Lecture 9 — 3 Oct 07

Last time: new idea $\boxed{\text{MIXED STRATEGIES}}$

e.g. $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ in RPS

Defn
- A mixed strategy p_i is a randomization over i's pure strategies
- $p_i(s_i)$ is the probability that p_i assigns to pure strategy s_i
- $p_i(s_i)$ could be zero eg $(\frac{1}{2}, \frac{1}{2}, 0)$
- $p_i(s_i)$ could be one ie a pure strategy

Payoffs from Mixed Strategy

The expected payoff of the mixed strategy p_i is the weighted average of the expected payoffs of each of the pure strategies in the mix

eg

		2 a	b	
1	A	2,1	0,0	1/5
	B	0,0	1,2	4/5
		1/2	1/2	

Suppose $p = (\frac{1}{5}, \frac{4}{5})$
$q = (\frac{1}{2}, \frac{1}{2})$

what is p's expected payoff?

(1) Ask $EU_1(A,q) = [2](\frac{1}{2}) + [0](\frac{1}{2}) = 1$
$EU_1(B,q) = [0](\frac{1}{2}) + [1](\frac{1}{2}) = \frac{1}{2}$

(2) $EU_1(p,q) = (\frac{1}{5})EU_1(A,q) + (\frac{4}{5})EU_1(B,q)$
$= (\frac{1}{5})[1] + (\frac{4}{5})[\frac{1}{2}]$
$\boxed{= 3/5}$

Lesson If a mixed strategy is a BR, then each of the pure strategies in the mix must themselves be a BR.
In particular, each must yield the same expected payoff.

Defn A mixed strategy profile $(p_1^*, p_2^*, \dots, p_N^*)$ is a mixed strategy NE if for each player i, p_i^* is a BR to p_{-i}^*

« **Defn** A mixed strategy profile... »

lesson ⟹ If $p_i^*(s_i) > 0$ then s_i is also a BR_i to p_{-i}^*

Example Tennis — Venus and Serena Williams

		S (at net) l	r	
V (passing shot)	L	50,50	80,20	p
	R	90,10	20,80	1-p
		q	1-q	

There is no pure-strategy NE.
Let's find a mixed-strategy NE.

- **Trick** To find Serena's NE mix $(q, 1-q)$ look at Venus's payoffs

V's payoffs against q: $L \to [50]q + [80](1-q)$
$=$
$R \to [90]q + [20](1-q)$

If Venus is mixing in NE then the payoffs to L and R must be equal

$50q + 80(1-q) = 90q + 20(1-q)$
$60(1-q) = 40q$
$60 = 100q$
$\boxed{0.6 = q}$ ← Serena's mix

- To find Venus' NE mix, $(p, 1-p)$ use Serena's payoffs

S's payoffs: $l \to [50]p + [10](1-p)$
$=$
$r \to [20]p + [80](1-p)$

$30p = 70(1-p)$
$100p = 70$
$\boxed{p = 0.7}$ ← Venus' mix

$$NE = [\underset{L\ \ R}{\overset{V}{(.7, .3)}}, \underset{l\ \ r}{\overset{S}{(.6, .4)}}]$$

the changed box

S

V	l	r	
L	30, 70	80, 20	p
R	90, 10	20, 80	1-p
	q	1-q	

Two effects (1) Direct Effect Serena should lean l more | $q\uparrow$

(2) Strategic Effect Venus hits L less often, so Serena should $q\downarrow$ | $q\downarrow$

To find the new q for Serena, use Venus' payoffs

$$V: L \to [30]q + [80](1-q)$$
$$=$$
$$R \to [90]q + [20](1-q)$$

$$60q = 60(1-q)$$

$$\boxed{q = .5}$$ q went $\downarrow$

Strategic effect is bigger

$$S: l \to 70p + 10(1-p)$$
$$=$$
$$r \to 20p + 80(1-p)$$

$$50p = 70(1-p)$$

$$\boxed{p = 7/12} < \tfrac{7}{10}$$

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