SOLUTIONS

14

Competitive Market Equilibrium

Solutions for *Microeconomics: An Intuitive Approach with Calculus*

Apart from end-of-chapter exercises provided in the student *Study Guide*, these solutions are provided for use by instructors. (End-of-Chapter exercises with solutions in the student *Study Guide* are so marked in the textbook.)

The solutions may be shared by an instructor with his or her students at the instructor's discretion.

They may not be made publicly available.

If posted on a course web-site, the site must be password protected and for use only by the students in the course.

Reproduction and/or distribution of the solutions beyond classroom use is strictly prohibited.

In most colleges, it is a violation of the student honor code for a student to share solutions to problems with peers that take the same class at a later date.

- Each end-of-chapter exercise begins on a new page. This is to facilitate maximum flexibility for instructors who may wish to share answers to some but not all exercises with their students.
- If you are assigning only the A-parts of exercises in *Microeconomics: An Intuitive Approach with Calculus*, you may wish to instead use the solution set created for the companion book *Microeconomics: An Intuitive Approach*.
- *Solutions to Within-Chapter Exercises are provided in the student* Study Guide.

14.1 *In Table 14.1, the last column indicates the predicted change in the number of firms within an industry when economic conditions change.*

A: *In two cases, the table makes a definitive prediction, whereas in two other cases it does not.*

(a) *Explain first why we can say definitively that the number of firms falls as a recurring fixed cost (i.e. license fee) increases? Relate your answer to what we know about firm output and price in the long run.*

Answer: When a fixed cost increases, the long run *MC* curve does not change but the long run *AC* curve shifts up. Since the *MC* curve always crosses the lowest point of the *AC* curve, we know that this implies that the lowest point of the long run *AC* curve shifts to the right — i.e. to a higher level of output. This implies that the output level of firms that remain in the industry will increase in the new equilibrium — as will the price (since the *AC* curve has shifted up). But an increase in price means that, for any downward sloping demand curve, consumers will demand less of the good. The industry therefore produces less at a higher price — with each firm in the industry producing more. The only way this is possible is if some firms have exited — i.e. the number of firms in the industry has decreased.

(b) *Repeat (a) for the case of an increase in demand.*

Answer: When demand increases, none of the cost curves for firms change — with the long run *AC* curve in particular remaining unchanged. Thus, each firm in the new equilibrium will be producing at the same lowest point of its *AC* curve — and at the same price. The only thing that has changed is that demand has shifted — which implies that, at the same price, more output will be produced in the industry. With each firm in the industry producing the same output quantity, the only way more can be produced in the industry is for more firms to have entered — i.e. the number of firms in the industry has increased.

(c) *Now consider an increase in the wage rate and suppose first that this causes the long run AC curve to shift up without changing the output level at which the curve reaches its lowest point. In this case, can you predict whether the number of firms increases or decreases?*

Answer: In this case, the output level produced by each firm in the industry will remain the same but will be sold at a higher price. When price increases, however, less will be demanded (assuming a downward sloping market demand curve) — which implies the industry produces less. With each firm in the industry producing the same as before, the only way for the industry to produce less is for some firms to have exited. Thus, the number of firms in the industry falls in this case.

(d) *Repeat (c) but assume that the lowest point of the AC curve shifts up and to the right.*

Answer: If the lowest point of the *AC* curve shifts up and to the right, it means that firms that remain in the industry will produce *more* at a higher price — but the higher price implies that less will be demanded and thus the industry produces *less.* The only way each firm can produce more while the industry produces less is if some firms exited — and the number of firms in the industry therefore declines.

(e) *Repeat (c) again but this time assume that the lowest point of the AC curve shifts up and to the left.*

Answer: In this case, the lowest point of the *AC* curve occurs at a lower level of output and higher price — which means that firms in the industry will produce less and sell at that higher price. A higher price in turn means that consumers will demand less. Thus, each firm produces *less* as does the industry. Whether this implies more or fewer firms now depends on how much less each firm produces relative to how much the quantity demanded falls with the increase in price. Suppose each firm produces x% less and the industry as a whole produces y% less. Then if $x = y$, the number of firms stays exactly the same; if $x < y$, the number of firms falls and if $x > y$, the number of firms in the industry has to increase.

(f) *Is the analysis regarding the new equilibrium number of firms any different for a change in r?*

Answer: No, the analysis is no different for a change in r. Even if capital is fixed in the short run, it is variable in the long run — and treated just like the input labor.

(g) *Which way would the lowest point of the AC curve have to shift in order for us not to be sure whether the number of firms increases or decreases when w* falls*?*

Answer: When w falls, we know the long run AC curve will shift down — which implies that the long run equilibrium price will fall. At a lower price, the quantity demanded will increase — which implies that industry output will *increase*. Were each firm to continue to produce the same amount as before — or were each firm to produce less, then the only way for the industry to produce more would be for the number of firms to increase. Thus, in order for us not to be sure of whether the number of firms increases, it would have to be that each firm produces more (just as the industry produces more) — and this only happens if the lowest point of the AC curve shifts to the right as w falls.

B: *Consider the case of a firm that operates with a Cobb-Douglas production function $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ where $\alpha,\beta > 0$ and $\alpha+\beta < 1$.*

(a) *The cost function for such a production process — assuming no fixed costs — was given in equation (13.45) of exercise 13.5. Assuming an additional recurring fixed cost F, what is the average cost function for this firm?*

Answer: Including the fixed cost F, the total cost function becomes

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + F \tag{14.1}$$

which gives us an AC function

$$AC(w,r,x) = \frac{C(w,r,x)}{x} = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{F}{x}. \tag{14.2}$$

(b) *Derive the equation for the output level x^* at which the long run AC curve reaches its lowest point.*

Answer: The AC curve reaches its lowest point where its derivative with respect to x is zero — i.e. where

$$\frac{\partial AC(w,r,x)}{\partial x} = \left[(1-\alpha-\beta)\left(\frac{w^{\alpha}r^{\beta}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} x^{(1-2(\alpha+\beta))/(\alpha+\beta)}\right] - \frac{F}{x^2} = 0. \tag{14.3}$$

Solving this for x, we then get the output level at the lowest point of the long run AC curve:

$$x^* = \left(\frac{A\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(\alpha+\beta)}. \tag{14.4}$$

(c) *How does x^* change with F, w and r?*

Answer: Given the expression for x^* above, it is easy to see that x^* increases with F and decreases with w and r. Put differently, the lowest point of the AC curve occurs at higher output levels as the fixed cost increases and at lower output levels when input prices increase.

(d) True or False*: For industries in which firms face Cobb-Douglas production processes with recurring fixed costs, we can predict that the number of firms in the industry increases with F but we cannot predict whether the number of firms will increase or decrease with w or r.*

Answer: This is true. As F increases, output price rises as does output by each firm. The higher output price, however, means that the quantity demanded — and thus the quantity supplied by the industry — decreases. The only way the industry output can decrease when firm output increases is if some firms have left the industry. When input prices increase, the equilibrium output price similarly rises (as the AC shifts up) — causing the industry to produce less. But, in the case analyzed here, each firm also produces less — so we cannot immediately tell whether the number of firms will increase or decrease.

14.2 *Table 14.1 was constructed under the assumption that all firms in the industry are identical.*

A: *Suppose that all firms in an industry have U-shaped long run average cost curves.*

(a) *Leaving aside the column labeled "Firm Output", what would change in the table if firms have different cost structures — i.e. some firms have lower marginal and average costs than others?*

Answer: Nothing would change in the first two columns — i.e. which cost curves are affected by the various changes. In the market price column, everything would stay the same — except for the long run change in market price which would now be positive. This is because, when demand increases and thus firms enter, it will be higher cost firms that enter, with the new marginal firm having a higher minimum point to its *AC* curve. The industry output column would remain the same, as would conclusions about the number of firms in the industry.

(b) *Industries such as those described in (a) are sometimes called* increasing cost industries *compared to* constant cost industries *where all firms are identical. Can you derive a rationale for these terms?*

Answer: When all firms are identical, an increase in demand will, in the long run, simply cause more firms with the same cost structure to enter the market — with each producing at the lowest point of its *AC* curve. Thus, as the industry expands, the average cost of output remains *constant* — thus the term "constant cost industry." When firms differ in their cost structure, an increase in demand will cause entry of firms that produce at greater average cost — which implies that the average cost of output produced in the industry *increases* with industry output; thus the term "increasing cost industry."

(c) *It has been argued that, in some industries, the average and marginal costs of* all *firms decline as more firms enter the market. For instance, such industries might make use of an unusual labor market skill that becomes more plentiful in the market as more workers train for this skill when many firms demand it. How would the long run industry supply curve differ in this case from that discussed in the text as well as that described in (a)?*

Answer: As demand increases, the lowest point of the average cost curves of all firms in the industry would therefore fall — implying that, as the industry expands to meet increased demand, output is produced at lower costs. The zero profit condition of long run equilibrium then implies that output price must fall as the industry expands — leading to a downward sloping industry supply curve.

(d) *Industries such as those described in (c) are sometimes referred to as* decreasing cost industries. *Can you explain why?*

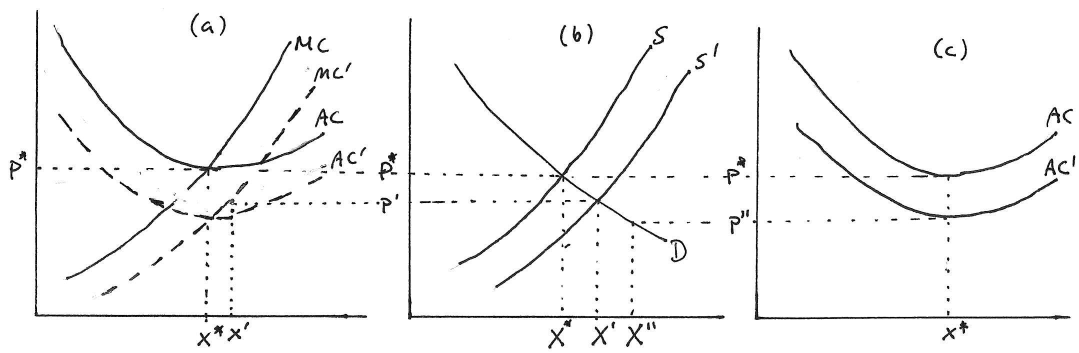
Answer: This is because the industry experiences lower costs as it expands.

14.3 Everyday and Business Application: *Fast Food Restaurants and Grease (cont'd)*: *In exercise 12.8, you investigated the impact of hybrid vehicles that can run partially on grease from hamburger production on the number of hamburgers produced by a fast food restaurant. You did so, however, in the absence of considering the equilibrium impact on prices — assuming instead that prices for hamburgers are unaffected by the change in demand for grease.*

A: *Suppose again that you use a decreasing returns to scale production process for producing hamburgers using only labor and that you produce 1 ounce of grease for every hamburger. In addition, suppose that you are part of a competitive industry and that each firm also incurs a recurring fixed cost F every week.*

(a) *Suppose that the cost of hauling away grease is $q > 0$ per ounce. Illustrate the shape of your marginal and average cost curve (given that you also face a recurring fixed cost.)*

Answer: These are illustrated in panel (a) of Graph 14.1 as the solid curves labeled MC and AC. The marginal curve is upward sloping as it was in exercise 12.8 because it is unaffected by the recurring fixed cost. The average cost curve, however, is U-shaped as a result of F.



Graph 14.1: Hamburgers and Hybrid Vehicles

(b) *Assuming all restaurants are identical, illustrate the number of hamburgers you produce in long run equilibrium.*

Answer: In long run equilibrium, you will be making zero profit — which means the long run price is p^* as illustrated in panel (a) of Graph 14.1. As a result, you produce x^*.

(c) *Now suppose that, as a result of the increased use of hybrid vehicles, the company you previously hired to haul away your grease is now willing to pay for the grease it hauls away. How do your cost curves change?*

Answer: The marginal and average cost curves will now shift down by the change in q. This is illustrated in panel (a) of Graph 14.1 with the dashed MC and AC curves.

(d) *Describe the impact this will have on the equilibrium price of hamburgers and the number of hamburgers you produce in the short run.*

Answer: In panel (b) of Graph 14.1, we illustrate the initial short run market supply curve S that intersects the demand curve D at the original price p^*. As a result of each firm's marginal cost curve shifting down, the short run market supply curve shifts down to S' — resulting in a decrease in the price to p'. The industry ends up producing more (X' rather than X^* in panel (b)) — which means each restaurant is producing more in the short run when the number of restaurants is fixed. This is illustrated as x' in panel (a).

(e) *How does your answer change in the long run?*

Answer: In the long run, the equilibrium price must settle at a point where all restaurants again make zero profit — i.e. to the lowest point of the new AC' curve. This is illustrated in panel (c) of Graph 14.1. Because the cost of each hamburger produced decreases by the same amount, the average cost curve shifts down in a parallel way — leaving the lowest point at that same output quantity x^* as before. Thus, in the long run, each restaurant will produce the same number of hamburgers as originally (x^*) and will sell them at price p''. The price has in essence fallen by the full decrease in the marginal cost. In the new equilibrium, there will be more restaurants than before.

(f) *Would your answers change if you instead assumed that restaurants used both labor and capital in the production of hamburgers?*

Answer: No — the cost curves would shift in exactly the same way.

(g) *In exercise 12.8, you concluded that the cholesterol level in hamburgers will increase as a result of these hybrid vehicles if restaurants can choose more or less fatty meat. Does your conclusion still hold?*

Answer: Yes, this conclusion still holds — if firms can profit from using fattier beef, they will all do so in equilibrium. If a firm did not do so, it would make negative profit.

