US-China two-country stock flow consistent model



- First, we present a model of an open economy under a regime of fixed exchange rates, with no private international capital flows. It can be interpreted as a small open economy.
- In addition, this model will show that reductions or increases in foreign exchange reserves, as a result of foreign exchange interventions by the central bank to keep the exchange rate fixed, have no effect on the money supply. In other words, foreign exchange interventions by central banks are "automatically" sterilize.
- The way in which this model works will prepare the ground for a more complex model, presented in the second part, in which the exchange rate is determined by demand and supply for internationally tradable financial assets and in which changes in the exchange rate feed back to help determine trade and all other flows as sequences in real time.
- It will also be shown how this more complex model can be extended to a fixed exchange regime.

Transaction matrix in US-China two-country economy



	US					China				
	1.HH\$	2.Frm\$	3.CB\$	4.Gvt\$	xr\$	5.HH¥	6.Frm¥	7.CB ¥	8.Gvt¥	Sum
1. Consumption	-C\$	+C\$				-C¥	+ C ¥			0
2. Government expenditure		+ G \$		-G\$			+G¥		-G¥	0
3. Exports/imports		+X\$			xr\$		-IM¥			0
4. Imports/exports		-IM\$			xr\$		+ X ¥			0
5. Output/income	+Y\$	-Y\$				+ Y ¥	-Y¥			0
6. Taxes	-T\$			+T\$		-T¥			+T¥	0
7. Δ Money	-ΔΗ\$		+H\$			-ΔH¥		+∆H¥		0
8. Δ T-bills	-ΔB\$\$		-ΔBcb\$	+ΔB\$	xr\$	-ΔB¥\$		-ΔBcb¥\$		0
9. Δ T-bills	-ΔB\$¥				xr\$	-ΔB ¥¥		-ΔBcb¥	+ΔB¥	0
Sum	0	0	0	0		0	0	0	0	0

The GDP identity:

$$Yu = Cu + Gu + Xu - IMu$$
 (1)

$$\triangleright Yc = Cc + Gc + Xc - IMc$$
 (2)



The wealth identity and tax:

$$> Tu = \theta u Yu$$
 (5)

$$> Tc = \theta c Yc$$
 (6)

With the two countries forming a single system, exports now become endogenous. Exports by each country are thus equal to imports by the other, converted to a common rate of exchange. Imports are determined in each country by the relevant income and price elasticities, with lowercase boldface letters denoting logs.

$$> Xu = IMc/xru$$
 (7)

$$> Xc = IMuxru$$
 (8)

$$\triangleright imu = \mu_{0u} + \mu_{1u}yu + \mu_{2u}xru$$
 (9)

$$\triangleright imc = \mu_{0c} + \mu_{1c}yc + \mu_{2c}xrc$$
 (10)



The consumption and the supply of Treasury bills:

$$> Cu = \alpha_{1u} Yu(1 - \theta u) + \alpha_{2u} Vu_{-1}$$
 (11)

$$\triangleright \Delta B c_s = G c - T c$$
 (14)

The array of asset demands for u residents, where all asset demands are valued in u currency:

$$> \frac{Buu_d}{vu} = \lambda_{10u} + \lambda_{11u}.ru - \lambda_{12u}.rc$$
 (15)

$$> \frac{Buc_d}{Vu} = \lambda_{20u} + \lambda_{21u}.ru - \lambda_{22u}.rc$$
 (16)

$$\geq \frac{Hu_d}{Vu} = \lambda_{30u} - \lambda_{31u}.ru - \lambda_{32u}.rc \quad (17B)$$



The array of asset demands for c residents, where all asset demands are valued in c currency:

$$\geq \frac{Bcc_d}{Vc} = \lambda_{10c} - \lambda_{11c}ru + \lambda_{12c}rc$$
 (18)

$$\geq \frac{B \operatorname{cu}_d}{V \operatorname{c}} = \lambda_{20 \operatorname{c}} + \lambda_{21 \operatorname{c}} r \operatorname{u} - \lambda_{22 \operatorname{c}} r \operatorname{c}$$
 (19)

$$\geq \frac{Hc_d}{Vc} = \lambda_{30c} - \lambda_{31c}ru - \lambda_{32c}rc$$
 (20*B*)

