Lecture 3   12 Sep 07

Last time iterative deletion of dominated strategies

Today an application
model of politics
2 candidates <<players>>
choose positions on political spectrum

1	2	3	4	5	6	7	8	9	10

10% votes at each position
voters vote for closest candidate
if tie, split $\frac{1}{2}$ $\frac{1}{2}$

payoffs candidates aim to maximize share of vote

2 dominates 1?

test does 2 dominate 1?

vs 1 $u_1(1,1) = 50\% < u_1(2,1) = 90\%$ ✓
vs 2 $u_1(1,2) = 10\% < u_1(2,2) = 50\%$ ✓
vs 3 $u_1(1,3) = 15\% < u_1(2,3) = 20\%$ ✓
vs 4 $u_1(1,4) = 20\% < u_1(2,4) = 25\%$ ✓
⋮

Conclude 2 strictly dominates 1
9 strictly dominates 10 <<same argument>>

What about 2: is it dominated by 3? ✗ No

vs 1 $u_1(2,1) = 90\% > u_1(3,1) = 85\%$ ✗

But if we delete strategies 1 & 10, then does 3 dominate 2?

vs 2 $u_1(2,2) = 50\% < u_1(3,2) = 80\%$ ✓
vs 3 $u_1(2,3) = 20\% < u_1(3,3) = 50\%$ ✓
vs 4 $u_1(2,4) = 25\% < u_1(3,4) = 30\%$ ✓
vs 5 $u_1(2,5) = 30\% < u_1(3,5) = 35\%$ ✓
⋮

2 and 9 are not dominated,
but they are dominated once we realize 1 & 10 won't be chosen

x	xx	xxx	xxxx			xxxx	xxx	xx	x
1	2	3	4	5	6	7	8	9	10

Prediction: candidates around the center
$\boxed{\text{Median Voter Theorem}}$

Downs 1957 <<political science>>
Hotelling 1929 <<economics>>

Missing

✓ • voters not evenly distributed
problem set • many candidates / not voting
do later • position not believed (commit to policy)
• primaries
• high dimensions
→ <<take in advanced poly sci courses>>

Different Approach

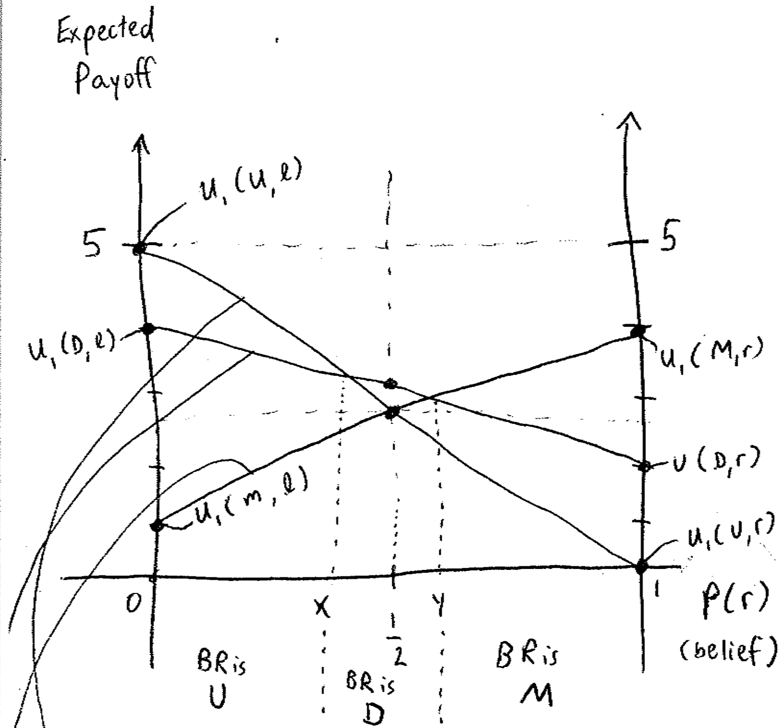
1 \ 2	l	r
U	5,1	0,2
M	1,3	4,1
R	4,2	2,3

Best Response

<< Nothing dominated.
So I can't stop at teaching dominated strategies. >

U does best against l
M does best against r

Expected Payoff of U vs $(\frac{1}{2},\frac{1}{2}) = (\frac{1}{2})(5) + (\frac{1}{2})0 = 2\frac{1}{2}$
Expected Payoff of M vs $(\frac{1}{2},\frac{1}{2}) = (\frac{1}{2})(4) + (\frac{1}{2})1 = 2\frac{1}{2}$
Expected Payoff of D vs $(\frac{1}{2},\frac{1}{2}) = (\frac{1}{2})4 + (\frac{1}{2})2 = 3$



$E u_1 (U, p(r)) = (1-p(r))[5] + (p(r))[0]$

$E U_1 (D, p(r)) = (1-p(r))[4] + (p(r))[2]$

$E U_1 (M, p(r)) = (1-p(r))[1] + (p(r))[4]$

$X = \frac{1}{3}$ replace $p(r)$ with x, equate D, U