Lecture 10 — 8 Oct 07

<< Williams Sister Tennis >>

		S: l	S: r	
V	L	50, 50	80, 20	p^*
	R	90, 10	20, 80	$1-p^*$
		q^*	$1-q^*$	

$(p^*, 1-p^*) = (.7, .3)$

$(q^*, 1-q^*) = (.6, .4)$

$\boxed{p^* = .7 \quad q^* = .6}$

Check p^* is a BR(q^*)

Venus' payoffs: $L \to 50(.6) + 80(.4) \to .62$

$R \to 90(.6) + 20(.4) \to .62$

Venus' payoffs from p^* $\to (.7)[.62] + (.3)[.62] \to = \boxed{.62}$

We can see that Venus has no strictly profitable pure-strategy deviation.

<< this implies there's no strictly profitable mixed-strategy deviation, either >>

Lesson: We only ever have to check for strictly profitable pure-strategy deviation

<< Dating >> "Battle of the Sexes"

		D: AP	D: REP	
N	AP	2, 1	0, 0	p
	REP	0, 0	1, 2	$(1-p)$
		q	$(1-q)$	

pure-strategy << (Nash Eq.) >> (AP, AP)
(REP, REP)

Find a mixed NE of this game ...

To find NE q, use Nina's payoffs

N: $AP \to 2q + 0(1-q)$, $REP \to 0q + 1(1-q)$ (equal) $\Rightarrow 2q = 1(1-q)$ $\boxed{q = \frac{1}{3}}$ $\boxed{(1-q) = \frac{2}{3}}$

To find NE p, use David's payoffs

D: $AP \to 1p + 0(1-p)$, $REP \to 0p + 2(1-p)$ (equal) $\Rightarrow 1p = 2(1-p)$ $\boxed{p = \frac{2}{3}}$ $\boxed{(1-p) = \frac{1}{3}}$

Check that $p = \frac{2}{3}$ is BR for Nina

N: $AP \to 2(\frac{1}{3}) + 0(\frac{2}{3})$, $REP \to 0(\frac{1}{3}) + 1(\frac{2}{3})$ $\Big\} = \frac{2}{3}$

$p \to \frac{2}{3}\left[\frac{2}{3}\right] + \frac{1}{3}\left[\frac{2}{3}\right] = \frac{2}{3}$

<< no strictly profitable pure deviation. $\Rightarrow$ no strictly profitable mixed deviation, either >>

NE $= \left[\left(\frac{2}{3}, \frac{1}{3}\right), \left(\frac{1}{3}, \frac{2}{3}\right) \right]$ (N: p, $1-p$; D: q, $1-q$) $\to$ payoffs N: $\frac{2}{3}$, D: $\frac{2}{3}$

<< payoffs are low because they fail to meet sometimes >>

Prob(meet) $= \frac{2}{3}\frac{1}{3} + \frac{1}{3}\frac{2}{3} = \boxed{\frac{4}{9}}$

<< meaning Prob(not meet) $= \frac{5}{9}$, over half the time! >>

<< Interpretations of mixing probabilities

1. People literally randomizing
2. Beliefs of others' actions (that make you indifferent between things you'd do)
3. ... "Proportions of Players" >>

		taxpayer: H	taxpayer: C	
Auditor	A	2, 0	4, -10	p
	N	4, 0	0, 4	$1-p$
		q	$1-q$	

<< No pure NE >>

Find (mixed) NE here $\left[\left(\frac{2}{7}, \frac{5}{7}\right), \left(\frac{2}{3}, \frac{1}{3}\right) \right]$ (A N; H C)

Aud: $A \to 2q + 4(1-q)$, $N \to 4q + 0(1-q)$ $\Rightarrow 2q = 4(1-q)$ $\boxed{q = \frac{2}{3}}$

Tax: $H \to 0$, $C \to -10p + 4(1-p)$ (equal) $\Rightarrow 4 = 14p$ $\boxed{p = \frac{2}{7}}$

Open Yale courses

<< So think of $\frac{2}{3}$ as proportion of people being honest on their taxes. >>

Policy Lets raise the fine to -20

$$
\begin{array}{cc|c|c|c}
 & & \multicolumn{2}{c}{T} & \\
 & & H & C & \\
\hline
\text{Aud} & A & 2,0 & 4,-20 & p \\
\hline
 & N & 4,0 & 0,4 & 1-p \\
\hline
 & & q & 1-q &
\end{array}
$$

What happens to tax compliance q?

$$\text{Aud} \quad \left.\begin{array}{l} A \rightarrow 2q + 4(1-q) \\ \qquad = \\ N \rightarrow 4q + 0(1-q) \end{array}\right\} \quad \boxed{q = \frac{2}{3}}$$

$$T \quad \left.\begin{array}{l} H \rightarrow 0 \\ \qquad = \\ C \rightarrow -20p + 4(1-p) \end{array}\right\} \quad \begin{array}{l} 24p = 4 \\ \boxed{p = \frac{1}{6}} < \frac{2}{7} \end{array}$$

<< In equilibrium, rich will be audited more >>
(but will cheat with same [equilibrium] rate)

<< To get higher compliance rate:
- Change payoffs to auditor
  - make it less costly to do an audit
  - give a bigger gain for catching a cheater
- or set audit rates higher, by Congress
  - but Congressmen are wealthy and may have a conflict of interest >>

Lesson 1: Can interpret proportions of people playing

Lesson 2: Check only for pure deviations

Lesson 3: Row v. column payoffs + incentives

Next time: evolution...