Tobin adding-up constraints apply once again. To obtain solutions to the whole model, Equations (17B) and (20B) are dropped, and the demand for cash is written as:

$$> H \mathbf{u}_d = V \mathbf{u} - B \mathbf{u} \mathbf{u}_d - B \mathbf{u} \mathbf{c}_d$$
 (17)



In order to keep interest rates fixed, the central bank must exchange bills for cash, and vice versa, on demand, making the supply of both cash and bills endogenous:

$$> H \mathbf{u}_s = H \mathbf{u}_d \quad (21)$$

$$Hc_s = Hc_d (22)$$

$$\triangleright$$
 Buu_s = Buu_d (23R_flexible)

$$\triangleright Bcc_s = Bcc_d$$
 (24)

Hence, the supply of domestic bills to their own CB is also endogenous:

$$\triangleright Bcbu_s = Bcbu_d$$
 (25)

$$\triangleright Bcbc_s = Bcbc_d$$
 (26R_fixed)



We further recall that the u currency is the international currency, so that the u central bank does not hold any foreign reserves, while the c country is on a pure flexible exchange rate regime and does not intervene in exchange markets, which implies that the c central bank does not acquire new reserves $\Delta Bcbcus = 0$, so that Bcbcu, is a historically given constant. This implies that changes in central banks' stocks of domestic Treasury bills are equal to changes in the liabilities of each central bank:

$$\triangleright \Delta B c b u_d = \Delta H u_s$$
 (27)

$$\triangleright \Delta Bcbc_d = \Delta Hc_s$$
 (28 * flexible)

All bill supplies must go somewhere as can be seen from the balance sheet identity (lines 8 and 9 of Table). Treasury bills (Bus) issued by the U.S. government can be held by c foreign residents (Bcus), U.S. residents (Buus), the u central bank (Bcbus), and the c central bank (Bcbcus).

Similarly, Treasury bills issued by the c government can be held by foreign u residents, c domestic residents, or the c central bank.



As all supplies of assets to domestic residents have been demand determined in Equations (21)-(26), the supply of assets abroad must, in each case, equal the gap between total supplies and supplies that meet domestic demand.

$$\triangleright Bcu_S = Bu_S - Buu_S - Bcbcu_S - Bcbu_S$$
 (29B * flexible)

$$\triangleright Buc_s = Bc_s - Bcc_s - Bcbcs$$
 (30 * flexible)

But now we have a sharp confrontation. Demand in each country for assets issued abroad, **denominated in the currency of the country where they are held**, has been determined in Equations (16) and (19). At the same time, supplies of assets that must be sold abroad, **denominated in the currency of the country where they have been issued**, have been determined in Equations (29) and (30). The exchange rate must be such that it equalizes the demand and supply for internationally traded assets that now confront one another in each country. That is, it must simultaneously be the case that:

- $\rightarrow xru = Bcu_d/Bcu_s(31B)$
- $> xru = Buc_s/Buc_d$ (32 * flexible)



For both of these conditions to be met, rather more has to happen than is immediately obvious. When the model comes to be solved, the exchange rate, must satisfy, and be satisfied by, not only the asset demand/supply equivalences but every other equation in which it (the exchange rate) appears.

The whole process is further complicated because the response of the trade variables (Equations (9) and (10)) to changes in the exchange rate will normally be completely different as between the two countries.

The two countries will also exhibit different responses of consumption as a result of capital gains, which may now be identified as the change in the value of the opening stock of foreign issued bills due to a change in the exchange rate within the period.

$$\triangleright CGu = \Delta xrcBuc_{s-1}$$
 (33)

$$\triangleright CGc = \Delta x r u B c u_{s-1}$$
 (34)



To check that we have enough equations to determine a single exchange rate that is capable of doing all the work that is required of it, we must write out the model with each variable appearing not more than once on the left-hand side of an equation.

