Lecture 22 28 Nov 07

last time repeated interaction

need: gain if cheat today $\leq$ [value of relationship after cooperation] $-$ [value of relationship after cheating] (tomorrow)

promise | threat

Credibility : focus of SPE

Prisoner's dilemma repeated with prob δ of continuing

	C	D
C	2, 2	-1, 3
D	3, -1	0, 0

grim trigger : play C, then

play { C if no one has ever defected; D otherwise }

temptation today $\overset{?}{\leq}$ [(value of promise) $-$ (value of threat)] tomorrow

weight by δ

3 − 2

1

(value of (C,C) forever) $-$ (value of (D,D) forever)

(value of 2 forever) $-$ (value of 0 forever)

$2 + \delta 2 + \delta^2 2 + \delta^3 2 + \ldots = X$

$-\ 2\delta + \delta^2 2 + \delta^3 2 + \delta^4 2 + \ldots = \delta X$

$X - \delta X = 2$

0

$X = \frac{2}{1-\delta}$

<< Is grim trigger an equilibrium [when both play it] ? >>

need: $1 \overset{?}{\leq} \left[\frac{2}{1-\delta} - 0\right]\delta$

$\Leftrightarrow 1 - \delta \overset{?}{\leq} 2\delta$

$\Leftrightarrow \boxed{\delta \geq \frac{1}{3}}$

- How about playing D now, then C, then D forever ?

  → (D,C), (C,D), (D,D), (D,D) → $3 + \delta(-1) + 0 + 0 \ldots$

  $= 3 - \delta$

  this defection is even worse (than the previous defection of D, D, D, ...)

  punishment (D,D) forever is a SPE

- How about cheating, not in the first period but in the second ?

  the same analysis says this is not profitable if $\delta \geq \frac{1}{3}$

Lesson we can get cooperation in PD (prisoners' dilemna) using Grim Trigger (as a SPE) provided $\delta \geq \frac{1}{3}$

Lesson For an ongoing relationship to provide incentives for good behavior today, it helps for there to be a high probability that the relationship will continue.

weight you put on the future

<< what about a less draconian strategy ? >>

one-period punishment ...

one period punishment

play C to start, then

play { C if either (C,C) or (D,D) were played last
     { D if either (C,D) or (D,C) were played last

is this an SPE?

temptation (today) $\overset{?}{\leq}$ [(value of promise) − (value of the threat)] (tomorrow)

$3-2 \overset{?}{\leq} \left[\left(\text{value of } \frac{2}{\text{"forever"}}\right) - \left\{\begin{array}{l}\text{value of 0 tomorrow}\\ \text{then 2 forever}\\ \text{starting the}\\ \text{next day}\end{array}\right\}\right] \quad \delta$

$1 \overset{?}{\leq} \left[\left(\frac{2}{1-\delta}\right) - \delta\left(\frac{2}{1-\delta}\right)\right] \quad \delta$

$\Leftrightarrow \quad 1 \leq \frac{2\delta}{1-\delta}[1-\delta]$

$\Leftrightarrow \quad \frac{1}{2} \leq \delta$

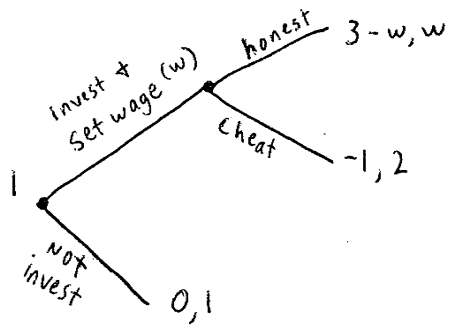
Trade off

Shorter punishments need more weight (δ) on future

<< Example to show repeated interaction works >>

Repeated Moral Hazard

\+ labor cheap
− contracts hard to enforce



if set w=1 (the going wage in Fredonia)
then the agent will cheat

to make him be honest,
need $w \geq 2$ — incentive design

In equilibrium, $w^* = 2$, the agent works

Wage premium in this emerging market is 100%

- Consider repeated interaction with prob δ

what wage (w^{**}) will you pay?

temptation to cheat today $\leq \delta$ (tomorrow) [(value of continuing the relationship) − {value of ending the relationship}]

"Continuing" "firing"

$2 - w^{**} \leq \left[\left(\text{value of } \frac{w^{**}}{\text{forever}}\right) - \left(\text{value of } \frac{1}{\text{forever}}\right)\right]\delta$

$2 - w^{**} \leq \left[\frac{w^{**}}{1-\delta} - \frac{1}{1-\delta}\right]\delta$

$(1-\delta)2 - (1-\delta)w^{**} \leq w^{**}\delta - [1]\delta$

$(1-\delta)2 + \delta[1] \leq w^{**}$

<< or: $2 - \delta \leq w^{**}$ >>

if $\delta = 0$, $w^{**} = 2$ one-shot wage
if $\delta = 1$, $w^{**} = 1$ going wage
if $\delta = \frac{1}{2}$, $w^{**} = 1\frac{1}{2}$ wage premium is now only 50%

<< to get good behavior, must be a reward >>

<< size of reward related to prob. of future >>