Foreign Competition and Innovation*

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Abstract

Empirical studies have found that enhanced foreign competition can encourage or discourage innovation. To address this relationship, I examine a market structure in which a small number of large multi-product oligopolists compete with a large number of small single-product firms in the same industry. The single-product firms are short-lived while the multi-product firms live forever, and the large firms invest in innovation in order to enlarge their product spans. All firms export. I show that an increase in the competitiveness of foreign firms can increase or reduce innovation efforts of a large multi-product firm. Moreover, changes in the incentives to innovate can be different for more-productive and less-productive oligopolists. As a result, aggregate sectoral innovation may rise or decline, depending on the productivity distribution of the oligopolists. I also show that changes in short-term operating profits may not be aligned with changes in the incentives to invest in innovation.

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JEL Classification: D43, F1, L1

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1 Introduction

The impact of international trade on innovation and growth was extensively studied in the 1990s, by scholars who embraced the then new endogenous growth theory. Of particular interest has been the finding that trade impacts growth through multiple channels, some leading to acceleration, other to deceleration. In a basic model of expanding product variety, where the stock of knowledge reduces innovation costs and rises with cumulative investment in R&D, the finding was that trade raises growth as long as this knowledge flows freely across countries. In contrast, when countries do not share such knowledge, some countries may experience growth deceleration, although this does not necessarily lead to lower welfare (see Grossman and Helpman (1991a) ch. 3).

Trade encourages innovation and growth through a demand effect, consisting of market expansion. Namely, the option to sell to buyers in foreign countries raises rents on R&D and leads to more investment in innovation.[1] At the same time, trade discourages innovation through a supply effect, due to foreign firms gaining access to the domestic market (a business stealing effect). With constant elasticity of substitution preferences, this supply effect equals in size to the demand effect, leading to a nil net impact. But trade accelerates innovation and growth on account of knowledge accumulation, as long as knowledge flows freely across national borders (see Grossman and Helpman (1991a) ch. 3). As a consequence, trade leads to more innovation and faster growth.[2]

Bloom et al. (2016) used firm-level data to study the impact of China's accession to the WTO in 2001 and its subsequent export expansion on European firms. They found that this occurrence accelerated innovation and adoption of new technology by these firms, and lead to faster productivity growth. Companies more exposed to Chinese imports created more patents in the European Patent Office, increased R&D, increased management quality, raised IT intensity, and raised their productivity levels. The China shock also lead to a reallocation of resources from low- to high-productivity firms, thereby raising sectoral productivity levels. In short, both within and between firm effects brought about aggregate technological upgrading.

Autor et al. (2020) studied the response of patenting and R&D spending of US firms to import competition from China. They found that imports from China curtailed global sales of US firms, their purchases of inputs, their R&D spending, and their patent grants. In other words, it brought about a decline in US innovation.

To reconcile these US findings with the opposite findings for the UK, Autor et al. (2020) invoked an inverted-U relationship between competition and innovation across sectors, originally proposed by Aghion et al. (2005).[3] Unlike the above discussed models of economic growth, where innovation is carried out by new entrants, in Aghion et al. (2005) it is undertaken by incumbents. This

[1] In a review of the empirical literature on trade and innovation, Shu and Steinwender (2019) report extensive evidence on a positive market size effect.

[2] In the Grossman and Helpman (1991b) model of growth based on quality ladders, the demand effect can be larger or smaller than the supply effect, while in the model of Baldwin and Robert-Nicoud (2008) with expanding variety, which features heterogeneous firms, the supply effect is larger than the demand effect. Four mechanisms that link trade to innovation and growth are reviewed in Melitz and Redding (2023).

[3] The empirical analysis in Aghion et al. (2005), which demonstrated this inverted-U relationship, used UK patent data, weighted by citation.

modification shifts the incentives to innovate from future rents to the difference between future and current rents of incumbents.[4] While more competition can harm future rents, thereby reducing the incentives to innovate, it also reduces current rents through a business-stealing effect, which encourages innovation. The net effect depends on whether the difference between future and current rents rises. Competition may raise the incremental profits from innovation, and particularly in sectors with "neck-and-neck" competition, where firms at comparable technology levels seek to escape the competitive pressure. In contrast, in sectors with high dispersion of productivity, where innovation is carried out mostly by laggard firms, intensified competition has little effect on current rents, yet it depresses future rents of the laggards and thereby discourages them from innovating. Autor et al. (2020) suggested that this theory may explain the contradicting findings in the US and UK, because productivity dispersion across firms was larger in the US than in the UK.[5]

Aghion et al. (2022a) examined the impact of firm-level export shocks on innovation of French firms. They found heterogeneous outcomes. While a positive export shock raised patenting of highly-productive firms (with some lag), it had little impact on patenting of low-productivity firms, despite the fact that sales and employment rose in all of them. To interpret these findings, they proposed a model in which—due to a market size effect—a positive export shock encourages innovation by all incumbent firms. But this same shock also encourages entry of new firms that compete with the incumbents in export markets, thereby discouraging their innovation. In the model, this competition effect is more pronounced among low-productivity incumbents, leading them to innovate less. Firm heterogeneity appears to be important for assessing the impact of foreign trade shocks on domestic innovation in France.[6]

I propose in this paper a mechanism that links foreign competition to domestic innovation of long-lived multi-product heterogeneous oligopolists, who compete with small short-lived single-product firms. The latter, from home and abroad, engage in monopolistic competition, limiting the power of the oligopolists. The rate of turnover of the small firms is very high, and they rapidly respond with entry or exit to shifts in market conditions. The large, long-lived multi-product firms, invest in R&D and thereby increase their product spans, as in Klette and Kortum (2004).[7]

This structure captures salient features of the US economy. Data from Hottman et al. (2016) supports the view that large multi-product firms face competition from small single-product firms.

[4] Arrow (1962) directed attention to the canibalization effect of innovation by incumbents. This is sometimes referred to as the "Arrow replacement effect".

[5] See Akcigit and Melitz (2022) for a review of trade and innovation in the presence of business stealing and escape competition.

[6] In a companion paper, Aghion et al. (2022b) studied the impact of the China shock on innovation of French firms. They found that it had a positive effect on firms who competed with China in output markets and a negative effect on firms that competed with China in input markets. The former relation was concentrated, however, among low-productivity firms. Moreover, the output and input shocks were highly correlated at the industry level. As a result, the overall effect was small.

[7] Shimomura and Thisse (2012) studied interactions between a monopolistically competitive fringe of single-product firms and oligopolistic large firms in a static closed economy, while Parenti (2018) studied such interactions in a static economy with foreign trade. Impullitti and Licandro (2017) studied a world of two symmetric countries and a continuum of sectors. Within a sector (product line) there was oligopolistic competition and cost-reducing innovation. They showed that in this framework dynamic welfare gains from trade liberalization are large.

In addition, Cao et al. (2022) reported that 95% of US firms had single establishments in 2014, while Cao et al. (2022) and Kehrig and Vincent (2019) reported that growth of large firms took place mostly through the extensive margin. Finally, Bernard et al. (2007) found that large multi-product firms carried out an overwhelming share of US exports.

To study the impact of foreign competition on domestic innovation, I consider a two-country world with two sectors. One sector produces homogeneous goods with labor, using a constant returns to scale technology. The other sector employs labor to produce varieties of a differentiated product. In the foreign country varieties are produced by short-lived single-product firms. In the home country they are produced by both short-lived single-product firms and by long-lived multi-product (multi-variety) oligopolists. The large oligopolists invest in R&D to add product lines to their collection of merchandise.

Using this model, I investigate the impact of productivity improvements in the technology of foreign exporters on the innovation efforts of home-country oligopolists. This type of technical change is relevant for the ascent of China and its expansion into foreign markets. Moreover, it may have played a bigger role in the "China shock" than China's accession to the WTO, because the rapid productivity growth started much earlier than the accession to the WTO. In the decomposition of China's growth of real output per worker, Brandt et al. (2022) estimated that total productivity growth contributed 3.2 percentage points per annum during 1980-1989, 2.9 percentage points per annum during 1990-1999, and 3.1 percentage points per annum during 2000-2009 (see their Figure 1). In the first two of these periods, TFP growth explains (in a growth accounting sense) approximately one half of the growth rate of real output per worker, while in the third period it explains about one third. Capital deepening also played a large role in all three periods, although in the first one its contribution was smaller than the contribution of TFP. Clearly, productivity growth in China played a prominent role in exerting competitive pressure on UK and US firms.

