SOLUTIONS

15

The "Invisible Hand" and the First Welfare Theorem

Solutions for *Microeconomics: An Intuitive Approach with Calculus*

Apart from end-of-chapter exercises provided in the student *Study Guide*, these solutions are provided for use by instructors. (End-of-Chapter exercises with solutions in the student *Study Guide* are so marked in the textbook.)

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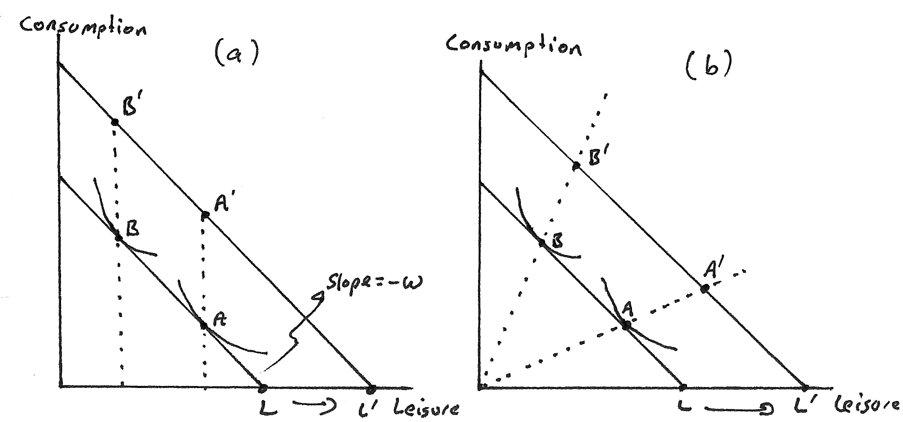
- Each end-of-chapter exercise begins on a new page. This is to facilitate maximum flexibility for instructors who may wish to share answers to some but not all exercises with their students.
- If you are assigning only the A-parts of exercises in *Microeconomics: An Intuitive Approach with Calculus*, you may wish to instead use the solution set created for the companion book *Microeconomics: An Intuitive Approach*.
- *Solutions to Within-Chapter Exercises are provided in the student* Study Guide.

15.1 Everyday Application: *Labor Saving Technologies: Consider inventions such as washing machines or self-propelled vacuum cleaners. Such inventions reduce the amount of time individuals have to spend on basic household chores — and thus in essence increase their leisure endowments.*

A: *Suppose that we wanted to determine the aggregate impact such labor saving technologies will have on a particular labor market in which the wage is w.*

(a) *Draw a graph with leisure on the horizontal axis and consumption on the vertical and assume an initially low level of leisure endowment for worker A. For the prevailing wage w, indicate this worker's budget constraint and his optimal choice.*

Answer: This is illustrated in panel (a) of Graph 15.1 as the lower of the two parallel budgets where worker A optimizes at bundle A.



Graph 15.1: Labor Saving Household Technologies

(b) *On the same graph, illustrate the optimal choice for a second worker B who has the same leisure endowment and the same wage w but chooses to work more.*

Answer: This is also illustrated in panel (a) where worker B optimizes at bundle B — consuming less leisure and thus working more.

(c) *Now suppose that a household-labor saving technology (such as an automatic vacuum cleaner) is invented and both workers experience the same increase in their leisure endowment. If leisure is quasilinear for both workers, will there be any impact on the labor market?*

Answer: The increase in leisure endowment is indicated as an increase from L to L' in panel (a) of the graph. Since wage remains the same, this results in a parallel shift out of the budget constraints. If tastes are quasilinear in leisure, then worker A will optimize at A' and worker B will optimize at B'. Since their leisure consumption remains unchanged, this implies that workers will increase their labor supply by exactly the increase in leisure $(L' - L)$.

(d) *Suppose instead that tastes for both workers are homothetic. Can you tell whether one of the workers will increase his labor supply by more than the other?*

Answer: This is illustrated in panel (b) of Graph 15.1 where worker A will choose bundle A' and worker B will choose bundle B'. Worker A will therefore increase his leisure consumption by more than worker B — with neither worker committing the entire increase in leisure $(L' - L)$ to increased work hours. However, because worker B increases his leisure consumption by less than worker A, we know that worker B will increase his labor supply by more than worker A.

(e) *How does your answer suggest that workers in an economy cannot generally be modeled as a single "representative worker" even if they all face the same wage?*

Answer: In order for us to be able to use a "representative worker", it would have to be the case that, when leisure endowments are redistributed between workers, the overall amount of labor supplied remains unchanged. We can see in panel (a) of Graph 15.1 that, when leisure is quasilinear, leisure demand remains unchanged as leisure endowments are changed. Thus, were we to redistribute leisure endowments between individuals, the one who gets more leisure endowment would supply all of it as labor while the one who loses it would reduce his labor hours by the same amount. Thus, the actions of the two workers would exactly offset each other. The same is not, however, true in panel (b) where tastes are homothetic. Thus, a redistribution of leisure among workers would cause an increase in labor hours for the worker who receives more leisure endowment and reduce the labor hours of the worker who receives less — but the two would not offset each other unless the tastes were also identical.

B: *Consider the problem of aggregating agents in an economy where we assume individuals have an exogenous income.*

(a) *In a footnote in this chapter, we stated that, when the indirect utility for individual m can be written as $V^m(p_1,p_2,I^m)=\alpha^m(p_1,p_2)+\beta(p_1,p_2)I^m$, then demands can be written as in equation (15.2). Can you demonstrate that this is correct by using Roy's Identity?*

Answer: Applying Roy's identity, we get

$$x_i^m(p_1,p_2,I)=-\frac{\partial V/\partial p_i}{\partial V/\partial I}=-\frac{\left(\partial\alpha^m(p_1,p_2)/\partial p_i\right)+I^m\left(\partial\beta(p_1,p_2)/\partial p_i\right)}{\beta(p_1,p_2)}. \tag{15.1}$$

If we now define

$$a_i^m(p_1,p_2)=-\frac{\partial\alpha^m(p_1,p_1)/\partial p_i}{\beta(p_1,p_2)} \text{ and } b_i(p_1,p_2)=-\frac{\partial\beta(p_1,p_2)/\partial p_i}{\beta(p_1,p_2)}, \tag{15.2}$$

we can write the demand function for good i by consumer m as

$$x_i^m(p_1,p_2,I)=a_i^m(p_1,p_2)+I^m b_i(p_1,p_2). \tag{15.3}$$

Note that we can do this because the first term on the right hand side of equation (15.1) contains both an m superscript and an i subscript — thus causing the a function to contain both. But the second term contains (aside from I^m) only an i subscript (and no m superscript) — thus allowing us to write the b function without the m superscript.

(b) *Now consider the case of workers who choose between consumption (priced at 1) and leisure. Suppose they face the same wage w but different workers have different leisure endowments. Letting the two workers be superscripted by n and m, can you derive the form that the leisure demand equations $l^m(w,L^m)$ and $l^n(w,L^n)$ would have to take in order for redistributions of leisure endowments to not impact the overall amount of labor supplied by these workers (together) in the labor market?*

Answer: In order for redistributions in leisure endowments to have offsetting effects, it must be the case that the first derivative of $l^m(w,L^m)$ with respect to L^m is equal to the first derivative of $l^n(w,L^n)$ with respect to L^n and that the second derivative of each is zero. (This gives us the parallel linear (and offsetting) changes in consumption bundles as endowments are redistributed.) In order for this to be the case, the functions have to take the form

$$l^m(w,L^m)=a^m(w)+b(w)L^m \text{ and } l^n(w,L^n)=a^n(w)+b(w)L^n. \tag{15.4}$$

The first derivatives with respect to the leisure endowments are then equal to $b(w)$, and the second derivatives are zero. Were the b functions superscripted by m and n, this would not be the case, nor would it be the case if leisure entered the b or a functions directly.

(c) *Can you re-write these in terms of labor supply equations $\ell^m(w,L^m)$ and $\ell^n(w,L^n)$?*

Answer: Since labor supply is just the leisure endowment minus leisure demand, we get

$$\ell^m(w,L^m)=L^m-\left(a^m(w)+b(w)L^m\right)=(1-b(w))\,L^m-a^m(w) \tag{15.5}$$

and

$$\ell^n(w,L^n) = L^n - \left(a^n(w) + b(w)L^n\right) = (1-b(w))\,L^n - a^n(w). \tag{15.6}$$

(d) *Can you verify that these labor supply equations have the property that redistributions of leisure between the two workers do not affect overall labor supply?*

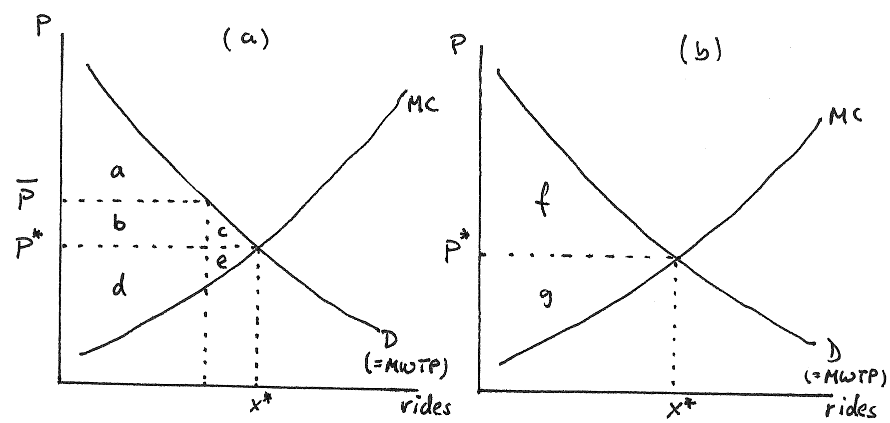
Answer: The first derivative of the labor supply functions with respect to the leisure endowments are now equal to $(1 - b(w))$ and thus equal to each other — and the second derivatives are zero. Thus, a redistribution of endowments indeed causes an increase in labor supply by the worker who receives more endowment which is exactly offset by the decrease in labor supply by the worker who receives less endowment.

