be used by the hegemon in joint threats, and therefore $H \notin S_i^D$. As there are no externalities from any input apart from H, by Proposition 7 the optimal wedges on all inputs apart from H are zero. By Proposition 8, the participation constraints of all foreign firms bind.

[24] That is, $\mathcal{S}_i = \{\emptyset\} \cup \{\{j\}\}_{j \in \mathcal{J}_i}$.

Figure 3: **Application: National Security Externality**



Notes: Figure depicts the model set-up for the application on national security as described in Section 4.2.

Using separability of the value function across elements of the action set, we can therefore write the participation constraint as

$$v_i(\{H\}, 0, 0, z^H) - v_i(\{H\}, 0, \tau_{iH}, z^H) = V_i(\overline{\mathcal{S}}_i' \backslash \{H\}, \overline{T}_i, 0) - V_i(\mathcal{S}_i \backslash \{H\}). \tag{10}$$

Equation 10 characterizes an intuitive trade-off between managing externalities and asking for side payments. The right-hand side measures the gain in value for firm i from the joint threat provided by the hegemon, accounting for the required side payment (if any). The left-hand side measures the cost to the firm of accepting a nonzero wedge τ_{iH}, which reduces its profits from production using H. Therefore, the hegemon can induce firm i to reduce its usage of input H only to the extent it provides value via the joint threat, and this value is not extracted via side payments.

Input-Output Structure of Externalities. Consider a rest-of-world firm that the hegemon cannot contract with, $i \notin \mathcal{D}_m$. It is simplest to asume that these firms have a nonbinding incentive constraint for good H, so that their demand for good H is given by the first-order condition[25]

$$p_i A_{iH}(z^H) g_{iH}'(x_{iH}) = 1. \tag{11}$$

[25] If instead the firm faces a binding incentive constraint, we can write an analogous equation $p_i A_{iH}(z^H) \hat{g}_{iH}'(x_{iH}) = 1$, where we have defined $\hat{g}_{iH}'(x_{iH}) = \frac{g_{iH}(x_{iH}) - g_{iH}(\theta_{iH}(x_{iH}) x_{iH})}{x_{iH}}$, and proceed with similar analysis.

From here, let $z^{H,-m}$ be the subset of allocations z_{iH} of firms that do not contract with the hegemon, and z^{Hm} for those that contract with the hegemon. Employing Proposition 6, we construct the endogenous response $\frac{dz^{H,-m}}{da} = \Psi^{z,-m} \frac{\partial x^{H,-m}}{\partial a}$ of rest-of-world firms that the hegemon cannot contract with to changes in a resulting from a change in the hegemon's contract. Since from Proposition 6 we have $\Psi^{z,-m} = \left(\mathbb{I} - \frac{\partial x^{H,-m}}{\partial z^{H,-m}} \right)^{-1}$, the key objects of interest take the form $\frac{\partial x^*_{iH}}{\partial z_{jH}}$. Differentiating equation 11 in z_{jH}, we obtain

$$\frac{\partial x^*_{iH}}{\partial z_{jH}} = \frac{x^*_{iH}}{z_{jH}} \frac{\xi_{ij}}{\gamma_i},$$

where $\xi_{ij} \equiv \frac{z_{jH}}{A_{iH}(z^H)} \frac{\partial A_{iH}(z^H)}{\partial z_{jH}}$ is the elasticity of productivity A_{iH} with respect to the externality z_{jH}, and where $\gamma_i = \frac{-x^*_{iH} g''_{iH}(x^*_{iH})}{g'_{iH}(x^*_{iH})}$.

Optimal Contract. The hegemon chooses side payments $\overline{T}_i$ and wedges τ_{iH} for all firms $i \in \mathcal{D}_m$ to maximize its utility, equation (9), subject to the participation constraints, equation (10). In doing so, the hegemon must account for the endogenous response of rest-of-world firms the hegemon does not contract with, $i \notin \mathcal{D}_m$, as derived above.

We can capture interesting economics of the application with only two firms in the rest of world: one firm, i, that the hegemon can contract with; and one firm, j, that the hegemon cannot contract with. In this environment, Proposition 7 yields an optimal tax formula (see Appendix A.12) given by[26]

$$\tau_{iH} = - \overbrace{\frac{1}{\eta_i} \frac{\partial u_m}{\partial z_{iH}}}^{\text{Direct Externality}} - \overbrace{\frac{1}{\eta_i} \frac{\partial u_m}{\partial z_{jH}} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}}}^{\text{Network Amplification}} + \underbrace{p_i A_{iH}(z^H) \Big[g_{iH}(x^*_{iH}(z^H)) - g_{iH}(x_{iH}) \Big] \left(\xi_{ii} + \xi_{ij} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \right) \frac{1}{z_{iH}}}_{\text{Participation Constraint}}$$

If there are no externalities from z^H, then this tax formula collapses to $\tau_{iH} = 0$, consistent with Proposition 4. In the presence of national security externalities, the optimal tax is positive, $\tau_{iH} > 0$, reflecting the hegemon's desire to mitigate the negative externality. Three key forces underlie the tax formula.

The first term in the tax formula is the direct externality from an increase in z_{iH} on representative consumer m. The negative externality contributes to a positive tax. This tax is upweighted when η_i is higher, that is when the marginal value of slack in the participation constraint is lower. When side payments are positive, $\eta_i = 1 - \lambda_{iS^D_i}$, and so is decreasing in the tightness of the incentive constraint involving the joint threat. If this incentive constraint is tighter, the opportunity cost of managing externalities is higher because side payments are more distortive, and hence the hegemon

[26]For simplicity, we assume that firm i is unconstrained in its purchase of good H if it does not accept the contract. The tax formula is qualitatively unchanged if i is constrained.

pushes for larger corrective wedges.

The second term is the indirect effect from the network externality: as z_{iH} falls, productivity A_{jH} of firm j usage of H input falls, prompting firm j to reduce its usage of H. This leads to a fall in z_{jH}, which has a positive externality on the consumer. The effect $\frac{\xi_{ji}}{\gamma_j - \xi_{jj}}$ captures the magnitude of this response. This effect contributes towards an even higher tax rate, since reducing demand of firm i for good H has a positive externality to the hegemon of also reducing demand of firm j for good H.

Finally, the third term captures the effect on changes in externalities on the participation constraint of firm i. In particular, it captures the change in profits of the outside option for the firm relative to the inside option. This effect is positive, with $g_{iH}(x^*_{iH}(z^H)) - g_{iH}(z_{iH}) \geq 0$ representing the cost of foregone production from accepting the positive tax. Intuitively, a corrective tax reduces productivity, which in turn reduces the optimal scale of an untaxed firm. Because productivity and optimal scale have fallen, the temptation of the firm to deviate to the outside option also falls. As a result, the hegemon wishes to overshoot simple Pigouvian correction, and employ a higher-magnitude tax in order to reduce incentives of individual firms to reject the contract.

5 Hegemonic Competition for Dominance

We now consider the possibility that multiple countries can become hegemons. For simplicity, we focus on the case in which two countries, m_1 and m_2, can become hegemons, and assume that there are no externalities from the aggregate vector z.

Hegemon competition unfolds in two stages. In the first stage, each hegemon $m \in \{m_1, m_2\}$ chooses simultaneously whether or not to pay its fixed cost $F_m \geq 0$ to become a hegemon. In the second stage, any hegemons that enter can offer a contract as described in Section 3, taking as given the contract offered by the other hegemon (if it entered). There are four environments in the second stage: (i) neither hegemon has entered, and the equilibrium is as in Section 2; (ii) exactly one hegemon enters, and its optimal contract is as in Section 3; (iii) both hegemons enter. We now turn to characterizing the equilibrium of the second stage when both hegemons enter, and then turn back to the entry choice in the first stage.

5.1 Competition Setup

Consider the second stage, and assume that both m_1 and m_2 have paid the fixed cost and become hegemons. Let $\mathcal{C} = \mathcal{C}_{m_1} \cup \mathcal{C}_{m_2}$ be the set of firms that contract with at least one hegemon. Hegemon $m \in \{m_1, m_2\}$ offers a contract $\{\Gamma_i^m\}_{i \in \mathcal{C}_m}$, where $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, \tau_i^m\}_{i \in \mathcal{C}_m}$ denotes the contract offered to firm $i \in \mathcal{C}_m$. It is convenient to define a trivial contract $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ offered by hegemon m to firms $i \in \mathcal{C} \backslash \mathcal{C}_m$, and let $\Gamma^m = \{\mathcal{S}_i'^m, \mathcal{T}_i, \tau_i^m\}_{i \in \mathcal{C}}$ be the hegemon's contract, including trivial contracts offered to firms $i \notin \mathcal{C}_m$. As in Section 3, the joint threat $\mathcal{S}_i'$ must be feasible under direct transmission.

Firm i faces revenue-neutral wedges and side payments from both hegemons that are added together when both contracts are accepted.[27] Anticipating that a best response to hegemon $-m$ setting $\tau_i^{-m} = 0$ is for hegemon m to set $\tau_i^m = 0$, we will solve the model assuming all wedges to be zero, and then verify that neither hegemon has an incentive to deviate to nonzero wedges. Therefore, we write the contract $\Gamma_i = \{\mathcal{S}'_i, \mathcal{T}_i^{m_1} + \mathcal{T}_i^{m_2}, 0\}$ as the combined contract when firms accepts both contracts. It remains to characterize the joint threat action set $\mathcal{S}'_i$ that arises when both contracts are accepted.

The joint threat $\mathcal{S}'_i$ that arises when both hegemon's contracts are accepted is constructed by taking the joint trigger sets R_{ij}^m underlying each hegemon's joint threat, defining the combined joint trigger set $R_{ij} = R_{ij}^{m_1} \cup R_{ij}^{m_2}$, and then applying Lemma 1 to this configuration of joint triggers to obtain $\mathcal{S}'_i$. Given our model has only tracked action sets, and not joint triggers directly, we provide an equivalent method of constructing $\mathcal{S}'_i$ in the definition below.