My main findings are that an increase in foreign competition can encourage or discourage domestic innovation, and that the impact can vary across firms with disparate productivity levels. High-productivity domestic firms may respond by innovating more and low-productivity firms may respond by innovating less, or, conversely, high-productivity firms may respond by innovating less and low-productivity firms may respond by innovating more. The outcomes depend on the relative demand levels and market shares of oligopolists at home and abroad.

An interesting point that emerges from this analysis is that the direction of change of short-term operating profits is not necessarily indicative of the direction of change in the profitability of innovation. And this relationship can differ across firms with different productivity levels.

Basic features of the model are described in Section 2. An instantaneous equilibrium, which has to hold at every point in time, is studied in Section 3. The profits that emerge in an instantaneous equilibrium are studies in Section 4. Section 5 describes the optimal control problems of large firms and characterizes their dynamics. The impact of foreign competition on innovation is then studied in Section 6.

2 Preliminaries

I consider a two-country world, consisting of a home country H and a foreign country F. Every country is populated by a continuum of identical individuals of mass one. Labor is the only input and labor markets are competitive.

There are two sectors. One sector produces a tradable homogeneous good with one unit of labor per unit output in every country. This good is traded at no cost and it serves as numeraire. Demand for the homogeneous good is high enough to ensure production in both countries. For these reasons the price of the homogeneous good equals one and so does the wage rate in every country. The other sector produces tradable varieties of a differentiated product with technologies and trade costs described below.[8]

Every individual supplies a fixed amount of labor, l, and has a utility function[9]

$$u = x_0 + \frac{\varepsilon}{\varepsilon - 1} X^{\frac{\varepsilon-1}{\varepsilon}},\ \varepsilon > 1, \tag{1}$$

where x_0 is consumption of the homogeneous good and X is the real consumption index of varieties of the differentiated product. This real consumption index is a CES aggregator of individual varieties that has a price index

$$P = \left[\int_{\omega \epsilon \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}},\ \sigma > \varepsilon,$$

where Ω is the set of available brands, σ is the elasticity of substitution between them, and $p(\omega)$ is the price of variety ω.

A country-J individual maximizes utility subject to the budget constraint $x_0 + P_J X = l + y_J$, where P_J is the price index and y_J is non-wage income in country J, $J = H, F$. This yields $X_J = P_J^{-\varepsilon}$, as long as consumers purchase the homogenous good and varieties of the differentiated product, which is assumed to be satisfied.[10] The demand for variety ω is then independent of y_J and equal to

$$x_J(\omega) = P_J^{\delta} p_J(\omega)^{-\sigma},\ \delta := \sigma - \varepsilon > 0,\ J = H, F. \tag{2}$$

Assume that F can produce brands of the differentiated product with single-product firms only, and that these firms live only one instant. To enter the industry, a single-product firm has to spend f_F units of labor and this investment provides it with a unique brand of the product. After entry, a firm needs a_F units of labor per unit output to manufacture its product. These firms serve their home market and export to H.

There are variable export costs of the melting iceberg type, denoted by $\tau > 1$. After entry, a

[8] It is easy to generalize the analysis to multiple differentiated products sectors, or to differences in labor productivity in the homogeneous sector of the two countries. The latter would lead to different wage rates in H and F.

[9] I can allow l to vary across countries, but this variation will not affect the results.

[10] This requires l to be large enough.

single-product firm ω choses prices $p_{F,F}(\omega)$ and $p_{F,H}(\omega)$ to maximize operating profits

$$\pi_F := P_F^{\delta} p_{F,F}(\omega)^{-\sigma} \left[p_{F,F}(\omega) - a_F\right] + P_H^{\delta} p_{F,H}(\omega)^{-\sigma} \left[p_{F,H}(\omega) - \tau a_F\right],$$

subject to the demand functions (2), taking as given the price indexes P_F and P_H. Here $p_{F,F}$ denotes the price charged by an F-country firm in F and $p_{F,H}$ denotes the price charged by an F-country firm in H. The resulting prices are

$$p_{F,F}(\omega) = p_F := \frac{\sigma}{\sigma - 1} a_F,\ p_{F,H}(\omega) = \tau p_F,\ \text{for all}\, \omega. \tag{3}$$

Denote by n_F the number of single-product firms in F.

Country H has two types of firms in the differentiated product sector: single-product firms that live one instance and large multi-product firms that live forever. Every large firm has a positive measure of product lines, n_i, $i = 1, 2, ..., I$.

All country-H single-product firms share the same technology. It requires f_H units of labor for entry and a_H units of labor per unit output. These firms serve the home market and export to F. After entry, a single-product firm ω chooses prices $p_{H,F}(\omega)$ and $p_{H,F}(\omega)$ to maximize operating profits

$$\pi_H := P_F^{\delta} p_{H,F}(\omega)^{-\sigma} \left[p_{H,F}(\omega) - \tau a_H\right] + P_H^{\delta} p_{H,H}(\omega)^{-\sigma} \left[p_{H,H}(\omega) - a_H\right],$$

subject to the demand functions (2), taking as given P_F and P_H. As a consequence, the pricing strategy of a country-H single-product firm is

$$p_{H,H}(\omega) = p_H := \frac{\sigma}{\sigma - 1} a_H,\, p_{H,F}(\omega) = \tau p_H,\ \text{for all}\, \omega. \tag{4}$$

Denote by n_H the number of single-product firms in H.

At every point in time the firms play a two-stage game, in which the product spans $\{n_i\}_{i=1}^{I}$ are given. In the first stage, single-product firms enter, yielding n_F and n_H. Assume $\{n_F, n_H\} >> 0$ at every point in time. In the second stage, all firms play a Bertrand price game.

Unlike single-product firms, large multi-product firms do not view the price indexes P_F and P_H as given. A large firm recognizes the relationship between its own prices and these price indexes. Due to symmetry across products within a firm, a multi-product firm understands the functional relationships

$$P_F = \left(n_F p_F^{1-\sigma} + n_H \tau^{1-\sigma} p_H^{1-\sigma} + \sum_{i=1}^{I} n_i p_{F,i}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \tag{5}$$

$$P_H = \left(n_F \tau^{1-\sigma} p_F^{1-\sigma} + n_H p_H^{1-\sigma} + \sum_{i=1}^{I} n_i p_{H,i}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \tag{6}$$

where $p_{J,i}$ is the price firm i charges for each one of its products in country J.

The subgame perfect equilibrium of this two-stage game constitutes an instantaneous equilibrium.

3 Instantaneous Equilibrium

To characterize an instantaneous equilibrium, begin with the second stage of the game in which $\{n_F, n_H\}$ has already been determined. At this stage every firm chooses its prices, taking as given prices of all other firms. In this event large firm i maximizes profits by solving

$$\max_{p_{H,i},p_{F,i}} n_i P_H^{\delta} p_{H,i}^{-\sigma} (p_{H,i} - a_i) + n_i P_F^{\delta} p_{F,i}^{-\sigma} (p_{F,i} - \tau a_i),$$

subject to (5)-(6). The solution yields prices

$$p_{J,i} = \frac{\sigma - \delta s_{J,i}}{\sigma - \delta s_{J,i} - 1} \tau_J a_i, \ \tau_H := 1, \ \tau_F := \tau, \ J = F, H, \tag{7}$$

where $s_{J,i}$ is the market share of firm i in country J and

$$s_{J,i} = \frac{n_i p_{J,i}^{1-\sigma}}{P_J^{1-\sigma}}, \ J = F, H. \tag{8}$$

The markup factor in country J, $\frac{\sigma - \delta s_{J,i}}{\sigma - \delta s_{J,i} - 1}$, is increasing in the market share $s_{J,i}$, and therefore it is lager than the markup factor of single-product firms, $\frac{\sigma}{\sigma - 1}$. Equations (7)-(8) jointly determine prices and market shares of large firms, given product spans $\{n_i\}_{i=1}^{I}$ and price indexes P_F and P_H.[11]

We can express the relationships embodied in (7)-(8) by means of price functions $p_{J,i}(n_i, P_J)$ and market share functions $s_{J,i}(n_i, P_J)$. I prove in the appendix the following

Lemma 1. *(i) Let* $\beta_{J,i} = \frac{\delta s_{J,i}}{(\sigma - \delta s_{J,i} - 1)(\sigma - \delta s_{J,i})} > 0$. *Then the elasticities of* $p_{J,i}(n_i, P_J)$ *with respect to its two arguments are* $\frac{\beta_{J,i}}{1 + (\sigma-1)\beta_{J,i}}$ *and* $\frac{(\sigma-1)\beta_{J,i}}{1+(\sigma-1)\beta_{J,i}}$, *respectively; and the elasticities of* $s_{J,i}(n_i, P_J)$ *with respect to its two arguments are* $\frac{1}{1+(\sigma-1)\beta_{J,i}}$ *and* $\frac{\sigma-1}{1+(\sigma-1)\beta_{J,i}}$, *respectively.*