B: *Suppose, as in exercise 12.8, that your production function is given by $f(\ell) = A\ell^{\alpha}$ (with $0 < \alpha < 1$) and that the cost of hauling away grease is q. In addition, suppose now that each restaurant incurs a recurring fixed cost of F.*

(a) *Derive the cost function for your restaurant.*

Answer: Solving the production function $x = A\ell^{\alpha}$ for ℓ, we get the conditional labor demand function

$$\ell(w,x) = \left(\frac{x}{A}\right)^{1/\alpha}. \tag{14.5}$$

Multiplying this by w and adding the cost of hauling grease as well as the fixed cost, we get

$$C(w,x,q) = w\left(\frac{x}{A}\right)^{1/\alpha} + qx + F. \tag{14.6}$$

(b) *Derive the marginal and average cost functions.*

Answer: Taking the derivative of the cost function with respect to x, we get

$$MC(w,x,q) = \left(\frac{w}{\alpha A^{1/\alpha}}\right)x^{(1-\alpha)/\alpha} + q. \tag{14.7}$$

Dividing the cost function by x, we get the average cost function

$$AC(w,x,q) = \left(\frac{w}{A^{1/\alpha}}\right)x^{(1-\alpha)/\alpha} + q + \frac{F}{x}. \tag{14.8}$$

(c) *How many hamburgers will you produce in the long run?*

Answer: In the long run, you produce at the lowest point of your average cost curve. To determine the output quantity at which this occurs, we take the first derivative of our AC function, set it to zero and solve for x. This gives us

$$x^* = A\left(\frac{\alpha F}{(1-\alpha)w}\right)^{\alpha}. \tag{14.9}$$

(d) *What is the long run equilibrium price of hamburgers?*

Answer: To determine the long run equilibrium price, we plug x^* back into the average cost function and solve it; i.e.

$$p^* = AC(w,x^*,q) = \left(\frac{w}{A^{1/\alpha}}\right)\left[A\left(\frac{\alpha F}{(1-\alpha)w}\right)^{\alpha}\right]^{(1-\alpha)/\alpha} + q + \frac{F}{\left[A\left(\frac{\alpha F}{(1-\alpha)w}\right)^{\alpha}\right]} \tag{14.10}$$

which, after some algebra, reduces to

$$p^* = \frac{w^{\alpha}F^{(1-\alpha)}}{A(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}} + q. \tag{14.11}$$

(e) *From your results, determine how the long run equilibrium price and output level of each restaurant changes as q changes.*

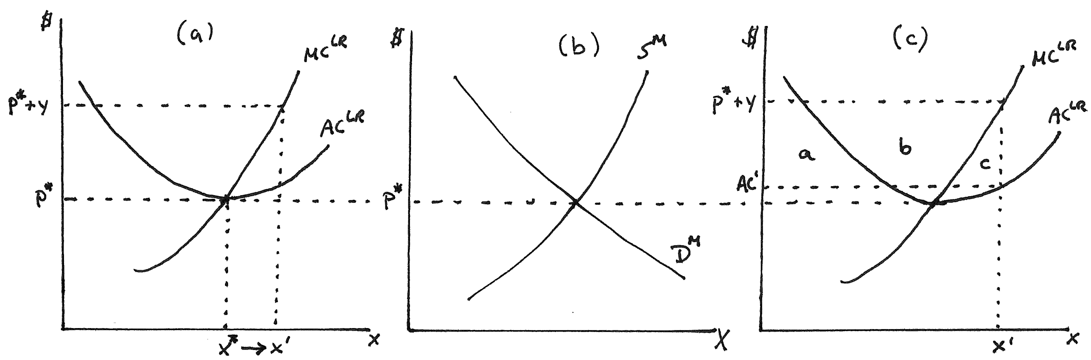
Answer: The term q does not appear in our equation for x^* — which implies that it has no impact on the number of hamburgers produced by each restaurant in the long run. This is consistent with what we determined graphically. Our equation for p^*, on the other hand, has q simply entering as an additive term. Thus, in long run equilibrium, q is simply passed onto the consumer — as it falls (and even becomes negative), consumers therefore get the entire benefit.

14.4 Business Application: *Brand Names and Franchise Fees: Suppose you are currently operating a hamburger restaurant that is part of a competitive industry in your city.*

A: *Your restaurant is identical to others in its homothetic production technology which employs labor ℓ and capital k and has decreasing returns to scale.*

(a) *In addition to paying for labor and capital each week, each restaurant also has to pay recurring weekly fees F in order to operate. Illustrate the average weekly long run cost curve for your restaurant.*

Answer: This is illustrated in panel (a) of Graph 14.2 where the long run average cost curve for your restaurant is U-shaped because of the combination of a recurring fixed cost and decreasing returns to scale in production.



Graph 14.2: MacWendy's Franchise Deal

(b) *On a separate graph, illustrate the weekly demand curve for hamburgers in your city as well as the short run industry supply curve assuming that the industry is in long run equilibrium. How many hamburgers do you sell each week?*

Answer: This is illustrated in panel (b) where the market supply curve S^M is simply all short run restaurant supply curves added together. This has to intersect demand at p^* which lies at the lowest point of the AC curve in panel (a). It is only at that price that long run profits for restaurants are zero and thus no incentives for entering or exiting the industry exist. At this price, you will sell hamburgers so long as p^* is greater than long run average cost — and we know that MC crosses the AC curve at its lowest point. Thus, you will produce x^* as indicated in panel (a) of Graph 14.2.

(c) *As you are happily producing burgers in this long run equilibrium, a representative from the national MacWendy's chain comes to your restaurant and asks you to convert your restaurant to a MacWendy's. It turns out, this would require no effort on your part — you would simply have to allow the MacWendy's company to install a MacWendy's sign, change some of the furniture and provide your employees with new uniforms — all of which the MacWendy's parent company is happy to pay for. MacWendy's would, however, charge you a weekly franchise fee of G for the privilege of being the only MacWendy's restaurant in town. When you wonder why you would agree to this, the MacWendy's representative pulls out his marketing research that convincingly documents that consumers are willing to pay \$y more per hamburger when it carries the MacWendy's brand name. If you accept this deal, will the market price for hamburgers in your city change?*

Answer: The market price for hamburgers in your city would remain unchanged at p^* since the industry is competitive and thus a single firm's actions cannot change prices. However, the price you can charge for a MacWendy's hamburger will be $p^* + y$.

(d) *On your average cost curve graph, illustrate how many hamburgers you would produce if you accepted the MacWendy's deal.*

Answer: You would produce where $p^* + y$ intersects your marginal cost curve. In the long run, we would use the long run marginal cost curve which is illustrated in panel (a) of Graph 14.2. Output at your restaurant would then increase from x^* to x'. (If capital is fixed in the short run, your initial increase in output would be less since the short run MC curve is steeper — and you would fully adjust to x' only once you can adjust your level of capital.)

(e) *Next, for a given y, illustrate the largest that G could be in order for you to accept the MacWendy's deal.*

Answer: This is done in panel (c) of Graph 14.2 where the long run average and marginal cost curves are drawn once again. We know that you will be able to sell your MacWendy's hamburgers at the price $p^* + y$, and we know you will sell x'. That makes your revenue equal to the box $(p^* + y)x'$. We also know you will incur average costs AC' — or total long run costs AC' times x', the smaller rectangular box. The difference between these two boxes $(a+b+c)$ is the long run profit that you can make (per time period) from being a MacWendy's *not counting the franchise fee G*. Thus, $a+b+c$ is the most you would be willing to pay per time period for the franchise fee.

(f) *If you accept the deal, will you end up hiring more or fewer workers? Will you hire more or less capital?*

Answer: Since neither the wage nor the rental rate has changed, and since the production technology is homothetic, we know your labor to capital ratio will not change as you produce more output (because all cost minimizing input bundles lie on the same ray from the origin in the isoquant graph). We know from what we did above that you will produce more — thus you will hire more labor and more capital.

(g) *Does your decision on how many workers and capital to hire under the MacWendy's deal depend on the size of the franchise fee G?*

Answer: No — once you accept the Wendy's deal, it is a fixed cost that has no impact on the MC curve. It therefore has no impact on how much you will produce — and thus no impact on how many workers and capital you hire.

(h) *Suppose that you accepted the MacWendy's deal and, because of the increased sales of hamburgers at your restaurant, one hamburger restaurant in the city closes down. Assuming that the total number of hamburgers consumed remains the same, can you speculate whether total employment (of labor) in the hamburger industry went up or down in the city? (*Hint*: Think about the fact that all restaurants operate under the same decreasing returns to scale technology.)*

Answer: If my increased production drove one restaurant out of business and the overall number of hamburgers sold in the city remains unchanged, it must mean that production by the other restaurant that was driven out of business went down by x^* while your production went from x^* to $2x^*$. With decreasing returns to scale, however, the other restaurant needed to use less labor and capital to produce x^* than you need to use to increase your output from x^* to $2x^*$. Thus, overall employment in the restaurant sector of the city increases.

B: *Suppose all restaurants in the industry use the same technology that has a long run cost function $C(w,r,x) = 0.028486(w^{0.5}r^{0.5}x^{1.25})$ which, as a function of wage w and rental rate r, gives the weekly cost of producing x hamburgers.*[1]

(a) *Suppose that each hamburger restaurant has to pay a recurring weekly fee of $4,320 to operate in the city in which you are located and that $w = 15$ and $r = 20$. If the restaurant industry is in long run equilibrium in your city, how many hamburgers does each restaurant sell each week?*

Answer: If the industry is in long run equilibrium, each restaurant makes zero long run profit and thus operates at the lowest point of its AC curve. Given $w = 15$ and $r = 20$, the average cost function is

[1] For those who find unending amusement in proving such things, you can check that this cost function arises from the Cobb-Douglas production function $f(\ell,k) = 30\ell^{0.4}k^{0.4}$.

$$AC(x) = \frac{0.028486(15^{0.5}20^{0.5}x^{1.25})}{x} + \frac{4320}{x} = 0.4934x^{0.25} + \frac{4320}{x}. \qquad (14.12)$$

The lowest point of this AC curve occurs where the derivative with respect to x is zero — i.e. where

$$\frac{dAC(x)}{dx} = \frac{0.12335}{x^{0.75}} - \frac{4320}{x^2} = 0. \qquad (14.13)$$

Solving this for x, we get $x = 4320$. Thus, each restaurant sells 4,320 hamburgers per week.

(b) *At what price do hamburgers sell in your city?*

Answer: Plugging 4,320 into the AC function in equation (14.12), this gives us the lowest point of the AC curve on the vertical axis — which is also equal to the long run equilibrium price. That price is $p = 5$.

(c) *Suppose that the weekly demand for hamburgers in your city is $x(p) = 100,040 - 1000p$. How many hamburger restaurants are there in the city?*

Answer: At a price of \$5 per hamburger, the total demand will be $x = 100,040 - 1000(5) = 95,040$ hamburgers per week. With each hamburger restaurant producing 4,320 per week, this implies that the number of such restaurants in the city is $95040/4320 = 22$.

(d) *Now consider the MacWendy's offer described in A(c) of this exercise. In particular, suppose that the franchise fee required by MacWendy's is $G = 5,000$ and that consumers are willing to pay 94 cents more per hamburger when it carries the MacWendy's brand name. How many hamburgers would you end up producing if you accept MacWendy's deal?*

Answer: Since you would be able to sell your MacWendy's hamburgers for \$5.94 instead of \$5, we need to determine how many you would produce from your long run marginal cost curve. Given the cost function $C(w,r,x) = 0.028486(w^{0.5}r^{0.5}x^{1.25})$ that becomes $C(x) = 0.493392x^{1.25}$ when evaluated at the input prices $w = 15$ and $r = 20$, this is

$$MC(x) = \frac{\partial C(x)}{\partial x} = 0.61674x^{0.25}. \qquad (14.14)$$

(Note that the fixed cost is irrelevant for the marginal cost curve which is why we did not need to include it in the $C(x)$ function we differentiated.) You will produce until price is equal to marginal cost — i.e. until $p = 5.94 = 0.61674x^{0.25}$. Solving this for x, we get that you will produce $x = 8,605$ hamburgers per week.

(e) *Will you accept the MacWendy's deal?*

Answer: Yes, you will — because your long run profit is now positive. We just determined that you will sell 8,605 hamburgers per week at a price of \$5.94 — which gives you total revenue of about \$51,114. Your weekly cost is given by the cost function that includes the fixed cost and franchise fee:

$$C = 0.493392\left(8605^{1.25}\right) + 4320 + 5000 \approx 50,211. \qquad (14.15)$$

Subtracting costs from revenues, we get a long run profit of \$903 per week.

(f) *Assuming that the total number of hamburgers sold in your city will remain roughly the same, would the number of hamburger restaurants in the city change as a result of you accepting the deal?*

Answer: Since you are selling roughly twice as many hamburgers (8,605 versus 4,320) as a MacWendy's hamburger restaurant, you will in effect drive one restaurant out of the market. The total number of hamburger restaurants therefore falls to 21 — 20 of the general kind and 1 MacWendy's.

(g) *What is the most that the MacWendy's representative could have charged you for you to have been willing to accept the deal?*

Answer: Since you are making \$903 in weekly profit when you are paying a \$5,000 weekly franchise fee, the most that the MacWendy's representative could have charged is \$5,903 per week.

(h) *Suppose the average employee works for 36 hours per week. Can you use Shephard's Lemma to determine how many employees you hire if you accept the deal? Does this depend on how high a franchise fee you are paying?*

Answer: Applying Shephard's Lemma to the function $C(w,r,x) = 0.028486(w^{0.5}r^{0.5}x^{1.25})$, we get the conditional labor demand function

$$\ell(w,r,x) = \frac{\partial C(w,r,x)}{\partial w} = 0.014243\left(\frac{r^{0.5}x^{1.25}}{w^{0.5}}\right). \tag{14.16}$$

After becoming a MacWendy's, you are producing 8,605 hamburgers per week. Plugging in $x = 8605$ and the input prices $w = 15$ and $r = 20$, we therefore get that you will hire

$$\ell = 0.014243\left(\frac{20^{0.5}8605^{1.25}}{15^{0.5}}\right) \approx 1,363 \tag{14.17}$$

hours of labor. With the average employee working 36 hours per week, this implies 37.86 workers. Note that the franchise fee is irrelevant to this because it is a fixed cost — as long as you accept it, you will hire about 38 workers.

(i) *How does this compare to the number of employees hired by the competing non-MacWendy's hamburger restaurants? In light of your answer to (f), will overall employment in the hamburger industry increase or decrease in your city as a result of you becoming a MacWendy's restaurant?*

Answer: Your competitors face the same input prices $w = 15$ and $r = 20$ but produce only 4,320 hamburgers per week. Plugging these values into equation (14.16), we get

$$\ell = 0.014243\left(\frac{20^{0.5}4320^{1.25}}{15^{0.5}}\right) = 576 \tag{14.18}$$

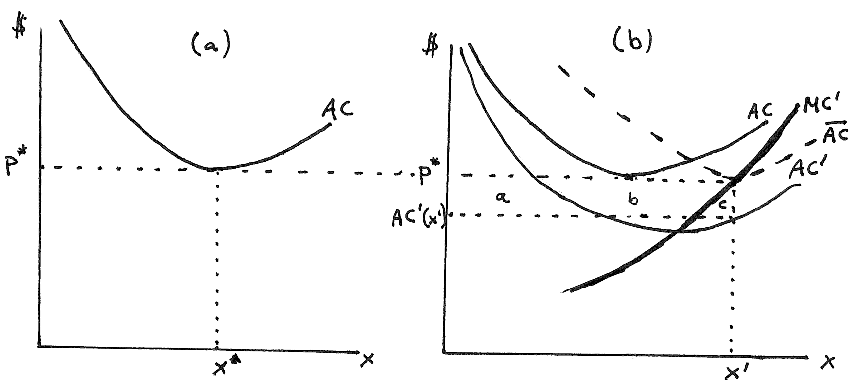
hours of labor per week. At an average work week of 36 hours, this implies 16 workers per week. Given that the number of restaurants will decrease by 1 as a result of you becoming a MacWendy's, the city will lose 16 hamburger restaurant jobs — but it will gain about 22 jobs in your MacWendy's (since you employed 16 workers before becoming a MacWendy's and about 38 afterwards). Thus, there is a net increase of 6 hamburger restaurant jobs in the city as a result of you becoming a MacWendy's restaurant.