Definition 6 *Let $\hat{R}_{ij}^m$ be the unique element of $\mathcal{S}_i'^m$ with $j \in \hat{R}_{ij}^m$. Define $\hat{R}_{ij} = \hat{R}_{ij}^{m_1} \cup \hat{R}_{ij}^{m_2}$. Then, $\mathcal{S}'_i$ is the restricted action set characterized by Lemma 1 under the configuration of joint triggers $\hat{R}_i$.*

Observe that $\mathcal{S}'_i$ is a joint threat of $\mathcal{S}_i'^m$ for $m \in \{m_1, m_2\}$. The focus in the analysis below is on the (combined) maximal joint threats, $\overline{\mathcal{S}}'_i$, which arises when both hegemons offer maximal joint threats, $\mathcal{S}_i'^m = \overline{\mathcal{S}}_i'^m$. Recall that $S_i^{Dm} = \bigcup_{S \in \mathcal{S}_i^{Dm}} S$ and $\overline{\mathcal{S}}_i'^m = \{S_i^{Dm}\} \cup (\mathcal{S}_i \backslash \mathcal{S}_i^D)$, where we define $S_i^{Dm} = \emptyset$ if $i \notin \mathcal{C}_m$. Then applying Definition 6, we obtain $\overline{\mathcal{S}}'_i$ given by

$$\overline{\mathcal{S}}'_i = (\mathcal{S}_i \backslash (\mathcal{S}_i^{Dm_1} \cup \mathcal{S}_i^{Dm_2})) \cup \mathcal{X}_i, \quad \mathcal{X}_i = \begin{cases} \{S_i^{Dm_1}, S_i^{Dm_2}\} & S_i^{Dm_1} \cap S_i^{Dm_2} = \emptyset \\ \{S_i^{Dm_1} \cup S_i^{Dm_2}\} & \text{otherwise} \end{cases} \tag{12}$$

Intuitively, $\overline{\mathcal{S}}'_i$ combines both hegemon's maximal joint threats into a single maximal joint threat if the two have any common inputs. If there are no common inputs, the two hegemon's maximal joint threats are separate actions within $\overline{\mathcal{S}}'_i$.

An important property, made clear under Definition 6, is that because $\overline{\mathcal{S}}_i'^m$ is a joint threat of any alternate $\mathcal{S}_i'^m$ that is feasible under direct transmission, then $\overline{\mathcal{S}}'_i$ is also a joint threat of any alternate feasible $\mathcal{S}'_i$ formed from alternative feasible joint threats $\mathcal{S}_i'^m$. Therefore, $V_i(\overline{\mathcal{S}}'_i) \geq V_i(\mathcal{S}'_i)$ for any feasible $\mathcal{S}'_i$. Note, further, that we can apply the usual definition of pressure points to $\overline{\mathcal{S}}'_i$.

Finally, we define the participation constraints of all firms. In particular, hegemon m's contract is accepted by firm i if

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{13}$$

[27] Each hegemon takes as given the other hegemon's equilibrium rebates when both contracts are accepted. If firm i chooses to only accept one contract, equilibrium rebates by the hegemon whose contract is accepted are those that maintain revenue neutrality under the single contract, while there are no rebates by the hegemon whose contract was rejected. If neither contract is accepted, there are no rebates.

Both contracts are accepted by firm i if

$$V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}. \tag{14}$$

Case of Disjoint Hegemon Threats. The case in which hegemons' threats have no common inputs, $S_i^{Dm_1} \cap S_i^{Dm_2} = \emptyset$, is straightforward. For every $i \in \mathcal{C}_m$, Γ_i^{-m} is the trivial contract, so that $\Gamma_i = \Gamma_i^m$. Equation 14 reduces to $V_i(\Gamma_i^m) \geq V_i(\mathcal{S}_i)$. Therefore, the optimal contract of hegemon m is the same as in Proposition 4.

For the remainder of this section, we assume hegemons's threats are not disjoint for at least one firm i. For such firms, defining $\mathcal{S}_i^D = \mathcal{S}_i^{Dm_1} \cup \mathcal{S}_i^{Dm_2}$, then $\overline{\mathcal{S}}_i'$ is the joint threat formed using $\mathcal{S}_i^D$.

5.2 Existence of an Equilibrium

We show existence of an equilibrium when hegemon threats are not disjoint, in which both hegemons offer maximal joint threats, and both hegemon's contracts are accepted. We then discuss how competition shapes the side payments extracted.

The model with two hegemons has to account for the fact that if hegemon m's contract is rejected by firm i, then hegemon m can no longer use firm i in joint threats.[28] This is important because a best response of hegemon m to a contract Γ^{-m} might involve offering a contract to firm i that leads firm i to reject the contract of hegemon $-m$. To make progress, we restrict the form of the network structure as follows. Let $\mathcal{P} = \{i \in \mathcal{C} \mid V_i(\overline{\mathcal{S}}_i') > V_i(\mathcal{S}_i)\}$ denote the set of firms for which the two hegemons can, possibly only jointly, generate a pressure point.

Definition 7 *Hegemon pressure points are **isolated** if: $i \in \mathcal{P} \Rightarrow \mathcal{J}_i \cap \mathcal{P} = \emptyset$.*

Definition 7 states states that if the two hegemons can generate a pressure point on i, then the two hegemons cannot generate a pressure point on any firm $j \in \mathcal{J}_i$ that is immediately upstream from i. It ensures that two firms with pressure points from the set of hegemons they contract with are not directly linked to one another. Using this condition we can prove the following result.

Lemma 3 *Suppose that hegemon pressure points are isolated. Fix a contract Γ^{-m} of hegemon $-m$:*

1. *For all $i \notin \mathcal{P}$, $\Gamma_i^m = \{\mathcal{S}_i, 0, 0\}$ is part of an optimal contract for hegemon m.*
2. *For all $i \in \mathcal{P}$, $\mathcal{S}_i'^m$ is feasible if and only if it is feasible under direct transmission.*
3. *For all $i \in \mathcal{P}$, it is weakly optimal for hegemon m to offer maximal joint threats, $\mathcal{S}_i' = \overline{\mathcal{S}}_i'^m$.*

[28]This was not an issue in the model with a single hegemon because that hegemon always ensured its contract satisfied the participation constraint.

Lemma 3 isolates the competition model to a separable problem of analyzing how hegemons compete at each pressure point $i \in \mathcal{P}$. It does so in three steps. First, since a non-pressure-point firm cannot be incentivized to accept a nontrivial contract, hegemon m offers a trivial contract $\{\mathcal{S}_i, 0, 0\}$ to every $i \notin \mathcal{P}$ to ensure its contract is accepted and its ability to use these firms to transmit threats is preserved. The second part verifies that our notion of feasibility under direct transmission remains relevant, which follows from Definition 7: feasibility of joint threats of a hegemon $i \in \mathcal{P}$ is not reliant on the decisions of another firm $j \in \mathcal{P}$ over which contract(s) to accept. The third part of Lemma 3 extends optimality of maximal joint threats (Lemma 2) to the competition model.

Finally, we can show that an equilibrium of the model of competition exists, in which both hegemons offer maximal joint threats with zero wedges.

Proposition 10 *Suppose that hegemon pressure points are isolated. An equilibrium of the model with competition exists in which each hegemon m offers a contract featuring maximal joint threats and no wedges,* $\Gamma_i^m = \{\overline{\mathcal{S}}_i'^m, \overline{T}_i^{m*}, 0\}$*, to each* $i \in \mathcal{C}_m$*. Each firm* $i \in \mathcal{C}$ *accepts the contract(s) it is offered.*

The proof of Proposition 10 proceeds by constructing side payments $\overline{T}_i^{m*}$ such that each contract Γ_i^m is a best response to contract Γ_i^{-m}, and such that both contracts are accepted, that is $V_i(\Gamma_i) \geq \max\{V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2}), V_i(\mathcal{S}_i)\}$.

The side payments extracted by each hegemon from a foreign firm $i \notin \mathcal{I}_{m_1} \cup \mathcal{I}_{m_2}$ depend on the degree to which they can provide different threats. In the limit where hegemon threats have no overlap, $\mathcal{S}_i^{Dm_1} \cap \mathcal{S}_i^{Dm_2} = \emptyset$, there is no competition: both hegemons offer a contract identical to that of Proposition 4. Despite the multipolar world, firms receive no surplus and do not benefit from competition. By contrast when threats have full overlap, $\mathcal{S}_i^{Dm_1} = \mathcal{S}_i^{Dm_2}$, the two hegemons offer the same set of threats, and so bid each other down to zero side payments, $\overline{T}_i^m = 0$. In this case, firms receive full surplus from the relationships. This result is reminiscent of the Bertrand paradox, in which two firms competing on prices bid each other down to the perfect competition price. This outcome is also efficient ex post, since all joint threats are supplied and no side payments are extracted.

For a firm that is domestic to hegemon m, that is $i \in \mathcal{I}_m$, it remains optimal for hegemon m to demand no side payments, $\overline{T}_i^{m*} = 0$. Hegemon $-m$ then extracts the largest side payment that leaves firm i indifferent between accepting both contracts and accepting only that of hegemon m: $V_i(\overline{S}_i', \overline{T}_i^{-m*}) = V_i(\overline{\mathcal{S}}_i'^m)$. Thus the joint threats that the firm's own hegemon can provide become that firm's outside option, to which that firm is held by the other hegemon.

Entry Decision in First Stage. Entry by both hegemons is a Nash equilibrium in the first stage if hegemon m entering is a best response to hegemon $-m$ entering. If hegemon m enters when hegemon $-m$ enters, Proposition 10 characterizes existence of an equilibrium. If hegemon m does not enter when hegemon $-m$ enters, then $-m$ is a single hegemon, and so by Proposition 4 every firm $i \in \mathcal{I}_m$ receives value equivalent to outside option $V_i(\mathcal{S}_i)$. Therefore, given equilibrium $(\Gamma_i^m, \Gamma_i^{-m})$ if both hegemons enter, then hegemon m enters, given entry by hegemon $-m$, if

$$\sum_{i \in \mathcal{I}_m} V_i(\Gamma_i) + \sum_{i \in \mathcal{D}_m} \overline{T}_i^* - F_m \geq \sum_{i \in \mathcal{I}_m} V_i(\mathcal{S}_i). \tag{15}$$

Entry by both hegemons is an equilibrium of the first stage if equation (15) holds for $m \in \{m_1, m_2\}$. Since $V_i(\Gamma_i) \geq V_i(\mathcal{S}_i)$, entry by both hegemons is an equilibrium for sufficiently small (possibly zero) entry costs F_m.

6 Conclusion

We provide a framework to understand geoeconomic power. Hegemon countries use their existing financial and trade network to exert power on foreign firms and government. They extract surplus from the part of the world production network that they can pressure by asking for costly actions that can take the form of markups, increases in lending rates, but also import subsidies or restrictions on specific activities. The framework can be used as a foundation for future analysis and extensions of a rich set of issues in geoeconomics.

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A Proofs

A.1 Proof of Lemma 1

A strategy of suppliers in j in their relationship with individual firm i in the End takes the form

$$\sigma_{ij}(S) = \begin{cases} NC, & \exists k \in K_{ij} \; s.t. \; k \in S \\ C, & o.w. \end{cases},$$

where $K_{ij} \subset \mathcal{J}_i$ is the set of goods that, when any of which are stolen by individual firm i, lead suppiers in j to choose Not Complete. Letting $R_{ij} \subset \mathcal{J}_i$ be the joint trigger set of suppliers in j for individual firm i, then

$$K_{ij} = \{j\} \cup \bigcup_{x \in R_{ij}} K_{ix}, \tag{16}$$

where $j \in K_{ij}$ reflects the presence of an individual trigger, and $\bigcup_{x \in R_{ij}} K_{ix}$ reflects the presence of joint triggers. It is important to remember that joint triggers are symmetric: $k \in R_{ij}$ if and only if $j \in R_{ik}$. Thus if $k \in R_{ij}$, then $K_{ij} \subset K_{ik}$ and $K_{ik} \subset K_{ij}$, and therefore $K_{ij} = K_{ik}$.