15.2 Business Application: *Disneyland Pricing Revisited*: *In end-of-chapter exercise 10.10, we investigated different ways that you can price the use of amusement park rides in a place like Disneyland. We now return to this example. Assume throughout that consumers are never at a corner solution.*

A: *Suppose again that you own an amusement park and assume that you have the only such amusement park in the area — i.e. suppose that you face no competition. You have calculated your cost curves for operating the park, and it turns out that your marginal cost curve is upward sloping throughout. You have also estimated the downward sloping (uncompensated) demand curve for your amusement park rides, and you have concluded that consumer tastes appear to be identical for all consumers and quasilinear in amusement park rides.*

(a) *Illustrate the price you would charge per ride if your aim was to maximize the overall surplus that your park provides to society.*

Answer: This is illustrated in panel (a) of Graph 15.2 where demand intersects the MC curve at output x^* and price p^*. The price p^* maximizes the total surplus — because, for all quantities below x^*, the marginal benefit (measured by the $MWTP$ curve) exceeds the marginal cost of production. (The demand curve is equal to the $MWTP$ curve because of the quasilinearity of rides in consumer tastes.)



Graph 15.2: Amusement Park Rides

(b) *Now imagine that you were not concerned about social surplus and only about your own profit. Illustrate in your graph a price that is slightly higher than the one you indicated in part (a). Would your profit at that higher price be greater or less than it was in part (a)?*

Answer: This is illustrated in panel (a) using the price $\overline{p}$. At the original price p^*, profit is equal to $(d+e)$. At the new price $\overline{p}$, profit is equal to $(b+d)$. Since b is greater than e, profit is higher at the higher price. This is because, while you give up some profit from not producing as much, you more than make up for it by being able to sell what you do produce at a higher price.

(c) True or False*: In the absence of competition, you do not have an incentive to price amusement park rides in a way that maximizes social surplus.*

Answer: This is true (as already illustrated above). Note that social surplus falls from $(a+b+c+d+e)$ under p^* to $(a+b+d)$ under $\overline{p}$. Thus, while profit increases, social surplus falls because consumers lose more than the increase in profit.

(d) *Next, suppose that you decide to charge the per-ride price you determined in part (a) but, in addition, you want to charge an entrance fee into the park. Thus, your customers will now pay that fee to get into the park — and then they will pay the per-ride price for every ride they* *take. What is the most that you could collect in entrance fees without affecting the number of rides consumed?*

Answer: This is illustrated in panel (b) of Graph 15.2. At price p^*, consumers get a total consumer surplus of area f. That is the most they would be willing to pay to enter the park and face a price per ride of p^*. You could therefore charge a per-consumer entrance fee of area f divided by the number of consumers.

(e) *Will the customers that come to your park change their decision on how many rides they take? In what sense is the concept of "sunk cost" relevant here?*

Answer: No, once they pay the fee, they will continue to consume as many rides as before. In essence, once they pay the entrance fee, that fee is a sunk cost — and it does not affect decisions in the park. (This is technically true in this example only because of the quasilinearity of rides in consumer tastes. If rides were not quasilinear, then the entrance fee would alter the income available for consumption of rides and other goods — which would produce an income effect in addition to the substitution effect. But, since rides are quasilinear, there is no income effect.)

(f) *Suppose you collect the amount in entrance fees that you derived in part (d). Indicate in your graph the size of consumer surplus and profit assuming you face no fixed costs for running the park?*

Answer: By charging the maximum entrance fee derived in (d), you are in essence taking all the consumer surplus. Thus, in addition to earning the profit g from selling rides in the park, you are also earning the revenue f from the entrance fees — causing your total profit (in the absence of fixed costs) to be the area $(f+g)$.

(g) *If you do face a recurring fixed cost FC, how does your answer change?*

Answer: If you face recurring fixed costs FC, your (long run) profit will be $(f+g-FC)$.

(h) True or False*: The ability to charge an entrance fee in addition to per-ride prices restores efficiency that would be lost if you could only charge a per-ride price.*

Answer: This is true — total surplus will now be back to what it was in panel (a) of the Graph when price p^* was charged — except that the consumer surplus appears as profit instead of consumer surplus.

(i) *In the presence of fixed costs, might it be possible that you would shut down your park if you could not charge an entrance fee but you keep it open if you do?*

Answer: Yes. Suppose that recurring fixed costs were greater than the area g in panel (b) of the graph. Then you would earn profit if you could charge an entrance fee and possibly not if you cannot charge an entrance fee. (We don't know yet until we talk about monopolies later on in the text exactly how high fixed costs would have to be in order for you not to be able to operate without an entrance fee — because, as we illustrated earlier in the problem, you can in fact increase profit by charging a price higher than p^* if you cannot charge an entrance fee.)

B: *Suppose, as in exercise 10.10, tastes for your consumers can be modeled by the utility function $u(x_1,x_2)=10x_1^{0.5}+x_2$, where x_1 represents amusement park rides and x_2 represents dollars of other consumption. Suppose further that your marginal cost function is given by $MC(x)=x/(250,000)$.*

(a) *Suppose that you have 10,000 consumers on any given day. Calculate the (aggregate) demand function for amusement park rides.*

Answer: Each consumer's demand function is calculated by solving the problem

$$\max_{x_1,x_2} 10x_1^{0.5}+x_2 \text{ subject to } I=px_1+x_2 \tag{15.7}$$

which gives the demand function $x_1(p)=25/p^2$. Multiplying by 10,000, we get the aggregate demand function

$$X(p)=\frac{250,000}{p^2}. \tag{15.8}$$

(b) *What price would you charge if your goal was to maximize total surplus? How many rides would be consumed?*

<u>Answer</u>: Inverting the aggregate demand function gives us the aggregate demand curve $p = 500/(x^{0.5})$. Total surplus is maximized where this intersects the marginal cost curve. This implies that we have to solve the equation

$$\frac{500}{x^{0.5}} = \frac{x}{250,000} \tag{15.9}$$

which gives us $x = 250,000$. Plugging this back into the demand curve $p = 500/(x^{0.5})$, we get a per-ride price of $p = 1$ at which each consumer would demand 25 rides.

(c) *In the absence of fixed costs, what would your profit be at that price?*

<u>Answer</u>: At that price, your total revenues would be \$250,000. Your costs would be the area under the MC curve — which is \$125,000. (This is easy to calculate since the MC curve is linear and thus just half the total revenues. More generally, you would calculate this as the integral of the MC curve.) Thus, your profit (in the absence of fixed costs) is \$125,000.

(d) *Suppose you charged a price that was 25% higher. What would happen to your profit?*

<u>Answer</u>: If you charged a price of \$1.25 per ride (rather than the \$1 per ride you just calculated), the number of rides demanded would be

$$X(2) = \frac{250,000}{1.25^2} = 160,000. \tag{15.10}$$

This implies that your revenue would be $160,000(1.25) = \$200,000$. Your marginal cost at 160,000 would be $MC(160,000) = 160,000/250,000 = 0.64$. This implies that your total costs would be $0.64(160,000)/2) = \$51,200$ — which implies your profit would be $200,000 - 51,200 = \$148,800$. This is an increase in profit from the \$125,000 we calculated for the per-ride price of \$1.

(e) *Derive the expenditure function for your consumers.*

<u>Answer</u>: We first have to solve the expenditure minimization problem

$$\min_{x_1,x_2} px_1 + x_2 \text{ subject to } u = 10x_1^{0.5} + x_2 \tag{15.11}$$

which gives us the compensated demand functions

$$x_1(p) = \frac{25}{p^2} \text{ and } x_2(p,u) = u - \frac{50}{p}. \tag{15.12}$$

Using these, we can then derive the expenditure function

$$E(p,u) = p\left(\frac{25}{p^2}\right) + \left(u - \frac{50}{p}\right) = u - \frac{25}{p}. \tag{15.13}$$

(f) *Use this expenditure function to calculate how much consumers would be willing to pay to keep you from raising the price from what you calculated in (b) to 25% more. Can you use this to argue that raising the price by 25% is inefficient even though it raises your profit?*

<u>Answer</u>: The amount that each consumer is willing to pay to avoid this price increase is

$$E(1.25,\overline{u}) - E(1,\overline{u}) = \left(\overline{u} - \frac{25}{1.25}\right) - \left(\overline{u} - \frac{25}{1}\right) = \$5.00, \tag{15.14}$$

where we can leave the utility amount $\overline{u}$ unspecified since it cancels out. (You could also assume some sufficiently high income, calculate the utility the consumer gets at the initial price, and then us it to derive the amount the consumer is willing to pay to not incur the price increase. Your answer would be the same.)

Thus, each consumer is willing to pay \$5 to avoid the increase in per-ride prices from \$1 to \$1.25 — which implies that the 10,000 consumers are willing to pay \$50,000. Above, we calculated that profit increases from \$125,000 to \$148,800. Thus, the increase in price causes an increase of \$23,800 in profit but consumers would have been wiling to pay \$50,000 to avoid that increase — causing a social loss of \$26,200.

(g) *Next, determine the amount of an entrance fee that you could charge while continuing to charge the per-ride price you determined in (b) without changing how many rides are demanded.*

Answer: At a price of $1 per ride, each consumer demands 25 rides. The total willingness to pay for these rides is the area under the demand curve $p = 5/x^{0.5}$; i.e.

$$\int_0^{25} \frac{5}{x^{0.5}}\,dx = 10x^{0.5}|_0^{25} = 50 - 0 = 50. \qquad (15.15)$$

At a price of $1 per ride, the consumer has to pay $25 for the 25 rides she consumes once in the park — which means her consumer surplus is 50 − 25 = 25. Thus, you could charge an entry fee of $25.

(h) *How much is your profit now? What happens to consumer surplus? Is this efficient?*

Answer. Your profit now be the previously calculated profit of $125,000 plus the revenue from the entrance fee which is 25(10,000) = $250,000 — for a total profit of $375,000. There would now be no consumer surplus left. And yes, this would be efficient — because you are producing the efficient number of rides but simply transferred the consumer surplus to yourself.