The price functions $p_{J,i}(n_i, P_J)$ can also be used to express operating profits of firm i in country J as

$$\pi_{J,i}(n_i, P_J) := n_i P_J^{\delta} p_{J,i}(n_i, P_J)^{-\sigma} [p_{J,i}(n_i, P_J) - a_{J,i}], \ a_{J,i} = \tau_J a_i, \ J = F, H. \tag{9}$$

Now turn to stage one of the game. At entry, a single-product firm correctly forecasts the price indexes P_F and P_H in the second stage of the game, and the price it will charge for its own product. It is shown in the appendix that for small variable trade costs, single-product firms enter in at most one country. For high trade costs, they enter in both. I assume that τ is large enough to ensure positive entry of single-product firms in both countries. In this event the free entry conditions are

$$\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(P_F^{\delta} + P_H^{\delta}\tau^{1-\sigma}\right) = z_F := f_F a_F^{\sigma-1}, \tag{10}$$

[11] In combination with (5)-(6), these equations also provide solutions to the price indexes for given values of $\{n_i\}_{i=1}^{I}$ and $\{n_F, n_H\}$.

$$\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(P_F^{\delta}\tau^{1-\sigma}+P_H^{\delta}\right)=z_H:=f_H a_H^{\sigma-1}, \tag{11}$$

and the price indexes are

$$P_F^{\delta}=\frac{z_F-\tau^{1-\sigma}z_H}{\left[1-\tau^{2(1-\sigma)}\right]\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}, \tag{12}$$

$$P_H^{\delta}=\frac{z_H-\tau^{1-\sigma}z_F}{\left[1-\tau^{2(1-\sigma)}\right]\frac{1}{\sigma}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}. \tag{13}$$

Both price indexes are positive if and only if $\tau^{1-\sigma}<\min\left\{\frac{z_F}{z_H},\frac{z_H}{z_F}\right\}$. I adopt this assumption.

Let a hat over a variable x represents a proportional rate of change; that is, $\hat{x}=\mathrm{d}\log x$. Then (12) and (13) deliver

Lemma 2. *In an instantaneous equilibrium*

$$\delta\hat{P}_F=\frac{z_F}{z_F-\tau^{1-\sigma}z_H}\hat{z}_F-\frac{\tau^{1-\sigma}z_H}{z_F-\tau^{1-\sigma}z_H}\hat{z}_H,$$

$$\delta\hat{P}_H=\frac{z_H}{z_F-\tau^{1-\sigma}z_H}\hat{z}_H-\frac{\tau^{1-\sigma}z_F}{z_F-\tau^{1-\sigma}z_H}\hat{z}_F.$$

The statistics $z_J=f_J a_J^{\sigma-1}$, $J=F,H$, are important determinants of the price indexes. A lower value of z_J represents more competitiveness of J's single-product firms, either because they have lower entry costs or lower unit production costs. It results in a lower price index in country J and a higher price index in the trade partner country. In short, more efficient single-product firms raise competition in their own country and reduce it abroad.

4 Profits

We have seen in the previous section that an improvement in the technology of foreign single-product firms reduces the price index in the foreign country and raises it in the home country. These changes reduce profits of every multi-product firm in country F and raises its profits in country H. Do the overall profits of such a firm rise or decline? This is the question addressed in this section.

Overall operating profits of firm i are

$$\pi_i\left(n_i,P_F,P_H\right):=\sum_{J=F,H}\pi_{J,i}\left(n_i,P_J\right),\ i=1,2,...,I, \tag{14}$$

where $\pi_{J,i}\left(n_i,P_J\right)$ is given in (9). As is evident, a change in costs of foreign country single-product firms impacts profits in country J through the price indexes P_J. Using (7), (27) and (9), this impact can be expressed as[12]

$$\hat{\pi}_{J,i} = \delta \left[1 - s_{J,i} \frac{(\sigma-1)\beta_{J,i}}{1+(\sigma-1)\beta_{J,i}} \right] \hat{P}_J, \;\; J = F, H. \tag{15}$$

Now suppose that foreign exporters (i.e., foreign single-product firms) become more competitive, either due to a decline in their entry costs, f_F, or due to a decline in their manufacturing costs, a_F. In response, z_F declines ($\hat{z}_F < 0$). Lemma 2 then implies that the price index declines in the foreign country and rises in the home country, which, according to (15), reduces profits abroad and raises profits at home. The impact on profits can vary across firms, depending on their marginal costs, $1/a_i$, and their initial number of product lines, n_i. What are the circumstances in which overall profits rise or decline?

To answer this question, first note that a large firm's profits can be larger in the home or the foreign market, depending on the competitiveness of single-product firms in the two countries and the size of the trade costs. I prove in the appendix the following

Lemma 3. *(i) If $z_H > z_F$ then $P_F < P_H$ and $\pi_{H,i} > \pi_{F,i}$ for $i = 1, 2, ..., I$; and (ii) if $z_H < z_F$ then $P_F > P_H$ and for every i there exists a $\tau_{c,i} > 1$ such that $\pi_{H,i} > \pi_{F,i}$ for $\tau > \tau_{c,i}$ and $\pi_{H,i} < \pi_{F,i}$ for $\tau \epsilon [1, \tau_{c,i})$.*

This lemma provides conditions under which a large firm's profits are higher in one country or the other. When competitiveness of single-product firms is larger in country F, every large firm has higher profits in the home market independently of trade costs. And when competitiveness of single-product firms is larger in country H, a large firm has higher profits in the home market only when trade costs are high enough and lower profits in the home market when trade costs are low.

To see whether an increase in the efficiency of foreign exporters raises or reduces a multi-product firm's overall profits, I prove in the appendix the following

Proposition 1. *A decline in z_F raises firm i's overall profits if and only if*

$$P_H^{\delta} \Phi(s_{H,i}) > P_F^{\delta} \Phi(s_{F,i}), \tag{16}$$

where

$$\Phi(s) := \frac{(\sigma - \delta s - 1)(\sigma - \delta s) + (\sigma - 1)(1 - s)\delta s}{(\sigma - \delta s - 1)(\sigma - \delta s) + (\sigma - 1)\delta s} \cdot \frac{(\sigma - \delta s)^{-\sigma}}{(\sigma - \delta s - 1)^{1-\sigma}}, \tag{17}$$

and $\Phi(s)$ is a declining function.

There are two points worth noting about this proposition. First, each side of inequality (16) includes a combination of a price index and a market share, which are not independent of each

[12] This follows from

$$\hat{\pi}_{J,i} = \delta \hat{P}_J + \left(-\sigma + \frac{\frac{p_{J,i}}{a_{J,i}}}{\frac{p_{J,i}}{a_{J,i}} - 1} \right) \hat{p}_{J,i} = \delta \left(\hat{P}_J - s_{J,i} \hat{p}_{J,i} \right) = \delta \left[1 - s_{J,i} \frac{(\sigma-1)\beta_{J,i}}{1+(\sigma-1)\beta_{J,i}} \right] \hat{P}_J.$$

other. As shown in Lemma 1, a higher price index in a country raises the market share of every large firm in that country. Therefore, since $\Phi(s)$ is a declining function, it is not apparent whether a higher price index in the home country or a lower price index in the foreign country make this inequality more likely. Second, this inequality depends on the productivity of firm i, $1/a_i$, and on its product span, n_i. For this reason it may be satisfied for some firms but not other.
Nevertheless, we have

Corollary. *A decline in z_F reduces firm i's overall profits when* $P_F = P_H = P$, for $i = 1, 2, ..., I$.

In other words, when the competitive pressure is similar in the two countries, an improvement in the competitiveness of foreign exporters reduces overall profits of every large firm, independently of its productivity or product span. The reason is that in this case a firm's market share is larger in the home country, i.e., $s_{H,i} > s_{F,i}$ (see (19) below), because the foreign country is more expensive to serve due to the transport costs.

