Lecture 11 10 Oct 07

Evolution and Game Theory

(1) Influence of GT on bio animal behavior

strategies ⟷ genes
payoffs ⟷ genetic fitness

« good strategies "grow"
but the strategies are not chosen »
hard-wired

(2) Influence from bio ⟶ social sciences

« firms with rules of thumb decisions, and markets selecting/surviving the fittest »

Simplified Model

- within species competition
- symmetric 2-player games
- large pop, random matching – avg payoffs
- relatively successful strategies grow

no gene redistribution
asexual reproduction

	C	D
C coop	2, 2	0, 3
D defect	3, 0	1, 1

$1-\varepsilon$ ε (for C being majority)
ε $1-\varepsilon$ (for D being majority)

e.g. lions on a hunt
ants defending a nest

Is cooperation Evolutionarily Stable?

C vs $[(1-\varepsilon)C + \varepsilon D] \to (1-\varepsilon)[2] + \varepsilon[0] = 2(1-\varepsilon)$

$\wedge$

D vs $[(1-\varepsilon)C + \varepsilon D] \to (1-\varepsilon)[3] + \varepsilon[1] = 3(1-\varepsilon) + \varepsilon$

so conclude C is not ES (evolutionarily stable)

Is D ES?

D vs $[(1-\varepsilon)D + \varepsilon C] \to (1-\varepsilon)[1] + \varepsilon[3] = (1-\varepsilon) + 3\varepsilon$

$\vee$

C vs $[(1-\varepsilon)D + \varepsilon D] \to (1-\varepsilon)[0] + \varepsilon[2] = 2\varepsilon$

D is ES (any mutation from D gets wiped out)

Lesson (1) Nature can suck

« sexual reproduction can change this »

(2) If a strategy is strictly dominated then it is not ES.

« the strictly dominant strategy will be a successful mutation »

	a	b	c
a	2, 2		
b			1, 1
c		1, 1	

Is c ES? – No

c vs $[(1-\varepsilon)c + \varepsilon b] \to (1-\varepsilon)[0] + \varepsilon[1] = \varepsilon$

$\wedge$

b vs $[(1-\varepsilon)c + \varepsilon b] \to (1-\varepsilon)[1] + \varepsilon[0] = 1-\varepsilon$ (b/c ε small)

« b will grow from small proportion (ε) to $\frac{1}{2}$ »

- Note: b, the invader, is itself not ES
  « but it still avoids dying out »

Is c a NE?
No, because b is a profitable deviation

Lesson If s is not Nash, (s,s) is not NE, Then s is not ES.

⇕

If s is ES ⇒ (s,s) is NE

Open Yale courses

	a	b
a	1, 1	0, 0
b	0, 0	0, 0
	ε	$1-\varepsilon$

$NE = (a,a)$, (b,b)

Is b ES?

$b \rightsquigarrow [0] \qquad\qquad = 0$

$a \rightsquigarrow (1-\varepsilon)[0] + \varepsilon[1] = \varepsilon$

<< so b,b was Nash, but was not ES >>

<< reason is because b is a weak Nash >>

If (s,s) is a "strict" NE,
then s is ES

BIO

1) **Formal Definition** (BIO - Maynard Smith 1972)

In a symmetric, 2 player game,
the pure strategy $\hat{s}$ is ES (in pure strategies) if

there exists an $\bar{\varepsilon} > 0$

$$(1-\varepsilon)[u(\hat{s},\hat{s})] + \varepsilon[u(\hat{s},s')] > (1-\varepsilon)u(s',\hat{s}) + \varepsilon u(s',s')$$

for all possible deviations s'
and for all mutation sizes $\varepsilon < \bar{\varepsilon}$

<< payoff to ES $\hat{s}$ > payoff to mutant >>
"for all small mutations"

ECON

② In a symmetric, 2 player game,
A strategy $\hat{s}$ is ES (in pure strategies) if

(a) $(\hat{s},\hat{s})$ is a (symmetric) NE
ie $u(\hat{s},\hat{s}) \geq u(s',\hat{s})$ for all s'

AND (b) if $u(\hat{s},\hat{s}) = u(s',\hat{s})$
then $u(\hat{s},s') > u(s',s')$] "It better beat up on the mutant"

"you're better against the [illegible] than it is against itself"

BIO ① ⟺ ② ECON

Fix a $\hat{s}$ and suppose $(\hat{s},\hat{s})$ is NE
↳ ie $u(\hat{s},\hat{s}) \geq u(\hat{s},s')$ for all s'

Two cases

(a) $u(\hat{s},\hat{s}) > u(\hat{s},s')$ for all s'

the mutant dies out because she meets $\hat{s}$ oft.

(b) $u(\hat{s},\hat{s}) = u(s',\hat{s})$ but
$u(\hat{s},s') > u(s',s')$
the mutant does "okay" against $\hat{s}$ (the masses)
but badly against s' (itself)

ie:
<< (a) The mutant does poorly against the masses
(b) The mutant does equally against the masses
but gets clobbered against itself >>