(i) *Suppose the recurring fixed cost of operating the park is $200,000. Would you operate it if you had to charge the efficient per-ride price but could not charge an entrance fee? What if you could charge an entrance fee?*

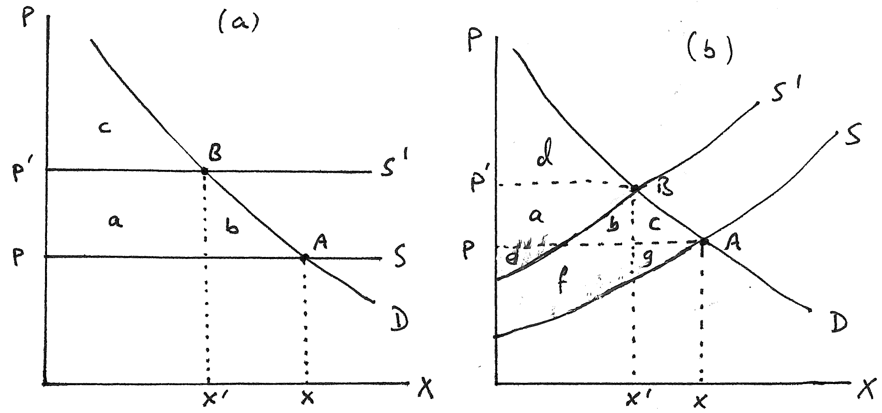
Answer: If you could not charge an entrance fee and had to charge the price of $1 per ride, you would earn a profit before accounting for fixed costs of $125,000. Once you take the fixed cost into account, you would therefore operate at a loss of $75,000 and would not operate the park. However, if you could charge an entrance fee, you can earn up to $375,000 in profit before accounting for fixed costs — which still leaves you a maximum profit of $175,000 if you did charge the highest possible entrance fee. Thus, you would operate the park if you could also charge an entrance fee.

15.3 Business and Policy Application: *License Fees and Surplus without Income Effects: In previous chapters, we explored the impact of recurring license fees on an industry's output and price. We now consider their impact on consumer and producer surplus.*

A: *Suppose that all firms in the fast food restaurant business face U-shaped average cost curves prior to the introduction of a recurring license fee. The only output they produce is hamburgers. Suppose throughout that hamburgers are a quasilinear good for all consumers.*

(a) *First, assume that all firms are identical. Illustrate the long run market equilibrium and indicate how large consumer and long run producer surplus (i.e. profit) are in this industry.*

Answer: This is illustrated in panel (a) of Graph 15.3. The long run supply curve S is flat because of entry and exit decisions by identical firms — leading to equilibrium price p and equilibrium output level x at point A. Producer surplus (or long run profit) is simply zero. Consumer surplus can be measured on the uncompensated demand curve because of the quasilinearity of x (that causes compensated demand curves to lie on top of the uncompensated demand curve). Thus, long run consumer surplus is $(a+b+c)$.



Graph 15.3: License Fees and Quasilinear Tastes

(b) *Illustrate the change in the long run market equilibrium that results from the introduction of a license fee.*

Answer: The long run supply curve shifts up (to S') because the long run average cost curves of each firm shift up. At the new equilibrium, price is p' and output is x' at point B. (Each firm ends up producing more, but the industry produces less as firms exit.)

(c) *Suppose that the license fee has not yet been introduced. In considering whether to impose the license fee, the government attempts to ascertain the cost to consumers by asking a consumer advocacy group how much consumers would have to be compensated (in cash) in order to be made no worse off. Illustrate this amount as an area in your graph.*

Answer: Ordinarily, this would be measured along the compensated demand curve that goes through A. Since x is quasilinear, however, the compensated demand curves lie on the uncompensated demand curve that goes through A and B. Thus, the compensation is given by area $(a+b)$.

(d) *Suppose instead that the government asked the consumer group how much consumers would be willing to pay to avoid the license free. Would the answer change?*

Answer: Ordinarily this would be measured on the compensated demand curve that goes through B. However, all compensated demand curves lie on the uncompensated demand curve because of the quasilinearity of x. Thus, the amount consumers would be willing to pay is $(a+b)$.

(e) *Finally, suppose the government simply calculated consumer surplus before and after the license fee is imposed and subtracted the latter from the former. Would the government's conclusion of how much the license fee costs consumers change?*

Answer: No. The consumer surplus at A is $(a+b+c)$ and the consumer surplus at B is c — making the difference area $(a+b)$. (Again, this is only true because consumer surplus can, because of the quasilinearity of x, be measured on the uncompensated demand curve.)

(f) *What in your answers changes if, instead of all firms being identical, some firms had higher costs than others (but all have U-shaped average cost curves)?*

Answer: Not very much would be different — except for the fact that producer surplus would now be positive given that firms who are more cost effective can earn positive profit. This is illustrated in panel (b) of Graph 15.3 where supply shifts from S to S'. The change in consumer surplus is $(a+b+c)$ (which is equivalent to $(a+b)$ in panel (a)) — and the same measure would give the compensation required to consumers or the amount consumers would be willing to pay to prevent the fee from going into effect. Producer surplus, or long run profit, would be $(e+f+g)$ before and $(e+a)$ after.

B: *Suppose that each firm's cost function is given by $C(w,r,x) = 0.047287w^{0.5}r^{0.5}x^{1.25} + F$ where F is a recurring fixed cost.*[1]

(a) *What is the long run equilibrium price for hamburgers x (as a function of F) assuming wage $w = 20$ and rental rate $r = 10$?*

Answer: Each firm's cost function would then be

$$C(x,F) = 0.047287(20)^{0.5}(10)^{0.5}x^{1.25} + F = 0.66873917x^{1.25} + F. \quad (15.16)$$

From this, we can derive the long run average cost function as

$$AC(x,F) = 0.66873917x^{0.25} + \frac{F}{x}. \quad (15.17)$$

To find the lowest point of this average cost function, we take the derivative with respect to x, set it to zero and solve for x to get $x = 4.18256389F^{0.8}$. Plugging this back into the average cost function, we get the long run equilibrium price (as a function of F):

$$p(F) = 0.66873917\left(4.18256389F^{0.8}\right)^{0.25} + \frac{F}{4.18256389F^{0.8}} = 1.195439F^{0.2}. \quad (15.18)$$

(b) *Suppose that, prior to the imposition of a license fee, the firm's recurring fixed cost F was \$1,280. What is the pre-license fee equilibrium price?*

Answer: Using the equation $p(F)$, we can determine the initial equilibrium price

$$p(1280) = 1.195439\left(1280^{0.2}\right) = 5. \quad (15.19)$$

(c) *What happens to the long run equilibrium price for hamburgers when a \$1,340 recurring license fee is introduced?*

Answer: Again, using the equation $p(F)$ and substituting the new fixed cost $F = 1280 + 1340 = 2620$, we get

$$p(2620) = 1.195439\left(2620^{0.2}\right) = 5.77. \quad (15.20)$$

(d) *Suppose that tastes for hamburgers x and a composite good y can be characterized by the utility function $u(x,y) = 20x^{0.5} + y$ for all 100,000 consumers in the market, and assume that all consumers have budgeted \$100 for x and other goods y. How many hamburgers are sold before and after the imposition of the license fee?*

[1] You can check for yourself that this is the cost function that arises from the production function $f(\ell,k) = 20\ell^{0.4}k^{0.4}$.

Answer: The demand function derived from this utility function is $x(p) = 100/p^2$. Summing over 100,000 consumers, we get a market demand function of

$$X(p) = \frac{10,000,000}{p^2}. \quad (15.21)$$

Substituting the before and after prices of \$5 and \$5.77, this implies that 2,000,000 hamburgers were sold before the license fee and about 1,733,100 hamburgers are sold afterwards.

(e) *Derive the expenditure function for a consumer with these tastes.*

Answer: We need to solve the expenditure minimization problem

$$\min_{x,y}\ px + y \ \text{ subject to } \ u = 20x^{0.5} + y. \quad (15.22)$$

This gives us the compensated demand functions

$$x(p) = \frac{100}{p^2} \ \text{ and } \ y(p,u) = u - \frac{200}{p}. \quad (15.23)$$

Substituting this into the expenditure equation $px + y$, we get the expenditure function

$$E(p,u) = p\left(\frac{100}{p^2}\right) + u - \frac{200}{p} = u - \frac{100}{p}. \quad (15.24)$$

(f) *Use this expenditure function to answer the question in A(c).*

Answer: First, we have to figure out how much utility consumers get in the absence of the license fee when $p = 5$. In that case, they consume 4 of x and 80 of y (given that they have budgeted \$100 for both goods) — which gives utility $u = 20(4^{0.5})+80 = 120$. In order to reach this utility level at the higher price $p = 5.77$, we have to evaluate the expenditure function $E(p,u)$ at $p = 5.77$ and $u = 120$; i.e.

$$E(5.77,120) = 120 - \frac{100}{5.77} \approx 102.67. \quad (15.25)$$

Since each consumer has \$100 budgeted to start with, this implies that the government would have to compensate each consumer by \$2.67 — or a total of \$267,000 for the 100,000 consumers.

(g) *Use the expenditure function to answer the question in A(d).*

Answer: If consumers are asked how much they are willing to pay to not have the license fee implemented, they would first need to know how much utility they will get if the license fee in fact does get implemented. At $p = 5.77$, each consumer demands approximately 3 hamburgers (x) — down from 4 — and consumes \$82.67 of other goods (y) — up from 80 before. This implies that each consumer gets utility $u(3,82.67) = 20(3^{0.5}) + 82.67 = 117.31$ if the license fee is implemented. (Had we not rounded a bit, this would actually be 117.33.) If the fee is not implemented, price falls to $p = 5$ — thus, in order to determine how much of a budget each consumer will need to be as well off without the fee as they are with it, we need to evaluate the expenditure function $E(p,u)$ at $p = 5.77$ and $u = 117.31$. This gives us

$$E(5.77,117.31) = 117.31 - \frac{100}{5} = 97.31. \quad (15.26)$$

Thus, a consumer with a current budget of \$100 would be willing to pay \$2.69 — or, had we not rounded the utility figure and used 117.33, we would get that they are willing to pay \$2.67 each. Thus, the answer is the same as what we derived in the previous part — and consumers overall would be willing to pay approximately \$267,000 to avoid the license fee being implemented.

(h) *Take the integral of the demand function that gives you the consumer surplus before the license fee and repeat this to get the integral of the consumer surplus after the license fee is imposed.*

Answer: The consumer surplus before the license fee is

$$\int_5^{\infty} \frac{100}{p^2}\,dp = -\frac{100}{p}\Big|_5^{\infty} = 0 - \left(-\frac{100}{5}\right) = 20, \tag{15.27}$$

and the consumer surplus after the license fee is

$$\int_{5.77}^{\infty} \frac{100}{p^2}\,dp = -\frac{100}{p}\Big|_{5.77}^{\infty} = 0 - \left(-\frac{100}{5.77}\right) = 17.33. \tag{15.28}$$

You could of course also have used the aggregate demand curve — and you would then have gotten the same answers (multiplied by 100,000).