First note that one of the two Equations (31B) and (32) must be modified, because we cannot let the exchange rate, xru, appear on the left hand side of two equations. We shall retain Equation (32) and write Equation (31B) as

$$\triangleright B cu_s = B cu_d / xru(31)$$

But now we have two equations with Bcus on the left-hand side, Equations (31) and (29B). Hence, we rewrite Equation (29B) as

$$\triangleright Buu_s = Bu_s - Bcu_s - Bcbcu_s - Bcbu_s$$
 (29)

And close the model by recalling that

$$\rightarrow xrc = 1/xru$$
 (35)



We now have an equation in every endogenous variable, save the two interest rates that remain exogenous.

We still have two equations with Buus on the left-hand side (Equations (23R) and (29)), so we shall drop Equation (23R).

It would then seem that Buus (Equation (29)) and Buud (Equation (15)) are independent of each other. However, since the accounting of the whole system is comprehensive, the system guarantees that Buus = Buud, which is Equation (23R).

This is the "redundant" equation, which must be dropped. The two terms of this equation are equivalent without the need for any equation to make that happen-so long as every other equation is satisfied.



The model is now complete. Besides the stock of foreign reserves, $Bcbcu_s$, held by the c central bank, the exogenous variables are G, theta, and r (for each country).

Output in each country together with consumption, imports, exports, wealth, and its allocation between the available assets and the exchange rate are all endogenously determined.

When the exchange rate changes, this changes the import propensity, disposable income, and hence output in each country-and hence (all still within one period) the budget deficit/surplus and changed supplies of assets, hence back to the exchange rate, and so on.

And having reached a kind of temporary equilibrium in each short period, the imaginary economies evolve further in sequences through time on their way toward a full steady state.



- Imagine that the whole system is in a full stationary steady state with no change taking place in any stock or any flow.
- First assume that there is an increase, which persists, in Gu, government expenditure in the u country, and next trace through the consequences.
- Output by the u country rises. As a result, imports IMu by the u country rise, and the u balance of trade becomes negative.
- There is a small increase in the output of the c country because its exports have increased.
- Taxes Tu increase but less than Gu, so the u government budget goes into deficit.
- This means that there has to be an increase in the outstanding stock of bills Bus issued by the u Treasury.



- Because Buud/Vu and Hud/Vu are fixed (there being no change in interest rates by assumption), and because the one-period change in Vu is small, there **has to be an increase in Bcus**, the amount of u Treasury bills that are supplied abroad.
- But a similar situation is occurring in the c country. Because interest rates are fixed, Bcud/Vc and Bccd/V# are also both fixed. In addition, because the one-period change in Vc is small, the demand by c households for Treasury bills issued by the u government, **Bcud**, **hardly changes**.
- The increase Bcus must thus be absorbed through a **depreciation** in the dollar exchange rate (xru).
- Next, the change in the exchange rate feeds into both import functions, reducing the import propensity in the u country and raising it in the c country, thus eventually generating balanced trade in the u country.