To further study the likelihood of (16), use (7) and (8) to obtain

$$s_{J,i}^{\frac{1}{1-\sigma}} \left(\frac{\sigma - \delta s_{J,i}}{\sigma - \delta s_{J,i} - 1} \right)^{-1} = \tau_J P_J^{-1} b_i, \; J = F, H, \; b_i := n_i^{\frac{1}{1-\sigma}} a_i. \tag{18}$$

In each one of these equations, the left-hand side is declining in the market share.[13] Therefore each one of them describes an implicit relationship between a market share and the expression on the right-hand side,

$$s_{J,i} = s\left(\tau_J P_J^{-1} b_i\right), \; J = F, H, \tag{19}$$

where $s(\cdot)$ is a declining function (and recall that $\tau_F = \tau$ and $\tau_H = 1$). Using this function, define

$$\Theta\left(b_i; P_F, P_H\right) := \frac{P_H^{\delta} \Phi\left[s\left(P_H^{-1} b_i\right)\right]}{P_F^{\delta} \Phi\left[s\left(\tau P_F^{-1} b_i\right)\right]}.$$

From Proposition 1 we know that for $\Theta\left(b_i; P_F, P_H\right) > 1$ a decline in z_F raises overall profits of firm i while for $\Theta\left(b_i; P_F, P_H\right) < 1$ it reduces the firm's overall profits. Firm i has a larger b_i the higher are its manufacturing costs (the less productive it is) or the smaller is its product mix.

The dashed orange curve in Figure 1 plots $\Theta\left(b; P_F, P_H\right)$ as a function of b for $(P_F, P_H) = (1, 0.95)$. The values of z_F and z_H that generate these price indexes can be computed from the free entry conditions (10) and (11), respectively. As expected from the corollary to Proposition 1, for P_H close to one the curve is everywhere below one, implying that every large firm suffers a profit

[13] For market shares to be between zero and one requires $b_i \geq \frac{\sigma-1}{\sigma} \max\left\{P_H, \frac{P_F}{\tau}\right\}$.

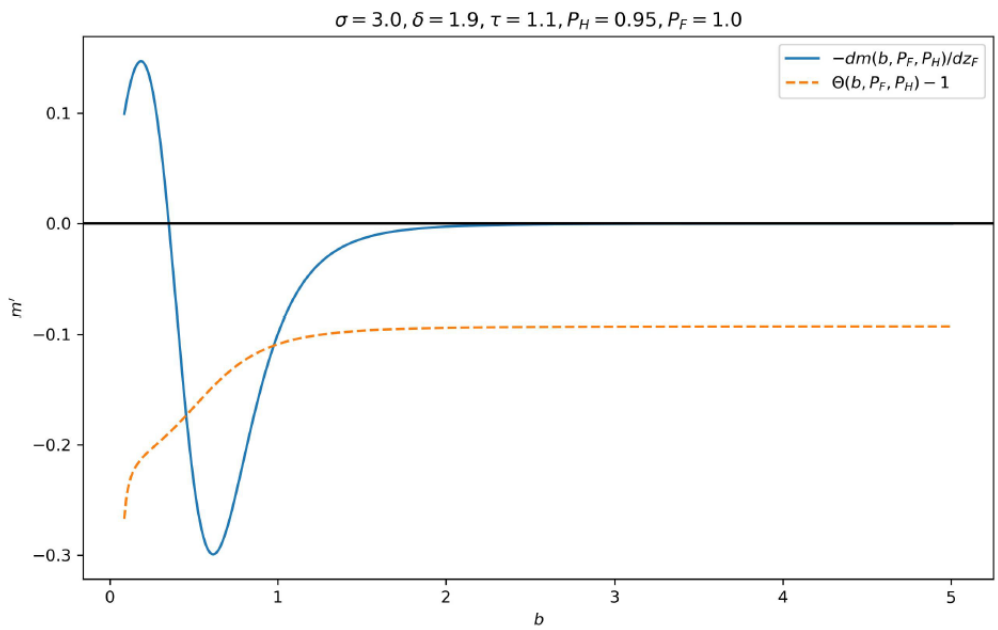

Figure 1: $-dm\left(b, P_F, P_H\right)/dz_F$ and $\Theta\left(b, P_F, P_H\right) - 1$ for $P_H = 0.95$

loss from an improvement in the competitiveness of foreign exporters. But if instead the price index in the home country is high and equal to two, as depicted in Figure 2, firms with low values of b_i lose from having to deal with more competitive foreign exporters, while firms with high values of b_i gain. The statistic b_i is larger the lower the firm's labor productivity $1/a_i$ is, or the fewer its product lines n_i are. Therefore firms with larger product spans are qualitatively influenced by foreign competition in similar fashion as firms with higher labor productivity.[14]

I conclude from this analysis that technical change that reduces entry costs or raises labor productivity of foreign exporters, can raise or reduce overall operating profits of large multi-product firms. Moreover, lower-productivity large firms or large firms with smaller product spans can benefit or lose from fiercer import competition.

There are two additional observations worth making. First, if a firm can invests in R&D in order to expand the assortment of its product lines, the product span of such a firm becomes endogenous and dependent on its productivity. Furthermore, the relationship between a firm's productivity and its long-run number of product lines is not monotonic; it has an inverted-U shape.[15] For these reasons we need to study firm dynamics in order to establish the relationship between productivity, $1/a_i$, and the long-run value of b_i. Second, investment in innovation is driven by the benefit of expanding the product range, and this benefit is different from the gain in short-run operating

[14] As shown the appendix, the market shares as functions of b_i are higher in the home country than in the foreign country for both $P_H = 0.95$ and $P_H = 2$, and lower for $P_H = 0.5$.

[15] See Helpman and Niswonger (2022)) for a closed economy that has this feature, and this property also holds in the current case. Feenstra and Ma (2009) were the first to derive an inverted-U shape relationship between product span and productivity.

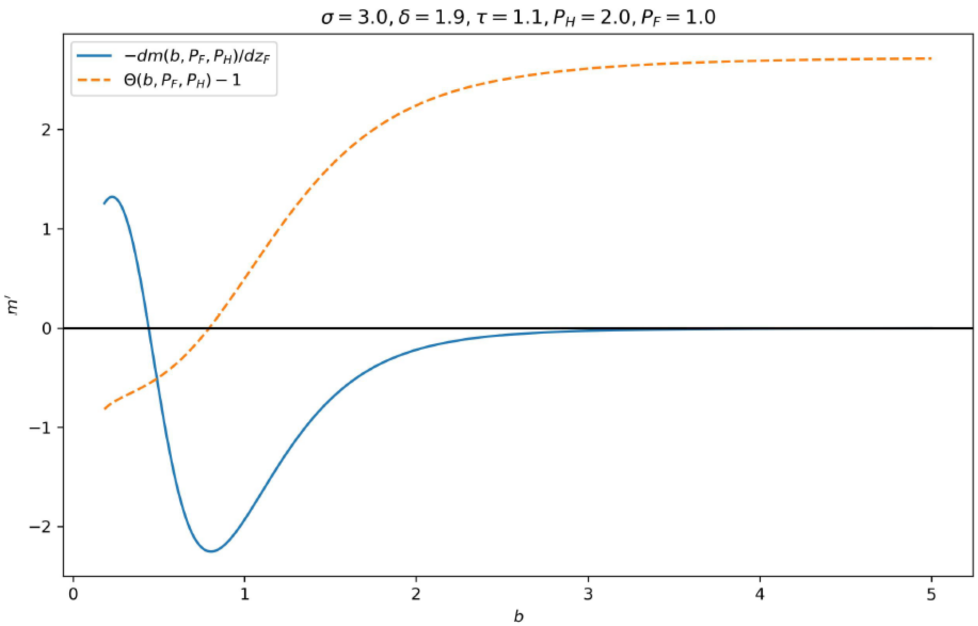

Figure 2: $-dm\left(b, P_F, P_H\right)/dz_F$ and $\Theta\left(b, P_F, P_H\right) - 1$ for $P_H = 2.0$

profits. For these reasons it is necessary to study firm dynamics in order to understand how a large firm's benefits from innovation depend on its productivity. This task is undertaken in the next section.

5 Dynamics

Time is continuous and starts at $t = 0$. The product span of firm i is time dependent, denoted by $n_i\left(t\right)$. The initial condition is $n_i\left(0\right) = n_i^0$, with n_i^0 given, $i = 1, 2, ..., I$. At every point in time firm i can invest in R&D in order to expand its product mix. An investment flow of ι_i labor units per unit time expands n_i by $\phi\left(\iota_i\right)$ units per unit time. The function $\phi\left(\iota\right)$ is increasing, concave, $\phi\left(0\right) = 0$, and it satisfies the Inada conditions $\lim_{\iota \searrow 0} \phi'\left(\iota\right) = +\infty$ and $\lim_{\iota \to \infty} \phi'\left(\iota\right) = 0$. In addition, n_i depreciates at the rate θ per unit time. As a result, n_i evolves over time according to

$$\dot{n}_i = \phi(\iota_i) - \theta n_i, \text{ for all } t \geq 0, \tag{20}$$

where the time index t has been suppressed for simplicity.