(i) *How large is the change in consumer surplus from the price increase? Compare your answer to what you calculated in parts (f) and (g).*

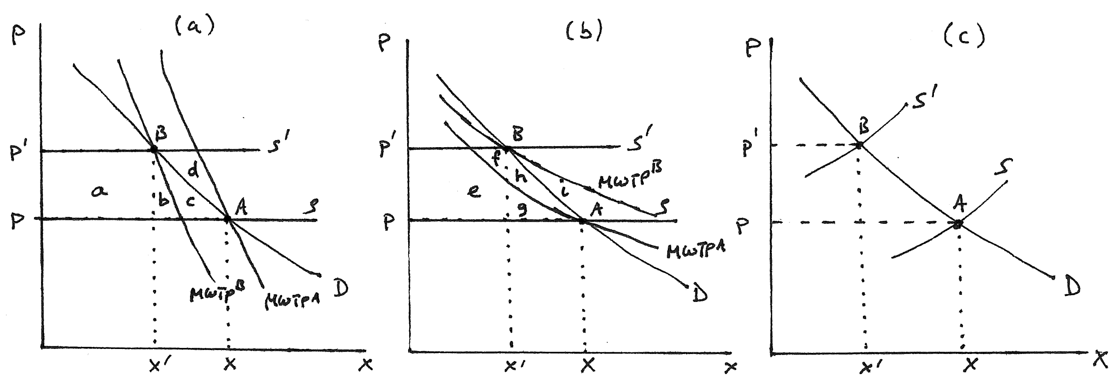
Answer: The change in consumer surplus is therefore 20 − 17.33 = 2.66 or (up to rounding errors) identical to what we calculated in parts (f) and (g). This is because, under quasilinear tastes, the (uncompensated) demand curve lies on top of the compensated demand curves — and we can thus use the (uncompensated) demand curve to measure changes in consumer surplus. (It furthermore implies that the two measures of changes in consumer surplus derived in (f) and (g) are identical because, even though they are measured on different compensated demand curves, they are identical because the compensated demand curves for different utility levels lie on top of one another.)

15.4 Business and Policy Application: *License Fees and Surplus with Income Effects: In this exercise, assume the same set-up as in exercise 15.3 except that this time we will assume that hamburgers are a normal good for all consumers.*

A: *As in exercise 15.3, we'll consider the long run impact of a license fee for fast food restaurants on consumer surplus. In (a) and (b) of exercise 15.3, you should have concluded that the long run price increases as a result of the license fee.*

(a) *Consider your graph from part (c) of exercise 15.3. Does the area you indicated over- or under-estimate the amount consumers would have to be compensated (in cash) in order to accept the license fee?*

Answer: The area measured along the uncompensated demand curve is illustrated in panel (a) of Graph 15.4 as $(a+b+c)$. The compensation required is measured along the compensated demand curve (or $MWTP$) curve that goes through A. When hamburgers are normal goods, this $MWTP^A$ curve is steeper than the demand curve. Thus, the measure of the required compensation is $(a+b+c+d)$ — or d greater than what you would estimate using the demand curve. Thus, the demand-curve measure underestimates the compensation required.



Graph 15.4: License Fees and Income Effects

(b) *Does the area over- or under-estimate the amount consumers are willing to pay to avoid the license fee?*

Answer: The area that measures the payment consumers are willing to make to avoid the fee is measured along the compensated demand curve that goes through B — i.e. $MWTP^B$ in panel (a) of Graph 15.4. This area is $(a+b)$ — or c less than what you would estimate using the regular demand curve. Thus, you would be over-estimating the amount people are willing to pay to avoid the fee if you used the regular demand curve to measure this.

(c) *How would your answers to (a) and (b) differ if hamburgers were instead an inferior good for all consumers?*

Answer: This is illustrated in panel (b) of Graph 15.4. The graph differs from panel (b) in that the marginal willingness to pay curves are now shallower than the regular demand curve. Thus, the compensation required is given by $(e+g)$ and the amount consumers are willing to pay to avoid the fee is given by $(e+f+g+h+i)$. The area measured on the regular demand curve is $(e+f+g+h)$. Thus, the compensation would now be over-estimated (by $(f+h)$) while the amount consumers are willing to pay to avoid the fee is under-estimated (by area i) when the regular demand curve is used instead of the correct compensated demand curve.

(d) *Do any of your conclusions depend on the assumption (made explicitly in exercise 15.3) that all firms are identical?*

Answer: No — none of the conclusions would change. In panel (c) of Graph 15.4, the only thing that changes is that the supply curves have positive slope — but the consumers still move from A to B and all the previous applies.

B: *Suppose that tastes by consumers are characterized by the utility function $u(x,y) = x^{0.25}y^{0.75}$ and that each consumer had \$100 budgeted for hamburgers x and other goods y.*

(a) *Calculate how many hamburgers each consumer consumes — and how much utility (as measured by this utility function) each consumer obtains — when the price of hamburgers is \$5 (and the price of "other goods" is \$1).*

Answer: The demand function for hamburgers in this case is $x(p) = 0.25(100)/p$. Thus, the consumer consumes $x = 25/5 = 5$ hamburgers, leaving \$75 for other consumption. This allows the consumer to get utility $u(5,75) = 5^{0.25}75^{0.75} \approx 38.11$.

(b) *Derive the expenditure function for a consumer with such tastes.*

Answer: In Chapter 10, we derived the expenditure function for Cobb-Douglas utility functions of the form $u(x_1,x_2) = x_1^{\alpha}x_2^{(1-\alpha)}$ as

$$E(p_1,p_2,u) = \frac{up_1^{\alpha}p_2^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}. \quad (15.29)$$

Substituting the appropriate values for this problem (given that the price of other goods is 1), we get

$$E(p,u) = \frac{up^{0.25}}{0.25^{0.25}(1-0.25)^{(1-0.25)}} = 1.75476535up^{0.25}. \quad (15.30)$$

(c) *Suppose that the license fee causes the price to increase to \$5.77 (as in exercise 15.3). How does your answer to (a) change?*

Answer: The number of hamburgers falls to $x = 25/5.77 = 4.3327556$, leaving \$75 for other consumption. This implies that utility falls to $u = 4.3327556^{0.25}75^{0.75} \approx 36.77$.

(d) *Using the expenditure function, calculate the amount the government would need to compensate each consumer in order for them to agree to the imposition of the license fee?*

Answer: In order for the government to compensate the consumer, it has to ensure the consumer has enough of a budget to reach the pre-license fee utility level of 38.11 at the post-license fee price of 5.77. Plugging these into $E(p,u)$, we get

$$E(5.77, 38.11) = 1.75476535(38.11)(5.77)^{0.25} \approx 103.65. \quad (15.31)$$

Since the consumer begins with a budget of \$100, this implies the government needs to compensate each consumer by \$3.65.

(e) *Calculate the amount that consumers would be willing to pay to avoid the license fee.*

Answer: In order to calculate this, we need to know how much less of a budget the consumer could have to reach post-license fee utility 36.77 at the pre-license free price of 5. Plugging these into the expenditure function, we get

$$E(5, 36.77) = 1.75476535(36.77)(5)^{0.25} \approx 96.48. \quad (15.32)$$

Given that the consumer begins with a budget of \$100, this implies she is willing to give up approximately \$3.52 to avoid the license fee.

(f) *Suppose you used the demand curve to estimate the change in consumer surplus from the introduction of the license fee. How would your estimate compare to your answers in (d) and (e)?*

Answer: We would be taking the integral of the demand function and evaluating it between the two prices — i.e.

$$\int_5^{5.77} \frac{25}{p}\,dp = 25\ln(p)|_5^{5.77} = 25(\ln(5.77) - \ln(5)) \approx 3.58. \quad (15.33)$$

Thus, the estimate falls in between the answers to (d) and (e) — just as predicted in part A of the exercise.

(g) *Can you use integrals of compensated demand curves to arrive at your answers from (d) and (e)?*

Answer: For a Cobb-Douglas utility function of the form $u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$, we calculated in Chapter 10 that the compensated demand functions for x_1 is

$$h_1(p_1, p_2, u) = \left(\frac{\alpha p_2}{(1-\alpha)p_1}\right)^{(1-\alpha)} u. \quad (15.34)$$

Plugging in $\alpha = 0.25$ and $p_2 = 1$ for this problem, this gives us

$$h_x(p, u) = \left(\frac{1}{3p}\right)^{0.75} u. \quad (15.35)$$

The compensation amount in part (d) is measured on the compensated demand curve for the pre-license fee utility level 38.11, while the payment in part (e) is measured on the post-license fee utility level 36.77. Thus, the amount in (d) is

$$\int_5^{5.77} \left(\frac{1}{3p}\right)^{0.75} (38.11)\,dp = 4\left(\frac{1}{3}\right)^{0.75} (38.11)p^{0.25}|_5^{5.77} \approx 3.65 \quad (15.36)$$

and the amount in (e) is

$$\int_5^{5.77} \left(\frac{1}{3p}\right)^{0.75} (36.77)\,dp = 4\left(\frac{1}{3}\right)^{0.75} (36.77)p^{0.25}|_5^{5.77} \approx 3.52. \quad (15.37)$$

These are exactly the same answers we derived using the expenditure function in (d) and (e).

15.5 Policy Application: *Redistribution of Income without Income Effects*: *Consider the problem a society faces if it wants to both maximize efficiency while also insuring that the overall distribution of "happiness" in the society satisfies some notion of "equity".*

A: *Suppose that everyone in the economy has tastes over x and a composite good y, with all tastes quasilinear in x.*

(a) *Does the market demand curve (for x) in such an economy depend on how income is distributed among individuals (assuming no one ends up at a corner solution)?*

Answer: If everyone's tastes are quasilinear in x, this means that each person's demand for x is independent of income (unless someone is at a corner solution). Thus, the aggregate demand curve in the market for x does not depend on the distribution of income in the population. Since the supply curve also does not depend on the distribution of income, the market equilibrium in the x market is independent of the income distribution.

