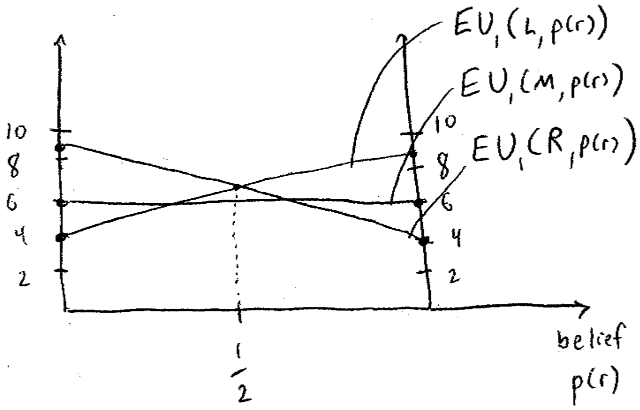
Lecture 4 17 Sep 07

Penalty Kick Game

Portsmouth v. Liverpool

	goalie l	r
L	4, -4	9, -9
M	6, -6	6, -6
R	9, -9	4, -4

$u_1(L,l) = 4$, ie 40% chance of scoring



M is not a BR to any belief

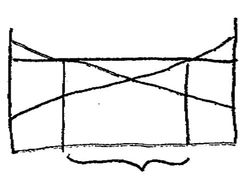
Lesson Do not shoot to middle (unless you are German)

Lesson Do not choose a strategy that is never a BR to any belief

real numbers

	"l"	"r"
"L"	63.6	94.4
"R"	89.3	43.7

"L" = natural



"no middle if you kick hard but not accurately"

(i) **Defn** Player i's strategy $\hat{s}_i$ is a $\boxed{BR}$ ← "best response" to the strategy s_{-i} of other players if

$$u_i(\hat{s}_i, s_{-i}) \geq u_i(s_i', s_{-i}) \text{ for all } s_i' \text{ is } S_i$$

or $\hat{s}_i$ solves $\max_{s_i} u_i(s_i, s_{-i})$

(ii) **Defn** Player i's strategy $\hat{s}_i$ is a $\boxed{BR}$ to the belief p about the others' choices if

$$E u_i(\hat{s}_i, p) \geq E u_i(s_i', p) \text{ for all } s_i' \text{ is } S_i$$

or $\hat{s}_i$ solves $\max_{s_i} E u_i(s_i, p)$

Example $EU_1(L, p) = p(l)\,u_1(L, l) + p(r)\,u_1(L, r)$

Partnership Game

- 2 agents own firm jointly, share 50% of profit each
- each agent chooses effort level to put into the firm

  $S_i = [0, 4]$ "Continuum of strategies"
- firm profit is given by $4[s_1 + s_2 + b s_1 s_2]$

  Complementarity / synergy

  $0 \leq b \leq \frac{1}{4}$
- payoffs $U_1(s_1, s_2) = \frac{1}{2}[4(s_1 + s_2 + b s_1 s_2)] - s_1^2$

  $U_2(s_1, s_2) = \frac{1}{2}[4(s_1 + s_2 + b s_1 s_2)] - s_2^2$

  effort cost

$$\max_{s_1} \; 2(s_1 + s_2 + b s_1 s_2) - s_1^2$$

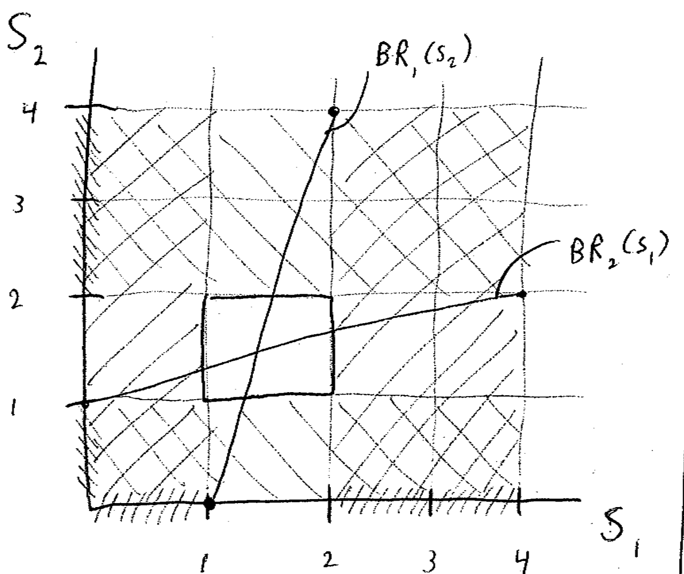
differentiate

f.o.c. $\boxed{2(1+bs_2) - 2\hat{s}_1 = 0}$

s.o.c. $-2 < 0$ ✓

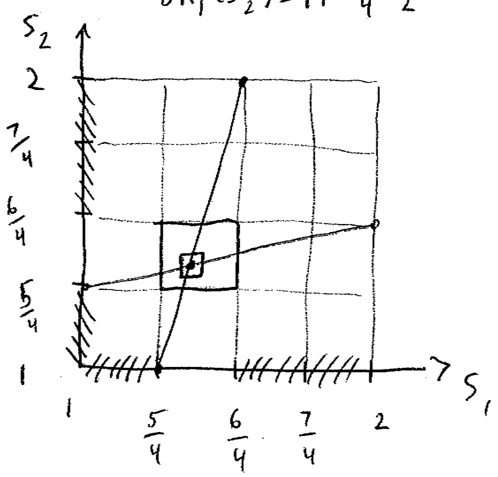
$\boxed{\hat{s}_1 = 1 + b s_2} = BR_1(s_2)$

similarly, $\boxed{\hat{s}_2 = 1 + b s_1} = BR_2(s_1)$



draw BR_1, BR_2 for the case $b = \frac{1}{4}$

$BR_1(s_2) = 1 + \frac{1}{4}s_2$

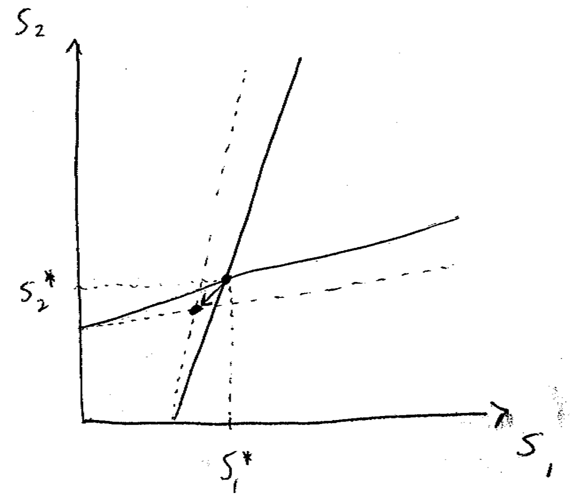


$$\left.\begin{array}{l} s_1^* = 1 + b s_2^* \\ s_2^* = 1 + b s_1^* \end{array}\right\} \longrightarrow s_1^* = s_2^*$$

$(1-b)s_1^* = 1 \quad \ldots \quad \boxed{s_1^* = s_2^* = \frac{1}{1-b}}$

<< inefficiently low effort, because at the margin I only capture $\frac{1}{2}$ the benefit I put in, but I absorb all the cost of the effort >>

EXTERNALITY



Nash Equilibrium << intersection of lines, (in this graph) >>

The players are playing a best response to each other

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