Assume that the interest rate is constant and equal to r, and firm i maximizes the discounted present value of profits net of investment costs. It therefore solves the optimal control problem

$$\max_{\{\iota_i(t), n_i(t)\}_{t \geq 0}} \int_0^\infty e^{-\rho t} \left\{\pi_i\left[n_i\left(t\right), P_F, P_H\right] - \iota_i\left(t\right)\right\} dt$$

subject to (14), (20), $n_i(0) = n_i^0$, and a transversality condition (see below). Every firm i correctly forecasts the instantaneous equilibria that evolve over time. This includes birth and death of short-lived single-product firms and the emergence of price indexes from the two-stage game in every instant of time. The current-value Hamiltonian of this problem is

$$\mathcal{H}(\iota_i, n_i, \eta_i) = [\pi_i(n_i, P_F, P_H) - \iota_i] + \eta_i[\phi(\iota_i) - \theta n_i],$$

where η_i is the co-state variable of constraint (20). The first-order conditions of this problem are:

$$\frac{\partial \mathcal{H}}{\partial \iota_i} = -1 + \eta_i \phi'(\iota_i) = 0,$$

$$-\frac{\partial \mathcal{H}}{\partial n_i} = -\frac{\partial \pi_i(n_i, P_F, P_H)}{\partial n_i} + \theta\eta_i = \dot{\eta}_i - r\eta_i,$$

and the transversality condition is:

$$\lim_{t\to\infty} e^{-rt}\eta_i(t)\, n_i(t) = 0.$$

The optimal path of (ι_i, n_i) has also to satisfy (20). These first-order conditions can be expressed as:

$$\eta_i\phi'(\iota_i) = 1, \tag{21}$$

$$\dot{\eta}_i = (r+\theta)\eta_i - \pi_{i,n}(n_i, P_F, P_H), \tag{22}$$

where

$$\begin{aligned}\pi_{i,n}(n_i, P_F, P_H) &:= \frac{\partial \pi_i(n_i, P_F, P_H)}{\partial n_i} \qquad (23)\\ &= \sum_{J=F,H} (\tau_J a_i)^{1-\sigma} P_J^{\delta} \frac{\sigma\left[\frac{\sigma - \delta s_{J,i}(n_i,P_J)}{\sigma-\delta s_{J,i}(n_i,P_J)-1}\right]^{-\sigma}}{[\sigma - \delta s_{J,i}(n_i, P_J) - 1]\sigma + s_{J,i}(n_i, P_J)^2\delta^2},\end{aligned}$$

represents marginal profits of n_i. This marginal profit function is declining in n_i.[16]

To understand the first-order conditions (21) and (22), first note that the co-state variable η_i measures the marginal value of n_i. Namely, it measures the addition to the present value of profits of an extra product line. Bearing this in mind, (21) simply states that the cost of an additional unit of R&D, represented by the right-hand side, just equals the marginal benefit, represented by the left-hand side. An additional unit of ι_i raises n_i by $\phi'(\iota_i)$ units, and every additional unit of n_i is valued at η_i.

Condition (22) represents asset pricing. The asset in question is the product mix n_i. Rewrite this condition as

$$\frac{\pi_{i,n}(n_i, P_F, P_H)}{\eta_i} + \frac{\dot{\eta}_i}{\eta_i} = r + \theta.$$

[16]Helpman and Niswonger (2022) showed that $\frac{\left(\frac{\sigma-\delta s}{\sigma-\delta s-1}\right)^{-\sigma}}{(\sigma-\delta s-1)\sigma+s^2\delta^2}$ is a declining function of s. Together with Lemma 1 this implies that $\pi_{i,n}(n_i, P_F, P_H)$ is declining in n_i.

We can then interpret the first term on the left-hand side as the inverse of the price-earnings ratio, where the marginal profit $\pi_{i,n}$ represents earnings and η_i represents the asset price. The second term on the left-hand side represents capital gains, while the right-hand side represents a risk-adjusted interest rate. This is therefore a standard asset pricing equation: the inverse of the price-earnings ratio plus expected capital gains equal the risk adjusted interest rate.

From (21) we obtain the investment level ι_i as an increasing function of η_i, which is represented by $\iota(\eta_i)$. Substituting this function into (20) yields the autonomous differential equation

$$\dot{n}_i = \phi\left[\iota\left(\eta_i\right)\right] - \theta n_i. \tag{24}$$

A time varying vector (n_i, η_i) that satisfies the autonomous system of differential equations (22) and (24), and the transversality condition, solves the firm's optimal control problem.[17]

The steady state of these differential equations is characterized by

$$\phi\left[\iota\left(\eta_i\right)\right] = \theta n_i, \tag{25}$$

$$\left(r + \theta\right)\eta_i = \pi_{i,n}\left(n_i, P_F, P_H\right). \tag{26}$$

The left-hand side of (25) is increasing in η_i. Therefore the curve in (n_i, η_i) space along which n_i is constant is upward sloping. The right-hand side of (26) is declining in n_i, because $\pi_{i,n}\left(n_i, P_F, P_H\right)$ is declining in n_i. Therefore the curve in (n_i, η_i) space along which η_i is constant is downward sloping. These curves are depicted in Figure 3, which also portrays the dynamics that result from (22) and (24). The vector (n_i, η_i) evolves over time along a stable saddle-path that converges to a steady state. The transversality condition is satisfied in this steady state. Therefore this saddle-path solves the firm's optimal control problem. Product span n_i rises over time when n_i^0 is below its steady state value and n_i declines over time when n_i^0 is above its steady state value.

6 Foreign Exporters and Innovation

We have seen in the previous section how the product span of a large firm evolves over time as a result of investment in innovation. In this section, I study the response of such a firm to improvements in the technology of foreign exporters. Of particular interest is whether the large firm counters a rise in foreign competition with expansion of R&D or contraction. In the former case its product span rises, in the latter case it declines.

Consider an initial state in which all multi-product firms are in a steady state, of the type described in Figure 3. From Helpman and Niswonger (2022) we know that the location of curve $\dot{\eta}_i = 0$ may not vary monotonically with a_i (see their Proposition 5). Namely, this curve can be higher or lower for a more-productive firm (i.e., a firm with a lower a_i). If it is higher, the

[17] Note that $\mathcal{H}(\iota_i, n_i, \eta_i)$ is concave in the first two arguments.

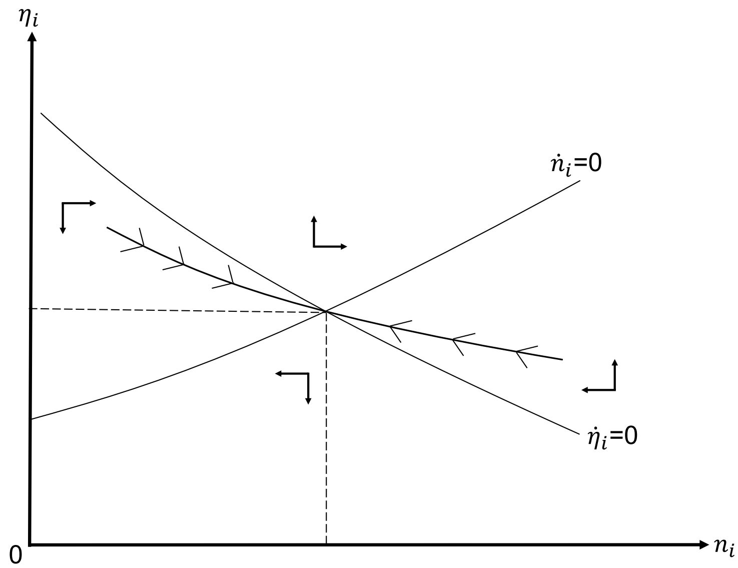

Figure 3: Transition Dynamics

more-productive firm has a larger steady-state collection of products. If it is lower, the more-productive firm has a smaller steady-state product mix. In particular, the relationship between n_i and productivity can have an inverted-U shape.