(b) *Suppose you are asked for advice by a government that has the dual objective of maximizing efficiency as well as insuring some notion of "equity". In particular, the government considers two possible proposals: Under proposal A, the government redistributes income from wealthier individuals to poorer individuals before allowing the market for x to operate. Under proposal B, on the other hand, the government allows the market for x to operate immediately and then redistributes money from wealthy to poorer individuals after equilibrium has been reached in the market. Which would you recommend?*

Answer: Since the market outcome in the x market is independent of the distribution of income, it does not matter whether income is redistributed before or after the market equilibrium has been reached. The end result will be exactly the same. Thus, you should tell the government it does not matter which policy is put in place.

(c) *Suppose next that the government has been replaced by an omniscient social planner who does not rely on market processes but who shares the previous government's dual objective. Would this planner choose a different output level for x than is chosen under proposal A or proposal B in part (b)?*

Answer: No, the social planner would do exactly what the government would do under either of the two policies. This is because the social planner is not restricting his ability to achieve different notions of equity by allowing surplus in the x market to be maximized — which happens when the competitive equilibrium quantity of x is produced.

(d) True or False*: As long as money can be easily transferred between individuals, there is no tension in this economy between achieving many different notions of "equity" and achieving efficiency in the market for x.*

Answer: This is true (as already explained in the previous part).

(e) *To add some additional realism to the exercise, suppose that the government has to use distortionary taxes in order to redistribute income between individuals. Is it still the case that there is no tradeoff between efficiency and different notions of equity?*

Answer: In this case, a tradeoff does emerge — because redistribution through distortionary taxes implies the creation of deadweight losses as income is transferred between individuals. Thus, more redistribution implies a loss of social surplus — thus the tension between "equity" and efficiency.

B: *Suppose there are two types of consumers: Consumer type 1 has utility function $u^1(x,y) = 50x^{1/2} + y$, and consumer type 2 has utility function $u^2(x,y) = 10x^{3/4} + y$. Suppose further that consumer type 1 has income of 800 and consumer type 2 has income of 1,200.*

(a) *Calculate the demand functions for x for each consumer type assuming the price of x is p and the price of y is 1.*

Answer: Using the utility function $u(x,y) = Ax^{\alpha} + y$, we can solve for the demand function for x as

$$2x(p) = \left(\frac{\alpha A}{p}\right)^{1/(1-\alpha)}. \tag{15.38}$$

Substituting for the terms in the two utility functions for the two types, this implies demand functions

$$x^1(p) = \left(\frac{0.5(50)}{p}\right)^{1/(1-0.5)} = \frac{625}{p^2} \text{ and } x^2(p) = \left(\frac{0.75(10)}{p}\right)^{1/(1-0.75)} = \frac{3,164.0625}{p^4} \quad (15.39)$$

for type 1 and 2 respectively.

(b) *Calculate the aggregate demand function when there are 32,000 of each consumer type.*

Answer: Multiplying each demand function by 32,000 and adding, we get

$$X(p) = \frac{32,000(625)}{p^2} + \frac{32,000(3,164.0625)}{p^4} = \frac{20,000,000p^2 + 101,250,000}{p^4}. \quad (15.40)$$

(c) *Suppose that the market for x is a perfectly competitive market with identical firms that attain zero long run profit when $p = 2.5$. Determine the long run equilibrium output level in this industry.*

Answer: Substituting $p = 2.5$ into the equation $X(p)$, we get $X(2.5) = 5,792,000$.

(d) *How much x does each consumer type consume?*

Answer: Type 1 consumers consume $625/(2.5^2) = 100$ units of x and type 2 consumers consume $3,164.0625/(2.5^4) = 81$ units of x.

(e) *Suppose the government decides to redistribute income in such a way that, after the redistribution, all consumers have equal income — i.e. all consumers now have income of 1,000. Will the equilibrium in the x market change? Will the consumption of x by any consumer change?*

Answer: Income does not enter any demand function (because the good x is quasilinear) — which implies that the income distribution does not enter the aggregate demand function $X(p)$. Thus, redistributing income in this way does not change either the equilibrium level of output in the market or the level of x consumption of any individual.

(f) *Suppose instead of a competitive market, a social planner determined how much x and how much y every consumer consumes. Assume that the social planner is concerned about both the absolute welfare of each consumer as well as the distribution of welfare across consumers — with more equal distribution more desirable. Will the planner produce the same amount of x as the competitive market?*

Answer: Yes — social surplus is still maximized at the same output level regardless of how the planner decides to redistribute income (so long as no one ends up at a corner solution). Thus, the planner would want to maximize the surplus in the x market by picking the same output level as the market — and he can then worry about redistributing income to the desired level.

(g) True or False*: The social planner can achieve his desired outcome by allowing a competitive market in x to operate and then simply transferring y across individuals to achieve the desired distribution of happiness in society.*

Answer: This is true. In other words, in an economy where all tastes are quasilinear in x, the planner does not actually have to calculate the optimal quantity of x but can rather allow the market to determine that quantity since it is unaffected by how income is distributed. By shifting y from some people to others, the planner can then achieve whatever desired level of "equity" he desires.

(h) *Would anything in your analysis change if the market supply function were upward sloping?*

Answer: Since the market demand curve is unaffected by redistribution of income, the market demand would continue to intersect market supply at the same point regardless of whether or not the supply curve slopes up. Thus, nothing changes fundamentally in the problem if we assume an upward sloping supply curve.

(i) *Economists sometimes refer to economies in which all individuals have quasilinear tastes as "transferable utility economies" — which means that in economies like this, the government can transfer happiness from one person to another. Can you see why this is the case if we were using the utility functions as accurate measurements of happiness?*

Answer: If we use the two utility functions in this problem as accurate measurements of happiness, then the planner will increase utility by 1 unit for a person of type 1 and lower it by 1 unit for a person of type 2 if he transfers one unit of y from person 1 to person 2. Thus, he is in essence able to transfer utility between individuals.

15.6 Policy Application: *Redistributing Income with Income Effects*: *Consider again, as in exercise 15.5, the problem faced by a society that wants to both maximize efficiency and achieve some notion of "equity".*

A: *Suppose again that everyone has tastes over x and a composite good y, but now suppose that tastes are homothetic.*

(a) *Does the market demand curve (for x) depend on how income is distributed among individuals?*

Answer: Yes, it does (with the exception of the case dealt with in part (b)). When tastes are homothetic, consumers choose their consumption bundle on a ray from the origin for any given set of prices. But if their tastes differ — even if they are all homothetic — they optimize on different rays when they face the same prices. Thus, as income is redistributed (without changing prices), the consumption bundles for each individual change along the ray relevant for that individual. And since the rays for different individuals have different slopes, the movements along rays as income is redistributed do not offset one another. Thus, the market demand curve depends on the distribution of income in the population.

(b) *Would your answer to (a) be different if you thought that everyone had identical (homothetic) tastes?*

Answer: If everyone had identical homothetic tastes, then everyone's optimal consumption bundle would move along the same ray from the origin as income changes. Thus, changes in consumption resulting from redistribution of income would exactly offset each other — and the market demand curve would therefore only depend on overall income in the economy but not the distribution of income across individuals.

(c) *Suppose you are again asked for the same advice as in exercise 15.5A(b). What is your answer now?*

Answer: If income is redistributed before the market for x unfolds, then everyone knows how much income they have and therefore everyone optimizes at the point where MRS equals the (negative) price ratio. The market demand curve would be composed of the sum of all the individual demand curves derived from individual optimizing behavior — and the market equilibrium would maximize social surplus. If the market operates before income redistribution, however, individuals use their pre-redistribution income to optimize — and those decisions go into the formation of the market equilibrium. When income is redistributed after the equilibrium has been reached, however, individuals would discover that they have in fact not chosen their utility maximizing bundle because they used the wrong income to optimize. As a result, the market equilibrium followed by income redistribution would be inefficient. You should therefore advise the government to choose the policy that redistributes before, not after, the market operates.

(d) *Repeat part A(c) from exercise 15.5 for this economy.*

Answer: This planner would choose the output level under the efficient policy that redistributes income before, not after the market operates.

(e) *Recall that we defined a situation as "efficient" if there is no way to change the situation and make someone better off without making someone else worse off. In general (i.e. not just within the context of the example in this exercise), is it possible to have two efficient outcomes where some individuals prefer the first outcome while others prefer the second?*

Answer: Yes. Consider two possible ways we could divide a pie: In the first case, I get 80% and you get 20%, and in the second case I get 20% and you get 80%. If all we have is a pie to divide, both of these are efficient ways of dividing the pie — because in both cases there is no way to make someone better off without taking pie away from the other guy and therefore make him worse off. I would clearly prefer the first case, and you would clearly prefer the second — but both are efficient. Moving from the first case to the second is not a move toward greater efficiency (and in fact makes me worse off), but once we changed to the second case, we are at an efficient situation in the sense that we cannot make anyone better off without making someone else worse off.

(f) True or False*: If the government redistributes income between individuals prior to the market for x operating, the outcome is efficient so long as income can be redistributed without cost.*

Answer: This is true. It does not matter how the government divides the income across individuals — if the market is permitted to operate afterwards, output will be set by the intersection of demand and supply. And that output level will be efficient for the distribution of income the government achieved before the market opened. Of course it matters to individuals whether the benefit or are hurt by the redistribution — which implies that individuals will typically care which way income is redistributed. But the market can achieve the efficient outcome for any income distribution once that distribution has been achieved.

(g) True or False*: In the quasilinear example of exercise 15.5, all efficient outcomes (excluding those that involve corner solutions) will involve the same level of production of x, but in the example of the current exercise this is no longer the case.*

Answer: This is also true. We demonstrated in the previous exercise that the first part is true — and it is true because there are no income effects when tastes are quasilinear. Thus, as the government redistributes income, demand for x never changes — which implies the intersection of demand and supply never changes. In the case of homothetic tastes, however, we have concluded that the market demand curve does depend on how income is distributed — which implies the intersection of demand and supply will be affected by how income is distributed. Thus, for different redistribution policies, the market will produce different levels of x — but for any initial income distribution, the market will end up producing the quantity of x that is efficient for *that* initial distribution of income.

(h) True or False*: Assuming redistribution takes place before the market opens, a tradeoff between efficiency and equity only emerges in this economy if redistributing money between individuals involves the use of distortionary taxes.*

Answer: This is again true. We concluded that the market will reach an efficient outcome for any initial income distribution — and thus the government can achieve different notions of “equity” by redistributing initial income however it wishes. Once it is done, the market selects the efficient level of output for that income distribution. But if redistributing income involves distortionary taxes, then redistribution carries a dead weight loss — which implies that a tradeoff between “equity” and efficiency emerges.