Constructing a Candidate K_{ij}. We construct sets consistent with equation (16) that involve minimal retaliation (i.e., the smallest such sets). Let $\{X_{ij}^n\}_{n=0}^{\infty}$ be a sequence of sets constructed iteratively as follows. Let $X_{ij}^0 = \{j\}$ and, for $n \geq 1$, let $X_{ij}^n = X_{ij}^{n-1} \cup \bigcup_{x \in X_{ij}^{n-1}} R_{ix}$. To understand this sequence, the first element $X_{ij}^0 = \{j\}$ is the individual trigger. The second element, $X_{ij}^1 = \{j\} \cup R_{ij}$, adds in the fact that joint triggers of suppliers in j with suppliers in their joint trigger set, R_{ij}, adds in the individual triggers of these suppliers. The next step then adds in the individual triggers associated with the joint triggers of the suppliers that were added in the previous step, and so on.

Since $\mathcal{J}_i$ is a finite set, since $X_{ij}^{n-1} \subset X_{ij}^n \subset \mathcal{J}_i$, and since $X_{ij}^n = X_{ij}^{n-1} \Rightarrow X_{ij}^{n+1} = X_{ij}^n$, then $\exists \overline{N}_{ij} > 0$ such that $X_{ij}^{\overline{N}_{ij}} = X_{ij}^n$ for all $n \geq \overline{N}_{ij}$. We now define the minimum retaliation set, intuitively the smallest set consistent with equation (16).

Definition 8 *The* ***minimum relation set*** *of suppliers in j for firm i is $X_{ij}^* = X_{ij}^{\overline{N}_{ij}}$.*

We can now show that all members of X_{ij}^* have the same minimum retaliation set.

Lemma 4 *$k \in X_{ij}^*$ if and only if $X_{ik}^* = X_{ij}^*$.*

Proof of Lemma 4. The if statement is immediate since $k \in X_{ik}^*$ by construction. Consider then only if and let $k \in X_{ij}^*$. Since $k \in X_{ij}^*$, then by construction of the sequence we have $X_{ik}^* \subset X_{ij}^*$.[29] Moreover since $k \in X_{ij}^*$, by construction there is a sequence $x_0, .., x_N$, with $x_0 = j$ and $x_N = k$, such that $x_n \in R_{ix_{n-1}}$ for $n = 1, ..., N$. Reversing that sequence and using symmetry of joint triggers, we have a sequence $x_N, ..., x_0$ such that $x_{n-1} \in R_{ix_n}$. Hence, $j \in X_{ik}^N$, and hence $j \in X_{ik}^*$. But then by construction we also have $X_{ij}^* \subset X_{ik}^*$, and hence $X_{ij}^* = X_{ik}^*$. $\square$

[29]Observe that if $k \in X_{ij}^*$, then there is a step N with $k \in X_{ij}^N$. Given construction of the sequence, all elements X_{ik}^1 are then added at step $N+1$, and so on.

Defining End Subperiod Strategies. Let $K_{ij} = X^*_{ij}$ for all j. Since $R_{ij} \subset X^*_{ij}$ by construction, then Lemma 4 implies $X^*_{ix} = X^*_{ij}$ for all $x \in R_{ij}$. Since further $j \in X^*_{ij}$, then

$$\{j\} \cup \bigcup_{x \in R_{ij}} K_{ix} = \{j\} \cup \bigcup_{x \in R_{ij}} X^*_{ij} = X^*_{ij} = K_{ij},$$

consistent with equation (16). Thus we have strategies $\sigma_{ij}(S)$ consistent with equation (16). Finally, we let the strategy ς_{ij} of individual firm i with regards to suppliers in j be $\varsigma_{ij}(S) = \sigma_{ij}(S)$. Observe finally that the strategies σ_i, ς_i are a Nash equilibrium of the End game for every $S \in P(\mathcal{J}_i)$.

Incentive Compatibility and Action Sets. We now have strategies of firms and suppliers that result in end subperiod Nash equilibria. We now turn to characterizing a minimal action set, $\mathcal{S}^*_i$, with the property that any allocation x_i is incentive compatible with respect to $\mathcal{S}^*_i$ if and only if it is incentive compatible with respect to $P(\mathcal{J}_i)$. Our candidate action set is given by

$$\mathcal{S}^*_i \equiv \{\emptyset\} \cup \bigcup_{j \in \mathcal{J}_i} \{X^*_{ij}\}$$

Observe that $|\mathcal{S}^*_i| \leq |\mathcal{J}_i| + 1$. Observe further that if $X^*_{ij} = X^*_{ik}$, only one copy is kept in the set $\mathcal{S}^*_i$. Thus we obtain the following properties, which underpin Definition 1.

Lemma 5 *$\mathcal{S}^*_i$ has the properties: (i) $\emptyset \in \mathcal{S}^*_i$; (ii) $\bigcup_{S \in \mathcal{S}^*_i} S = \mathcal{J}_i$; (iii) $\forall S, S' \in \mathcal{S}^*_i, S \cap S' = \emptyset$ if $S \neq S'$.*

Proof of Lemma 5. The first property follows by construction. The second property follows because $\bigcup_{j \in \mathcal{J}_i} X^*_{ij} = \mathcal{J}_i$. The third property follows because for all $j, k \in \mathcal{J}_i$, either $X^*_{ij} = X^*_{ik}$ or $X^*_{ij} \cap X^*_{ik} = \emptyset$ (Lemma 4). $\square$

We prove the following Lemma, which completes the proof of Lemma 1.

Lemma 6 *The allocation x_i is incentive compatible with respect to $P(\mathcal{J}_i)$ if and only if it is incentive compatible with respect to $\mathcal{S}^*_i$.*

Proof of Lemma 6. The only if statement holds trivially since $\mathcal{S}^*_i \subset P(\mathcal{J}_i)$. Thus consider the if statement. Suppose that x_i is incentive compatible with respect to $\mathcal{S}^*_i$. Let $S \in P(\mathcal{J}_i)$. If $S \in \mathcal{S}^*_i$ then incentive compatibility holds by assumption, so let $S \notin \mathcal{S}^*_i$. Given a stealing action S and given the strategies σ_i, ς_i, the equilibrium (NC, NC) is selected in all end games of suppliers $k \in \bigcup_{j \in S} X^*_{ij}$ while the equilibrium (C, C) is selected in all end games of suppliers $k \notin \bigcup_{j \in S} X^*_{ij}$.

Given Lemma 5, there is a unique subset $\mathcal{X}_i(S) \subset \mathcal{S}^*_i$ of nonempty elements such that $\bigcup_{X \in \mathcal{X}_i(S)} X = \bigcup_{j \in S} X^*_{ij}$. We begin by showing that for any $S \in P(\mathcal{S}_i)$, the stealing choice S is weakly dominated by the stealing choice $\Xi_i(S) \equiv \bigcup_{X \in \mathcal{X}_i(S)} X$.

When firm i chooses stealing decision S, the set of firms that select NC is $\bigcup_{j \in S} X^*_{ij}$, which is by definition $\Xi_i(S)$. Thus the payoff to firm i from S is

$$\hat{\Pi}_i(S) = \sum_{j \notin \Xi_i(S)} \pi_{ij}(x_{ij}) + \sum_{j \in \Xi_i(S)} \pi^D_{ij}(x_{ij}) - \sum_{j \in \Xi_i(S) \backslash S} p_j \theta_{ij}(x_{ij}) x_{ij}$$

where the final term accounts for the fact firm i has paid for the subset of goods $\Xi_i(S) \backslash S$ that were nevertheless subsequently cut off by Not Complete due to joint triggers.

Next consider the payoff from the strategy $\Xi_i(S)$. Since every $j \in \Xi_i(S)$ belongs to a unique element $X \in \mathcal{X}_i(S)$ and since every $X \in \mathcal{X}_i(S)$ is a subset of $\Xi_i(S)$, then $\bigcup_{j \in \Xi_i(S)} X^*_{ij} = \bigcup_{X \in \mathcal{X}_i(S)} X = \Xi_i(S)$. Thus we have

$$\hat{\Pi}_i(\Xi_i(S)) = \sum_{j \notin \Xi_i(S)} \pi_{ij}(x_{ij}) + \sum_{j \in \Xi_i(S)} \pi^D_{ij}(x_{ij})$$

But then we have $\hat{\Pi}_i(\Xi_i(S)) \geq \hat{\Pi}_i(S)$, so that stealing choice $\Xi_i(S)$ weakly dominates stealing choice S. Thus if x_i is incentive compatible with respect to $\Xi_i(S)$, it is also incentive compatible with respect to S.

Finally, we show that incentive compatibility with respect to $\mathcal{S}^*_i$ implies incentive compatibility with respect to $\Xi_i(S)$ for any $S \in P(\mathcal{J}_i)$. Recall that $\Xi_i(S) = \bigcup_{X \in \mathcal{X}_i(S)} X$. Incentive compatibility with respect to $X \in \mathcal{X}_i(S)$ is given by

$$\sum_{j \in X} \pi^D_{ij}(x_{ij}) \leq \sum_{j \in X} \pi_{ij}(x_{ij})$$

From Lemma 5, $X \cap X' = \emptyset$ for all distinct $X, X' \in \mathcal{X}_i(S)$. Thus summing the previous constraint over $X \in \mathcal{X}_i(S)$, we have

$$\sum_{j \in \Xi_i(S)} \pi^D_{ij}(x_{ij}) \leq \sum_{j \in \Xi_i(S)} \pi_{ij}(x_{ij}),$$

which is the incentive compatibility constraint for stealing action $\Xi_i(S)$. Thus if x_i is incentive compatible with respect to $\mathcal{S}^*_i$, it is incentive compatible with respect to $\Xi_i(S)$ and, since stealing $\Xi_i(S)$ weakly dominates S, is also incentive compatible with respect to S. But since S was generic, then incentive compatibility with respect to $\mathcal{S}^*_i$ implies incentive compatibility with respect to $P(\mathcal{J}_i)$, completing the proof. □

A.2 Proof of Proposition 1

The proof is provided in text, except for the Lagrange multiplier. If the constraint binds, then the critical point of the firm's Lagrangian for $S = \{j\}$ is $0 = \frac{\partial \pi_{ij}}{\partial x_{ij}} + \lambda_{ij}\left[\frac{\partial \pi_{ij}}{\partial x_{ij}} - \frac{\partial \pi^D_{ij}}{\partial x_{ij}}\right]$. Subsituting in elasticities, we have

$$0 = \sigma_{ij}\frac{\pi_{ij}}{x_{ij}} + \lambda_{ij}\left[\sigma_{ij}\frac{\pi_{ij}}{x_{ij}} - \sigma^D_{ij}\frac{\pi^D_{ij}}{x_{ij}}\right].$$

Finally, using that a binding constraint implies $\pi_{ij} = \pi^D_{ij}$ and rearranging yields the result.

