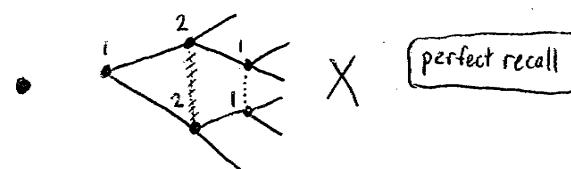
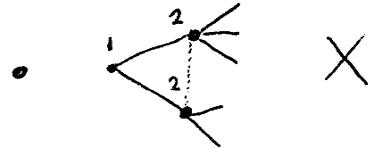


Formal Defn An information set of player  $i$  is a collection of player  $i$ 's nodes among which  $i$  cannot distinguish.

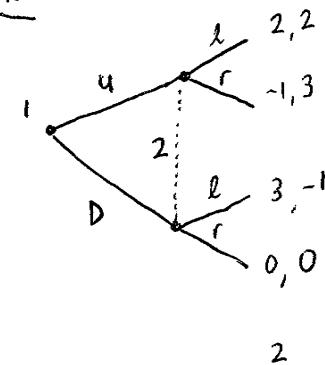
Rules not allowed



Defn Perfect Information: all information sets in the tree have just one node

ImPerfect Information: NOT perfect information

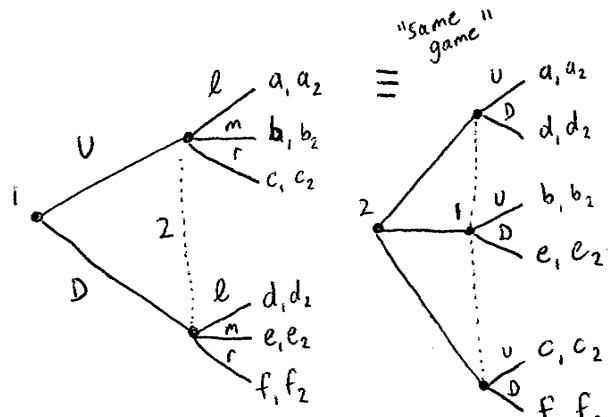
Example



	$l$	$r$
U	2, 2	-1, 3
D	3, -1	0, 0

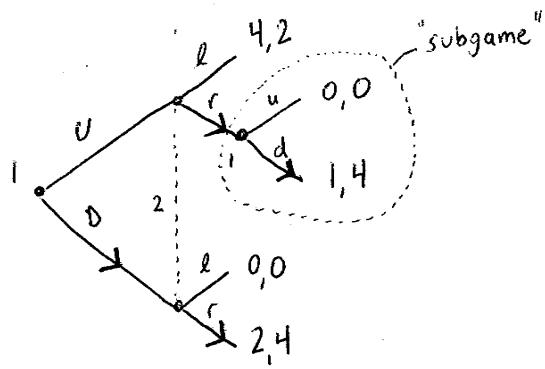
Prisoner's Dilemma

Defn A pure-strategy of player  $i$  is a complete plan of action: it specifies what player  $i$  will do at each of its information sets



	$l$	$m$	$r$
U	a <sub>1</sub> , a <sub>2</sub>	b <sub>1</sub> , b <sub>2</sub>	c <sub>1</sub> , c <sub>2</sub>
D	d <sub>1</sub> , d <sub>2</sub>	e <sub>1</sub> , e <sub>2</sub>	f <sub>1</sub> , f <sub>2</sub>