(i) *Does your conclusion in (h) hold more generally for non-homothetic tastes as well?*

Answer: Yes — nothing we have said with the exception to the answer to (b) changes if tastes are not homothetic.

B: *Suppose again, as in exercise 15.5, that there are two types of individuals in the economy. Type 1 has utility function $u^1(x,y) = x^{\alpha}y^{(1-\alpha)}$ and type 2 has utility function $u^2(x,y) = x^{\beta}y^{(1-\beta)}$ (with both α and β falling between 0 and 1). Suppose further that type 1 individuals have income I and type 2 individuals have income I'.*

(a) *What is each type's demand function for x assuming price p for x and a price of 1 for y.*

Answer: The demand functions for type 1 and 2 consumers are, respectively,

$$x^1(p,I) = \frac{\alpha I}{p} \text{ and } x^2(p,I') = \frac{\beta I'}{p}. \tag{15.41}$$

(b) *What is the market demand function for x if there are an equal number N of each type in the economy.*

Answer: The market demand function is then

$$X(p,I,I') = N\left(\frac{\alpha I}{p}\right) + N\left(\frac{\beta I'}{p}\right) = \frac{N(\alpha I + \beta I')}{p}. \tag{15.42}$$

(c) *Suppose $\alpha = \beta$ and money can be transferred across individuals without cost. Will the equilibrium output level in the x market be affected by income redistribution policies? Will individual consumption levels of x be affected by such policies?*

Answer: When $\alpha = \beta$, the market demand function from equation (15.42) becomes

$$X(p,I,I') = \frac{\alpha N(I+I')}{p}. \tag{15.43}$$

Since the total income in the economy is constant, the term $N(I+I')$ remains constant as income is redistributed between individuals. Thus, the market demand is just a function of price p and total income $N(I+I')$ — which implies that the market demand does not depend on how income is distributed. As a result, the output quantity in the market is unaffected by redistribution of income. However, since each individual's demand function depends on that individual's income, consumption of x will fall for those who lose income under any redistribution policy, and consumption of x will increase for anyone who gains income. Overall, the effects cancel each other out.

(d) *Next, suppose $\alpha \neq \beta$. Will the equilibrium output level in the x market be affected by income redistribution policies?*

Answer: Equation (15.42) cannot then be reduced to anything simpler and therefore remains

$$X(p,I,I') = \frac{N(\alpha I + \beta I')}{p}. \tag{15.44}$$

Since I and I' enter separately, the market demand therefore does not just depend on total income in the economy but also on the distribution of income. This implies that the equilibrium output in the economy will be affected by redistribution across types of individuals.

(e) *Suppose again that you are asked for your advice on the two alternative policies described in exercise 15.5A(b) (assuming again that the government has the dual objective of maximizing efficiency and achieving some notion of "equity".) What is your advice now assuming that individuals cannot trade goods with one another after they have purchased x?*

Answer: If the government redistributes before decisions are made in the market, then each individual will optimize given the market price that emerges. While the precise price that emerges will depend on the distribution of income across individuals, the equilibrium in the market will be efficient for the same reasons as always. However, if the government were to allow the market to reach equilibrium before it redistributes income, then individuals are making consumption decisions in the x market based on their pre-redistribution income. Since individual demands depend on individual income, this implies that, once the government redistributes, individuals will not be at their utility maximizing output bundle. The resulting situation will therefore not be efficient. It is therefore preferable to redistribute before individuals make decisions in the market.

(f) *Assume again that the government is replaced by an omniscient social planner who shares the previous government's dual objective. Will his decision on how much x to produce mirror the outcome of either of the two policies you considered in part (e)?*

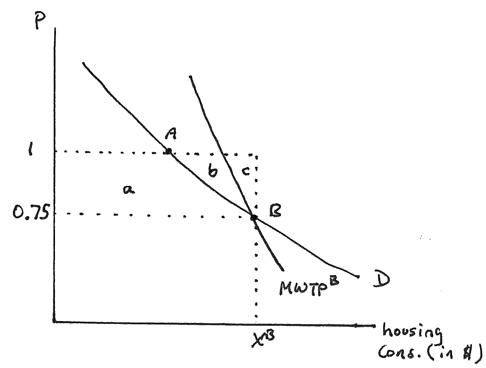
Answer: The planner would want to mirror the outcome under the policy that redistributes before the market operates.

15.7 Policy Application: *Dead Weight Loss from Subsidy of Mortgage Interest: The U.S. tax code subsidizes housing through a deduction of mortgage interest. For new homeowners, mortgage interest makes up the bulk of their housing payments which tend to make up about 25% of a household's income. Assume throughout that housing is a normal good.*

A: *For purposes of this problem, we will assume that all housing payments made by a household represent mortgage interest payments. If a household is in a 25% tax bracket, allowing the household to deduct mortgage interest on their taxes then is equivalent to reducing the price of $1 worth of housing consumption to $0.75.*

(a) *Illustrate a demand curve for a consumer, indicating both the with- and without-deductibility housing price.*

Answer: This is illustrated in Graph 15.5 where the demand curve is given by D. Without tax deductibility of housing costs, the consumer would locate at A where the price of a dollar of housing consumption is $1. Under deductibility, however, the consumer faces a price of $0.75 for every dollar in housing consumption — which implies she will locate at B.



Graph 15.5: Tax Deductibility of Housing Costs

(b) *On the same graph, illustrate the compensated (or $MWTP$) curve for this consumer assuming that housing costs are deductible.*

Answer: If housing costs are deductible, the consumer locates at B. Thus, we would need to draw the $MWTP$ curve that runs through B — indicated as $MWTP^B$ in the graph. This is steeper than the uncompensated demand curve because housing is assumed to be a normal good.

(c) *On your graph, indicate where you would locate the amount that a consumer would be willing to accept in cash instead of having the subsidy of housing through the tax code.*

Answer: The consumer is equally happy all along $MWTP^B$ — which implies that we would need to give her the area $(a+b)$ in cash in order for her to be indifferent between the cash and the price subsidy.

(d) *On your graph, indicate the area of the deadweight loss.*

Answer: Under the subsidy implicit in the tax deductibility provision of the tax code, the government in essence pays $0.25 for every $1 in housing the consumer chooses. Under the subsidy the consumer chooses x^B — which implies that the total cost of the subsidy to the government is $0.25x^B$ — which is equal to the area $(a+b+c)$. Thus, the tax deductibility costs the government c more than the cash subsidy that would make the consumer just as well off — which implies c is the deadweight loss.

(e) *If you used the regular demand curve to estimate the deadweight loss, by how much would you over- or under-estimate it?*

Answer: If we used the regular demand curve to estimate the cash amount necessary to make the consumer just as happy, we would implicitly assume that housing is quasilinear (which it is not). As a result, we would conclude that the area a is how much cash the consumer would accept instead of tax deductibility of housing — which would lead us to conclude that the deadweight loss from tax deductibility is $(b+c)$ when it is actually just c. Thus, we would over-estimate the deadweight loss by area b.

B: *Suppose that a household earning \$60,000 (after taxes) has utility function $u(x,y)=x^{0.25}y^{0.75}$, where x represents dollars worth of housing and y represents dollars worth of other consumption. (Thus, we are implicitly setting the price of x and y to \$1.)*

(a) *How much housing does the household consume in the absence of tax deductibility?*

Answer: Letting p equal the price of housing, the demand function for this consumer is $x(p)=0.25(60,000)/p$. When $p=1$, this implies that $x=15,000$.

(b) *If the household's marginal tax rate is 25% (and if all housing payments are deductible), how much housing will the household consume?*

Answer: The housing price for this household now falls to \$0.75 — which implies the household will choose $x=0.25(60,000)/0.75=20,000$ in housing.

(c) *How much does the implicit housing subsidy cost the government for this consumer?*

Answer: Since the government effectively pays a quarter of the housing bill, it costs the government \$5,000.

(d) *Derive the expenditure function for this household (holding the price of other consumption at \$1 but representing the price of housing as p.)*

Answer: For a Cobb-Douglas utility function of the form $u(x_1,x_2)=x_1^{\alpha}x_2^{(1-\alpha)}$, we showed in Chapter 10 that the expenditure function takes the form

$$E(p_1,p_2,u)=\frac{up_1^{\alpha}p_2^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}. \tag{15.45}$$

Setting $p_1=p$, $p_2=1$ and $\alpha=0.25$, this gives us

$$E(p,u)=1.75476535p^{0.25}u. \tag{15.46}$$

(e) *Suppose the government contemplates eliminating the tax deductibility of housing expenditures. How much would it have to compensate this household for the household to agree to this?*

Answer: At the subsidized housing price, the household consumes \$20,000 in housing and \$45,000 in other goods. This gives utility of $u(20000,45000)\approx 36,742$. To reach this level of utility at a non-subsidized price, the household's budget would have to be

$$E(1,36742)=1.75476535(36742)\approx 64,474. \tag{15.47}$$

Since the consumer begins with \$60,000, this means the government would have to pay the household \$4,474.

(f) *Can you derive the same amount as an integral on a compensated demand function?*

Answer: For a Cobb-Douglas utility function of the form $u(x_1,x_2)=x_1^{\alpha}x_2^{\beta}$, we calculated in Chapter 10 that the compensated demand functions for x_1 is

$$h_1(p_1,p_2,u)=\left(\frac{\alpha p_2}{(1-\alpha)p_1}\right)^{(1-\alpha)}u. \tag{15.48}$$

Plugging in $\alpha=0.25$, $p_2=1$ and $p_1=p$, we get the compensated demand function

$$h_x(p,u)=\left(\frac{1}{3p}\right)^{0.75}u. \tag{15.49}$$

Evaluating the integral of this between the prices 0.75 and 1 when utility is 36,742, we get

$$\int_{0.75}^{1}\left(\frac{1}{3p}\right)^{0.75}(36742)\,dp = 4(36742)\left(\frac{1}{3}\right)^{0.75} p^{0.25}|_{0.75}^{1}$$
$$= 64474\left(1-0.75^{0.25}\right) \approx 4,474. \quad (15.50)$$

(g) *Suppose you only knew this household's (uncompensated) demand curve and used it to estimate the change in consumer surplus from eliminating the tax deductibility of housing expenditures. How much would you estimate this to be?*

Answer: You would take the integral of the uncompensated demand curve between prices 0.75 and 1 to get

$$\int_{0.75}^{1}\frac{15000}{p}\,dp = 15000\ln(p)|_{0.75}^{1} = 15000(\ln(1)-\ln(0.75)) \approx 4,315. \quad (15.51)$$

(h) *Are you over- or under-estimating the deadweight loss from the subsidy if you use the (uncompensated) demand curve?*

Answer: If we use the uncompensated demand curve, we estimate the dead-weight loss from the subsidy as $5,000-4,315$ = \$685. If we use the compensated demand curve, we get $5,000-4,474$ = \$526. Thus, we are over-estimating the deadweight loss if we use the uncompensated demand function.