A.3 Proof of Proposition 2

Let $|S| \geq 2$. The first part of the Proposition is immediate, so assume the IC constraint binds. The corresponding Lagrangian is $\mathcal{L}_{iS} = \sum_{j \in S}\left[\pi_{ij} + \lambda_{iS}[\pi_{ij} - \pi^D_{ij}]\right]$, so that we have critical points $0 = \frac{\partial \pi_{ij}}{\partial x_{ij}} + \lambda_{iS}\left[\frac{\partial \pi_{ij}}{\partial x_{ij}} - \frac{\partial \pi^D_{ij}}{\partial x_{ij}}\right]$. Following the proof of Proposition 1, we can write

$$0 = \frac{\sigma_{ij}}{\sigma^D_{ij}}\pi_{ij} + \lambda_{iS}\left[\frac{\sigma_{ij}}{\sigma^D_{ij}}\pi_{ij} - \pi^D_{ij}\right].$$

Summing over $j \in S$ and using the binding constraint, we have

$$0 = \sum_{j \in S} \frac{\sigma_{ij}}{\sigma_{ij}^D} \pi_{ij} + \lambda_{iS} \left[\sum_{j \in S} \frac{\sigma_{ij}}{\sigma_{ij}^D} \pi_{ij} - \sum_{j \in S} \pi_{ij} \right].$$

Dividing through $\sum_{j \in S} \pi_{ij}$ and rearranging, we have $\lambda_{iS} = \frac{1}{1 - \sum_{j \in S} \frac{\sigma_{ij}}{\sigma_{ij}^D} \omega_{ij}} - 1$, which yields the result.

A.4 Proof of Proposition 3

We break the proof into the if and only if statements.

If. Suppose that there exist $S', S'' \in \{S_1, \ldots, S_n\}$ such that $\lambda_{iS'} > \lambda_{iS''}$ (without loss of generality). Suppose that we augment the incentive compatibility constraint for S to be

$$\sum_{j \in S} \pi_{ij}^D(x_{ij}) \leq \sum_{j \in S} \pi_{ij}(x_{ij}) + \tau_S,$$

where τ_S is a constant that is equal to zero. Observe that since $S' \cap S'' = \emptyset$, then joint threat constructed from S' and S'' yields the incentive constraint

$$\sum_{j \in S' \cup S''} \pi_{ij}^D(x_{ij}) \leq \sum_{j \in S' \cup S''} \pi_{ij}(x_{ij}) + \tau_S + \tau_{S'}.$$

Therefore, a weaker expansion of incentive compatible allocations than achieved by a joint threat is to instead increase $\tau_{S'}$ and decrease $\tau_{S''}$ in such a manner that $\tau_{S'} + \tau_{S''} = 0$. If such a perturbation strictly increases value, then creating a joint threat also strictly increases value.

Because the decision problem of firm i is separable across elements of S, we can write the Lagrangian for element S as

$$\mathcal{L}(x_i, \lambda | S) = \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}) + \lambda_{iS} \Big[\tau_S + \pi_{ij}(x_{ij}) - \pi_{ij}^D(x_{ij}) \Big].$$

Because $v_i(S, \tau_S) = \mathcal{L}(x_i, \lambda | S)$ when evaluated at optimal quantities, then by Envelope Theorem we have

$$\frac{\partial v_i(S, \tau_S)}{\partial \tau_S} = \lambda_{iS}.$$

Therefore, the total profit impact of firm i of the perturbation $\tau_{S'} = \epsilon$ and $\tau_{S''} = -\epsilon$ is

$$\frac{\partial v_i(S', \tau_{S'})}{\partial \tau_{S'}} \epsilon - \frac{\partial v_i(S'', \tau_{S''})}{\partial \tau_{S''}} \epsilon = \lambda_{S'} - \lambda_{S''} > 0.$$

Therefore, there is an $\epsilon > 0$ such that when defining τ by $\tau_{S'} = \epsilon$, $\tau_{S''} = -\epsilon$, and $\tau_S = 0$ otherwise, we have $V_i(\mathcal{S}_i, \tau) > V_i(\mathcal{S}_i, 0)$. But since $V_i(\mathcal{S}_i') \geq V_i(\mathcal{S}_i, \tau)$, then $V_i(\mathcal{S}_i') > V_i(\mathcal{S}_i)$, and hence $(S_1, \ldots, S_n)$ is a pressure point on i.

Only If. Because the decision problem of firm i is separable across elements of the action set, and because elements $S \notin \{S_1, \ldots, S_n\}$ are unchanged, the same allocations x_{ij}^* for $j \in \bigcup S \in \mathcal{S}_i \backslash \{S_1, \ldots, S_n\} S$ remain optimal. It remains to show that optimal allocations are unchanged for $j \in \bigcup_{S \in \{S_1, \ldots, S_n\}} S$.

Suppose first that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} = 0$. Then, $x^*_{ij} = x^u_{ij}$ for all $j \in \bigcup_{x=1}^n S_x$. But then since $x^*_{ij} = x^u_{ij}$ is also implementable under joint threats, then the optimal allocation under joint threats is again $x^*_{ij} = x^u_{ij}$, and hence $(S_1, \ldots, S_n)$ is not a pressure point on i.

Suppose next that $\lambda_{iS_1} = \ldots = \lambda_{iS_n} > 0$ and let x^*_i be optimal production under $\mathcal{S}_i$. Because the decision problem of firm i is separable across elements of the action set, let us focus on the subset $\mathcal{X} = \{S_1, \ldots, S_n\}$ of elements in the joint threat. Denoting $\mathcal{L}(x_i, \hat{\lambda}|\mathcal{X})$ the Lagrangian associated with elements $\mathcal{X}$,

$$\mathcal{L}(x_i, \hat{\lambda}_i|\mathcal{X}) = \sum_{j \in \bigcup_{S\in\mathcal{X}} S} \pi_{ij}(x_{ij}) + \sum_{S\in\mathcal{X}} \hat{\lambda}_{iS} \sum_{j\in S} \Big[\pi_{ij}(x_{ij}) - \pi^D_{ij}(x_{ij})\Big].$$

Recalling that the firm's objective function is concave while each constraint is convex, the Lagrangian has a saddle point at (x^*_i, λ_i).

Next, consider the decision problem of firm i when faced with a joint threat, so that $\mathcal{S}'_i$ has an element $S' = \bigcup_{S\in\mathcal{X}} S$. As again the decision problem of the firm is separable across elements of $\mathcal{S}'_i$, then we can define the Lagrangian of firm i with respect to element S' by

$$\mathcal{L}(x_i, \mu_i| \bigcup_{S\in\mathcal{X}} S) = \sum_{j \in \bigcup_{S\in\mathcal{X}} S} \pi_{ij}(x_{ij}) + \mu_i \sum_{j \in \bigcup_{S\in\mathcal{X}} S} \Big[\pi_{ij}(x_{ij}) - \pi^D_{ij}(x_{ij})\Big].$$

Observe that once again, the objective function is concave while the constraint is convex. Since $S \cap S' = \emptyset$ for all $S, S' \in \mathcal{X}$, then we can write

$$\mathcal{L}(x_i, \mu_i| \bigcup_{S\in\mathcal{X}} S) = \sum_{j \in \bigcup_{S\in\mathcal{X}} S} \pi_{ij}(x_{ij}) + \sum_{S\in\mathcal{X}} \mu_i \sum_{j\in S} \Big[\pi_{ij}(x_{ij}) - \pi^D_{ij}(x_{ij})\Big].$$

Finally, let us define $\mu_i = \lambda_{iS_1}$. Since $\lambda_{iS_1} = \ldots = \lambda_{i,S_n}$, then we have

$$\mathcal{L}(x_i, \mu_i| \bigcup_{S\in\mathcal{X}} S) = \sum_{j \in \bigcup_{S\in\mathcal{X}} S} \pi_{ij}(x_{ij}) + \sum_{S\in\mathcal{X}} \lambda_{iS} \sum_{j\in S} \Big[\pi_{ij}(x_{ij}) - \pi^D_{ij}(x_{ij})\Big].$$

As a result, we have $\mathcal{L}(x_i, \mu_i|\bigcup_{S\in\mathcal{X}} S) = \mathcal{L}(x_i, \lambda_i|\mathcal{X})$ for all x_i. More generally since for any μ'_i there is a corresponding vector $\lambda'_{iS} = \mu'_i$, then since $\mathcal{L}(x_i, \hat{\lambda}_i|\mathcal{X})$ has a saddle point at (λ_i, x^*_i), then $\mathcal{L}(x_i, \hat{\mu}_i|\bigcup_{S\in\mathcal{X}} S)$ has a saddle point at (μ_i, x^*_i). Therefore, x^*_i is also an optimal policy under joint threat $\mathcal{S}'_i$. Therefore, $V_i(\mathcal{S}'_i) = V_i(\mathcal{S}_i)$ and hence $(S_1, \ldots, S_n)$ is not a pressure point. This concludes the proof.

A.5 Proof of Lemma 2

Consider a hypothetical optimal contract $\{\mathcal{S}'_i, \mathcal{T}_i, \tau_i\}_{i\in\mathcal{C}_m}$ that is feasible and satisfies firms' participation constraints, and suppose that $\mathcal{S}'_i \neq \overline{\mathcal{S}_i}'$. Let $z^* = x^*$ denote optimal firm production and the equilibrium externality vector under this contract. The proof strategy is to show that the hegemon can achieve the same allocation x^* and side payments $\mathcal{T}_i$, and hence value, using an implementable contract featuring maximal joint threats.

Suppose all firms $i \in \mathcal{C}_m$ face maximal joint threats, and conjecture that equilibrium externalities are z^* $(= x^*)$. We begin by constructing a vector of taxes τ^* that implements the allocation x^*

when externalities are z^* and side payments are $\mathcal{T}_i$. In particular, we define τ^* by

$$\tau_{ij}^* = \frac{\partial \pi_{ij}(x_{ij}^*, z^*)}{\partial x_{ij}}.$$

Considering the relaxed problem (not imposing incentive compatibility) of firm i,

$$\max_{x_i} \sum_{j \in \mathcal{J}_i} \Big[\pi_{ij}(x_{ij}, z^*) - \tau_{ij}^*(x_{ij} - x_{ij}^*) - T_{ij} \Big],$$

which yields solution $\frac{\partial \pi_{ij}(x_{ij}, z^*)}{\partial x_{ij}} = \tau_{ij}^*$, that is $x_{ij} = x_{ij}^*$ for all $j \in \mathcal{J}_i$. Provided this allocation is incentive compatible, it is also a solution to firm i's decision problem subject to incentive compatibility.