14.5 Business Application: *"Economic Rent" and Profiting from Entrepreneural Skill: Suppose, as in exercise 14.4, that you are operating a hamburger restaurant that is part of a competitive industry. Now you are also the owner, and suppose throughout that the owner of a restaurant is also one of the workers in the restaurant and collects the same wage as other workers for the time he/she puts into the business each week. (In addition, of course, the owner keeps any weekly profits.)*

A: *Again, assume that all the restaurants are using the same homothetic decreasing returns to scale technology, but now the inputs include the level of entrepreneural capital c in addition to weekly labor ℓ and capital k. As in exercise 14.4, assume also that all restaurants are required to pay a recurring weekly fixed cost F.*

(a) *First assume that all restaurant owners possess the same level of entrepreneural skill c. Draw the long run AC curve (for weekly hamburger production) for a restaurant and indicate how many weekly hamburgers the restaurant will sell and at what price assuming that the industry is in long run equilibrium.*

Answer: This is illustrated in panel (a) of Graph 14.3 where the long run average cost curve of each firm is U-shaped because of the recurring fixed cost. Each restaurant will produce x^* and sell it at p^*.



Graph 14.3: Rent on Entrepreneural Skill

(b) *Suppose next that you are special and possess more entrepreneural and management skill than all those other restaurant owners. As a result of your higher level of c, the marginal product of labor and capital is 20% greater for any bundle of ℓ and k than it is for any of your competitors. Will the long run equilibrium price be any different as a result?*

Answer: No, a single firm in a competitive industry is not large enough to affect market prices.

(c) *If your entrepreneural skill causes the marginal product of capital and labor to be 20% greater for any combination of ℓ and k than for your competitors, how does your isoquant map differ from theirs? For a given wage and rental rate, will you employ the same labor to capital ratio as your competitors?*

Answer: We know that the slopes of isoquants are $TRS = -MP\ell/MP_k$. If both marginal products increase by the same percentage, then the ratio is unchanged — which implies that your isoquant map looks exactly the same as your competitors' except that it is differently labeled because you can produce more with less capital and labor. Since the shapes of the isoquants are the same as those for your competitors', isocosts will be tangent along the same ray from the origin — which implies that you will employ the same labor to capital ratio as you minimize your costs. You will simply require less capital and labor for any given level of output.

(d) *Will you produce more or less than your competitors? Illustrate this on your graph by determining where the long run MC and AC curves for your restaurant will lie relative to the AC curve of your competitors.*

Answer: Since you need less labor and capital for any given level of output, your average costs are lower. Similarly, the lowest point of your AC curve will occur at a higher level of output than for your competitors. This is illustrated in panel (b) of Graph 14.3 where AC is your competitors' average cost curve and AC' is yours. Finally, we can put your long run MC' curve into the graph (making sure it crosses your average cost curve AC' at its lowest point. Since the market price is unchanged at p^*, we know you will profit maximize where p^* intersects MC' — at output level x'. You will therefore produce more than your competitors.

(e) *Illustrate in your graph how much weekly profit you will earn from your unusually high entrepreneural skill.*

Answer: In panel (b) of Graph 14.3, two rectangular boxes emerge from the dotted lines combined with the axes. The larger of these is equal to total revenues for your firm (price times output); the smaller one is your total cost (average cost times output); and the difference — area $a+b+c$ — is the difference between the two. Since those without your entrepreneural skill make zero profit, the profit you derive from your skill is therefore $a+b+c$.

(f) *Suppose the owner of MacroSoft, a new computer firm, is interested in hiring you as the manager of one of its branches. How high a weekly salary would it have to offer you in order for you to quit the restaurant business assuming you would work for 36 hours per week in either case and assuming the wage rate in the restaurant business is $15 per hour.*

Answer: It would have to offer you a salary equal to the level of compensation you currently get for spending your time in the restaurant business. Since you are one of the workers drawing an hourly wage $w = 15$, you are making $540 per week as one of the workers in your restaurant *plus* you earn a profit of $a+b+c$ as indicated in Graph 14.3. MacroSoft would therefore have to offer you a minimum weekly salary of $a+b+c+540$.

(g) *The benefit that an entrepreneur receives from his skill is sometimes referred to as the* economic rent *of that skill — because the entrepreneur could be renting his skill out (to someone like MacroSoft) instead of using it in his own business. Suppose MacroSoft is willing to hire you at the rate you determined in part (f). If the economic rent of entrepreneural skill is included as a cost to the restaurant business you run, how much profit are you making in the restaurant business?*

Answer: You would then be making zero profit because $a+b+c$ in Graph 14.3 would become an additional periodic fixed cost in your restaurant business.

(h) *Would counting this economic rent on your skill as a cost in the restaurant business affect how many hamburgers you produce? How would it change the AC curve in your graph?*

Answer: Since the economic rent is a recurring long run fixed cost, it does not impact the long run MC curve in panel (b) of Graph 14.3. Thus, price p^* continues to intersect MC' at x' — implying you will produce exactly the same amount as if we did not count economic rent on your skill as a cost. The only thing that would change in panel (b) of the graph is that the average cost curve would be higher because $a+b+c$ is now included as a recurring average cost — and this average cost curve (denoted $\overline{AC}$ in the graph) — would reach its lowest point as it intersects the unchanged MC'. Thus, price equals MC' at the lowest point of $\overline{AC}$ — giving us zero profit for the firm if economic rents on entrepreneural skill are counted as a cost for the firm (and a payment to the owner).

B: *Suppose that all restaurants are employing the production function $f(\ell,k,c) = 30\ell^{0.4}k^{0.4}c$ where ℓ stands for weekly labor hours, k stands for weekly hours of rented capital and c stands for the entrepreneural skill of the owner. Note that, with the exception of the c term, this is the same production technology used in exercise 14.4. The weekly demand for hamburgers in your city is, again as in exercise 14.4, $x(p) = 100{,}040 - 1{,}000p$.*

(a) *First, suppose that $c = 1$ for all restaurant owners, that $w = 15$ and $r = 20$, that there is a fixed weekly cost \$4,320 of operating a restaurant, and the industry is in long run equilibrium. Determine the weekly number of hamburgers sold in each restaurant, the price at which hamburgers sell, and the number of restaurants that are operating. (If you have done exercise 14.4, you should be able to use your results from there.)*

Answer: Since $c = 1$ for all restaurants, the production function becomes identical to that used in exercise 14.4. In parts (a) through (c) of exercise 14.4, you calculated that each restaurant will produce 4,320 hamburgers per week, that the long run equilibrium price will be \$5 per hamburger and that there will be 22 restaurants in your city.

(b) *Next, suppose that you are the only restaurant owner that is different from all the others in that you are a better manager and entrepreneur and that this is reflected in $c = 1.24573$ for you. Determine your long run AC and MC functions. (Be careful not to use the cost function given in exercise 14.4 since c is no longer equal to 1. You can instead rely on the cost function derived for Cobb-Douglas technologies given in equation (13.45) in exercise 13.5 (and remember to add the fixed cost).)*

Answer: Your production function can then be written as $f(\ell,k) = [30(1.24573)]\,\ell^{0.4}k^{0.4} = 37.3719\ell^{0.4}k^{0.4}$. The cost function for a Cobb-Douglas production process $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ is

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)}. \tag{14.19}$$

Substituting in $\alpha = \beta = 0.4$, $A = 37.3719$, $w = 15$ and $r = 20$, and adding the fixed cost of \$4,320, this gives us the cost function

$$C(x) = 0.374895x^{1.25} + 4320. \tag{14.20}$$

The marginal and average cost functions are then

$$MC(x) = \frac{dC(x)}{dx} = 0.468619x^{0.25} \text{ and } AC(x) = \frac{C(x)}{x} = 0.374895x^{0.25} + \frac{4320}{x}. \tag{14.21}$$

(c) *How many hamburgers will you produce in long run equilibrium?*

Answer: The long run equilibrium price must still be equal to \$5 per hamburger because the rest of the firms must be operating at zero long run profit. Your restaurant will, however, produce where price equals your MC curve which is different from those of the other firms. Thus, we set price equal to MC — i.e. $5 = 0.4686x^{0.25}$ and solve for x to get $x \approx 12,960$.

(d) *How many restaurants will there be in long run equilibrium given your higher level of c?*

Answer: Your production of 12,960 hamburgers per week is 3 times the production of the 4,320 hamburgers per week in the other restaurants. Before, there were 22 restaurants — which implies that now there will only be 20 including your restaurant. Thus, your entry into the restaurant market drives two of the other restaurants out of business.

(e) *How many workers (including yourself) and units of capital are you hiring in your business compared to those hired by your competitors? (Recall that the average worker is assumed to work 36 hours per week.)*

Answer: You can either solve the profit maximization problem to derive the labor and capital demand curves and use these to determine how many hours of labor and capital will be used. Alternatively, we could (as we did in part (h) of exercise 14.4) differentiate the cost function with respect to the input prices to get the conditional labor and capital demand functions — then plug in the input prices and output levels to get to the answer. Either way, we get that the other firms are hiring 576 labor hours and 432 capital hours per week, and your firm is hiring 1,728 labor hours and 1,296 hours of capital per week. At a work week of 36 hours, this implies that other firms hire 16 workers and your firm hires 36 workers.

(f) *How does your restaurant's weekly long run profit differ from that of the other restaurants?*

Answer: Other restaurants are selling 4,320 hamburgers at a price of \$5 to make total weekly revenues of \$21,600; and they pay a weekly fixed cost of \$4,320 and hire 576 worker hours at wage \$15 and 432 capital hours at rental rate \$20 for total cost of $4320+576(15)+432(20) = \$21,600$. Thus, profits of other restaurants are zero (as we know has to be in long run equilibrium). Your firm, on the other hand, is selling 12,960 hamburgers at a price of \$5 for a total revenue of \$64,800. Your costs include the \$4,320 weekly fixed cost plus the cost of 1,728 hours of labor hired at a wage of \$15 and 1,296 hours of capital hired at a rental rate of \$20 for a total cost of $4320+1728(15)+1296(20) = \$56,160$. This implies a profit for you of $64,800-56,160 = \$8,640$ per week.

(g) *Suppose Macrosoft is interested in hiring you as described in part A(f). How high a weekly salary would MacroSoft have to offer you in order for you to quit the restaurant business and accept the MacroSoft offer?*

Answer: Since you are also one of the workers who works 36 hours per week in your restaurant, your overall compensation is your labor income of 36(15) = \$540 plus the profit of \$8,640 per week — for a total of \$9,180 per week. This is the least that MacroSoft would have to offer you in weekly compensation in order to attract you away from your restaurant business.

(h) *If you decide to accept the MacroSoft offer and you exit the restaurant business, will total employment in the restaurant business go up or down?*

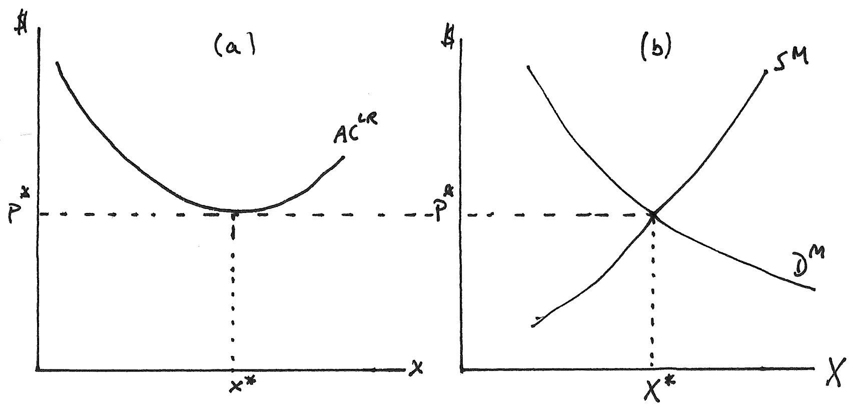
Answer: With you in the restaurant business, we have 36 workers working in your business and 16 in each of the 19 others — for a total of 340 restaurant workers. With you out of the business, there are 22 restaurants employing 16 workers each — for a total of 352 workers. Thus, if you accept the MacroSoft offer, the number of workers (including owners) in restaurants increases by 12.

14.6 Business and Policy Application: *Capital Gains Taxes: Taxes on capital gains are applied to income earned on investments that return a profit or "capital gain" and not on income derived from labor. To the extent that such capital gains taxes are taxes on the return on capital, they will impact the rental rate of capital in ways we will explore more fully in a later chapter. For now, we will simply investigate the impact of a capital-gains-tax-induced increase in the rental price of capital on firms within an industry.*

A: *Suppose you are running a gas station in a competitive market where all firms are identical. You employ weekly labor ℓ and capital k using a homothetic decreasing returns to scale production function, and you incur a weekly fixed cost of F.*

(a) *Begin with your firm's long run weekly average cost curve and relate it to the weekly demand curve for gasoline in your city as well as the short run weekly aggregate supply curve assuming the industry is in long run equilibrium. Indicate by x^* how much weekly gasoline you sell, by p^* the price at which you sell it, and by X^* the total number of gallons of gasoline sold in the city per week.*

Answer: This is illustrated in Graph 14.4 where the long run average cost curve for a gasoline station in panel (a) is U-shaped given the fixed cost. The market demand and supply curves must intersect at a price that causes gasoline stations to make zero long run profit — which occurs at the lowest point of the AC curve. This is indicated by the output price p^* and results in each firm producing x^* in panel (a) (because the marginal cost curve crosses AC at its lowest point) and in the industry producing X^* in panel (b).



Graph 14.4: A tax-induced increase in the rental rate

(b) *Now suppose that an increase in the capital gains tax raises the rental rate on capital k (which is fixed for each gas station in the short run). Does anything change in the short run?*

Answer: Since capital is fixed in the short run and the expense on capital is not a short run cost, it does not impact short run marginal cost curves and thus does not impact the market supply curve S^M. Thus neither the short run supply curve nor the demand curve change — implying p^* remains unchanged, as do x^* and X^*.