what matters is information, not time



strategies for 1:  $U_u, U_d, D_u, D_d$

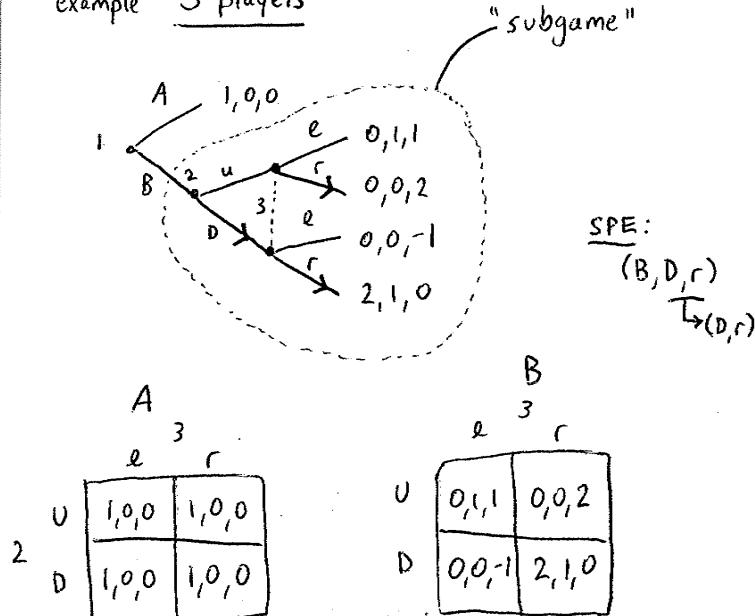
strategies for 2:  $l, r$  redundant

		2					
		l	r				
1	$U_u$	<table border="1"> <tr><td>4, 2</td><td>0, 0</td></tr> <tr><td>1, 4</td><td>0, 0</td></tr> </table>	4, 2	0, 0	1, 4	0, 0	$\overline{NE: (U_u, l)}$
4, 2	0, 0						
1, 4	0, 0						
$U_d$	<table border="1"> <tr><td>4, 2</td><td>1, 4</td></tr> <tr><td>1, 4</td><td>0, 0</td></tr> </table>	4, 2	1, 4	1, 4	0, 0	$\overline{(D_u, r)}$	
4, 2	1, 4						
1, 4	0, 0						
$D_u$	<table border="1"> <tr><td>0, 0</td><td>2, 4</td></tr> <tr><td>1, 4</td><td>0, 0</td></tr> </table>	0, 0	2, 4	1, 4	0, 0	$\overline{(D_d, l)}$	
0, 0	2, 4						
1, 4	0, 0						
$D_d$	<table border="1"> <tr><td>0, 0</td><td>2, 4</td></tr> <tr><td>1, 4</td><td>0, 0</td></tr> </table>	0, 0	2, 4	1, 4	0, 0	$\overline{(D_d, r)}$	
0, 0	2, 4						
1, 4	0, 0						

$$\begin{aligned} \overline{NE: (U_u, l)} & \quad \overline{(D_u, r)} \\ & \quad \overline{(D_d, l)} \\ & \quad \overline{(D_d, r)} \end{aligned}$$

SPE

example 3 players



SPE:  
 $(B, D, r)$   
 $\xrightarrow{(D, r)}$

		B					
		l	r				
2	A	<table border="1"> <tr><td>1, 0, 0</td><td>1, 0, 0</td></tr> <tr><td>1, 0, 0</td><td>1, 0, 0</td></tr> </table>	1, 0, 0	1, 0, 0	1, 0, 0	1, 0, 0	$\xrightarrow{(D, r)}$
1, 0, 0	1, 0, 0						
1, 0, 0	1, 0, 0						
B	<table border="1"> <tr><td>0, 1, 1</td><td>0, 0, 2</td></tr> <tr><td>0, 0, -1</td><td>2, 1, 0</td></tr> </table>	0, 1, 1	0, 0, 2	0, 0, -1	2, 1, 0		
0, 1, 1	0, 0, 2						
0, 0, -1	2, 1, 0						

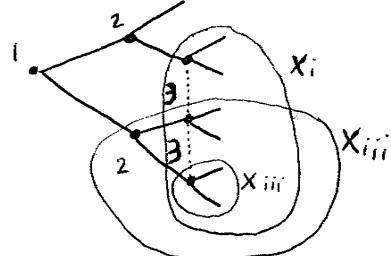
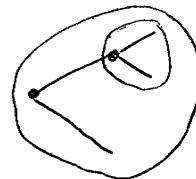
lots of NE: eg [A, u, l]

look at the green subgame:

matrix B ...

Defn A subgame is a part of the game that looks like a game within the tree. it satisfies:

- it starts from a single node
- it comprises all successors to that node
- it does not break up any information sets



Defn A NE  $(s_1^*, s_2^*, \dots, s_N^*)$  is a subgame perfect equilibrium ("SPE") if it induces a NE in every subgame of the game