Whenever a more productive firm has a larger product span, it has a lower value of $b_i = n_i^{\frac{1}{1-\sigma}} a_i$. Therefore it also has larger market shares in both countries, as can be seen from (19). But even if a more productive firm has a smaller product span in the steady state, its b_i is smaller.[18] In short, we have

Proposition 2. *In steady state, more productive multi-product firms (with lower values of a_i) have lower values of b_i and larger market shares $s_{F,i}$ and $s_{H,i}$.*

Now consider a technological improvement that either reduces entry costs or variable production costs of foreign exporters. As shown by Lemma 2, this reduces the foreign price index and raises the home country price index, yielding

$$\hat{P}_F = -\tau^{1-\sigma}\hat{P}_H < 0.$$

These changes alter the marginal profits of product span $\pi_{i,n}(n_i, P_F, P_H)$, according to

$$\hat{\pi}_{i,n} = \left(e_{(\pi_{i,n},P_H)} - \tau^{1-\sigma} e_{(\pi_{i,n},\,P_F)}\right)\hat{P}_H,$$

where $e_{(\pi_{i,n},P_J)}$ is the elasticity of $\pi_{i,n}(n_i, P_F, P_H)$ with respect to P_J, $J = F, H$. If $\hat{\pi}_{i,n} > 0$, the steady state condition (26) implies that curve $\dot{\eta}_i = 0$ in Figure 3 shifts upward, prompting an

[18]For suppose it is not. Then the more productive firm has smaller market shares in both countries and therefore a higher marginal profit $\pi_{i,n}(n_i, P_F, P_H)$. For this to be the case, it has to have a higher η_i in order to satisfy (26). Yet a higher η_i is consistent with (25) only if n_i is larger. A contradiction.

adjustment process that gradually raises n_i over time. And if $\hat{\pi}_{i,n} < 0$, curve $\dot{\eta}_i = 0$ shifts downward, prompting an adjustment process that gradually reduces n_i over time. That is, this trade shock encourages innovation in the former case and discourages it in the latter.

Using (23) and the share function $s(\cdot)$, these marginal profits can be expressed as

$$\pi_{i,n}(n_i, P_F, P_H) = \sum_{J=F,H} (\tau_J a_i)^{1-\sigma} P_J^{\delta} \frac{\sigma \left[\frac{\sigma - \delta s\left(\tau_J P_J^{-1} n_i^{\frac{1}{1-\sigma}} a_i\right)}{\sigma - \delta s\left(\tau_J P_J^{-1} n_i^{\frac{1}{1-\sigma}} a_i\right) - 1} \right]^{-\sigma}}{\left[\sigma - \delta s\left(\tau_J P_J^{-1} n_i^{\frac{1}{1-\sigma}} a_i\right) - 1\right] \sigma + s\left(\tau_J P_J^{-1} n_i^{\frac{1}{1-\sigma}} a_i\right)^2 \delta^2}.$$

It shows that an increase in P_J affects $\pi_{i,n}(\cdot)$ through two channels. First, there is a positive direct effect stemming from the rise in demand, P_J^{δ}. Second, there is a negative indirect effect through the market share, $s\left(\tau_J P_J^{-1} n_i^{\frac{1}{1-\sigma}} a_i\right)$, because a higher price index raises this share and a larger market share reduces marginal profits. For this reason the elasticity $e_{(\pi_{i,n}, P_J)}$ can be positive or negative, and so can the difference $e_{(\pi_{i,n}, P_H)} - \tau^{1-\sigma} e_{(\pi_{i,n}, P_F)}$. It therefore appears that technical change that improves the efficiency of foreign exporters, can bring about an upward or downward shift in curve $\dot{\eta}_i = 0$, and therefore can encourage or discourage innovation.

To gain additional insights, define

$$m(b_i, P_F, P_H) := n_i \pi_{i,n}(n_i, P_F, P_H) = \sum_{J=F,H} (\tau_J b_i)^{1-\sigma} P_J^{\delta} \frac{\sigma \left[\frac{\sigma - \delta s\left(\tau_J P_J^{-1} b_i\right)}{\sigma - \delta s\left(\tau_J P_J^{-1} b_i\right) - 1} \right]^{-\sigma}}{\left[\sigma - \delta s\left(\tau_J P_J^{-1} b_i\right) - 1\right] \sigma + s\left(\tau_J P_J^{-1} b_i\right)^2 \delta^2}.$$

Next, solve from the steady state condition (25) η_i as a function of n_i, $\eta_i = \eta(n_i)$, where $\eta(\cdot)$ is an increasing function representing curve $\dot{n}_i = 0$ in Figure 3. Now substitute this function together with $m(\cdot)$ into the steady state condition (26) to obtain

$$(r + \theta)\, \eta(n_i)\, n_i = m(b_i, P_F, P_H).$$

This equation links steady state values of n_i to steady state values of b_i, and from Proposition 2 we know that b_i is larger in steady state the larger a_i is.[19] The left-hand side of this equation is increasing in n_i, and therefore firm i has a larger product span in steady state when $m(b_i, P_F, P_H)$ is larger.[20] Importantly, if a rise in the competitiveness of foreign exporters raises $m(\cdot)$, then curve $\dot{\eta}_i = 0$ in Figure 3 shifts upward and firm i raises investment in R&D, leading to a gradual expansion of its product mix. If instead $m(\cdot)$ declines, curve $\dot{\eta}_i = 0$ shifts downward and firm i reduces investment in R&D, leading to a gradual contraction of its product mix.

Figure 1 exhibits two curves. The blue solid curve plots minus $dm(b, P_F, P_H)/dz_F$ as a function of b while the orange dashed curve plots $\Theta(b, P_F, P_H) - 1$ as a function of b, for $(P_F, P_H) =$

[19] Together with $b_i = n_i^{\frac{1}{1-\sigma}} a_i$, this equation provides a solution to the steady state values of (n_i, b_i) for a given a_i.
[20] Yet $m(\cdot)$ is not necessarily a monotonic function of b_i.

$(1, 0.95)$.[21] As can be seen in the figure, in this case a decline in z_F raises $m(b, P_F, P_H)$ for highly-productive firms (with low b_i), inducing them to increase investment in innovation and expanding product lines over time. In contrast, marginal values of product spans decline for low-productivity firms. They contract R&D efforts and shed products over time.

Figure 2 exhibits curve minus $dm(b, P_F, P_H)/dz_F$ for $(P_F, P_H) = (1, 2)$, which has qualitatively a similar shape to the curve in Figure 1. That is, the marginal values of product spans rise for highly-productive firms and decline for low-productivity firms. As a result, in this case too highly-productive firms respond by expanding product lines over time while low-productivity firms respond by contracting product lines over time. Note, however, that in the case $(P_F, P_H) = (1, 0.95)$, all firms suffer short-term operating profit losses (the orange dashed curve $\Theta(b, P_F, P_H) - 1$ is below zero for all b) and in the case $(P_F, P_H) = (1, 2)$ highly-productive firms suffer short-term operating profit losses while low-productivity firms enjoy short-term operating profit gains (the orange dashed curve $\Theta(b, P_F, P_H) - 1$ is below zero for low values of b and above zero for high values).

These examples show that short-term responses of operating profits can be poor indicators of changes in the profitability of innovation. While short-term operating profits of highly-productive firms decline in both examples, marginal values of their product spans rise. For low-productivity firms short-term operating profit changes are aligned with changes in the marginal values of product spans when $P_H = 0.95$ (both decline), yet they are not aligned when $P_H = 2$; in the latter case short-term operating profits of low-productivity firms rise while marginal values of their product spans decline. Evidently, there can be a disconnect between short-run changes in operating profits and changes in the incentives to innovate.

To better understand this disconnect, note that we can use (12) and (13) to express the price indexes as functions of the statistics z_F and z_H, $P_F(z_F, z_H)$ and $P_H(z_F, z_H)$. Substituting these functions into (14) yields overall profit functions

$$\widetilde{\pi}_i(n_i, z_F, z_H) \equiv \pi_i[n_i, P_F(z_F, z_H), P_H(z_F, z_H)], \ i = 1, 2, ..., I.$$

Using this transformation, the impact of a rise in the competitiveness of foreign exporters on operating profits can be expressed as

$$-\widetilde{\pi}_{i,z_F}(n_i, z_F, z_H) := -\frac{\partial \widetilde{\pi}_i(n_i, z_F, z_H)}{\partial z_F},$$

while the marginal value of product span n_i can be expressed as

$$\widetilde{\pi}_{i,n}(n_i, z_F, z_H) := \frac{\partial \widetilde{\pi}_i(n_i, z_F, z_H)}{\partial n_i}.$$

In Figures 1 and 2, $\Theta(b_i, P_F, P_H) - 1 > 0$ if and only if $-\widetilde{\pi}_{i,z_F}(n_i, z_F, z_H) > 0$, where $P_F = P_F(z_F, z_H)$, $P_H = P_H(z_F, z_H)$ and $b_i = n_i^{\frac{1}{1-\sigma}} a_i$ (see Proposition 1). The derivative $-\widetilde{\pi}_{i,z_F}(n_i, z_F, z_H)$ represents the rise in operating profits from a marginal improvement in the competitiveness of for-

[21] The derivative $dm(b, P_F, P_H)/dz_F$ accounts for the impact of z_F on the price indexes P_F and P_H.

eign exporters. As discussed in the Introduction, for an incumbent firm the incentive to innovate does not depend on a change in current rents (current profits), but rather on a change in the difference between future and current rents. For this reason $-\widetilde{\pi}_{i,z_F}(n_i, z_F, z_H)$ does not provide a guide to the profitability of innovation. Instead, the incentive to innovate is guided by the impact of an improvement in foreign competitiveness on $\widetilde{\pi}_{i,n}(n_i, z_F, z_H)$, the value of a marginal product in a firm's product mix. This marginal value rises if and only if $-\partial\widetilde{\pi}_{i,n}(n_i, z_F, z_H)/\partial z_F > 0$. However, in Figures 1 and 2, $-dm(b_i, P_F, P_H)/dz_F > 0$ if and only if $-\partial\widetilde{\pi}_{i,n}(n_i, z_F, z_H)/\partial z_F > 0$, where $P_F = P_F(z_F, z_H)$, $P_H = P_H(z_F, z_H)$ and $b_i = n_i^{\frac{1}{1-\sigma}} a_i$.