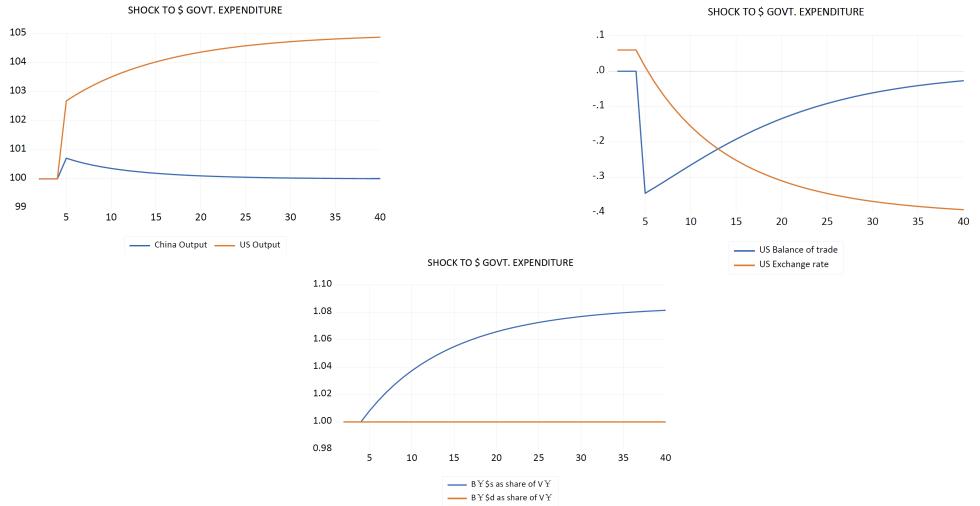


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- In addition, the falling dollar generates capital losses for residents in the c country where the value of the opening stock of bills issued abroad increases, and capital gains in the u country, which confronts a reduced value of bills issued in the c country.
- These revaluations of wealth stocks will feed into the asset demands in both countries in the same period, and affect consumption expenditures in the succeeding period, through a wealth effect. Although the responses in the two countries are symmetrical, they will not, in general, be identical. The coefficients in the asset demand functions will, in general, be entirely different as between the two countries, yet there has to be only a single exchange rate to satisfy all the relevant responses.
- The one-period solution that this model generates when shocked does not, in general, simultaneously generate a new overall steady state in which the balance-of-payments imbalance is eliminated. Rather, a new balance-of-payments deficit/surplus will occur, which will, in turn, generate a new, and similar, set of responses. So long as the exogenous variables do not change, the exchange rate will go on falling at a reducing rate until a new full steady state is achieved. Fiscal policy and also monetary policy in the form of interest rates are both under the full control of each government.
- MF model: Expansionary fiscal policy causes an increase in Y, an **appreciation** of the currency, and a decrease in the current account balance in a floating exchange rate system.

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- The rise in the u rate of interest immediately leads to a brisk hike in the u exchange rate, the value of the dollar in c units. In other words, there is a sudden appreciation of the dollar.
- The higher u rate of interest attracts net foreign capital, with all households now wanting to hold a larger proportion of u Treasury bills and a smaller proportion of c Treasury bills.
- The share of u bills in c portfolios immediately rises and that of c bills in c portfolios falls by an equivalent amount so long as both shares are measured in c currency.
- However, this conceals the fact that, because the exchange rate has changed, the share of u bills measured in u currency initially falls, rising only at a later stage. The initial fall is due to the fact that, since there is an approximately constant supply of u Treasury bills in the entire world, not all households will succeed in increasing their share of wealth held in the form of u Treasury bills, when measured in dollars.
- Households from the c country will thus initially hold less u Treasury bills, when measured in dollars, but they will succeed in holding more of them, when measured in their local currency. This will be achieved through an appreciated dollar.



- The stronger dollar will disturb the whole system by generating fiscal and trade imbalances. Because the stronger dollar will induce higher imports, the **u economy will run a trade** account deficit.
- The trade deficit, along with the capital losses of u households on their holdings of foreign Treasury bills due to the depreciation of the c currency, will slow the u economy and propel the u government budget position into a deficit.
- Because of this, u Treasury bills will have to be newly issued. The outstanding stock of Bus will rise gradually, and thus respond to the higher demand for this security. As a result, the value of the dollar will revert toward its original value, and so will the output of the u country.
- US: The stronger dollar will induce higher imports, the u economy will run a trade account deficit. Trade deficit + capital losses of u households→ u government deficit→ The outstanding stock of Bus will rise →xru and the output of the u country will revert to original values.