Since x_i^* is incentive compatible under $(\mathcal{S}_i', \mathcal{T}_i, \tau_i)$, then for all $S \in \mathcal{S}_i'$

$$\sum_{j \in S} \pi_{ij}^D(x_{ij}^*) \leq \sum_{j \in S} \Big[\pi_{ij}(x_{ij}^*, z^*) - T_{ij} - \tau_{ij}(x_{ij}^* - x_{ij}^*) \Big] = \sum_{j \in S} \Big[\pi_{ij}(x_{ij}^*, z^*) - T_{ij} \Big] \tag{17}$$

Let $S_i^D \equiv \bigcup_{S \in \mathcal{S}_i^D} S$ and let $\mathcal{X}_i = \mathcal{S}_i \backslash \mathcal{S}_i^D$. By Definition 4, $\mathcal{X}_i \subset \overline{\mathcal{S}}_i'$ and $\mathcal{X}_i \subset \mathcal{S}_i'$. Thus, $\bigcup_{S \in \mathcal{S}_i' \backslash \mathcal{X}_i} S = S_i^D$. If $\mathcal{X}_i$ is nonempty, then letting $S \in \mathcal{X}_i \subset \mathcal{S}_i'$, we have

$$\sum_{j \in S} \Big[\pi_{ij}(x_{ij}^*, z^*) - T_{ij} - \tau_{ij}^*(x_{ij}^* - x_{ij}^*) \Big] = \sum_{j \in S} \Big[\pi_{ij}(x_{ij}^*, z^*) - T_{ij} \Big] \geq \sum_{j \in S} \pi_{ij}^D(x_{ij}^*),$$

where the last line follows from equation 17. Hence the incentive constraint for $S \in \mathcal{X}_i$ is satisfied under contract $(\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*)$.

Next, since $S \cap S' = \emptyset$ for all $S, S' \in \mathcal{S}_i'$, and since $\bigcup_{S \in \mathcal{S}_i' \backslash \mathcal{X}_i} S = S_i^D$, then summing equation 17 over elements $S \in \mathcal{S}_i' \backslash \mathcal{X}_i$ yields

$$\sum_{j \in S_i^D} \pi_{ij}^D(x_{ij}^*) \leq \sum_{j \in S_i^D} \Big[\pi_{ij}(x_{ij}^*, z^*) - T_{ij} \Big].$$

Therefore, we have

$$\sum_{j \in S_i^D} \Big[\pi_{ij}(x_{ij}^*, z^*) - T_{ij} - \tau_{ij}^*(x_{ij}^* - x_{ij}^*) \Big] = \sum_{j \in S_i^D} \Big[\pi_{ij}(x_{ij}^*, z^*) - T_{ij} \Big] \geq \sum_{j \in S_i^D} \pi_{ij}^D(x_{ij}^*),$$

and thus the incentive constraint for S_i^D is satisfied under contract $(\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*)$.

Thus since $\overline{\mathcal{S}}_i' = \mathcal{X}_i \cup \{S_i^D\}$, x_i^* is incentive compatible under contract $(\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*)$. Thus since x_i^* is the solution to firm i's relaxed problem and is incentive compatible, it is firm i's optimal policy. Finally, every firm $i \notin \mathcal{C}_m$ faces the same decision problem as under the original contract, since the externality vector z^* is unchanged, and so has optimal policy x_i^*. Hence $x^* = z^*$ and externalities are consistent with their conjectured value.

Finally, given participation constraints are satisfied under contract $\{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}_{i \in \mathcal{C}_m}$, then we have

$$V_i(\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*, z^*) = \sum_{j \in \mathcal{J}_i} \Big[\pi_{ij}(x_{ij}^*, z^*) - T_{ij} \Big] = V_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i, z^*) \geq V_i(\mathcal{S}_i, 0, 0, z^*),$$

and hence the participation constraint of firm i is satisfied under contract $\{\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*\}_{i\in\mathcal{C}_m}$. Finally since $V_i(\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*, z^*) = V_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i, z^*)$ for all $i \in \mathcal{I}_m$, since z^* is unchanged, and since $\mathcal{T}_i$ is unchanged for all $i \in \mathcal{C}_m$, the hegemon's objective (equation 5) is also unchanged relative to the original contract. Thus the hegemon is indifferent between the implementable contracts $\{\mathcal{S}_i', \mathcal{T}_i, \tau_i\}_{i\in\mathcal{C}_m}$ and $\{\overline{\mathcal{S}}_i', \mathcal{T}_i, \tau_i^*\}_{i\in\mathcal{C}_m}$. Hence, it is weakly optimal for the hegemon to offer a contract involving maximal joint threats, concluding the proof.

A.6 Proof of Proposition 4

Given maximal joint threats and absence of externalities from z, the decision problem of the hegemon can be written as

$$\max_{\{\overline{T}_i,\tau_i\}_{i\in\mathcal{C}_m}} \sum_{i\in\mathcal{I}_m}\left[\Pi_i(\mathcal{S}_i', \mathcal{T}_i, \tau_i) + w_i\bar{\ell}_i\right] + \sum_{i\in\mathcal{D}_m}\overline{T}_i \quad s.t. \quad V_i(\overline{\mathcal{S}}_i', \overline{T}_i, \tau_i) \geq V_i(\mathcal{S}_i)\ \forall i \in \mathcal{C}_m.$$

First of all, observe that for any $\overline{T}_i \geq 0$,

$$0 \in \arg\max_{\tau_i} \Pi_i(\overline{\mathcal{S}}_i', \overline{T}_i, \tau_i).$$

$$0 \in \arg\max_{\tau_i} V_i(\overline{\mathcal{S}}_i', \overline{T}_i, \tau_i).$$

Therefore, for any $i \in \mathcal{C}_m$, setting $\tau_i = 0$ maximizes operating profits and maximally slackens the participation constraint. Therefore, $\tau_i = 0$ is an optimal policy for all $i \in \mathcal{C}_m$.

Consider next a domestic firm, $i \in \mathcal{I}_m$. By Envelope Theorem, $\frac{\partial V_i}{\partial \overline{T}_i} = -1 - \lambda_{iS_i^D} < 0$ and $\frac{\partial \Pi_i}{\partial \overline{T}_i} = -\lambda_{iS_i^D} \leq 0$. Therefore, $\overline{T}_i > 0$ weakly reduces operating profits and strictly tightens the participation constraint, so that $\overline{T}_i = 0$ is an optimal policy.

Finally, consider a foreign firm, $i \in \mathcal{D}_m$. As with a domestic firm, $\frac{\partial V_i}{\partial \overline{T}_i} = -1 - \lambda_{iS_i^D} < 0$. Since the hegemon's objective is strictly increasing in $\overline{T}_i$ for $i \in \mathcal{D}_m$, then the hegemon's optimal policy charges the largest side payment $\overline{T}_i^*$ such that the participation constraint just binds, $V_i(\overline{\mathcal{S}}_i', \overline{T}_i^*, 0) = V_i(\mathcal{S}_i)$.[30] Since $V_i(\overline{\mathcal{S}}_i', \overline{T}_i^*, 0)$ is a continuous and decreasing function of $\overline{T}_i$, then if $\mathcal{S}_i^D$ is not a pressure point on i, then $V_i(\overline{\mathcal{S}}_i', 0, 0) = V_i(\mathcal{S}_i)$ and hence $\overline{T}_i^* = 0$. By contrast if $\mathcal{S}_i^D$ is a pressure point, then $V_i(\overline{\mathcal{S}}_i', 0, 0) > V_i(\mathcal{S}_i)$ and hence $\overline{T}_i^* > 0$. This concludes the proof.

A.7 Proof of Proposition 5

The proof follows from Lemma 2 and Proposition 4 since the global planner's objective treats all firms $i \in \mathcal{C}_m$ as if they were domestic.

A.8 Proof of Proposition 6

The proof is presented in text.

[30]To see why such a value $\overline{T}_i^*$ exists, let us define $x_{ij}^{\max} = \arg\max_{x_{ij}} \pi_{ij}(x_{ij}) - \pi_{ij}^D(x_{ij})$, which maximizes slack. Then, let us define $\overline{T}_i = \sum_{j\in S_i^D}\left[\pi_{ij}(x_{ij}^{\max}) - \pi_{ij}^D(x_{ij}^{\max})\right]$. Since any private optimum without side payments must satisfy $\frac{\partial \pi_{ij}}{\partial x_{ij}} - \frac{\partial \pi_{ij}^D}{\partial x_{ij}} < 0$ (otherwise, the firm could increase profits without tightening the constraint), then clearly $V_i(\overline{\mathcal{S}}_i', \overline{T}_i, 0) < V_i(\mathcal{S}_i)$ and hence such a vlaue exists.

A.9 Proof of Proposition 7

To avoid confusion, we will adopt the notational shorthand $\lambda_{ij} \equiv \lambda_{iS}$ for the $S \in \overline{\mathcal{S}}_i'$ with $j \in S$, and similarly for Λ_{ij}. Therefore if $j, k \in S$, then $\lambda_{ik} = \lambda_{ik}$.

We begin with the case where the hegemon has a pressure point on every $i \in \mathcal{C}_m$ at equilibrium externalities z^*, and then below extend to include cases where the hegemon does not have a pressure point on a subset of firms.

We begin with the Lagrangian of firm i, given for S by

$$\mathcal{L}_i = \sum_{j \in S} \left[\pi_{ij} - \tau_{ij}(x_{ij} - x_{ij}^*) + \lambda_{ij} \left[\pi_{ij} - \pi_{ij}^D - \tau_{ij}(x_{ij} - x_{ij}^*) \right] \right] - \mathbf{1}_{S = S_i^D}(1 + \lambda_{ij})\overline{T}_i$$

Therefore, we have the FOC for x_{ij}

$$(1 + \lambda_{ij})\tau_{ij} = \frac{\partial \pi_{ij}}{\partial x_{ij}} + \lambda_{ij} \left[\frac{\partial \pi_{ij}}{\partial x_{ij}} - \frac{\partial \pi_{ij}^D}{\partial x_{ij}} \right]$$

Next, consider the Lagrangian of the hegemon. Since the hegemon has complete instruments for $i \in \mathcal{C}_m$, we adopt the primal approach of directly choosing allocations, and then back out the wedges that implement them from the firm's first order conditions. The hegemon's Lagrangian is

$$\mathcal{L}_m = \sum_{i \in \mathcal{I}_m} \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}, z^*) + u_m(z^*) + \sum_{i \in \mathcal{D}_m} \overline{T}_i + \sum_{i \in \mathcal{C}_m} \eta_i \left[\sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}, z^*) - \overline{T}_i - V_i(\mathcal{S}_i, z^*) \right]$$
$$+ \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_i} \Lambda_{ij} \left[\pi_{ij}(x_{ij}, z^*) - \pi_{ij}^D(x_{ij}, z^*) - \mathbf{1}_{S = S_i^D} \overline{T}_i \right]$$

We first construct the externality vector $\mathcal{E}_{ij} = \frac{d\mathcal{L}_m}{dz_{ij}}$. First define a basis of externalities,

$$e_{ij} = \sum_{i \in \mathcal{I}_m} \sum_{j \in \mathcal{J}_i} \frac{\partial \pi_{ij}(x_{ij}, z^*)}{\partial z_{ij}} + \sum_{i \in \mathcal{C}_m} \eta_i \left[\sum_{j \in \mathcal{J}_i} \frac{\partial \pi_{ij}(x_{ij}, z^*)}{\partial z_{ij}} - \frac{\partial V_i(\mathcal{S}_i, z^*)}{\partial z_{ij}} \right] + \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_i} \Lambda_{ij} \left[\frac{\partial \pi_{ij}(x_{ij}, z^*)}{\partial z_{ij}} - \frac{\partial \pi_{ij}^D(x_{ij}, z^*)}{\partial z_{ij}} \right],$$

which does not account for endogenous responses of agents the hegemon does not contract with (recall the hegemon is directly choosing the allocations of firms it contracts with). Define the subset of agents with whom the hegemon does not contract as $\mathcal{NC} = \mathcal{I} \backslash \mathcal{C}_m$, define $z_{NC} = \{z_i\}_{i \in \mathcal{NC}}$, and construct the inverse matrix Ψ_{NC}^z of this subset according to Proposition 6. Considering a shock dz_{ij} for $i \notin \mathcal{NC}$, then the response in equilibrium is $z_{NC}^* = \Psi_{NC}^z \frac{\partial z_{NC}^*}{\partial z_{ij}}$. Then, letting $e_{NC} = \{e_i\}_{i \in \mathcal{NC}}$ be a row vector, then we have

$$\mathcal{E}_{ij} \equiv \frac{\partial \mathcal{L}_m}{\partial z_{ij}} = e_{ij} + e_{NC} \Psi_{NC}^z \frac{\partial z_{NC}^*}{\partial z_{ij}},$$

which captures the total spillover from a change in z_{ij}, accounting for endogenous responses.