(c) *What happens to x^*, p^* and X^* in the long run? Explain how this emerges from your graph.*

Answer: Because of the increased rental rate, the AC curve will shift up and firms would make negative long run profits if they all continued to behave as before. Some firms will exit in the long run, which causes the supply curve S^M to shift to the left. That leftward shift raises output price until the price reaches the new point of the new average cost curve. Thus, we know price will increase — which implies that output in the industry as a whole will fall. Whether x^* — each gasoline station's output — increases or decreases depends on whether the lowest point of the new *AC* curve lies to the right or left of x^*. Without knowing more about the underlying technology of the firm, we can't be sure.

(d) *Is it possible for you to tell whether you will hire more or fewer workers as a result of the capital gains tax-induced increase in the rental rate? To the extent that it is not possible, what information could help clarify this?*

Answer: We can be sure that, for any given output level, each gasoline station will shift away from capital and toward labor. (Put differently, within the isoquant graph, the cost minimizing input bundles will lie on a shallower ray (assuming k is on the vertical axis) from the origin — with a higher labor to capital ratio regardless of how much the gasoline station sells.) We cannot, however, be sure whether the lowest point of the *AC* curve shifts to the right or left — and thus we cannot be sure whether output in each gasoline station will increase or decrease in the new equilibrium. Even if it decreases, the fact that we are switching to a higher labor to capital ratio may imply that the gasoline stations will hire more workers as they hire less capital — and the possibility that the lowest point of the *AC* curve could lie to the right of x^* only reinforces this possibility. It is more likely that the increase in the rental rate will cause an increase in the equilibrium number of workers in each gasoline station if capital and labor are relatively substitutable because, as we showed in chapter 13, it is then more likely that the cross-price labor demand curve (with respect to the rental rate) is upward sloping *even without an equilibrium increase in output price.*

(e) *Is it possible for you to be able to tell whether the number of gasoline stations in the city increases or decreases as a result of the increase in the rental rate? What factors might your answer depend on?*

Answer: Suppose the gasoline sold in the city decreases by x% and the gasoline sold by each firm decreases by y%. Then if $x = y$, the number of gasoline stations remains constant. If $x < y$, however, the number of gasoline stations increases and if $x > y$, the number of gasoline stations falls. Since we cannot be sure how x and y are related to one another from what is given in the exercise, we cannot be sure whether the total number of gasoline stations increases or decreases. It depends on how the lowest point of *AC* curves shifts and by how much.

(f) *Can you tell whether employment of labor in gasoline stations increases or decreases? What about employment of capital?*

Answer: It is not possible to tell from the information here whether labor employment in gasoline stations will increase or decrease. The fact that gas stations will substitute toward labor for any output level implies it is possible that labor employment will increase *if overall sales of gasoline do not fall sufficiently to offset this.* It is, however, possible to predict that less capital will be employed in the gasoline stations sector. This is because we know that overall output in the industry declines (because of the higher output price) *and* that firms will substitute away from capital for any output level.

B: *Suppose that your production function is given by* $f(\ell,k) = 30\ell^{0.4}k^{0.4}$, $F = 1080$ *and the weekly city-wide demand for gallons of gasoline is* $x(p) = 100{,}040 - 1{,}000p$. *Furthermore, suppose that the wage is* $w = 15$ *and the current rental rate is* 32.1568. *Gasoline prices are typically in terms of tenths of cents — so express your answer accordingly.*

(a) *Suppose the industry is in long run equilibrium in the absence of capital gains taxes. Assuming that you can hire fractions of hours of capital and produce fractions of gallons of gasoline, how much gasoline will you produce and at what price do you sell your gasoline? (Use the cost function derived for Cobb-Douglas technologies given in equation (13.45) in exercise 13.5 (and remember to add the fixed cost).)*

Answer: The cost function for a Cobb-Douglas production process $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ is

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)}. \qquad (14.22)$$

Evaluating this at $\alpha = \beta = 0.4$, $A = 30$, $w = 15$ and $r = 32.1568$, and adding the fixed cost of \$1,080, this gives us

$$C(x) = 0.62562x^{1.25} + 1080. \qquad (14.23)$$

From this, we can derive the AC function

$$AC(x) = 0.62562x^{0.25} + \frac{1080}{x}. \tag{14.24}$$

To determine where this reaches its lowest point (and where long run profit is thus zero), we set the derivative with respect to x to zero,

$$\frac{dAC(x)}{dx} = \frac{0.156405}{x^{0.75}} - \frac{1080}{x^2} = 0, \tag{14.25}$$

and then solve for x. This gives us a weekly output of $x \approx 1,178.524$ gallons of gasoline. Evaluating $AC(x)$ at this output level, we get a long run equilibrium price of \$4.582 per gallon.

(b) *How many gasoline stations are there in your city?*

Answer: To determine the total weekly gasoline demand in the city when $p = 4.582$, we evaluate the demand function $x(p) = 100,040 - 1,000p$ at $p = 4.582$ to get $x = 95,458$. With each gas station producing 1,178.524 gallons, this implies a total number of gas stations of $95,458/1178.524 = 81$.

(c) *Now suppose the government's capital gains tax increases the rental rate of capital by 24.39% to \$40. How will your sales of gasoline be affected in the new long run equilibrium?*

Answer: Evaluating the cost function in equation (14.22) at $\alpha = \beta = 0.4$, $A = 30$, $w = 15$ and $r = 40$, and adding the fixed cost of \$1,080, we get

$$C(x) = 0.6977554x^{1.25} + 1080. \tag{14.26}$$

This gives us an average cost function of

$$AC(x) = 0.6977554x^{0.25} + \frac{1080}{x} \tag{14.27}$$

whose first derivative set is

$$\frac{dAC(x)}{dx} = \frac{0.17443885}{x^{0.75}} - \frac{1080}{x^2}. \tag{14.28}$$

Setting this to zero and solving for x, we then get weekly sales of $x = 1,080$ gallons of gasoline, down from the previous 1,178.524.

(d) *What is the new price of gasoline?*

Answer: Evaluating the $AC(x)$ function at $x = 1080$, we get a long run equilibrium price of \$5 per gallon.

(e) *Will you change the number of workers you hire? How about the hours of capital you rent?*

Answer: We can either derive the conditional labor and capital demands by using Shephard's Lemma and evaluating the demands at the output level, or we can use the unconditional labor and capital demand functions derived from profit maximization. Either way, you should get that the quantity of labor hired remains constant at 144 worker hours, but the quantity of capital employed falls from 67.17 units to 54 units.

(f) *Will there be more or fewer gasoline stations in the city? How is your answer consistent with the change in the total sales of gasoline in the city?*

Answer: At the new price of \$5 per gallon, the total amount of gasoline demanded in the city is $x = 100,040 - 1,000(5) = 95,040$ — down from the initial 95,458. With each gas station producing 1,080 gallons per week, this implies that the total number of gas stations is 95040/1080=88 — up from the initial 81 gas stations. Thus, 7 new gas stations enter the market, but each gas station produces sufficiently less such that the total amount of gasoline sold in the city declines.

(g) *What happens to total employment at gasoline stations as a result of the capital gains tax? Explain intuitively how this can happen. Do you think this is a general result or one specific to the set-up of this problem?*

Answer: Initially we had 81 gasoline stations, each employing 144 worker hours. In the new equilibrium, we have 88 gasoline stations — each also employing 144 worker hours. Thus, the new equilibrium has 1,008 more labor hours employed in gasoline stations as a result of the capital gains tax. This occurs because each gasoline station, while employing the same number of workers as before, is now selling less gasoline and hiring less capital. Thus, the labor to capital ratio in each gasoline station has increased — as we would expect when firms substitute from the more expensive capital toward labor for any given output level. While the industry as a whole is selling slightly less gasoline at the new higher price, it is producing that gasoline using this higher labor to capital ratio — resulting in an increase in employment.

(h) *Which of your conclusions do you think is qualitatively independent of the production function used (so long as it is decreasing returns to scale), and which do you think is not?*

Answer: Since average costs increase, the conclusion that price will increase is independent of the production function employed in the analysis. This also implies that the conclusion that the industry will reduce output is independent of the production function. Because each firm will substitute away from the more expensive capital and toward labor for any given output level, we know that it must be the case that the labor to capital ratio increases for each firm. However, the lowest point of the average cost curve could in principle shift to the left — causing each firm to produce more rather than less, which further implies that the number of gasoline stations might decrease (rather than increase). If production in each firm were to increase, it is in principle possible for capital to also increase (rather than decrease as we calculated) even though the labor to capital ratio increases. Similarly, the amount of labor used by each gasoline station might increase or decrease — depending not only on whether overall output by each station increases but also on the relative substitutability of capital and labor in production. (Even if output decreases at each gas station, labor might also decrease if capital and labor are relatively complementary in production).

(i) *Which of your conclusions do you think is qualitatively independent of the demand function, and which do you think is not?*

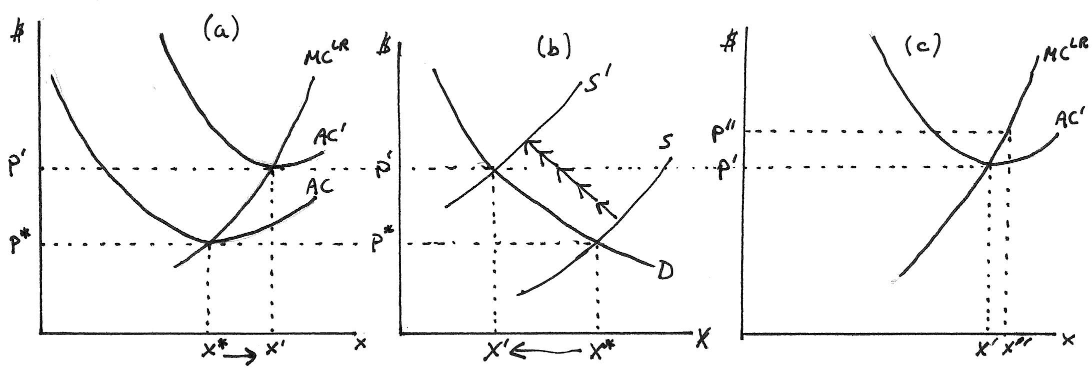
Answer: Since the price of gasoline will definitely increase, any downward sloping demand curve will give the result that overall gasoline consumption in the city will decline. How much the price increases is also independent of the demand function — because the long run equilibrium price is derived from the *AC* function. As long as the production function is the same, all our conclusions about what will happen to an individual gas station that continues to exist therefore remain unchanged regardless of what the demand function looks like. But if demand is more responsive to price than what we have modeled, it might fall sufficiently at the new price such that the number of gas stations falls (rather than rises) in the new equilibrium.

14.7 Business and Policy Application: *Using License Fees to Make Positive Profit: Suppose you own one of many identical pharmaceudical companies producing a particular drug x.*

A: *Your production process has decreasing returns to scale but you incur an annually recurring fixed cost F for operating your business.*

(a) *Begin by illustrating your firm's average long run cost curve and identify your output level assuming that the output price is such that you make zero long run profit.*

Answer: This is illustrated in panel (a) of Graph 14.5 where the initial long run average cost curve — AC — is U-shaped because of the fixed cost F. Zero long run profit implies you are producing at the lowest point of that AC curve — quantity x^* sold at price p^*.



Graph 14.5: A Large License Fee

(b) *Next to your graph, illustrate the market demand and short run market supply curves that justify the zero-profit price as an equilibrium price.*

Answer: This is done in panel (b) of the graph where the initial supply S intersects demand D at price p^*.

(c) *Next, suppose that the government introduces an annually recurring license fee G for any firm that produces this drug. Assume that your firm remains in the industry. What changes in your firm and in the market in both the short and long run as a result of the introduction of G and assuming that long run profits will again be zero in the new long run equilibrium?*

Answer: Since this is a fixed cost, it has no short run impact. In the long run, it does not shift the long run MC curve in panel (a) of Graph 14.5, only the long run AC curve which is illustrated as AC'. This implies that the lowest point of AC' lies to the right of the lowest point of the initial AC. Once the industry settles into a new long run equilibrium where all firms make zero profit, it must then be that the new equilibrium price p' falls at the lowest point of AC' causing each firm to produce x'. The license fee G therefore increases the output level in each firm that remains in the industry. However, overall production in the industry falls (in panel (b)) from X^* to X' as consumers demand less at the higher price p'. The fact that each firm is producing more but the industry is producing less implies that a number of firms must have exited on the way to the new long run equilibrium.

(d) *Now suppose that G is such that the number of firms required to sustain the zero-profit price in the new long run equilibrium is not an integer. In particular, suppose that we would require 6.5 firms to sustain this price as an equilibrium in the market. Given that fractions of firms cannot exist, how many firms will actually exist in the long run?*

Answer: If 7 firms produced, the price would be driven below p' and all firms would make negative long run profit. Thus, it must be that one more firm exits — and only 6 remain in the industry.

(e) *How does this affect the long run equilibrium price, the long run production level in your firm (assuming yours is one of the firms that remains in the market), and the long run profits for your firm?*

Answer: This is illustrated in panel (c) of Graph 14.5. We begin by replicating the new AC' as well as the (unchanged) long run MC curves from panel (a) — identifying again the zero profit price p' after the new fee G has been introduced. But if only 6 firms exist and it would have taken 6.5 to produce X' (in panel (b)) when each firm produces x', supply shifts further to the left as the 7th firm exits, driving price somewhat above p'. This reduces the quantity demanded and increases the quantity supplied by the remaining firms. In panel (c), the price p'' therefore lies above the zero profit price p' — with each of the remaining firms now producing $x'' > x'$. And, since price is now above the lowest point of the long run AC curve, each firm will make some positive economic profit.

(f) *True or False: Sufficiently large fixed costs may in fact allow identical firms in a competitive industry to make positive long run profits.*

Answer: This is, as we have just demonstrated, true.

(g) True or False*: Sufficiently large license fees can cause a competitive industry to become more* concentrated *— where by "concentrated" we mean fewer firms competing for each customer.*

Answer: This is true. As we have shown, an increase in fixed costs such as license fees will cause an upward and rightward shift of the long run average cost curves of firms — causing each firm to produce a larger quantity at a higher price in the new long run equilibrium. Because of the higher price, the industry produces less. Thus, we'll have fewer firms, with each firm attracting *more customers* — i.e. fewer firms compete for each given customer.