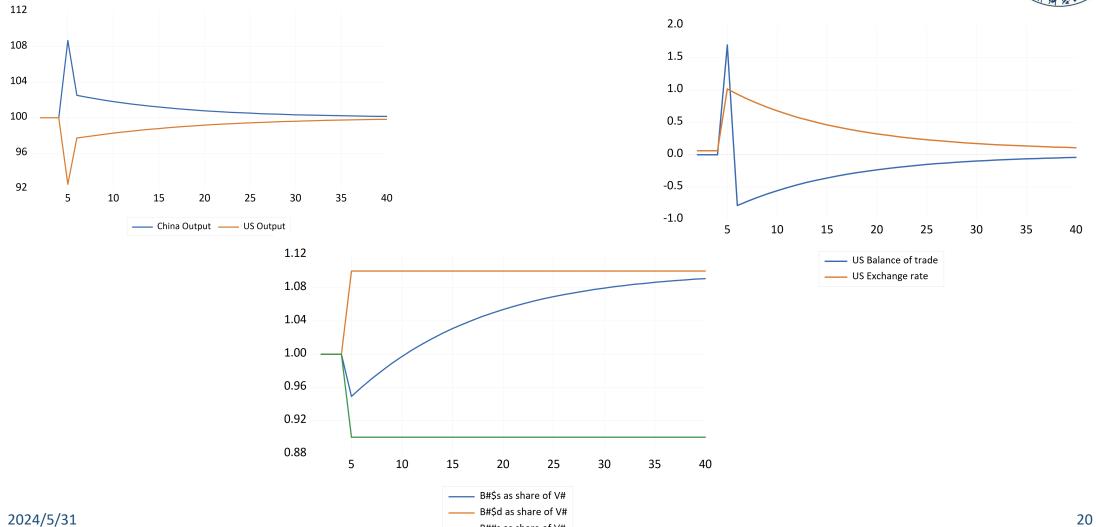


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- A symmetric process will occur in the c country. The appreciation of the dollar will lead to an increase in exports and capital gains for households holding dollar-denominated securities. Both of these effects will induce an initial boost in the output of the c country, as well as a trade surplus and a budget surplus. There will thus be a reduction in the outstanding stock of c Treasury bills, Bc s, which will correspond to the reduced demand for this security caused by the higher u interest rate. This will contribute to bringing back the value of the dollar to its original value.
- Thus, in this model, an increase in the interest rate leads to a slow down of the u economy through the exchange rate channel, but **this negative impact is only temporary**. In the new steady state, both economies are back to their initial flow levels, except that the u country, which imposed the higher interest rate, is now stuck with more substantial public debt and foreign debt.
- China: The stronger dollar will induce higher exports, the c economy will run a trade account surplus. Trade surplus + capital gains of c households → c government surplus → The outstanding stock of Bcs will fall →xru and the output of the c country will revert to original

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B##s as share of V#



- Changes in liquidity preference or speculative activity could be represented within the framework outlined above.
- An increase in the liquidity preference of asset holders in favor of u Treasury bills would lead to the same dynamics.
- This is because such a change in liquidity preference, just as an increase in the u interest rate, leads to an attempt by households to increase the share of u securities in their portfolios. Thus, such a change in liquidity preference would impose fluctuations in the exchange rate, and it would induce transitory changes in output and consumption.
- In the current case, it would lead to a monetary slow down of the u economy, through the exchange rate channel. The system, by inducing a u government deficit, would create the u government assets that the investors desire.



- To sum up, we see that monetary policy, defined as administered interest rate, is relatively less effective than fiscal policy, because its effect on output is only temporary, where fiscal policy has a permanent effect.
- This is in contrast with M-F model, where fiscal policy is less effective in flexible exchange rate.
- In addition, higher G here leads to a depreciation of u, because of the induced trade deficit.
- By contrast, the M-F model concludes that higher G leads to an appreciation of domestic currency (provided the BP curve is flat, when securities are perfect substitutes, or at least fatter than the LM curve), arising from a capital account surplus. This surplus is generated by higher interest rate, caused by crowding out, which results from unrealistic assumption that CB holds constant money supply despite an increased demand for money.



The model can be adapted to describe a fixed exchange rate world. First, of course, we must delete Equation (32) and make the exchange rate exogenous and constant.

If governments are to hold exchange rates fixed, they must, given any interest rates, be willing to buy or sell bills on any scale whatever at the chosen exchange rate. That is, among the other demand-determined asset supply functions, we must now have:

$$Buc_s = Buc_d x r u$$
 (32 * fixed)

But the inclusion of this particular equation would overdetermine the model, since Bu c s is already given by Equation (30)

$$Buc_s = Bc_s - Bcc_s - Bcbcs$$
 (30 * flexible)



There are three obvious possibilities if we imagine this system out of balance.

- 1. The fiscal policy of the deficit country must adjust to neutralize an ex ante excess supply of bills flowing into the market (in which case it must be endogenized);
- 2. The (endogenous) interest rate in the deficit country must rise indefinitely so that (in theory) a continuing increase in the relative supply of bills by the deficit country is always willingly held.
- 3. The remaining possibility is that the central bank of the surplus country acquires (while the deficit country disposes of) reserve assets on a limitless scale.