FOC for x_{ij} for a foreign firm. Consider the hegemon's FOC for x_{ij} for a foreign firm,

$$0 = \frac{d\mathcal{L}_m}{dx_{ij}} = \frac{\partial \mathcal{L}_m}{\partial x_{ij}} + \frac{\partial \mathcal{L}_m}{\partial z_{ij}} = \eta_i \frac{\partial \pi_{ij}(x_{ij}^*, z^*)}{\partial x_{ij}} + \Lambda_{ij} \left[\frac{\partial \pi_{ij}(x_{ij}^*, z^*)}{\partial x_{ij}} - \frac{\partial \pi_{ij}^D(x_{ij}^*, z^*)}{\partial x_{ij}} \right] + \mathcal{E}_{ij}.$$

For $\alpha > 0$, we define $\varepsilon_{ij} \equiv \mathcal{E}_{ij} - \alpha \frac{\partial \pi_{ij}(z^*_{ij}, z^*)}{\partial x_{ij}}$. Given $x^*_{ij} = z^*_{ij}$, we can rewrite the hegemon's FOC as

$$0 = (\eta_i + \alpha) \frac{\partial \pi_{ij}(x^*_{ij}, z^*)}{\partial x_{ij}} + \Lambda_{ij} \left[\frac{\partial \pi_{ij}(x^*_{ij}, z^*)}{\partial x_{ij}} - \frac{\partial \pi^D_{ij}(x^*_{ij}, z^*)}{\partial x_{ij}} \right] + \varepsilon_{ij}.$$

Define the firm's Lagrange multiplier to be $\lambda_{ij} = \frac{\Lambda_{ij}}{\eta_i + \alpha}$, which is finite and nonnegative given $\alpha > 0$, and is constant among elements $j \in S$. Therefore, substituting into the firm's FOC, we have

$$(\Lambda_{ij} + \eta_i + \alpha) \tau_{ij} = -\varepsilon_{ij}.$$

If $\eta_i > 0$, then setting $\alpha = 0$ provides a nonnegative Lagrange multiplier λ_{ij}, and hence we obtain the optimal tax formula. If instead $\eta_i = 0$, then define a sequence $\{\alpha_n\}_{n=0}^{\infty}$, $\alpha_n > 0$, with $\alpha_n \to 0$. Here, we have the limit

$$0 = \lim_{n \to +\infty} \left[(\Lambda_{ij} + \eta_i + \alpha_n) \tau_{ij} + \varepsilon_{ij}(\alpha_n) \right] = (\Lambda_{ij} + \eta_i) \tau^*_{ij} + \mathcal{E}_{ij}$$

which gives the result.

FOC for $\overline{T}_{ij}$ for a foreign firm. From the Lagrangian, we have $0 \geq 1 - \eta_i - \Lambda_{iS^D_i}$.

FOC for x_{ij} for a domestic firm. For a domestic firm, the hegemon's FOC is

$$0 = (1 + \eta_i) \frac{\partial \pi_{ij}(x^*_{ij}, z^*)}{\partial x_{ij}} + \Lambda_{ij} \left[\frac{\partial \pi_{ij}(x^*_{ij}, z^*)}{\partial x_{ij}} - \frac{\partial \pi^D_{ij}(x^*_{ij}, z^*)}{\partial x_{ij}} \right] + \mathcal{E}_{ij}.$$

Thus writing $\hat{\eta}_i \equiv 1 + \eta_i$, the exact same steps as for a foreign firm obtain $(\Lambda_{ij} + \eta_i + 1) \tau^*_{ij} = -\mathcal{E}_{ij}$. Since $\lambda_{ij} = \frac{\Lambda_{ij}}{1 + \eta_i}$, the limiting argument is not needed.

A.9.1 Case where a subset of firms do not have pressure points

Suppose that for a subset of firms $\mathcal{NP} \subset \mathcal{C}_m$ in a neighborhood of z^*, the hegemon does not have a pressure point on i. Then, $\tau^*_i = 0$ and $\overline{T}_i = 0$ for these firms. We redefine the contractable set as $\mathcal{C}^p_m = \mathcal{C}_m \backslash \mathcal{NP}$, and write the Lagrangian

$$\mathcal{L}_m = \sum_{i \in \mathcal{I}_m \backslash \mathcal{NP}} \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}, z^*) + u^p_m(z^*) + \sum_{i \in \mathcal{D}_m} \overline{T}_i + \sum_{i \in \mathcal{C}^p_m} \eta_i \left[\sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}, z^*) - \overline{T}_i - V_i(\mathcal{S}_i, z^*) \right]$$
$$+ \sum_{i \in \mathcal{C}^p_m} \sum_{j \in \mathcal{J}_i} \Lambda_{ij} \left[\pi_{ij}(x_{ij}, z^*) - \pi^D_{ij}(x_{ij}, z^*) - \mathbf{1}_{S = S^D_i} \overline{T}_i \right],$$

where $u^p_m(z^*) = u_m(z^*) + \sum_{i \in \mathcal{I}_m \cap \mathcal{NP}} V_i(\mathcal{S}_i, z^*)$. From here, analysis proceeds as before.

A.10 Proof of Proposition 8

Suppose that $\mathcal{E}_{ij} \leq 0$ for all $j \in S^D_i$, and conjecture that firm i's participation constraint does not bind. Recall that $v_i(S, 0, \tau_i, z) \leq v_i(S, 0, 0, z)$ for all $S \neq S^D_i$. Suppose that $\exists j \in S^D_i$ such that $\frac{\partial \pi_{ij}(x_{ij}) - \pi^D_{ij}(x_{ij})}{\partial x_{ij}} < 0$. Then since $\overline{E}_{ij} \leq 0$, a marginal decrease in x_{ij} slackens the incentive constraint and mitigates externalities. Because the participation constraint is not binding, this perturbation is feasible. Thus, $\frac{\partial \pi_{ij}(x_{ij})-\pi^D_{ij}(x_{ij})}{\partial x_{ij}} \geq 0$ for all $j \in S_i^D$. But then since $\overline{T}_i \geq 0$, we have $v_i(S_i^D, \overline{T}_i, \tau_i, z) < \sum_{S \in \mathcal{S}_i^D} v_i(S, 0, 0, z)$. Thus the participation constraint binds.

A.11 Proof of Proposition 9

The firm Lagrangian is the same as in the proof of Proposition 7. The global planner's Lagrangian under the primal approach (assuming pressure points on every firm) is

$$\mathcal{L} = \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}, z^*) + \sum_{i \in \mathcal{I} \setminus \mathcal{C}_m} V_i(\mathcal{S}_i, z^*) + \sum_{n=1}^{N} u_n(z^*) + \sum_{i \in \mathcal{D}_m} \overline{T}_i + \sum_{i \in \mathcal{C}_m} \eta_i \left[\sum_{j \in \mathcal{J}_i} \pi_{ij}(x_{ij}, z^*) - \overline{T}_i - V_i(\mathcal{S}_i, z^*) \right]$$
$$+ \sum_{i \in \mathcal{C}_m} \sum_{j \in \mathcal{J}_i} \Lambda_{ij} \left[\pi_{ij}(x_{ij}, z^*) - \pi^D_{ij}(x_{ij}, z^*) - \mathbf{1}_{S = S_i^D} \overline{T}_i \right]$$

Define the basis vector of externalities

$$e^p_{ij} = e_{ij} + \sum_{i \in \mathcal{C}_m \setminus \mathcal{I}_m} \sum_{j \in \mathcal{J}_i} \frac{\partial \pi_{ij}(x_{ij}, z^*)}{\partial z^*_{ij}} + \sum_{i \in \mathcal{I} \setminus \mathcal{C}_m} \frac{\partial V_i(\mathcal{S}_i, z^*)}{\partial z_{ij}} + \sum_{n \neq m} \frac{\partial u_n(z^*)}{\partial z_{ij}}$$

Then, $\mathcal{E}^p_{ij}$ is given analogously to before by

$$\mathcal{E}_{ij} = e^p_{ij} + e^p_{NC} \Psi^z_{NC} \frac{\partial z^*_{NC}}{\partial z_{ij}}.$$

Finally, the remainder of the proof is identical to the proof of Proposition 7, except that all firms (domestic and foreign) are treated as-if they were domestic. If there is a firm without a pressure point local to the optimum, then the analysis is modified as in the proof of Proposition 7.

A.12 Deriving the Tax Rate for National Security Application

With two firms as described in the rest of world, the hegemon's objective is $u_m(z^H) + \overline{T}_i$. The participation constraint is $V_i(\overline{\mathcal{S}}_i' \setminus \{H\}, \overline{T}_i, 0) + v_i(\{H\}, 0, \tau_{iH}, z^H) \geq V_i(\mathcal{S}_i \setminus \{H\}) + v_i(\{H\}, 0, 0, z^H)$, where

$$v_i(\{H\}, 0, 0, z) = \sup_{x_{iH}} \pi_{ij}(x_{ij}, z^H) \quad s.t. \quad \pi^D_{ij}(x_{ij}) \leq \pi_{ij}(x_{ij})$$

Assuming as in text the incentive constraint is slack, then the optimal policy satisfies $\frac{\partial \pi_{ij}(x^*_{ij}(z^H), z^H)}{\partial x_{iH}} = 0$. Since the incentive constraint does not bind when unconstrained, it also does not bind for any nonnegative tax rate $\tau_{iH} \geq 0$. Therefore we conjecture that the incentive constraint does not bind under the optimal contract, and verify the tax rate is nonnegative.