B: *Suppose that each firm in the industry uses the production function $f(\ell,k) = 10\ell^{0.4}k^{0.4}$ and each incurs a recurring annual fixed cost of $175,646.*

(a) *Determine how much each firm produces in the long run equilibrium if $w = r = 20$. (You can use the cost function derived for Cobb-Douglas technologies given in equation (13.45) in exercise 13.5 (and remember to add the fixed cost).)*

Answer: For a Cobb-Douglas function of $f(\ell,k) = A\ell^{\alpha}k^{\beta}$, the cost function (in the absence of fixed costs) is

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)}. \tag{14.29}$$

Dividing this by x and adding the average fixed cost FC/x, we then get the long run average cost function

$$AC(w,r,x) = \frac{C(w,r,x)}{x} + \frac{FC}{x} = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{FC}{x}. \tag{14.30}$$

Evaluating this at $\alpha = \beta = 0.4$, $A = 10$, $w = r = 20$ and $FC = 175,646$, this reduces to

$$AC(x) \approx 2.249x^{1/4} + \frac{175,646}{x}. \tag{14.31}$$

The lowest point of this AC curve occurs where its derivative is equal to zero; i.e. where

$$\frac{dAC(x)}{dx} = \frac{0.5623}{x^{3/4}} - \frac{175646}{x^2} = 0. \tag{14.32}$$

Solving this for x, we get $x \approx 24,873$. This is how much each firm is producing in long run equilibrium.

(b) *What price are consumers paying for the drugs produced in this industry?*

Answer: Substituting $x = 24,873$ back into the average cost function $AC(x)$ from equation (14.31), we get that the long run equilibrium price must be approximately $35.31.

(c) *Suppose consumer demand is given by the aggregate demand function $x(p) = 1,000,000 - 10,000p$. How many firms are in this industry?*

Answer: Plugging in the long run equilibrium price $p = 35.31$, we get consumer demand of $x = 1,000,000 - 10,000(35.31) = 646,900$. With each firm producing 24,873 units, this implies that there are $646,900/24873 = 26$ firms in the industry.

(d) *Suppose the government introduces a requirement that each company has to purchase an annual operating license costing \$824,354. How do your answers to (a), (b) and (c) change in the short and long run?*

Answer: Since this is an added fixed cost that is only a fixed expense in the short run, it does not affect anything in the short run. When added to the fixed cost of \$175,646, this annual fee increases the total fixed costs to \$1,000,000. This changes the long run AC curve to

$$AC(x) = 2.249x^{1/4} + \frac{1,000,000}{x} \tag{14.33}$$

and its derivative to

$$\frac{dAC(x)}{dx} = \frac{0.5623}{x^{3/4}} - \frac{1,000,000}{x^2}. \tag{14.34}$$

Setting the derivative to 0 and solving for x, we now get $x = 100,000$, up from the previous 24,873. Plugging this back into $AC(x)$, we get a long run equilibrium price of $p = 50$, up from the previous 35.31. At this price, consumers demand $x = 1,000,000 - 10,000(50) = 500,000$ units. With each firm producing 100,000 units, this leaves room for only 5 firms, down from the previous 26.

(e) *Are any of the firms that remain active in the industry better or worse off in the new long run equilibrium?*

Answer: No — they make zero profit before and again after the change to the new long run equilibrium.

(f) *Suppose instead that the government's annual fee were set at \$558,258. Calculate the price at which long run profits are equal to zero.*

Answer: When added to the fixed cost of \$175,646, this annual fee increases the total fixed costs to \$773,904. This changes the long run AC curve to

$$AC(x) = 2.249x^{1/4} + \frac{773,904}{x} \tag{14.35}$$

and its derivative to

$$\frac{dAC(x)}{dx} = \frac{0.5623}{x^{3/4}} - \frac{773,904}{x^2}. \tag{14.36}$$

Setting the derivative to 0 and solving for x, we now get approximately $x = 78,075$, up from the initial 24,873 but down from the 100,000 under the higher license fee. Plugging this back into $AC(x)$, we get a zero long run profit price of $p = 47$, up from the previous 35.31 but below the 50 under the higher license fee.

(g) *How many firms would this imply will survive in the long run assuming fractions of firms can operate?*

Answer: At a price of $p = 47$, consumers demand $x = 1,000,000 - 10,000(47) = 530,000$ units. With each firm producing 78,075 units, this would imply approximately $530,000/78,075 = 6.79$ firms in the market.

(h) *Since fractions of firms cannot operate, how many firms will actually exist in the long run? Verify that this should imply an equilibrium price of approximately \$48.2.(*Hint*: Use the supply function given for a Cobb-Douglas production process in equation (13.75) found in the footnote to exercise 13.7.)*

Answer: Since 7 firms cannot exist in the market, only 6 can survive. But if 6 firm produced 78,075 units at the zero long run profit price $p = 47$, only 468,450 units would be produced — which is 61,550 units less than the 530,000 units demanded by consumers at that price.

Thus, in order for the market to clear in the long run, price has to increase. In order to determine by how much, we have to first derive the long run supply curve for each firm. The supply function for a production process $f(\ell, k) = A\ell^{\alpha}k^{\beta}$ is

$$x(w,r,p) = \left(\frac{Ap^{(\alpha+\beta)}\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)^{1/(1-\alpha-\beta)} \quad (14.37)$$

which, when evaluated at $\alpha = \beta = 0.4$, $A = 10$, and $w = r = 20$, becomes

$$x(p) = 0.016p^4. \quad (14.38)$$

Multiplying this by 6 — which is the number of firms remaining in the industry, we get

$$X^{LR}(p) = 0.096p^4. \quad (14.39)$$

In an equilibrium with 6 firms, the equilibrium price then occurs where this supply function equals the demand function $x(p) = 1,000,000 - 10,000p$. The equation $0.096p^4 = 1,000,000 - 10,000p$ holds at $p = 48.197082195$ or approximately $p = 48.2$.

(i) *What does this imply for how much profit each of the remaining firms can actually make?*

Answer: At a price of \$48.2, equation (14.38) implies that the firm will produce output of approximately $x = 86,338$ which implies total revenues of $48.2(86,338) \approx 4,161,492$. Using equation (14.35), we can determine its long run average cost at output 86,338 to be

$$AC = 2.249(86,338^{1/4}) + \frac{773,904}{86,338} \approx 47.058. \quad (14.40)$$

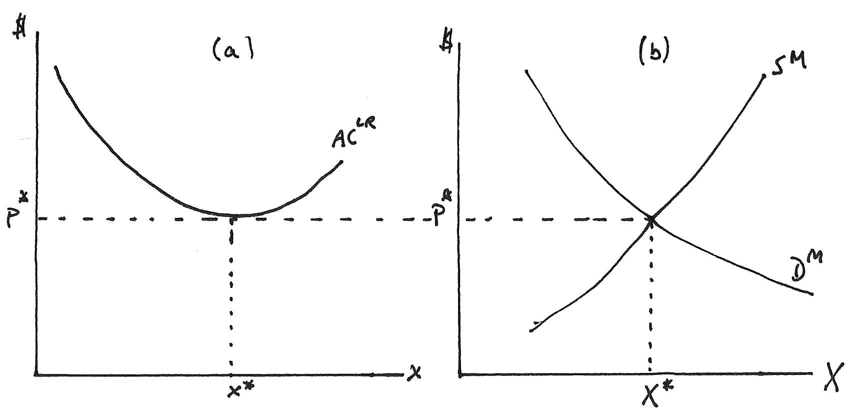
This implies total costs of $47.058(86,338) \approx 4,062,894$. Subtracting this from total revenues of 4,161,492, we get long run profit of approximately \$98,598.

14.8 Policy and Business Application: *Business Taxes (cont'd)*: *In exercise 13.11, we introduced a number of possible business taxes and asked what a firm's response would be* assuming that prices w, r and p remained unchanged. *Now that we have introduced the notion of equilibrium price formation, we can revisit the exercise.*

A: *Suppose the restaurant industry is in long run equilibrium, all restaurants use the same homothetic decreasing returns to scale technology and all have to pay a fixed annual franchise fee F.*

(a) *Illustrate the average cost curve for a restaurant and the related (short run) supply and demand graph for the industry.*

Answer: This is done in Graph 14.6.



Graph 14.6: Long Run Equilibrium

(b) *Revisit parts A(b) through A(h) of exercise 13.11 and explain whether the assumption that prices remained unchanged was warranted and, if not, why not.*

Answer: In (b) of exercise 13.11, a tax on last year's profits is not an economic cost of doing business today — so it does not change any of the curves in Graph 14.6 and therefore leaves the equilibrium unchanged. The assumption that prices do not change therefore holds.

(c) Answer: A tax on short run profits is equivalent to an increase in the recurring fixed cost F — which raises the long run AC curve without changing the long run MC curve. As a result, the lowest point of the long run AC curve shifts up and to the right — causing equilibrium output price to rise. The assumption that prices do not change therefore does not hold in the long run. (It does hold in the short run since a tax on short run profits is not a real economic cost in the short run.)

(d) Answer: Long run profits are zero — so taxing them does not change anything. The assumption that prices remain unchanged is therefore valid.

(e) Answer: Same answer as that to part (c).

(f) Answer: This shifts both the long run AC curve and the long run MC curve — which implies that the lowest point of the AC curve increases (but we cannot be sure whether it moves to the right or left). The higher AC implies long run price increases in equilibrium — thus the assumption that prices do not change is false. (Nothing changes in the short run if capital is fixed in the short run — so the assumption that prices remain unchanged is true in the short run.)

(g) Answer: The same holds in the long run as what we concluded for the tax on capital in part (f). In the short run, the short run MC curve shifts up — which implies the short run market supply curve shifts to the left — which implies output price increases. The assumption that prices do not change is therefore violated even in short run equilibrium.

(h) Answer: This shifts the long run AC up by t at every point — which implies the lowest point of the AC curve shifts up by t (but neither to the right or to the left). As a result, output price increases by t in the long run. In the short run, the short run MC curve increases — which implies the market supply curve shifts left — which implies output price rises. Thus, the assumption that prices do not change is invalid in both the short and long run in this case.

B: *Consider the same technology as the one used in exercise 13.11 as well as the recurring fixed cost F.*

(a) *Determine the long run equilibrium price p^* and output level x^* as a function of A, α, β w and r. (You can use the cost function given in equation (13.45) in exercise 13.5 as well as the profit function given equation (13.48) in exercise 13.7.)*

Answer: Adding the fixed cost F to the cost function and dividing by x, we get the average cost function

$$AC(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{F}{x} \qquad (14.41)$$

which reaches its lowest point when

$$\frac{\partial AC(w,r,x)}{\partial x} = \left[(1-\alpha-\beta)\left(\frac{w^{\alpha}r^{\beta}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} x^{(1-2(\alpha+\beta))/(\alpha+\beta)}\right] - \frac{F}{x^2} = 0. \qquad (14.42)$$

Solving this for x, we then get the output level at the lowest point of the long run AC curve:

$$x^* = \left(\frac{A\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(\alpha+\beta)}. \qquad (14.43)$$

If we plug this back into the AC function, we get the average cost at the lowest point of the AC curve — which has to be equal to the long run equilibrium price p^*. Alternatively, we can take the profit function, set it equal to F and solve for p to get

$$p^* = \left(\frac{w^{\alpha}r^{\beta}}{A\alpha^{\alpha}\beta^{\beta}}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(1-\alpha-\beta)} \qquad (14.44)$$

(b) *In exercise 13.11, we focused on the impact of policies from A(b) through A(h) on output supply and input demand functions. Now use your result from (a) to determine the impact of each of these policies on the long run equilibrium price and firm output level.*

Answer: For (b), a tax on last year's profits is not an economic cost of producing now (since nothing the firm does now has any impact on the size of the tax) — and it therefore enters neither the equation for x^* nor p^* — so nothing changes.

(c) Answer: A 50% tax on short run profits effectively increases the recurring fixed cost F. Thus the equations for x^* and p^* indicate that both firm output and the equilibrium price will rise as a result of this tax.

(d) Answer: Long run profits are zero — so a 50% tax on long run profits affects nothing and leaves the long run equilibrium price and firm quantity unchanged.

(e) Answer: This causes the same long run effect as the 50% tax on short run profits in part (c).

(f) Answer: The derivative of x^* with respect to r is negative — which implies that the equilibrium firm output level will fall as capital costs $(1+t)r$ rather than r. The derivative of p^* is positive — which implies that the equilibrium output price will increase.

(g) Answer: The answer is the same as for the tax on capital — except that now we would look at the derivatives of x^* and p^* with respect to w.

(h) Answer: The tax of t per hamburger takes a little more work. Since the tax is the same t for every hamburger produced, the average cost of each hamburger produced goes up by t — i.e.

$$AC(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha} r^{\beta} x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{F}{x} + t. \tag{14.45}$$

When we take the first derivative of this and set it to zero, however, t simply drops out — which leaves us with the same x^* as before. Thus, x^* — the equilibrium firm output level — is unaffected by t. Since every point on the AC curve shifts up by t, however, the lowest point must also shift up by t — which implies that the new p^* is

$$p^* = \left(\frac{w^{\alpha} r^{\beta}}{A\alpha^{\alpha}\beta^{\beta}}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(1-\alpha-\beta)} + t. \tag{14.46}$$

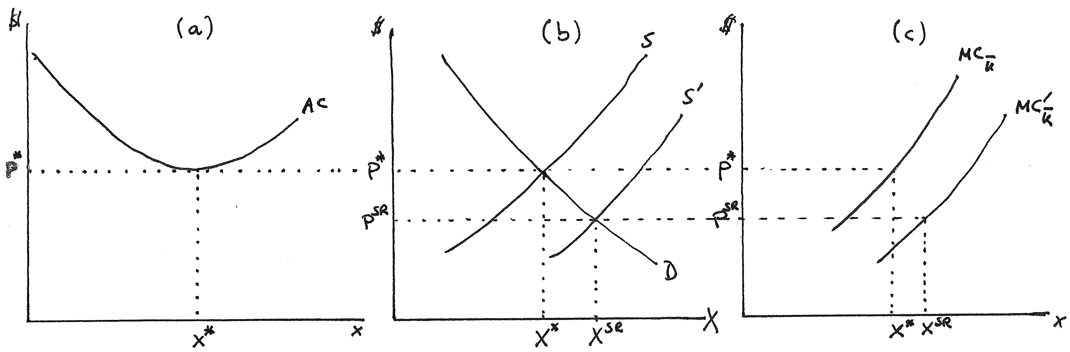
Thus, the equilibrium price rises by t — and the entire tax is passed onto consumers.

14.9 Policy and Business Application: *Minimum Wage Labor Subsidy (cont'd): In exercise 13.10, we investigated the firm's decisions in the presence of a government subsidy for hiring minimum wage workers. Implicitly, we assumed that the policy has no impact on the prices faced by the firm in question.*

A: *Suppose again that you operate a business that uses minimum wage workers ℓ and capital k. The minimum wage is w, the rental rate for capital is r and you are one of many identical businesses in the industry, each using a homothetic, decreasing returns to scale production process and each facing a recurring fixed cost F.*

(a) *Begin by drawing the average cost curve of one firm and relating it to the (short run) supply and demand in the industry assuming we are in long run equilibrium.*

Answer: This is done in panels (a) and (b) of Graph 14.7 where the long run equilibrium price p^* occurs at the intersection of the original market demand D and supply S curves (in panel (b)) and falls at the lowest point of each firm's average cost curve (in panel (a)).