Next note that

$$-\frac{\partial\widetilde{\pi}_{i,n}(n_i, z_F, z_H)}{\partial z_F} = -\frac{\partial\widetilde{\pi}_i(n_i, z_F, z_H)}{\partial z_F \partial n_i} = -\frac{\partial\widetilde{\pi}_{i,z_F}(n_i, z_F, z_H)}{\partial n_i}.$$

Therefore a rise in foreign competition raises the incentive to innovate if and only if more product lines raise the value of $-\widetilde{\pi}_{i,z_F}(n_i, z_F, z_H)$, independently of whether $-\widetilde{\pi}_{i,z_F}(n_i, z_F, z_H)$ is positive or negative. If, for example, a rise in foreign competition reduces operating profits, i.e., $-\widetilde{\pi}_{i,z_F}(n_i, z_F, z_H) < 0$, this per se provides no information on whether innovation has become more or less profitable. What matters is whether additional investment in R&D, which expands the firm's product span, will reduce or increase the operating profit loss or gain from enhanced foreign competition. For this reason short-term operating profit can rise, yet innovation will decline when it reduces the size of the profit gain from enhanced foreign competition. This case is illustrated by low-productivity firms in Figure 2. And the same logic works in reverse. Operating profits can decline, yet innovation will increase when it reduces the decline in operating profits. This is illustrated in the appendix for $P_H = 0.5$.

In summary, an increase in the competitiveness of foreign exporters can reduce or increase short-term operating profits of a large firm, and it can also increase or reduce the firm's profitability of R&D investment. Moreover, the shifts in these two profit margins can be positively or negatively correlated, and they may differ across heterogeneous firms with varying productivity levels. These findings provide a warning against simplistic conclusions about the relationship between operating profits and profitability of innovation.

Because changes in the competitiveness of foreign exporters impact large firms differently, depending on their productivity levels, the aggregate effects of such shocks depend on the distribution of productivity of multi-product firms. If, for example, the home country is populated by multi-product firms with low productivity, then in the case depicted in Figure 1 enhanced foreign competition reduces short-run aggregate operating profits and depresses R&D and innovation. Alternatively, in the case depicted in Figure 2, this type of shock depresses R&D and innovation of low-productivity firms, but raises aggregate short-term operating profits. Finally, in the case depicted in the appendix for $P_H = 0.5$, enhanced foreign competition raises aggregate innovation and reduces aggregate short-term operating profits of low-productivity firms. Naturally, predicting aggregate outcomes becomes even more challenging when some multi-product firms are highly productive and other have low productivity levels.

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Online Appendix

Appendix for Section 3

Let a hat over a variable x represents a proportional rate of change; that is, $\hat{x} = \mathrm{d}\log x$. Then differentiation of (7) and (8) yields

$$\hat{p}_{J,i} = \frac{\beta_{J,i}}{1+(\sigma-1)\beta_{J,i}}\hat{n}_i + \frac{1}{1+(\sigma-1)\beta_{J,i}}\hat{a}_{J,i} + \frac{(\sigma-1)\beta_{J,i}}{1+(\sigma-1)\beta_{J,i}}\hat{P}_J,\ J = F, H, \tag{27}$$

where

$$\beta_{J,i} = \frac{\delta s_{J,i}}{(\sigma - \delta s_{J,i} - 1)(\sigma - \delta s_{J,i})} > 0,$$

and

$$\hat{s}_{J,i} = \hat{n}_i - (\sigma-1)\hat{p}_{J,i} + (\sigma-1)\hat{P}_J,\ J = F, H. \tag{28}$$

For now, we note that (27) shows that a large firm charges higher prices in both markets the larger the number of its product lines or the larger the price index in that market. A higher price index represents lower competitive pressure. Equation (27) implies that the elasticity of $p_{J,i}(n_i, P_J)$ with respect to n_i is $\frac{\beta_{J,i}}{1+(\sigma-1)\beta_{J,i}}$ and its elasticity with respect to P_J is $\frac{(\sigma-1)\beta_{J,i}}{1+(\sigma-1)\beta_{J,i}}$. Combining (28) with (27) implies that the elasticities of $s_{J,i}(n_i, P_J)$ with respect to n_i is $\frac{1}{1+(\sigma-1)\beta_{J,i}}$ and its elasticity with respect to P_J is $\frac{\sigma-1}{1+(\sigma-1)\beta_{J,i}}$. This proves Lemma 1.

To see why in the absence of variable trade costs single-product firms enter only in one country, note that for $\tau = 1$operating profits of a single-product firm from country J are

$$\pi_J = \frac{1}{\sigma}\left(P_F^\delta + P_H^\delta\right)\left(\frac{\sigma}{\sigma-1}a_J\right)^{1-\sigma},\ J = H, F.$$

In this case the free entry conditions (10) and (11) can jointly be satisfied only in the knife-edge case where $f_J a_J^{\sigma-1}$ is the same for $J = F$ and $J = H$. Generally, the country with the higher value of $f_J a_J^{\sigma-1}$ has $n_J = 0$ in equilibrium (for $\tau = 1$). When $\tau \to \infty$, there are no exports, and free entry in country J leads to

$$\frac{1}{\sigma}P_J^\delta\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} = f_J a_J^{\sigma-1},\ J = H, F.$$

This equation determines the price index in each one of the countries when $\tau \to \infty$. Clearly, for high enough values of τ both free entry conditions are satisfied.

Appendix for Section 4

To prove Lemma 2, first note that from (12) and (13) the relative price indexes in the two countries satisfy

$$\left(\frac{P_F}{P_H}\right)^{\delta} = \frac{z_F - \tau^{1-\sigma} z_H}{z_H - \tau^{1-\sigma} z_F}.$$

Therefore $P_F > P_H$ when $z_H < z_F$ and $P_F < P_H$ when $z_H > z_F$. In other words, the competitive pressure—as measured by the price index—is higher in the country in which single-product firms are more competitive. Next note that if the price indexes were the same in both countries, we would have $\pi_{H,i} > \pi_{F,i}$ due to the positive trade costs. Furthermore, (15) implies that a higher price index in country J raises operating profits $\pi_{J,i}$. Therefore, whenever $z_H > z_F$ (which implies $P_H > P_F$), profits of multi-product firms are larger in the home country, because the demand level is higher there and it is cheaper to serve this market. If instead $z_H < z_F$ (which implies $P_F > P_H$), profits can be larger at home or abroad, because it is cheaper to serve market H while the demand is higher in market F. Which of these two effects dominates depends on the ratio $z_H/z_F < 1$ and the size of the trade costs $\tau > 1$. If the trade costs are very small (i.e., τ is close to one) and z_H/z_F is small too, foreign profits are larger. For τ very large, foreign profits have to be smaller. Using continuity of the profit functions, this implies that for $z_H < z_F$ there exists a value of τ, say $\tau_{c,i}$, such that for $\tau = \tau_{c,i}$ profits of firm i are the same in both countries. For higher trade costs profits are higher in H and for lower trade costs profits are higher in F.