We proceed to explore the last of these three possibilities, noting in advance that, under this assumption, both governments still retain full control over both fiscal and monetary policy.

Besides adopting Equation (32F), all we need to do to construct a fixed exchange rate version of our two-country model is invert a series of equations.

As we bump one equation, because its left-hand side variable is already found in a previous equation, we must be prepared to bump a series of other equations until all variables appear only once on the left-hand side. Thus, as already said, we first bump out Equation (30) and replace it with Equation (30F):

$$Bcbc_s = Bc_s - Bcc_s - Buc_s$$
 (30 * fixed)

But Bcbcs was already on the left-hand side of Equation (26). We decide to define Equation (26A) as the redundant equation (which ensures that the amount of domestic bills supplied to the c central bank is the amount demanded).

This means that Equation (23) cannot be the redundant equation any more and must be part of the new model, becoming (23F):

$$Buu_s = Buu_d$$
 (23 * fixed)



But now Equation (29) must get bumped, since Buus is also on its left-hand side. We thus rewrite it as:

$$Bcbcu_s = Bu_s - Bcu_s - Buu_s - Bcbu_s$$
 (29 * fixed)

Which defines the supply of foreign reserves to the c central bank.

And we modify Equation (28) $\Delta Bcbc_d = \Delta Hc_s$, the balance sheet constraint of the c central bank, to take into account possible changes in these foreign reserves:

$$\Delta Bcbc_d = \Delta Hc_s - \Delta Bcbcu_s xru$$
 (28 * fixed)

Knowing that the change in the value of these foreign reserves measured in c currency depends both on the addition to foreign reserves measured in dollars and to a possible revaluation of the dollar, so that the value of the foreign reserves of the c country measured in domestic currency is:

$$Bcbcu_d = Bcbcu_s xru$$
 (36 * fixed)

Model closure (flexible <u>VS</u> fixed exchange regime)

(1) Flexible exchange regime:

$$\triangleright B \operatorname{cu}_s = B \operatorname{cu}_d / x \operatorname{ru}$$
 (31)

$$ightharpoonup xru = Buc_s/Buc_d$$
 (32 * flexible)

$$\triangleright CGu = \Delta xrcBuc_{s-1}$$
 (33)

$$\triangleright CGc = \Delta x r u B c u_{s-1}$$
 (34)

$$\rightarrow xrc = 1/xru$$
 (35)

(2) Fixed exchange regime:

$$\triangleright Buc_s = Buc_d x r u \quad (32 * fixed)$$

$$\triangleright Bcbc_s = Bc_s - Bcc_s - Buc_s$$
 (30 * fixed)

$$\triangleright Bcbcu_s = Bu_s - Bcu_s - Buu_s - Bcbu_s$$
 (29 * fixed)

$$\triangleright \Delta Bcbc_d = \Delta Hc_s - \Delta Bcbcu_s xru$$
 (28 * fixed)

$$\triangleright Bcbcu_d = Bcbcu_s xru$$
 (36 * fixed)





The two-country fixed exchange regime model is now complete.

The case we want to illustrate is where a surplus country (China) wishes to maintain its surplus and, in so doing, purchases reserve assets (U.S. Treasury bills) on whatever scale is necessary to keep the exchange rate where it is.

The model says that there is no limit to this process.

We start from a full stationary state (with no external imbalance) and assume that the u propensity to import rises permanently.

The Chinese economy reaches a new quasi stationary state with a constant surplus in the trade account (and in the overall balance of payments).

All flows and all privately held stocks, including the stock of money, do not change at all.



Checking now the balance sheet of the Chinese central bank, we see that this is accompanied by an ever-rising stock of holdings of U.S. Treasury bills by the PBC (its foreign reserves, measured in the c currency), while the stock of domestic Treasury bills also held by PBC gets gradually depleted-this is the sterilization effect, which occurs endogenously as long as the PBC acts to keep the interest rate constant.

This phenomenon can occur without any forces leading to its reversal.

The surplus in the Chinese balance of payments is unaccompanied by the "increase in the money supply". We have thus recovered the result that was discussed earlier within our fixed exchange single economy.