Graph 14.7: Minimum Wage Subsidy

(b) *Now the government introduces a wage subsidy s that lowers the effective cost of hiring minimum wage workers from w to $(1-s)w$. What happens in the firm and in the industry in the short run?*

Answer: This is illustrated in panels (b) and (c) of Graph 14.7. Each individual firm's short run marginal cost curve (given the fixed level of capital $\overline{k}$) shifts down — from $MC_{\overline{k}}$ to $MC'_{\overline{k}}$ in panel (c). Since this is happening to all firms, this implies that the short run market supply curve (in panel (b)) shifts down — from S to S'. As a result, the new short run equilibrium price falls to p^{SR} (in panel (b)) — and each firm produces more (in panel (c)) at that price. (In principle it is possible to draw these graphs such that the price falls and each firm produces less. However, that is a logical impossibility — because the number of firms is fixed in the short run. If the market overall produces more, it must be that each firm produces more in the short run).

(c) *What happens to price and output (in the firm and the market) in the long run compared to the original quantities?*

Answer: In the long run, the AC curve in panel (a) of Graph 14.7 shifts down — but we cannot be sure whether the lowest point shifts right or left. (The more likely case is that it shifts to the right). Thus, in the new long run equilibrium, we know the price has to settle below p^* — with each firm producing more or less than originally depending on whether the lowest point of the AC curve falls to the right or left of x^*. Since price is below p^*, we can also be sure that the overall quantity produced in the market will increase.

(d) *Is it possible to tell whether there will be more or fewer firms in the new long run equilibrium?*

Answer: No, it is not. It could be that each firm produces sufficiently more in the new long run equilibrium and the overall quantity demanded increases relatively less such that fewer firms can be sustained in the new equilibrium. But the reverse is also possible.

(e) *Is it possible to tell whether the long run price will be higher or lower than the short run price? How does this relate to your answer to part (d)?*

Answer: No, it is not possible to tell for sure. This relates to (d) in the sense that it relates to whether additional firms will enter or existing firms will exit in the transition from the short run to the long run equilibrium. If conditions are such that the number of firms falls, this implies that the exit of firms from the industry will put upward pressure on price relative to its short run value. If, on the other hand, conditions are such that the number of firms increases, then this implies that the entry of new firms puts additional downward pressure on price — causing the long run price to fall below the short run price.

B: *Suppose that the firms in the industry use the production technology* $x = f(\ell, k) = 100\ell^{0.25}k^{0.25}$ *and pay a recurring fixed cost of* $F = 2,210$. *Suppose further that the minimum wage is $10 and the rental rate of capital is* $r = 20$.

(a) *What is the initial long run equilibrium price and firm output level?*

Answer: Plugging the production function values and input prices into the cost function for Cobb-Douglas production, we get a cost function of $C(x) = 0.00282843x^2$, and adding the fixed cost F, we get $C(x) = 0.00282843x^2 + 2210$. This gives us the average cost function $AC(x) = 0.00282843x + 2210/x$. Taking the first derivative, setting it to zero and solving for x then gives us the output level at the lowest point of the AC curve — which is $x \approx 884$. Plugging this back into the AC function, we then get that the long run equilibrium price is $5. Each firm therefore sells 884 output units at a price of $5 per unit.

(b) *Suppose that* $s = 0.5$ *— implying that the cost of hiring minimum wage labor falls to $5. How does your answer to (a) change?*

Answer: The new long run equilibrium is then derived exactly as it was in (a) except that $w = 5$ is substituted in the first step when we derive the cost function from the general Cobb-Douglas form of the cost function. This gives us $C(x) = 0.002x^2$ and, once we go through the remaining steps, $x \approx 1,051$ as the output quantity at the lowest point of the AC curve. The long run (zero profit) price is approximately $4.20.

(c) *How much more or less of each input does the firm buy in the new long run equilibrium compared to the original one? (The input demand functions for a Cobb-Douglas production process were previously derived and given in equation (13.50) of exercise 13.8.)*

Answer: Substituting $A = 100$, $\alpha = 0.25 = \beta$ and $r = 20$ into the labor and capital demand equations, we get

$$\ell(w,p) \approx 139.75\left(\frac{p^2}{w^{3/2}}\right) \text{ and } k(w,p) \approx 6.9877\left(\frac{p^2}{w^{1/2}}\right). \tag{14.47}$$

In the initial equilibrium, $(w,p) = (10,5)$ while in the new equilibrium, $(w,p) = (5,4.2)$. Substituting these into the above equations, we then get that an initial input bundle $(\ell,k) = (110.5, 55.25)$ and a new input bundle $(\ell,k) = (221, 55.25)$. (Answers may differ slightly due to rounding errors.) Labor input therefore doubled but capital input remained unchanged.

(d) *If price does not affect the quantity of* x *demanded very much, will the number of firms increase or decrease in the long run?*

Answer: If the quantity demanded (and thus the quantity produced by the industry) remains roughly the same, the number of firms in the industry must decline since each firm is now producing 1,051 units of output rather than the initial 884.

(e) *Suppose that demand is given by* $x(d) = 200,048 - 2,000p$. *How many firms are there in the initial long run equilibrium?*

Answer: In the initial long run equilibrium, $p = 5$. This implies that the total quantity demanded is $200,048 - 2,000(5) = 190,048$. Each firm produces 884 units initially, which implies that we have 215 firms operating. (Your answer may be slightly below 215 because of rounding error when we use 884 units per firm rather than 883.84 which is the more exact number returned by the math.)

(f) *Derive the short run market supply function and illustrate that it results in the initial long run equilibrium price.*

Answer: To derive the short run market supply function, we need to first determine the short run supply function of each of the 215 firms in the initial equilibrium. We concluded in (c) that each firm is using 55.25 units of capital in that initial equilibrium. In the short run, when capital is fixed, each firm is therefore operating on the slice

$$f_{k=55.25}(\ell) = \left[100(55.25)^{0.25}\right]\ell^{0.25} = 272.636\ell^{0.25}. \qquad (14.48)$$

Solving the short run profit maximization problem

$$\max_{\ell}\; p\left(272.636\ell^{0.25}\right) - w\ell, \qquad (14.49)$$

we get

$$\ell_{k=55.25}(w,p) = 278.42\left(\frac{p}{w}\right)^{3/4} \text{ and } x_{k=55.25}(w,p) = 1113.67\left(\frac{p}{w}\right)^{1/3}, \qquad (14.50)$$

the latter of which is the firm's short run supply function. Multiplying this by the number of firms in the industry (which we derived as 215), we get a short run market supply function

$$X^{SR}(w,p) = 239{,}440\left(\frac{p}{w}\right)^{1/3} \qquad (14.51)$$

which becomes

$$X^{SR}(p) = 111{,}138p^{1/3} \text{ when } w = 10. \qquad (14.52)$$

At the original equilibrium, it must be that demand is equal to this short run market supply — i.e.

$$200{,}048 - 2{,}000p = 111{,}138p^{1/3}, \qquad (14.53)$$

which holds (approximately, due to some rounding) for our initial long run equilibrium price $p = 5$.

(g) *Verify that the short run equilibrium price falls to approximately \$2.69 when the wage is subsidized.*

Answer: When wage falls to $w = 5$, the short run market supply curve in equation (14.51) becomes

$$X^{SR}(p) = 140{,}025p^{1/3}. \qquad (14.54)$$

The short run equilibrium then occurs where supply equals demand; i.e. where

$$140{,}025p^{1/3} = 200{,}048 - 2{,}000p \qquad (14.55)$$

which (approximately) holds when $p = 2.69$.

(h) *How much does each firm's output change in the short run?*

Answer: Plugging the price $p = 2.69$ and subsidized wage $w = 5$ into the short run supply function for each firm, we get $x_{k=55.25}(5, 2.69) \approx 906$. Note that this sums approximately to the overall quantity transacted in the market when there are (the initial) 215 firms in the market.

(i) *Determine the change in the long run equilibrium number of firms when the wage is subsidized and make sense of this in light of the short run equilibrium results.*

Answer: We previously determined that the long run equilibrium price will be roughly \$4.20 — which is lower than the initial price of \$5 but higher than the short run price of \$2.69. At \$4.20, we can determine the total level of output in the industry by substituting this price into the demand function to get $x = 191{,}648$ — and with each firm producing 1,051 in the new long run equilibrium, this implies approximately 182 firms — down from the initial 215.

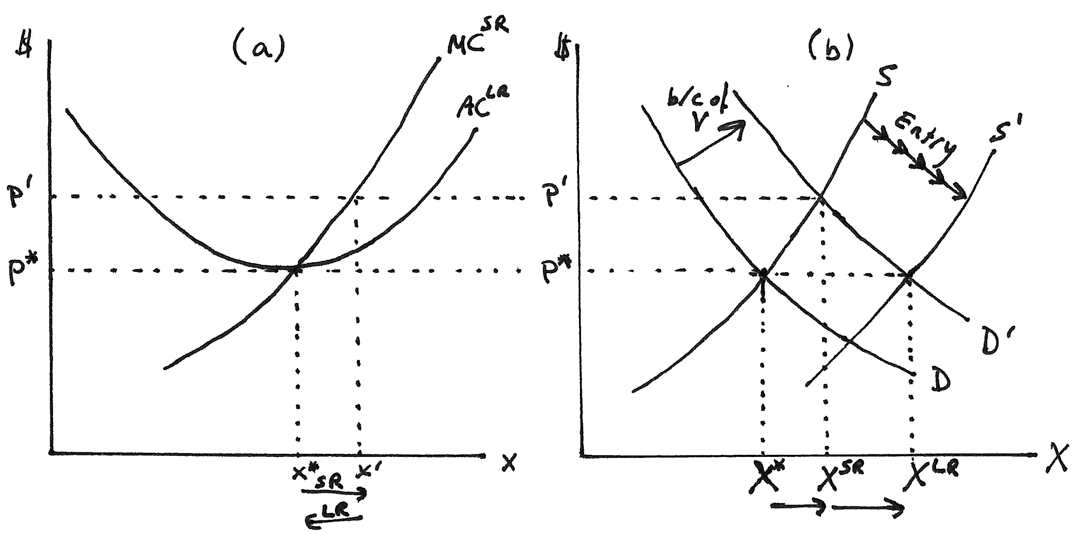
We can see the dynamics of what makes firms choose to exit by observing that the short run equilibrium price of \$2.69 lies below the long run zero-profit price of \$4.20 — thus firms are making negative long run profits in the short run equilibrium (while still making positive short run profits since the expense on capital and the fixed costs are not real costs in the short run).

14.10 Policy Application: *School Vouchers and the Private School Market: In the U.S., private schools charge tuition and compete against public schools that do not. One policy proposal that is often discussed involves increasing demand for private schools through* school vouchers. *A school voucher is simply a piece of paper with a dollar amount V that is given to parents who can pay for some portion of private school tuition with the voucher if they send their child to a private school. (Private schools can then redeem the vouchers for a payment of V from the government.) Assume throughout that private schools strive to maximize profit.*

A: *Suppose private schools have U-shaped average (long run) cost curves, and the private school market in a metropolitan area is currently in long run equilibrium (in the absence of private school vouchers).*

(a) *Begin by drawing a school's average long run cost curve (with the number of private school seats on the horizontal axis). Then, in a separate graph next to this, illustrate the city-wide demand curve for seats in private schools as a function of the tuition price p. Finally, include the short run aggregate supply curve that intersects with demand at a price that causes the private school market to be in long run equilibrium.*

Answer: This is done in Graph 14.8. In order for the private school market to be in long run equilibrium, each private school makes zero profit and thus operates on the lowest point of its long run AC curve. This implies that our initial supply S and demand D must intersect at price p^* and each private school educates x^* children.



Graph 14.8: Private School Markets and School Vouchers

(b) *Illustrate what happens to the demand curve as a result of the government making available vouchers in the amount of V to all families who live in the city. What happens to the number of seats made available in each existing private school, and what happens to the tuition level p in the short run?*

Answer: In panel (b) of Graph 14.8, we illustrate a shift out in the demand curve as parents are now more willing to pay for private schools. This shift in demand causes a short run increase in price to p' — with each school providing x' seats. Thus each school will open more seats in the short run at a higher tuition level, and more children in the city will attend private schools.

(c) *Next, consider the long run when additional private schools can enter the market. How does the tuition level p, the number of seats in each school and the overall number of children attending private schools change?*

Answer: In the short run, existing private schools will make profit as they operate at a price p' that lies above the lowest point on their AC curves. This will induce entry into the private school market — with new private schools entering. And this will in turn cause the short run supply curve in panel (b) of Graph 14.8 to shift out with each new entrant. The entry of new schools will continue so long as there are still positive profits in the private school market — i.e. so long as price remains above the original price p^*. (This assumes that entering schools have the same cost structure as existing schools — if they are less efficient, the process of entry would end before p^* is reached.) In the long run, with price falling back to p^*, we would then expect each private school to go back to supplying seats for x^* children, but the private school market will increase its supply of seats from the short run quantity of X^{SR} to X^{LR} (in panel (b) of the graph).

(d) *Opponents of private school vouchers sometimes express concern that the implementation of vouchers will simply cause private schools to increase their tuition level and thus cause no real change in who attends private school. Evaluate this concern from both a short and long run perspective.*

Answer: This concern is more valid in the short run than in the long run, but even in the short run it is not entirely correct. As we showed, it is certainly the case that tuition would increase in the short run — but we also showed that the number of children attending private schools would increase. So there would, even in the short run, be a real change in terms of who goes to school where — with the size of the change depending on just how steep short run MC curves are. In the long run, the concern appears largely invalid because — if schools can indeed increase tuition in response to voucher-induced demand, new schools will enter the market and eventually drive tuition price back down to its original level.

(e) *Proponents of private school vouchers often argue that the increased availability of private schools will cause public schools to offer higher quality education. If this is true, how would your answers to (b) and (c) change as a result?*

Answer: If proponents are correct, then parental demand for private schools should fall as private school competition increases and improves public schools. Thus, in the short run tuition levels would not rise as much. In the long run, tuition levels would fall back to p^*, but the overall increase of the market would be smaller (i.e. fewer private schools would enter.)