(i) *Suppose that all 50,000,000 home-owners in the U.S. are identical to the one you have just analyzed. What is the annual deadweight loss from the deductibility of housing expenses? By how much would you over- or under-estimate this amount if you used the aggregate demand curve for housing in this case?*

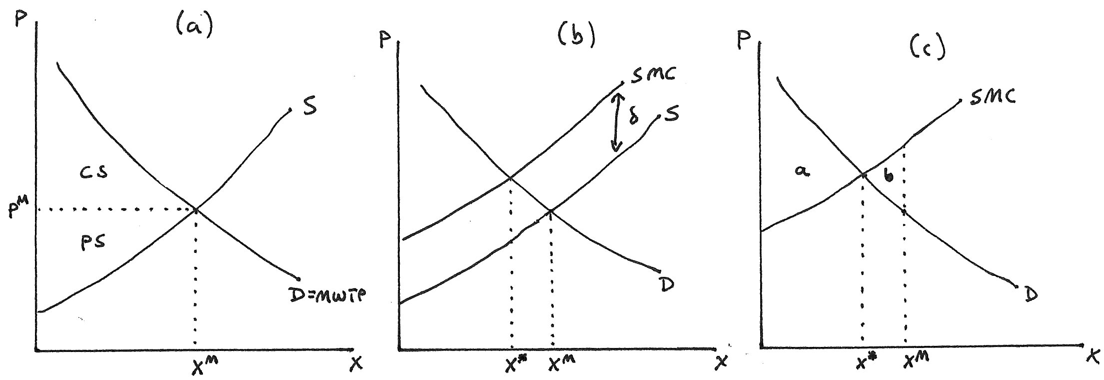
Answer: The annual deadweight loss is 526(50,000,000) = \$26,300,000,000 or \$26.3 billion dollars. If we used the uncompensated demand curve to estimate the deadweight loss, we would get \$34.25 billion instead. Thus, by using the wrong demand curve, we would over-estimate the deadweight loss by \$7.95 billion.

15.8 Policy Application: *Markets, Social Planners and Pollution: One of the conditions we identified as important to the first welfare theorem is that there are no* externalities. *One of the most important externalities in the real world is pollution from production (which we will explore in detail in Chapter 21).*

A: *Suppose that we consider the production of some good x and assume that consumers have tastes over x and a composite good y where x is quasilinear.*

(a) *Illustrate the market equilibrium in a graph with x on the horizontal and the price p of x on the vertical axis. Assume that the supply curve is upward sloping — either because you are considering the short run in the industry or because the industry is composed of firms that differ in their cost curves.*

Answer: This is illustrated in panel (a) of Graph 15.6 where the demand curve D is also equal to the $MWTP$ curve because of the quasilinearity of x. The market will produce output quantity x^M and price p^M.



Graph 15.6: Pollution in Production

(b) *On your graph, indicate the consumer surplus and producer profit (or producer surplus).*

Answer: Consumer surplus is indicated as CS and producer profit — or producer surplus — is indicated as area PS in panel (a) of the graph.

(c) *In the absence of externalities, why is the market equilibrium output level the same as the output level chosen by a social planner who wants to maximize social surplus?*

Answer: In the absence of externalities, the demand curve represents the marginal benefit society gets from each unit of x and the supply curve represents the marginal cost incurred by society. The difference between these is positive for all x less than x^M — implying that social surplus is produced for each unit of output until we reach x^M. For all output units above x^M, however, the marginal social benefit (as measured by the $MWTP$ curve) is less than the marginal social cost (as measured by the supply curve) — which implies we would incur a negative social surplus for any unit of output above x^M. Thus, a social planner that wishes to maximize social surplus would choose to produce x^M.

(d) *Now suppose that, for every unit of x that is produced, an amount of pollution that causes social damage of δ is emitted. If you wanted to illustrate not just the marginal cost of production (as captured in supply curves) but also the additional marginal cost of pollution (that is not felt by producers), where would that “social marginal cost” curve lie in your graph?*

Answer: This is illustrated in panel (b) of Graph 15.6 where the social marginal cost curve SMC lies δ above the supply curve — because, for every unit of output, society now incurs not only the marginal cost faced by producers (as represented in the supply curve) but also the additional marginal cost δ of pollution.

(e) *In the absence of any non-market intervention, do firms have an incentive to think about the marginal cost of pollution? Will the market equilibrium change as a result of the fact that pollution is emitted in the production process?*

Answer: Firms have no incentive to take the cost of pollution into account because they do not have to pay for it. Thus, the market equilibrium will remain unchanged and will continue to occur at the intersection of supply and demand — resulting in output x^M.

(f) *Would the social planner who wishes to maximize social surplus take the marginal social cost of pollution into account? Illustrate in your graph the output quantity that this social planner would choose and compare it to the quantity the market would produce.*

Answer: Since the social planner wants to maximize the total social surplus, he would want to take into account the cost of pollution. Thus, the social planner would want to produce so long as SMC is below $MWTP$ — or until x^* in panel (b) of Graph 15.6.

(g) *Re-draw your graph with the following two curves: The demand curve and the marginal social cost curve (that includes both the marginal costs of producers and the cost imposed on society by the pollution that is generated). Also, indicate on your graph the quantity x^* that the social planner wishes to produce as well as the quantity x^M that the market would produce. Can you identify in your graph an area that is equal to the deadweight loss that is produced by relying solely on the competitive market?*

Answer: This is drawn in panel (c) of Graph 15.6. If the social planner's production level x^* is produced, the total surplus would be equal to area a. If the market produces x^M instead, we would still get the area a of social surplus but we would incur a negative social surplus for the output units between x^* and x^M. That negative area is equal to b in the graph — giving us an overall surplus under the market of $(a-b)$. Thus, the deadweight loss from the overproduction in the market is equal to area b.

(h) *Explain how pollution-producing production processes can result in inefficient outcomes under perfect competition. How does your conclusion change if the government forces producers to pay δ in a per-unit tax?*

Answer: Pollution producing production processes result in inefficient market outcomes if firms are not forced to face the marginal cost of causing pollution. If the government charges the firms δ per output unit, it is in effect forcing firms to take the cost of pollution into account. As a result, the supply curve would shift up by δ (because the marginal cost of production has increased by δ). As a result, the new supply curve would intersect demand at x^* — thus restoring efficiency in the market.

B: *In exercise 15.2, you should have derived the aggregate demand function $X^D(p) = 250{,}000/p^2$ from the presence of 10,000 consumers with tastes that can be represented by the utility function $u(x,y) = 10x^{0.5} + y$. Suppose that this accurately characterizes the demand side of the market in the current problem. Suppose further that the long run market supply curve is given by the equation $X^S(p) = 250{,}000p$.*

(a) *Derive the competitive equilibrium price and quantity produced in the market.*

Answer: The competitive equilibrium occurs where demand intersects supply — i.e. where

$$X^D(p) = \frac{250{,}000}{p^2} = 250{,}000p = X^S(p). \tag{15.52}$$

Solving for p, we get the market price $p^M = 1$. At that price, both the supply and demand functions tell us that the competitive market output will be $x^M = 250{,}000$.

(b) *Derive the size of consumer surplus and profit (or producer surplus).*

Answer: The consumer surplus is

$$\int_1^\infty \frac{250{,}000}{p^2}\,dp = -\frac{250{,}000}{p}\Big|_1^\infty = 0 - \left(-\frac{250{,}000}{1}\right) = 250{,}000. \tag{15.53}$$

The producer surplus (or profit) is equal to total revenues minus costs. Total revenues are $1(250{,}000) = \$250{,}000$, and total costs are half that (given the linear supply curve). More generally, the costs are just the area under the supply curve — which is

$$\int_0^1 250000p\ dp = 125000p^2|_0^1 = 125000 - 0 = 125,000. \tag{15.54}$$

Producer surplus is then equal to $250,000 - 125,000 = \$125,000$.

(c) *Consider a social planner who wants to maximize the social surplus. How would this planner arrive at the same output quantity as the market?*

Answer: The social planner would want to find the production level at which the marginal social benefit of the output equals the marginal social cost. In the absence of externalities, the marginal social cost curve is given by the curve that captures the marginal costs of produces — i.e. the supply curve. The marginal social benefit is given by the aggregate marginal willingness to pay curve which in turn is equal to the demand curve given that x is quasilinear. Thus, the social planner solves the same problem we solved in part (a) — except that the social planner does not have to solve for the price since he is just interested in finding out the optimal quantity to produce.

(d) *Now suppose that each unit of x that is produced results in a pollution cost to society of \$0.61. What would be the market outcome in the absence of any non-market intervention?*

Answer: The market outcome would be unchanged since producers do not have to confront the pollution cost of production. Thus, $x^M = 250,000$ and $p^M = 1$ remains the competitive market equilibrium.

(e) *Verify that, when each unit of x results in \$0.61 pollution cost, the social planner would choose* $x = 160,000$ *as the optimal output quantity.*

Answer: The social planner would again want to determine where the social marginal benefit of production equals the social marginal cost. The social marginal benefit is unchanged and given by the marginal willingness to pay function (which is equal to the demand curve given the quasilinearity of x). The demand curve is the inverse of the demand function $X^D(p) = 250,000/p^2$ which is $MWTP(x) = 500/x^{0.5}$. The social marginal cost, however, is now \$0.61 higher than the marginal cost incurred by producers. The supply function is given as $X^S(p) = 250,000p$ — which implies that the underlying marginal cost function for production is the inverse; i.e. $MC(x) = x/250,000$. Adding the social marginal cost of pollution associated with each unit of output, the social marginal cost function is therefore $SMC(x) = (x/250,000) + 0.61$. The social planner then needs to solve the problem

$$MWTP(x) = \frac{500}{x^{0.5}} = \frac{x}{250,000} + 0.61 = SMC(x). \tag{15.55}$$

At $x = 160,000$, both the right and left hand side of this equation are equal to 1.25 — which verifies $x^* = 160,000$ as being the socially optimal output level.