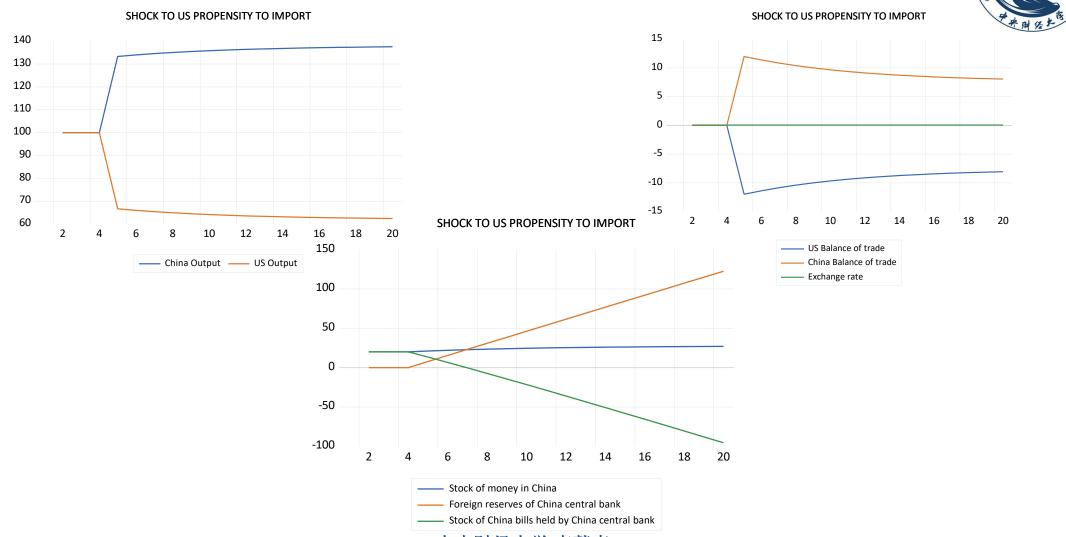
As to the US economy, it can face a balance-of-payments deficit as long as foreigners are willing to hold increasing amounts of u securities.

Simulation (Fixed Exchange Rate Model) Shock: an increase in the US propensity to import



- 1) freeze(chart1) chart1_g.line
- chart1.elem(1) legend(China Output)
- chart1.elem(2) legend(US Output)
- 2) freeze(chart2) chart2_g.line
- chart2.elem(1) legend(US Balance of trade)
- chart2.elem(2) legend(China Balance of trade)
- chart2.elem(3) legend(Exchange rate)
- 3) freeze(chart3) chart3_g.line
- chart3.elem(1) legend(Stock of money in China)
- chart3.elem(2) legend(Foreign reserves of China central bank)
- chart3.elem(3) legend(Stock of China bills held by China central bank)

Shock: an increase in the US propensity to import





In the fixed exchange rate regime, fiscal expansion leads to a permanently increased output but is accompanied by a twin deficit.

It would seem that fiscal and monetary policies are relatively more effective in the context of flexible exchange rate regimes.

Indeed, lower interest rates, unsurprisingly have only one effect-a temporary capital account deficit.

If we further assume that consumption depends negatively on interest rates, a reduction in interest rates leads to a sharp capital account deficit that quickly turns into a temporary surplus, a temporary increase in consumption and income, and a temporary budget surplus and trade deficit.

And it clearly illustrates the fact that the twin deficit proposition holds in the steady state, but not necessarily during the transition, when households accumulate or get rid of financial assets.



In the fixed exchange rate closure of the model, as well as its one-country version, we show why sterilization becomes endogenous when central banks fix interest rates.

Within the flexible exchange regime closure, still with monetary policy being represented by administered interest rates, we show that governments can achieve higher levels of activity by an appropriate choice of fiscal policy, at least within the limits imposed by the inflationary consequences of high activity levels (which have not been dealt with here).

This clearly contradicts the usual assertion, found in the Mundell-Fleming model, that fiscal policy has a weak or no effect in a flexible exchange regime. We have also shown that changes in interest rates, though they may have large and immediate consequences on the exchange rate and hence on levels of activity, seem to have effects that are self-reversing, thus inclining us to believe that the feedbacks tied to trade may still play a major role in the medium and long run.