(f) *If private school vouchers are made available to anyone who lives within the city boundaries (but not to those who live in suburbs), some families who previously chose to live in suburbs to send their children to suburban public schools might choose instead to live in the city and send their children to private schools. How would this affect your answers to (b) and (c)?*

Answer: This would exacerbate the increase in demand from the voucher as this demand would now come not only from parents in cities but also from parents that move to the city because of vouchers. Although it seems unlikely that such relocations of families will occur in the short run, to the extent that it does it would cause the short run increase in private school tuitions to be larger than otherwise predicted. In the long run, tuition prices would still have to fall to p^* due to entry of new schools — but the overall size of the private school market would now be larger than otherwise predicted (as more firms enter in response to the larger shift in demand.)

B: *In the following, all dollar values are expressed in thousands of dollars. Suppose that the total city-wide demand function for private school seats x is given by* $x(p) = 24710 - 2500p$ *and each private school's average long run cost function is given by* $AC(x) = 0.655x^{1/3} + (900/x)$.

(a) *Verify that* $AC(x)$ *arises from a Cobb-Douglas production function* $x = f(\ell, k) = 35\ell^{0.5}k^{0.25}$ *when* $w = 50$ *and* $r = 25$ *and when private schools face a fixed cost of 900. One unit of x is interpreted as one seat (or one child) in the school, and* ℓ *is interpreted here as a teacher. (Since dollar values are expressed in thousands, w represents a teacher's salary of \$50,000 and the fixed cost represents a recurring annual cost of \$900,000.)*

Answer: In Chapter 13 end of chapter exercises, we derived the AC function for a Cobb-Douglas production function of the form $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ when the firm faces a fixed cost of FC as

$$AC(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{FC}{x}. \tag{14.56}$$

Substituting in $\alpha = 0.5$, $\beta = 0.25$, $A = 35$, $w = 50$, $r = 25$ and $FC = 900$, we get the average cost function specified in the problem.

(b) *In order for the private school market to be in long run equilibrium, how many children are served in each private school? What is the tuition (per seat in the school) charged in each private school?*

Answer: The tuition has to settle at the lowest point of the AC curve — i.e. the point at which its slope is zero. We can thus take the first derivative of $AC(x)$ and set it to zero, which gives

$$\frac{dAC(x)}{dx} = \frac{1}{3}0.655x^{-2/3} - 900x^{-2} = 0. \tag{14.57}$$

Solving this for x, we get $x \approx 515$ — the number of children served in each private school. The average cost with this many children is then

$$AC(515) = 0.655(515^{1/3}) + \frac{900}{515} \approx 7. \tag{14.58}$$

The price for tuition in the private school market is therefore 7 — or $7,000 per student (since dollar amounts are in thousands.)

(c) *Given that you know the underlying production function, can you determine the class size in each private school? (*Hint*: You already determined the total number of children in part (a) and now need to determine the number of teachers in each private school.)*[2]

Answer: In chapter 13 end-of-chapter exercises, we determined that the long run labor demand function for a Cobb-Douglas production process $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ is

$$\ell(w,r,p) = \left(\frac{pA\alpha^{(1-\beta)}\beta^{\beta}}{w^{(1-\beta)}r^{\beta}}\right)^{1/(1-\alpha-\beta)}. \tag{14.59}$$

Substituting in $\alpha = 0.5$, $\beta = 0.25$, $A = 35$, $w = 50$, $r = 25$ and $p = 7$, we get $\ell \approx 36$. Thus, each private school has 36 teachers and admits about 515 students — giving us a class size of $515/36 \approx 14.3$ students per teacher.

(d) *How many private schools are operating?*

Answer: At a tuition price of $p = 7$, we can determine total demand from the demand function:

$$x = 24710 - 2500(7) = 7,210. \tag{14.60}$$

With each school serving about 515 students, this implies that there must be $7210/515 = 14$ private schools in the market.

(e) *Now suppose that the government makes private school vouchers in the amount of 5.35 (i.e. $5,350) per child available to parents. How will this change the demand function for seats in private schools? (*Hint*: Be careful to add the voucher in the correct way — i.e. to make the* demand *curve shift up.)*

Answer: This will shift the demand curve up by 5.35. The demand curve, however, has price on the vertical and x on the horizontal — so we need to write it as a function of x rather than as a function of p in order to add 5.35 to it. Taking the demand function $x(p) = 24710-2500p$ and solving for p, we get

[2] It may be helpful to check equation (13.50) in exercise 13.8. (Note: This equation is labeled (13.79) in the solutions set for exercise 13.8.)

$$p(x) = \frac{24710}{2500} - \left(\frac{1}{2500}\right)x = 9.884 - 0.0004x. \quad (14.61)$$

We can now add 5.35 to this given that the voucher will shift the demand curve up by this amount — which gives us $p(x) = 15.234 - 0.0004x$. Solving back for x, we can then get the new demand function

$$x(p) = 38,085 - 2500p. \quad (14.62)$$

(f) *Given this change in demand, what will happen to tuition and the number of children served in existing private schools in the short run assuming the number of schools is fixed and no new schools can enter in the short run? (*Hint*: You will need to know the current level of capital, derive the short run supply function for private schools, then aggregate them across the existing private schools.)*

Answer: In chapter 13 end-of-chapter exercises, we determined that the long run labor demand function for a Cobb-Douglas production process $f(\ell, k) = A\ell^{\alpha}k^{\beta}$ is

$$k(w,r,p) = \left(\frac{pA\alpha^{\alpha}\beta^{(1-\alpha)}}{w^{\alpha}r^{(1-\alpha)}}\right)^{1/(1-\alpha-\beta)}. \quad (14.63)$$

Substituting in $\alpha = 0.5$, $\beta = 0.25$, $A = 35$, $w = 50$, $r = 25$ and $p = 7$, we get $k \approx 36$. When voucher are introduced, each private school therefore has $\overline{k} = 36$ units of capital that are fixed in the short run. The short run production function is then

$$x = f_{\overline{k}}(\ell) = \left[35\left(36^{0.25}\right)\right]\ell^{0.5} \approx 85.73\ell^{0.5}. \quad (14.64)$$

The short run profit maximization problem is then

$$\max_{\ell}\ p\left(85.73\ell^{0.5}\right) - 50\ell. \quad (14.65)$$

Solving this, we get the short run labor demand function, and substituting it back into equation (14.72), we get the short run supply function:

$$\ell_{\overline{k}}(p) = 0.735p^2 \text{ and } x_{\overline{k}}(p) = 73.5p. \quad (14.66)$$

Since there are 14 private schools in the pre-voucher long run equilibrium, the aggregate short run supply function is then

$$X^{SR}(p) = 14(73.5p) = 1,029p. \quad (14.67)$$

We can now find the short run equilibrium price by setting the demand function (that includes the voucher) from equation (14.62) equal to this short run aggregate supply function; i.e.

$$38,085 - 2500p = 1,029p \quad (14.68)$$

and solve for p. This gives us the short run equilibrium tuition price of $p = 10.792$, which is (given we are measuring prices in thousands) $10,792, up from the initial $7,000 pre-voucher price.

The number of children served can now be determined from the demand function that includes the voucher (equation (14.62)) by substituting in $p = 10.792$ — which gives us

$$x = 38085 - 2500(10.792) \approx 11,105. \quad (14.69)$$

This is the total number of children in private schools in the short run equilibrium. Since the number of schools is unchanged at 14 in the short run, there will be $11,105/14 \approx 793$ students per school, up from the initial 515. (You can also verify this by plugging the short run equilibrium price $p = 10.792$ into the school's short run supply function in equation (14.66) to get $x_{\overline{k}} = 73.5(10.792) \approx 793$.)

(g) *What happens to private school class size in the short run?*

Answer: We already know from the previous part that the number of children in each private school will be 793 in the short run. To determine class size, we just have to determine the number of teachers working in each school in the short run. For this, we can use the short run labor demand function from equation (14.66), substitute the short run equilibrium price $p = 10.792$ and get

$$\ell_{\overline{k}} = 0.735p^2 = 0.735(10.792)^2 \approx 85.6. \quad (14.70)$$

With an average of 85.6 teachers and 793 students per school, class size in the short run falls to $793/85.6 \approx 9.3$ (from the original 14.3).

(h) *How do your answers change in the long run when new schools can enter?*

Answer: In the long run, new schools will enter the private school market until each school again operates at the lowest point of its long run average cost curve (and thus makes zero profit). This means that the tuition price will be driven back down to $p = 7$ in the new long run equilibrium, with each private school again serving about 515 students. (We calculated this in part (a) where we simply used the long run AC curve (which has not changed) to determine these values.) The number of teachers in each school will again go to 36, giving us a long run equilibrium class size of 14.3. But because of the voucher, more children will be attending private schools. We can determine exactly how many more by using the demand function that includes the voucher (equation (14.62)) and substituting in the long run equilibrium price $p = 7$ to get

$$x = 38085 - 2500(7) = 20,585. \quad (14.71)$$

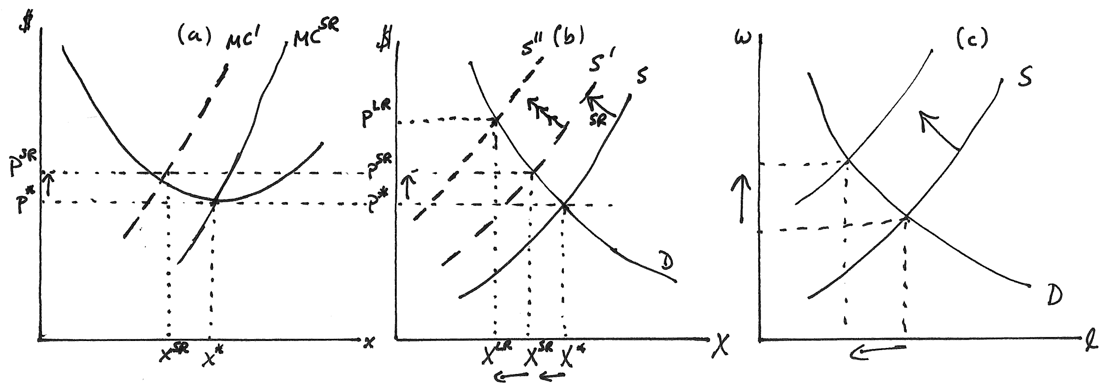
With 515 students per school, this implies the market will expand to 40 schools (from the initial 14) as a result of new schools forming to meet increased demand.

14.11 Policy Application: *Public School Teacher Salaries, Class Size and Private School Markets*: *In exercise 14.10, we noted that private schools that charge tuition operate alongside public schools in U.S. cities. There is much discussion in policy circles regarding the appropriate level of public school teacher salaries (which are set by the local or state government) as well as the appropriate number of public school teachers (that determines class size in public schools).*

A: *Suppose again that private schools face U-shaped long-run AC curves for providing seats to children and that the private school market is currently in long run equilibrium.*

(a) *Begin by drawing two graphs — one with the long run AC curve for a representative private school and a second with the demand and (short run) aggregate supply curves (for private school seats) that are consistent with the private school market being in long run equilibrium (with private school tuition p on the vertical axis).*

Answer: This is done in panels (a) and (b) of Graph 14.9. In order for the private school market to be in long run equilibrium, each school makes zero profit and thus operates on the lowest point of its long run *AC* curve. Thus, the equilibrium price is p^* as reflected by the intersection of demand *D* and supply *S* in panel (b).



Graph 14.9: Public School Teacher Salaries and Private School Markets

(b) *Now suppose the government initiates a major investment in public education by raising public school teacher salaries. In the market for private school teachers (with private school teacher salaries on the vertical and private school teachers on the horizontal), what would you expect to happen as a result of this public school investment?*

Answer: This is illustrated in panel (c) of Graph 14.9. I would expect the supply of private school teachers (of a given quality) to decrease as they are more attracted to the public schools. Thus, the equilibrium teacher wage in the private school market should increase.

(c) *How will this impact private school tuition levels, the number of seats in private schools and the overall number of children attending private schools in the short run?*

Answer: This increase in teacher salaries causes the short run *MC* curve of each private school to immediately shift to the left — indicated by MC' in panel (a) of Graph 14.9. Since this happens for all existing private schools, the market supply curve in panel (b) shifts immediately from S to S'. As a result, tuition price increases to p^{SR}, with the number of children attending private schools decreasing from the initial X^* to X^{SR} in panel (b). Each individual school will also admit fewer children (as shown in pane (a)). We know this because the same number of schools is admitting fewer students — and all schools are assumed to be roughly identical.

(d) *How does your answer change in the long run as private schools can enter and exit the industry?*

Answer: In principle, it could be that private schools either enter or exit as a result of these changes. Graph 14.9 illustrates the (more likely) case of schools exiting. If some private schools exit the market (because the short run increase in price is insufficient to cover long run costs), the supply curve in panel (b) of Graph 14.9 will shift further to S'', causing tuition price to increase further to p^{LR}. Thus, the market will serve fewer children in the long run — down from the initial drop to X^{SR} to X^{LR} in panel (b) of the graph. How many students are admitted by individual schools relative to the short run change is ambiguous — it depends on where the lowest point of the new long run AC falls, but it would typically be at a size smaller than the initial x^* (though it is in principle possible for it to be larger). If the lowest point of long run AC shifts sufficiently far to the left, private schools become very small — and if the demand curve is not too flat, this implies that the number of schools would actually *increase* in the long run. (This seems less likely and is not pictured in the graph.)

(e) *Suppose that instead of this teacher salary initiative, the city government decides to channel its public school investment initiative into hiring more public school teachers (as the city government is simply recruiting additional teachers from other states) and thus reducing class size. Assuming that this has no impact on the equilibrium salaries for teachers but does cause parents to feel more positively about public schools, how will the private school market be impacted in the short and long run?*

Answer: This would cause a decrease in demand for private schools — i.e. a leftward shift. The result would be the mirror image of what we concluded for vouchers in exercise 14.10: The lower demand would cause an initial drop in tuition levels in the short run, with each private school serving fewer children. In the long run, some private schools would exit, shifting the market supply curve to the left and raising tuition prices back to their original zero-profit level. In the long run, each remaining private school would therefore admit as many children as it did initially and would charge the same tuition it initially charged, but the private school market as a whole would serve fewer children.

(f) *How will your long run answer to (e) be affected if the government push for more public school teachers also causes equilibrium teacher salaries to increase?*

Answer: The shift in demand we just analyzed would shrink the private school sector but not change what the remaining private schools do (in terms of how many children they serve and what tuition they charge). If, however, there is an additional upward pressure on private school teacher salaries, then the smaller private school sector would change along the same lines as illustrated in Graph 14.9 — it would, in the long run, experience a further decrease in size, tuition levels would increase and existing private schools might be somewhat smaller or larger depending on how the increase in teacher salaries affects the lowest point of the AC curve.