(f) *Calculate the total social cost of pollution under the competitive market outcome. How much is social surplus reduced from what it would be in the absence of pollution?*

Answer: In the market outcome, $x^M = 250,000$ — and society incurs a social marginal cost of \$0.61 for each output unit produced. Thus, the social cost of pollution is $250,000(0.61) = \$152,500$. We previously calculated producer surplus of \$125,000 and consumer surplus of \$250,000 — for a combined producer and consumer surplus of \$375,000. We now need to deduct the social cost of pollution — which implies an overall surplus of $375,000 - 152,500 = \$222,500$ when the market produces 250,000 units of x.

(g) *Calculate the overall social surplus (including the cost of pollution) under the social planner's preferred outcome.*

Answer: Under the social planner's preferred outcome, $x^* = 160,000$. The benefit to consumers is then the area under the $MWTP$ curve up to x^* — i.e.

$$\int_0^{160,000} MWTP(x)dx = \int_0^{160,000} \frac{500}{x^{0.5}}dx = 1000x^{0.5}|_0^{160,000} = \$400,000. \tag{15.56}$$

The cost to producers is the area under the marginal cost curve — i.e.

$$\int_0^{160,000} MC(x)dx = \int_0^{160,000} \frac{x}{250,000}dx = \frac{x^2}{500,000}|_0^{160,000} = \$51,200. \tag{15.57}$$

The overall surplus without taking into account the pollution cost is therefore $400,000 - 51,200 = \$348,800$. The pollution cost is \$0.61 for each of the 160,000 output units produced — for a total of $0.61(160,000) = \$97,600$. Subtracting this from the overall surplus in the absence of pollution, we get $348,800 - 97,600 = \$251,200$.

(h) *What deadweight loss is produced as a result of the market's overproduction?*

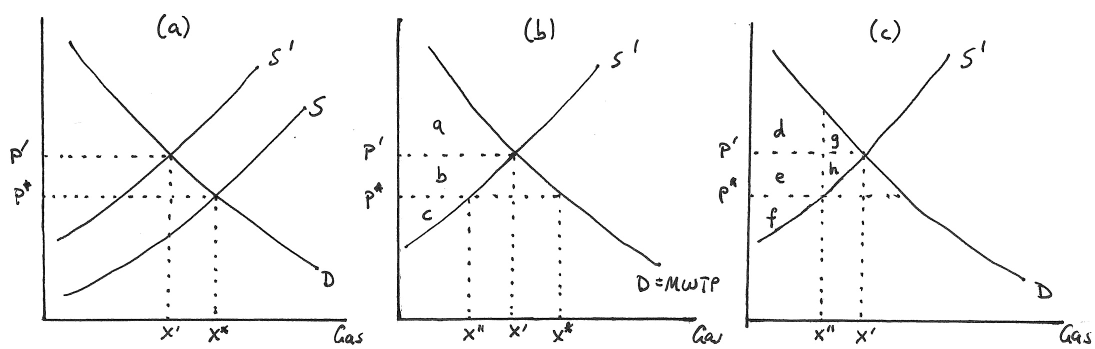
Answer: The deadweight loss is then the difference between the social surplus under the social planner and the social surplus at the market outcome — or $251,200 - 222,500 = \$28,700$.

15.9 Policy Application: *Anti-Price Gauging Laws: As we will discuss in more detail in Chapter 18, governments often interfere in markets by placing restrictions on the price that firms can charge. One common example of this is so-called "anti-price gauging laws" that restrict profits for firms when sudden supply shocks hit particular markets.*

A: *A recent hurricane disrupted the supply of gasoline to gas stations on the East Coast of the U.S. Some states in this region enforce laws that prosecute gasoline stations for raising prices as a result of natural disaster-induced drops in the supply of gasoline.*

(a) *On a graph with weekly gallons of gasoline on the horizontal and price per gallon on the vertical, illustrate the result of a sudden leftward shift in the supply curve (in the absence of any laws governing prices.)*

Answer: This is illustrated in panel (a) of Graph 15.7 where S is the original supply curve and S' is the new supply curve. The equilibrium shifts from one where price was p^* and gasoline consumption x^* to one where the price is p' and gasoline consumption is x'.



Graph 15.7: Anti-Price Gauging Laws

(b) *Suppose that gasoline is a quasilinear good for consumers. Draw a graph similar to the one in part (a) but include only the post-hurricane supply curve (as well as the unchanged demand curve). Illustrate consumer surplus and producer profit if price is allowed to settle to its equilibrium level.*

Answer: This is illustrated in panel (b) of Graph 15.7. Consumer surplus would be equal to area a and producer profit would be equal to area $(b+c)$.

(c) *Now consider a state that prohibits price adjustments as a result of natural disaster-induced supply shocks. How much gasoline will be supplied in this state? How much will be demanded?*

Answer: This is also illustrated in panel (b). At the pre-crisis price of p^*, firms would supply x'' — but consumers would want to buy x^*.

(d) *Suppose that the limited amount of gasoline is allocated at the pre-crisis price to those who are willing to pay the most for it. Illustrate the consumer surplus and producer profit.*

Answer: This is illustrated in panel (c) of Graph 15.7. If the limited amount of gasoline x'' is bought at p^* by those who value it the most, then consumer surplus is $(d+e)$. Producer profit is area f.

(e) *On a separate graph, illustrate the total surplus achieved by a social planner who insures that gasoline is given to those who value it the most and sets the quantity of gasoline at the same level as that traded in part (c). Is the social surplus different than what arises under the scenario in (d)?*

Answer: The social surplus would then be the same as in part (d) — equal to area $(d+e+f)$.

(f) *Suppose that instead the social planner allocates the socially optimal amount of gasoline. How much greater is social surplus?*

Answer: The socially optimal quantity is x'. If that much is produced, the total surplus is $(d+e+f+g+h)$ — which is greater than the surplus under the restricted quantity x'' by area $(g+h)$.

(g) *How does the total social surplus in (f) compare to what you concluded in (b) that the market would attain in the absence of anti-price gauging laws?*

Answer: It is identical.

(h) True or False*: By interfering with the price signal that communicates information about where gasoline is most needed, anti-price gauging laws have the effect of restricting the inflow of gasoline to areas that most need gasoline during times of supply disruptions.*

Answer: This is true, as demonstrated in the problem. The areas where gasoline would be most needed are those where the price would rise most in the absence of anti-price gauging laws. Thus, it is in these areas that the greatest shortages would emerge.

B: *Suppose again that the aggregate demand function $X^D(p) = 250{,}000/p^2$ arises from 10,000 local consumers of gasoline with quasilinear tastes (as in exercise 15.8).*

(a) *Suppose that the industry is in long run equilibrium — and that the short run industry supply function in this long run equilibrium is $X^S(p) = 3{,}906.25p$. Calculate the equilibrium level of (weekly) local gasoline consumption and the price per dollar.*

Answer: Setting $X^D(p) = X^S(p)$, we get $p = 4$. Substituting this back into either the demand or supply equation, we get $x = 15{,}625$.

(b) *What is the size of the consumer surplus and (short run) profit?*

Answer: The consumer surplus is

$$\int_4^\infty \frac{250{,}000}{p^2}\,dp = -\frac{250{,}000}{p}\Big|_4^\infty = 0-(-62{,}500) = \$62{,}500. \tag{15.58}$$

The firm (short run) profits are

$$\int_0^4 3{,}906.25p\,dp = 1{,}953.125p^2\Big|_0^4 = 31{,}250-0 = \$31{,}250. \tag{15.59}$$

(c) *Next suppose that the hurricane-induced shift in supply moves the short run supply function to $\overline{X}^S = 2{,}000p$. Calculate the new (short run) equilibrium price and output level.*

Answer: We solve for the new equilibrium price by setting $X^D(p) = \overline{X}^S(p)$ and solving for $p = 5$. Plugging this back into either the demand or supply functions, we get $x = 10{,}000$.

(d) *What is the sum of consumer surplus and (short run) profit if the market is allowed to adjust to the new short run equilibrium?*

Answer: Consumer surplus is now

$$\int_5^\infty \frac{250{,}000}{p^2}\,dp = -\frac{250{,}000}{p}\Big|_5^\infty = 0-(-50{,}000) = \$50{,}000. \tag{15.60}$$

Profits for firms are

$$\int_0^5 2{,}000p\,dp = 1{,}000p^2\Big|_0^5 = 25{,}000-0 = \$25{,}000. \tag{15.61}$$

Thus, the sum of consumer surplus and (short run) firm profits is $75,000.

(e) *Now suppose the state government does not permit the price of gasoline to rise above what you calculated in part (a). How much gasoline will be supplied?*

Answer: At a price of $p = 4$, the gallons of gasoline supplied will be

$$\overline{X}^S(4) = 2{,}000(4) = 8{,}000. \tag{15.62}$$

(f) *Assuming that the limited supply of gasoline is bought by those who value it the most, calculate overall surplus (i.e. consumer surplus and (short run) profit) under this policy.*

Answer: The easiest way to calculate this is to find the area under the demand *curve* that lies above the supply curve up to $x = 8,000$. The area under the demand curve is

$$\int_0^{8000} \frac{500}{x^{0.5}}\,dx = 1,000x^{0.5}|_0^{8000} \approx \$89,442.72. \quad (15.63)$$

The supply *curve* is the supply function solved for p — i.e. $p = 0.0005x$. The area under the supply curve up to $x = 8000$ is

$$\int_0^{8000} 0.0005x\,dx = 0.00025x^2|_0^{8000} = \$16,000. \quad (15.64)$$

Thus, the overall surplus is $89442.72 - 16000 = \$73,442.72$.

(g) *How much surplus is lost as a result of the government policy to not permit price increases in times of disaster-induced supply shocks?*

Answer: In the absence of the policy, total surplus was \$75,000 — which is \$1,557.28 greater than the total surplus under the policy.