I next prove Proposition 1. Consider the impact of z_F on the joint profits $\pi_i(n_i, P_F, P_H) = \sum_{J=F,H} \pi_{J,i}(n_i, P_J)$. In response to a change in z_F, holding z_H constant, Lemma 2 yields

$$\delta \hat{P}_F = \frac{z_F}{z_F - \tau^{1-\sigma} z_H} \hat{z}_F, \; \delta \hat{P}_H = -\frac{\tau^{1-\sigma} z_F}{z_F - \tau^{1-\sigma} z_H} \hat{z}_F.$$

From (14) and (15) I obtain

$$\hat{\pi}_i \cdot \pi_i = \sum_{J=F,H} \pi_{J,i} \hat{\pi}_{J,i} = \sum_{J=F,H} \pi_{J,i} \left[1 - s_{J,i} \frac{(\sigma-1)\beta_{J,i}}{1+(\sigma-1)\beta_{J,i}}\right] \delta \hat{P}_J,$$

and therefore

$$\hat{\pi}_i \cdot \pi_i \cdot \frac{z_F - \tau^{1-\sigma} z_H}{z_F} = \left\{\pi_{F,i}\left[1 - s_{F,i}\frac{(\sigma-1)\beta_{F,i}}{1+(\sigma-1)\beta_{F,i}}\right] - \tau^{1-\sigma}\pi_{H,i}\left[1 - s_{H,i}\frac{(\sigma-1)\beta_{H,i}}{1+(\sigma-1)\beta_{JH,i}}\right]\right\}\hat{z}_F.$$

Evidently, an improvement in competitiveness of foreign single-product firms, $\hat{z}_F < 0$, raises overall profits if and only if the expression in the curly brackets is negative; that is, if and only if

$$\tau^{1-\sigma}\pi_{H,i}\left[1 - s_{H,i}\frac{(\sigma-1)\beta_{H,i}}{1+(\sigma-1)\beta_{H,i}}\right] > \pi_{F,i}\left[1 - s_{F,i}\frac{(\sigma-1)\beta_{F,i}}{1+(\sigma-1)\beta_{F,i}}\right]. \tag{29}$$

Using the definition of $\beta_{J,i}$ above,

$$1 - s_{J,i}\frac{(\sigma-1)\beta_{J,i}}{1+(\sigma-1)\beta_{J,i}} = \frac{(\sigma-\delta s_{J,i}-1)(\sigma-\delta s_{J,i})+(\sigma-1)\delta\left(1-s_{J,i}\right)s_{J,i}}{(\sigma-\delta s_{J,i}-1)(\sigma-\delta s_{J,i})+(\sigma-1)\delta s_{J,i}}.$$

Therefore, with the aid of (7) and (9), inequality (29) is satisfied if and only if

$$P_H^{\delta}\Phi(s_{H,i}) > P_F^{\delta}\Phi(s_{F,i}),$$

which is (16) in the main text.

Next, decomposing the derivative of $\log\Phi(s)$ I obtain:

$$\begin{aligned}\delta^{-1}\cdot[\log\Phi(s)]' = \\ & -\frac{s\delta}{(\sigma-s\delta)(\sigma-s\delta-1))} \\ & -\frac{2(\sigma-1-\delta)s}{(\sigma-\delta s-1)(\sigma-\delta s)+(\sigma-1)(1-s)\delta s} \\ & -\frac{2\delta s}{(\sigma-\delta s-1)(\sigma-\delta s)+(\sigma-1)\delta s} \\ & -\frac{\sigma(\sigma-1)\delta s^2}{[(\sigma-\delta s-1)(\sigma-\delta s)+(\sigma-1)(1-s)\delta s][(\sigma-\delta s-1)(\sigma-\delta s)+(\sigma-1)\delta s]}.\end{aligned}$$

The first term on the right-hand side is negative, because $\sigma-1-\delta=\varepsilon-1>0$. The remaining three terms are also negative for this reason. It follows that $\Phi(s)$ is a declining function. This completes the proof of Proposition1.

Figure 4 depicts market shares as functions of b_i. The dashed curve portrays market shares in country F while the solid curves depict market shares in country H; an orange curve for $P_H=0.95$, a higher blue curve for $P_H=2$, and a lower green curve for $P_H=0.5$. In all cases market shares are lower for less productive firms or firms with fewer products. The downward slopes of these curves result from the fact that $s(\cdot)$ is a declining function.

Appendix for Section 6

In the examples with $P_H=0.95$ and $P_H=2$, discussed in the main text, import competition raises profitability of R&D investment and reduces short-run operating profits of highly-productive firms. This, however, is not always the case. When P_H is low enough, highly-productive firms gain higher operating profits from a decline in z_F in the short run, yet the marginal values of their product spans decline, as illustrated in Figure 5 for $(P_F,P_H)=(1,0.5)$. In contrast, short-term changes in operating profits of low-productivity firms and changes in the profitability of their investment in

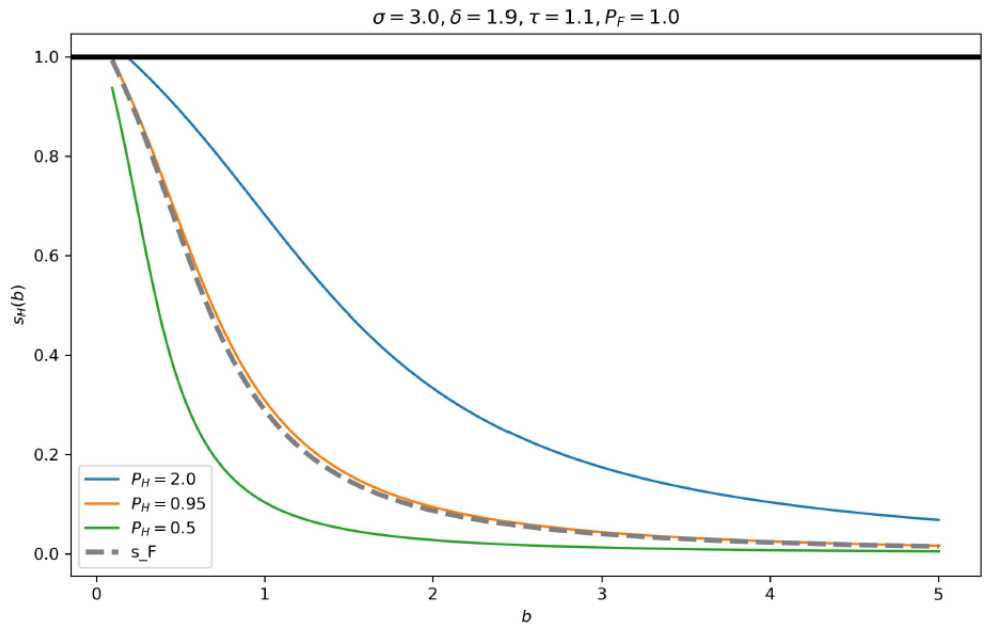

Figure 4: Market shares

innovation are negatively correlated; while operating profits decline, the marginal values of product spans rise.

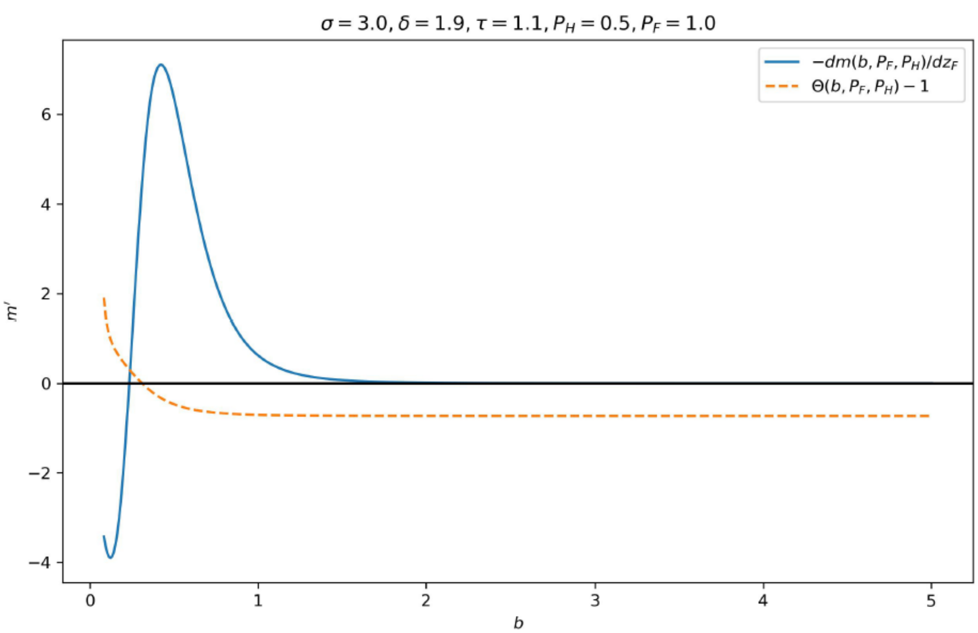

Figure 5: $-dm\,(b, P_F, P_H)\,/dz_F$ and $\Theta\,(b, P_F, P_H) - 1$ for $P_H = 0.5$