B: *As in exercise 14.10, assume a total city-wide demand function* $x(p) = 24,710 - 2500p$ *for private school seats and let each private school's average long run cost function be given by* $AC(x) = 0.655x^{1/3} + (900/x)$. *Again, interpret all dollar values in thousands of dollars.*

(a) *If you have not already done so, calculate the initial long run equilibrium size of each school, what tuition price they charge and how many private schools there are in the market.*

Answer: As demonstrated in exercise 14.10 (b) and (d), each school has 515 students and charges $7,000 in tuition. There are 14 private schools in the city.

(b) *If you did B(a) in exercise 14.10, you have already shown that this* $AC(x)$ *curve arises from the Cobb-Douglas production function* $x = f(\ell, k) = 35\ell^{0.5}k^{0.25}$ *when* $w = 50$ *and* $r = 25$ *and when private schools face a fixed cost of 900. If you have not already done so, use this information to determine how many teachers and how much capital each school hires.*

Answer: With the labor and capital demand functions from (c) and (f) in exercise 14.10, we calculated that the initial long run profit maximizing production plan includes approximately 36 teachers per school as well as 36 units of capital per school.

(c) *Suppose that the increased pay for public school teachers drives up the equilibrium wage for private school teachers from 50 to 60 (i.e. from $50,000 to $60,000 per year). What happens to the equilibrium tuition price in the short run?*

Answer: The short run production function for fixed capital $\overline{k} = 36$ is

$$x = f_{\overline{k}}(\ell) = \left[35\left(36^{0.25}\right)\right]\ell^{0.5} \approx 85.73\ell^{0.5}. \tag{14.72}$$

The short run profit maximization problem is then

$$\max_{\ell}\ p\left(85.73\ell^{0.5}\right) - w\ell. \tag{14.73}$$

Solving this, we get the short run labor demand function, and substituting it back into equation (14.72), we get the short run supply function:

$$\ell_{\overline{k}}(w,p) = 1,837.4\left(\frac{p}{w}\right)^2 \text{ and } x_{\overline{k}}(w,p) = 3674.8\left(\frac{p}{w}\right). \tag{14.74}$$

Setting w equal to the new level of 60 in the supply function $x_{\overline{k}}(w,p)$, we get each school's short run supply curve $x_{\overline{k}}(p) \approx 61.25p$, and multiplying it by 14 (i.e. the number of schools in the market), we get the market short run supply curve

$$X^{SR}(p) = 857.5p. \tag{14.75}$$

Setting this equal to the market demand curve $x(p) = 24710 - 2500p$ and solving for p, we get $p = 7.36$ for a tuition level of $7,360, up from the initial $7,000.

(d) *What happens to school size and class size?*

Answer: Plugging $w = 60$ and $p = 7.36$ into the expressions in equation (14.74), we get

$$\ell_{\overline{k}} = 1,837.4\left(\frac{7.36}{60}\right)^2 \approx 27.65 \text{ and } x_{\overline{k}} = 3674.8\left(\frac{7.36}{60}\right) \approx 450.8. \tag{14.76}$$

School size (i.e. the number of children in each school) therefore shrinks from the initial 515 to about 451, and the number of teachers per school shrinks from the initial 36 to around 27.65. This implies that class size increases from 14.3 to 16.3.

(e) *How will your answers on school size, tuition level and class size change in the long run? (*Hint*: You can use the cost function given in equation (13.35) of exercise 13.5 to derive the AC function — just make sure you keep track of the fixed cost of 900!)*

Answer: In the long run, schools again have to end up on the lowest point of their average cost curves. However, since w has increased, their average cost curves have shifted up. Dividing the cost function from exercise 13.5 by x and adding the average fixed cost $900/x$, we get

$$AC(w,r,x) = \frac{C(w,r,x)}{x} + \frac{FC}{x} = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{FC}{x}. \tag{14.77}$$

Substituting $\alpha = 0.5$, $\beta = 0.25$, $A = 35$, $w = 60$, $r = 25$ and $FC = 900$, we get

$$AC(x) = 0.74x^{1/3} + \frac{900}{x}. \tag{14.78}$$

The lowest point on this U-shaped average cost curve arises when the derivative of AC is zero; i.e. when

$$\frac{dAC(x)}{dx} = \frac{0.247}{x^{2/3}} - \frac{900}{x^2} = 0. \tag{14.79}$$

Solving this for x, we get $x \approx 469$. Thus, the school size, which started at 515 students and fell to 451 students in the short run, goes to 469 students in the long run. Plugging 469 back into the $AC(x)$ function in equation (14.78), we can then determine that the bottom of the U-shaped AC curve occurs at an average cost of approximately 7.668 — which has to be the tuition price in the new long run equilibrium. Thus, tuition, which started at $7,000 per student and went to $7,360 in the short run, rises to to $7,668 in the long run. Evaluating the long run labor demand equation at $p = 7.668$ and $w = 60$, we can determine that the number of teachers — which began at 36 and fell to 27.65 per school in the short run — goes to approximately 30 teachers per school. This causes class size — which began at 14.3 and rose to 16.3 in the short run — to go to 15.65.

(f) *How many private schools will remain in the market in the long run?*

Answer: We first have to determine the total quantity of school seats demanded — which we can do by evaluating the demand function at the new long run equilibrium price $p = 7.668$. This gives us

$$x(7.668) = 24710 - 2500(7.668) = 5,540. \quad (14.80)$$

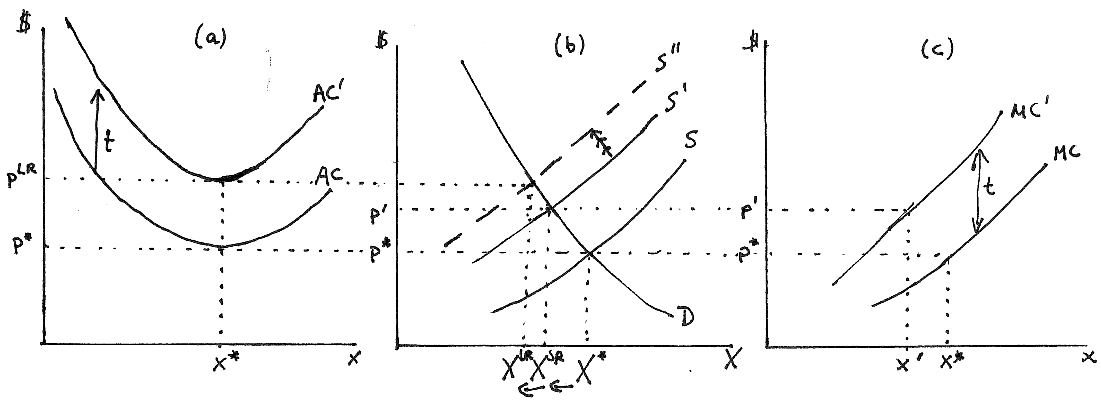
With each school serving about 469 children, this implies 11.8 or approximately 12 schools will remain. (Actually, given that 11.8 falls between 11 and 12, 11 schools would remain, with each producing at a slightly higher tuition price serving somewhat more students and making a small profit.)

14.12 Policy Application: *Pollution Taxes on Output: Suppose you are one of many firms that refine crude oil into gasoline. Not surprisingly, this process is one that creates pollution. The government therefore announces a new tax of \$t on each gallon of gasoline that leaves a refinery (to be paid by the refinery.)*

A: *For purposes of this exercise, assume that the refinement process of crude oil into gasoline has decreasing returns to scale but entails a recurring fixed cost.*

(a) *Begin by illustrating the industry in pre-tax equilibrium — showing one firm's average cost curve as well as the (short run) market supply and demand that supports an industry in long run equilibrium.*

Answer: This is done in panels (a) and (b) of Graph 14.10 where the pre-tax (long run) average cost curve curve is sketched as AC (in panel (a)) and where the market demand curve D and (short run) market supply curve S intersect at output quantity X^* and price p^* (in panel (b)).



Graph 14.10: Pollution Taxes on Gasoline

(b) *What changes for each firm and in the industry in the short run when the tax is introduced?*

Answer: When the tax of $\$t$ per gallon of gasoline is introduced, the marginal cost of each gallon of gasoline is increased, as is the average cost. The upward shift in the (short run) marginal cost curve is illustrated in panel (c) of Graph 14.10 as a shift from MC to MC'. The short run market supply curve is simply the sum of all the short run supply curves of firms — which are in turn made up of a portion of each firm's MC curve. Thus, when MC shifts to MC' in panel (c), the market supply curve shifts from S to S', an upward shift by amount t. This changes the market equilibrium in the short run to the intersection with D at output level X^{SR} at price p' — the new short run market price. At that price, each firm produces less (x' rather than x in panel (c)) — which has to be the case since in the short run the number of firms stays constant but the overall industry output drops.

(c) *What changes in the long run?*

Answer: In the long run, the average cost curve in panel (a) of Graph 14.10 shifts up from AC to AC' — with each point shifting up by t and thus x^* — the quantity at which the average cost curve reaches its lowest point — remaining unchanged. Thus, in the long run, each firm that remains in the industry produces the same amount of gasoline as it did in the original pre-tax equilibrium and up from x' (in panel (c)) to which firm output falls in the short run. This implies that some refineries must be exiting — with the long run equilibrium price rising to p^{LR} beyond the original increase to p' in the short run, and industry output falls to X^{LR}.

(d) True or False*: While refineries bear some of the burden of this tax in the short run, they will pass all of the tax on to consumers in the long run.*

Answer: This is true. In the short run, price does not rise by the full amount of the tax — which means consumers pay part of it and refinery owners pay the rest. But in the long run, the output price rises by the full amount of the per unit tax t — which implies that consumers pay the entire burden of the tax, with refineries making zero long run profit both before and after the tax is imposed.

(e) *I recently heard the following comment on one of the TV news shows (regarding a tax similar to the one we are analyzing here): "Regulators are particularly concerned about reports that companies in the industry managed to pass the pollution tax fully onto consumers and view this as a sign that the industry is not competitive but is rather engaged in strategic manipulation of gasoline prices." What do you make of this TV wisdom?*

Answer: This "wisdom" is garbage. As we have just shown, the assumption of perfect competition in the gasoline refinery industry leads to the prediction that the entire tax on gasoline will be passed onto consumers. The reason for this is that, assuming perfect competition, refineries make zero long run profit before and (in the long run) after the tax is imposed — and the mechanism that achieves this is the exiting of refineries on the way to the long run, not strategic manipulation of prices. (In later chapters we will see that firms that strategically manipulate price in markets that are not perfectly competitive will in fact not pass the entire tax onto consumers.)

(f) *Will refineries change the mix of labor and capital in the long run (assuming they continue operating)?*

Answer: No — they will produce the same output level x^* as before, and no input prices have changed. Thus, if they were producing x^* in a cost minimizing way before the tax was imposed, that same way of producing x^* will still be cost minimizing — i.e. there is no reason to change the input bundle.

(g) *Here is another quote from a recent TV analysis: "In talking to this refinery's owner, it seems that there are no plans in place to lay off any workers in response to the pollution tax on refined gasoline. Jobs in the industry therefore appear to be safe for now." Do you agree?*

Answer: It is true that, if a TV reporter talks to one of the refineries that continues operation, that refinery will not have altered the number of workers. But the tax has raised the price of gasoline by t per gallon — which implies consumers will demand less of it — which implies that some refineries will stop refining gasoline. It is in those refineries that jobs are lost in the industry — not in the refineries that continue to operate.

B: *Once again suppose that the production function used by firms in the gasoline refinery industry is $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ with $\alpha,\beta > 0$ and $\alpha+\beta < 1$, and suppose that each refinery pays a recurring fixed cost F.*

(a) *If you did not already do so in exercise 14.1, derive the expression for the output level x^* at which the long run AC curve reaches its lowest point. (This should be a function of A, α, β, w and r.)*

Answer: In exercise 14.1, we derived this by setting the first derivative of the AC function (with respect to x) to zero and solving for x. This gave us

$$x^* = \left(\frac{A\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(\alpha+\beta)}. \tag{14.81}$$

(b) *How does x^* change under the per-gallon tax on gasoline leaving the refinery?*

Answer: To see that x^* will not in fact change, we can go back to the Cobb-Douglas cost function, add the fixed cost F and also add the new cost of the tax which is tx. This gives us

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + F + tx. \tag{14.82}$$

From this, we get the average cost function

$$AC(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{F}{x} + t. \quad (14.83)$$

When we take the first derivative of this and set it to zero, however, t simply drops out — which leaves us with the same x^* as before. Thus, x^* — the equilibrium firm output level — is unaffected by t. In fact, the AC curve can be seen in equation (14.83) to shift up by t at every output level — i.e. to shift up in a parallel way by exactly t everywhere, which is why the lowest point remains at the same output level.

(c) *Can you use your answer to determine whether the number of gasoline refineries will increase or decrease as a result of the tax?*

Answer: Since the lowest point of the AC curve shifts up, we know that the long run equilibrium price will shift up. For any downward sloping aggregate demand curve, this implies that less gasoline will be demanded — and thus less gasoline will come out of refineries. But if the lowest point of the average cost curves remains at the same output level, we also know that each existing refinery will produce the same amount of gasoline as before. Thus, if the industry produces less but each refinery produces the same, it must be that some refineries have exited — i.e. the number of refineries decreases as a result of the tax.

(d) *If you have not already done so in exercise 14.1, determine the long run equilibrium price p^* before the tax (a function of A, α, β, w and r.) How does this change under the tax?*

Answer: We have already shown in exercise 14.1 that, in the absence of a tax on gasoline,

$$p^* = \left(\frac{w^{\alpha}r^{\beta}}{A\alpha^{\alpha}\beta^{\beta}}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(1-\alpha-\beta)}. \quad (14.84)$$

We also just showed in equation (14.83) that the AC function shifts up by t at every output level including x^*. This implies that price simply rises by t; i.e. in the new long run equilibrium,

$$p^* = \left(\frac{w^{\alpha}r^{\beta}}{A\alpha^{\alpha}\beta^{\beta}}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(1-\alpha-\beta)} + t. \quad (14.85)$$

(e) *Can you use your answer to determine who actually pays the tax?*

Answer: Refineries make zero economic profit before and after the tax (once the new long run equilibrium has been reached). The price paid by consumers, however, increases by the entire amount of the tax. Thus, consumers end up paying the tax in its entirety.

(f) *Will the tax result in less pollution? If so, why?*

Answer: Yes, it will result in less pollution to the extent to which consumers conserve on the use of gasoline due to the higher price. Note that refineries have no incentive to do anything differently — except that some will exit and thus less gasoline will be refined. The agents in the economy who have an incentive to change behavior are consumers who face the entire burden of the tax — and to the extent to which they respond by buying less gasoline, there should be less pollution.