Under the primal approach, the participation constraint is

$$V_i(\overline{\mathcal{S}}_i' \setminus \{H\}, \overline{T}_i, 0) + \pi_{iH}(z_{iH}, z^H) \geq \pi_{iH}(x^*_{iH}(z^H), z^H) + v_i(\{H\}, 0, 0, z^H).$$

Therefore, we have

$$\mathcal{E}_{iH} = \frac{du_m(z^H)}{dz_{iH}} + \eta_i \left[\frac{d\pi_{iH}(x_{iH}, z^H)}{dz_{iH}} - \frac{d\pi_{iH}(x^*_{iH}(z^H), z^H)}{dz_{iH}} \right],$$

where $\frac{du_m(z^H)}{dz_{iH}} = \frac{\partial u_m}{\partial z_{iH}} + \frac{\partial u_m}{\partial z_{jH}} \frac{\partial z_{jH}}{\partial z_{iH}}$ and so on. Using

$$\frac{\partial z_{jH}}{\partial z_{iH}} = \left(1 - \frac{\partial x^*_{jH}}{\partial z_{jH}}\right)^{-1} \frac{\partial x^*_{jH}}{\partial z_{iH}} = \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}},$$

then we have

$$\frac{du_m}{dz_{iH}} = \frac{\partial u_m}{\partial z_{iH}} + \frac{\partial u_m}{\partial z_{jH}} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}}$$

Since x^*_{iH} is the unconstrained optimum, we have

$$\frac{d\pi_{iH}}{dz_{iH}} = p_i \frac{dA_{iH}}{dz_{iH}} g_{iH}(x_{iH}) = \frac{1}{z_{iH}} p_i \left(\xi_{ii} + \xi_{ij} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}}\right) A_{iH} g_{iH}(x_{iH}).$$

Therefore, we have

$$\mathcal{E}_{iH} = \frac{\partial u_m}{\partial z_{iH}} + \frac{\partial u_m}{\partial z_{jH}} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}} + \eta_i p_i A_{iH}(z^H) \Big[g_{iH}(x_{iH}) - g_{iH}(x^*_{iH}(z^H)) \Big] \left(\xi_{ii} + \xi_{ij} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}}\right) \frac{1}{z_{iH}}.$$

Finally, since the participation constraint binds ($\eta_i > 0$) and conjecturing the incentive constraint for H is slack, then from Proposition 7

$$\begin{aligned} \tau_{iH} &= -\frac{1}{\eta_i} \mathcal{E}_{iH} \\ &= -\frac{1}{\eta_i} \frac{\partial u_m}{\partial z_{iH}} - \frac{1}{\eta_i} \frac{\partial u_m}{\partial z_{jH}} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}} \frac{z_{jH}}{z_{iH}} + p_i A_{iH}(z^H) \Big[g_{iH}(x^*_{iH}(z^H)) - g_{iH}(x_{iH}) \Big] \left(\xi_{ii} + \xi_{ij} \frac{\xi_{ji}}{\gamma_j - \xi_{jj}}\right) \frac{1}{z_{iH}}. \end{aligned}$$

Therefore given assumptions, the tax is nonnegative, and therefore the incentive constraint does in fact not bind. Finally, if the side payment is positive, then from Proposition 7 we have $\eta_i = 1 - \lambda_{iS_i^D}$, completing the argument.

A.13 Proof of Lemma 3

Fix a contract $\Gamma_i^{-m} = \{\mathcal{S}_i'^{-m}, \mathcal{T}_i^{-m}, 0\}$ of hegemon $-m$. The first part of Lemma 3 follows since for any $i \notin \mathcal{P}$, any contract with a positive side payment $\overline{T}_i^m > 0$ is rejected, and any contract that does not implement the firm's optimal allocation without wedges is rejected. The second part follows from the first part and from Definition 7: since for any $i \in \mathcal{P}$ we have $\mathcal{J}_i \cap \mathcal{P} = \emptyset$, then $S_i^D \subset \mathcal{J}_i \backslash \mathcal{P}$. Therefore, feasible joint threats are those that are feasible under direct transmission, given all firms $i \notin \mathcal{P}$ accept the contract.

We now turn to the third part. The proof strategy will be to show that if a contract $\Gamma_i^m \equiv \{\mathcal{S}_i'^m, \mathcal{T}_i^m, 0\}$ is accepted by firm i, then the contract $\Gamma_i^{m\prime} = \{\overline{\mathcal{S}}_i'^m, \mathcal{T}_i^m, 0\}$ is also accepted by firm i. Let $\Gamma_i = \{\mathcal{S}_i', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ be the joint contract if hegemon m offers Γ_i^m, and $\Gamma_i' = \{\mathcal{S}_i'', \mathcal{T}_i^m + \mathcal{T}_i^{-m}, 0\}$ the joint contract if hegemon m offers $\Gamma_i^{m\prime}$. Since the contract Γ_i^m is accepted by firm i, then

$$\max\{V_i(\Gamma_i), V_i(\Gamma_i^m)\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}.$$

Since $\overline{\mathcal{S}}_i'^m$ is a joint threat of $\mathcal{S}_i'^m$, then $\mathcal{S}_i''$ is a joint threat of $\mathcal{S}_i'$. Therefore, $V_i(\Gamma_i^{m\prime}) \geq V_i(\Gamma_i^m)$ and $V_i(\Gamma_i') \geq V_i(\Gamma_i)$. Therefore,

$$\max\{V_i(\Gamma_i'), V_i(\Gamma_i^{m\prime})\} \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\},$$

and hence contract $\Gamma_i^{m\prime}$ is also accepted by firm i. Finally, firm i is weakly better off (which is valued by hegemon m if firm i is domestic). Thus, maximal joint threats is a weak best response, concluding the proof.

A.14 Proof of Proposition 10

From Lemma 3, $\mathcal{S}_i' = \overline{\mathcal{S}}_i'$ is a best response to any contract Γ_i^{-m}, and therefore all side payments of m appear under the joint threat. Thus we will focus on the total side payment $\overline{T}_i$. Lemma 3 characterizes the contract for $i \notin \mathcal{P}$, so we focus here on a firm $i \in \mathcal{P}$. The optimal contract for firm i is characterized by Proposition 4 if only one hegemon contracts with i, so assume $i \in \mathcal{C}_{m_1} \cap \mathcal{C}_{m_2}$.

Let $\Gamma_i^m = \{\overline{\mathcal{S}}_i'^m, \overline{T}_i^m, 0\}$ be a candidate optimal contract of hegemon m, and let $\Gamma_i = \{\overline{\mathcal{S}}_i', \overline{T}_i^{m_1} + \overline{T}_i^{m_2}, 0\}$ be the joint contract.

A.14.1 Foreign Firms

Let $i \in \mathcal{P} \backslash (\mathcal{I}_{m_1} \cup \mathcal{I}_{m_2})$ be a firm foreign to both hegemons. We begin with the following intermediate result.

Lemma 7 $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium is which firm i accepts both contracts if and only if one of the following holds:*

1. *Firm i is held to its outside option, with*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{18}$$

2. *Firm i exceeds its outside option, with*

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i) \tag{19}$$

Proof of Lemma 7. Since both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

Suppose first that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then, since both contracts are accepted,

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}.$$

Finally, suppose that we have two contracts that satisfy this condition. Then, if either hegemon increased its side payment, the firm would reject both contracts and revert to the outside option. Likewise, a hegemon that lowered its side payment would have its contract accepted, but be strictly worse off. Therefore we have an equilibrium.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. Suppose, hypothetically, that

$$V_i(\Gamma_i) > \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}.$$

Then, hegemon m could increase its side payment without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = \max\{V_i(\Gamma_i^m), V_i(\Gamma_i^{-m})\}$. Suppose then that (without loss)

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) > V_i(\Gamma_i^{-m}).$$

Then again, hegemon m could increase its side payment without its contract being reject, and so be strictly better off. Therefore,

$$V_i(\Gamma_i) = V_i(\Gamma_i^{m_1}) = V_i(\Gamma_i^{m_2}) > V_i(\mathcal{S}_i).$$

Finally, supposing this condition holds, then if either hegemon increased its side payment, the firm would reject its contract and accept only that of the other hegemon. Likewise, a hegemon that lowered its side payment would have its contract accepted, but be strictly worse off. Therefore, neither hegemon deviates, and we have an equilibrium. This concludes the proof of Lemma 7. □

We use Lemma 7 to construct an equilibrium. Since $i \in \mathcal{P}$, $V_i(\overline{\mathcal{S}}_i') > V_i(\mathcal{S}_i)$. Observe that at least one hegemon m has a pressure point on i, that is $V_i(\overline{\mathcal{S}}_i'^m) > V_i(\mathcal{S}_i)$.[31] Without loss of generality, let $V_i(\overline{\mathcal{S}}_i'^m) \geq V_i(\overline{\mathcal{S}}_i'^{-m})$. We begin by constructing the minimal transfer $t_0^m \geq 0$ such that $V_i(\overline{\mathcal{S}}_i'^m, t_0^m) = V_i(\overline{\mathcal{S}}_i'^{-m}, 0)$. Since $\overline{S}_i'$ is a joint threat of $\overline{\mathcal{S}}_i'^m$, where therefore $V_i(\overline{\mathcal{S}}_i', t_0^m) \geq V_i(\overline{\mathcal{S}}_i'^m, t_0^m)$. If $V_i(\overline{\mathcal{S}}_i', t_0^m) = V_i(\overline{\mathcal{S}}_i'^m, t_0^m)$, then we have found contracts such that $V_i(\Gamma_i) = V_i(\Gamma_i^m) = V_i(\Gamma_i^{-m})$, and hence either equation (18) or (19) is satisfied. Thus we have an equilibrium.

Suppose instead $V_i(\overline{\mathcal{S}}_i', t_0^m) > V_i(\overline{\mathcal{S}}_i'^m, t_0^m)$. Then, we construct a function $t^{-m}(t)$ by

$$V_i(\overline{\mathcal{S}}_i'^m, t_0^m + t) = V_i(\overline{\mathcal{S}}_i'^{-m}, t^{-m}(t)).$$

We can construct this function from $t = 0$ to $t = \bar{t}$, where $\bar{t}$ solves $V_i(\overline{\mathcal{S}}_i'^m, t_0^m + t) = V_i(\mathcal{S}_i)$.

Suppose first $\exists t^* \in [0, t]$ such that

$$V_i(\overline{\mathcal{S}}_i'^m, t_0^m + t^* + t^{-m}(t^*)) = V_i(\overline{\mathcal{S}}_i'^m, t_0^m + t^*).$$

Then, equation (19) is satisfied if $t^* < \bar{t}$, and equation (18) is satisfied if $t^* = \bar{t}$. Therefore, by Lemma 7) we have found an equilibrium.

Suppose instead that no such t^* exists, and therefore $V_i(\overline{\mathcal{S}}_i^m, t_0^m + \bar{t} + t^{-m}(\bar{t})) > V_i(\mathcal{S}_i)$. Then, define T^* such that $V_i(\overline{\mathcal{S}}_i', T^*) = V_i(\mathcal{S}_i)$, and define $\overline{T}_i^m$ and $\overline{T}_i^{-m}$ such that $\overline{T}_i^m + \overline{T}_i^{-m} = T^*$, $\overline{T}_i^{-m} \geq t_0^m + \bar{t}$, and $\overline{T}_i^{-m} \geq t^{-m}(\bar{t})$. Then, equation (18) is satisfied, and hence we have found an equilibrium.

Therefore, an equilibrium exists as described, assuming both hegemons impose zero wedges. Observe that imposing nonzero wedges cannot increase the value of its objective, and leads to its contract being (weakly) rejected. Thus, zero wedges is a best response of each hegemon, concluding this portion of the proof.

A.14.2 Domestic Firms

Let $i \in \mathcal{P} \cap \mathcal{I}_m$ be a domestic firm of hegemon m. We obtain the following result, which parallels Lemma 7.

Lemma 8 $(\Gamma_i^m, \Gamma_i^{-m})$ *is part of an equilibrium is which firm* $i \in \mathcal{P} \cap \mathcal{I}_m$ *accepts both contracts if and only if one of the following holds:*

[31] Suppose that neither had a pressure point. Then by Proposition 3, all Lagrange multipliers in $\mathcal{S}_i^{Dm_1}$ are equal, and all Lagrange multipleirs in $\mathcal{S}_i^{Dm_2}$ are equal. But since $\mathcal{S}_i^{Dm_1} \cap \mathcal{S}_i^{Dm_2}$ is nonempty, then all Lagrange multipliers in $\mathcal{S}_i^D$ are equal. Thus by Proposition 3, $\mathcal{S}_i^D$ is not a pressure point, a contradiction.

1. *Firm i is held to its outside option, with $\overline{T}_i^m = 0$ and*

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\} \tag{20}$$

2. *Firm i exceeds its outside option, with $\overline{T}_i^m = 0$ and*

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\} \tag{21}$$

Proof of Lemma 8. Since both contracts are accepted, then

$$V_i(\Gamma_i) \geq \max\{V_i(\mathcal{S}_i), V_i(\Gamma_i^{m_1}), V_i(\Gamma_i^{m_2})\}.$$

Suppose first that firm i is held to its outside option, $V_i(\Gamma_i) = V_i(\mathcal{S}_i)$. Then, since both contracts are accepted,

$$V_i(\Gamma_i) = V_i(\mathcal{S}_i) \geq \max_{m \in \{m_1, m_2\}} \{V_i(\Gamma_i^m)\}.$$

Finally, suppose that we have two contracts that satisfy this condition and that $\overline{T}_i^m = 0$. If hegemon $-m$ increased its side payment, then its contract would be rejected. If hegemon m had a posititive side payment, it could decrease the side payment and increase value of its domestic firm i. Therefore, we have an equilibrium if $\overline{T}_i^m = 0$.

Suppose, second, that firm i exceeds its outside option, $V_i(\Gamma_i) > V_i(\mathcal{S}_i)$. Suppose, hypothetically, that

$$V_i(\Gamma_i) > V_i(\Gamma_i^m).$$

Then, hegemon $-m$ could increase its side payment without its contract being rejected, and so be strictly better off. Therefore, $V_i(\Gamma_i) = V_i(\Gamma_i^m)$, and therefore

$$V_i(\Gamma_i) = V_i(\Gamma_i^m) \geq \max\{V_i(\Gamma_i^{-m}), V_i(\mathcal{S}_i)\}.$$

If this condition holds, and $\overline{T}_i^m > 0$, then hegemon m could decrease its side payment for its domestic firm without its contract being rejected, and so be strictly better off. Therefore, $\overline{T}_i^m = 0$. Finally, suppose this condition holds and $\overline{T}_i^m = 0$. Then, if hegemon $-m$ increased its side payment, its contract would be rejected. Hegemon m cannot further decrease its side payment. Therefore, neither hegemon deviates, and we have an equilibrium. This concludes the proof. □

Lemma 8 first of verifies that $\overline{T}_i^m = 0$ in any equilibrium, that is a domestic firm is not charged a side payment by its hegemon. We can construct the side payment of hegemon $-m$ as follows. First, suppose that $V_i(\overline{\mathcal{S}}_i'^m) > V_i(\mathcal{S}_i)$. Then, $\overline{T}_i^{-m}$ solves $V_i(\overline{\mathcal{S}_i}, \overline{T}_i^{-m}) = V_i(\overline{\mathcal{S}}_i'^m)$, which is consistent with equation (21). Second, suppose that $V_i(\overline{\mathcal{S}}_i'^m) = V_i(\mathcal{S}_i)$. Then, $\overline{T}_i^{-m}$ solves $V_i(\overline{\mathcal{S}_i}, \overline{T}_i^{-m}) = V_i(\mathcal{S}_i)$, which is consistent with equation (21). In both cases, zero wedges is part of an optimal policy. Therefore, we have an equilibrium.

This concludes the proof of existence.

B Incentive Constraints with Non-Separable Production

In this appendix, we allow for non-separable production, and show that submodularity of production function is a sufficient condition for the reduction of incentive constraints underpinning the paper. We specialize this to provide a sufficient condition under CES production with decreasing returns.

Firm i has a subset of productive inputs $\mathcal{J}_i \subset \mathcal{I}$. Let $x_i \in \mathbb{R}_+^{|\mathcal{J}_i|}$ denote an input vector of firm i, where for simplicity we have only included productive inputs. Firm i's production function is $f_i(x_i)$.[32] For simplicity, we focus on the case where there are only individual triggers (i.e., no pre-existing joint triggers). Firm i's set of possible stealing actions is $P(\mathcal{J}_i)$.

Consider a stealing action $S \in P(\mathcal{J}_i)$. For notational convenience, we will define $\vartheta_{ij} = \theta_{ij}(x_{ij})x_{ij}$. Define the vector $\chi_i(S)$ element-wise by

$$\chi_{ij}(S) = \begin{cases} \vartheta_{ij}, & j \in S \\ x_{ij}, & j \notin S \end{cases}$$

If firm i chooses to steal S, it receives payoff $p_i f_i(\chi_i(S)) - \sum_{j \notin S} p_j x_{ij}$. Therefore, the incentive compatibility constraint for S is

$$p_i f_i(\chi_i(S)) \leq p_i f_i(x_i) - \sum_{j \in S} p_j x_{ij}.$$

As a preliminary to the coming result, recall that a function g is supermodular if for all x, y

$$g(x) + g(y) \leq g(x \vee y) + g(x \wedge y),$$

where $x \vee y$ is the component-wise maximum of x and y, while $x \wedge y$ is the component-wise minimum of x and y.[33] A function g is submodular if $-g$ is supermodular.[34]

We obtain the following result, paralleling Lemma 1.

Lemma 9 *Suppose that f_i is submodular. Suppose there are only individual triggers (i.e., no pre-existing joint triggers). Then, x_i is incentive compatible with respect to $P(\mathcal{J}_i)$ if and only if it is incentive compatible with respect to $\mathcal{S}_i^\circ$.*

Lemma 9 extends the underlying structure of the model to non-separable, submodular production functions: incentive compatibility with respect to isolated stealing decisions implies incentive compatibility for the joint stealing decision. This suggests a joint threat, forcing the firm to stealing both goods simultaneously, could be profit-improving by allowing the firm to choose a new allocation that was not previously incentive compatible.

A special case is CES production with decreasing returns,

$$f_i(x_i) = A_i \left(\sum_{j \in \mathcal{J}_i} \alpha_{ij} x_{ij}^\sigma \right)^{\beta/\sigma}.$$

This production function is submodular if $\beta \leq \sigma$.[35]

B.0.1 Proof of Lemma 9

Let x_i be an allocation that is incentive compatible with respect to $\mathcal{S}_i^\circ$, that is the incentive constraint is satisfied for each element $S = \{j\}$, $j \in \mathcal{J}_i$.

[32] For simplicity, we think of f_i as being constant in any inputs $j \notin \mathcal{J}_i$, which we therefore omit.

[33] That is, $(x \vee y)_n = \max\{x_n, y_n\}$ and $(x \wedge y)_n = \min\{x_n, y_n\}$.

[34] Recall that supermodularity can be verified by a nonnegative cross partial, $\frac{\partial^2 g}{\partial x_n \partial x_m} \geq 0$ for $n \neq m$, for a twice continuously differentiable function.

[35] See also Bocola and Bornstein (2023).

The proof proceeds by induction. The inductive step is to prove the following statement: If x_i is incentive compatible with respect to $S_1, S_2 \in \mathcal{S}_i$ with $S_1 \cap S_2 = \emptyset$, then x_i is incentive compatible with respect to $S_1 \cup S_2$. Since all elements of $P(\mathcal{J}_i)$ are unions of elements of $\mathcal{S}_i^\circ$ and elements of $\mathcal{S}_i^\circ$ are disjoint, proving the inductive set proves the result.

Inductive step. Suppose that x_i is incentive compatible with respect to $S_1, S_2 \in P(\mathcal{J}_i)$ with $S_1 \cap S_2 = \emptyset$, that is

$$p_i f_i(\chi_i(S_1)) \leq p_i f_i(x_i) - \sum_{j \in S_1} p_j x_{ij}$$

$$p_i f_i(\chi_i(S_2)) \leq p_i f_i(x_i) - \sum_{j \in S_2} p_j x_{ij}$$

Since $S_1 \cap S_2 = \emptyset$, then

$$\chi_{ij}(S_1) \vee \chi_{ij}(S_2) = x_i$$

$$\chi_{ij}(S_1) \wedge \chi_{ij}(S_2) = \chi_i(S_1 \cup S_2)$$

Since f_i is submodular, then

$$f_i(\chi_i(S_1 \cup S_2)) + f_i(x_i) \leq f_i(\chi_i(S_1)) + f_i(\chi_i(S_2))$$

$$p_i f_i(\chi_i(S_1 \cup S_2)) \leq p_i f_i(\chi_i(S_1)) + p_i f_i(\chi_i(S_2)) - p_i f_i(x_i)$$

Since $S_1 \cap S_2 = \emptyset$, then

$$\begin{aligned} p_i f_i(\chi_i(S_1 \cup S_2)) &\leq p_i f_i(\chi_i(S_1)) + p_i f_i(\chi_i(S_2)) - p_i f_i(x_i) \\ &\leq p_i f_i(x_i) - \sum_{j \in S_1} p_j x_{ij} + p_i f_i(x_i) - \sum_{j \in S_2} p_j x_{ij} - p_i f(x_i) \\ &= p_i f_i(x_i) - \sum_{j \in S_1 \cup S_2} p_j x_{ij} \end{aligned}$$

and therefore, x_i is incentive compatible with respect to $S_1 \cup S_2$. This completes the inductive step, and hence the proof.

Figure 4: **Extensive Form Representation of Firm-Supplier Game**