SOLUTIONS

4

Tastes and Indifference Curves

Solutions for *Microeconomics: An Intuitive Approach with Calculus*

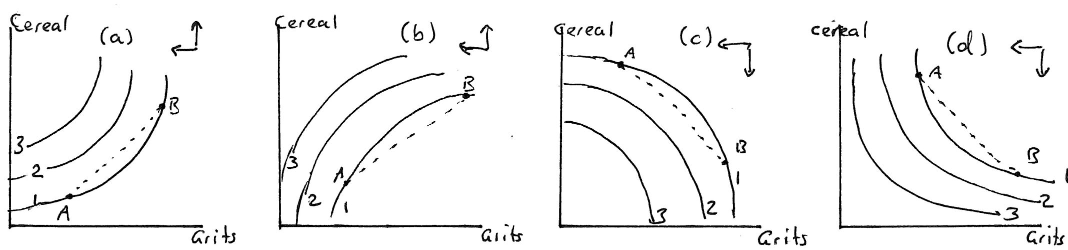
- Each end-of-chapter exercise begins on a new page. This is to facilitate maximum flexibility for instructors who may wish to share answers to some but not all exercises with their students.
- If you are assigning only the A-parts of exercises in *Microeconomics: An Intuitive Approach with Calculus*, you may wish to instead use the solution set created for the companion book *Microeconomics: An Intuitive Approach*.
- *Solutions to Within-Chapter Exercises are provided in the student* Study Guide.

4.1 *I hate grits so much that the very idea of owning grits makes me sick. I do, on the other hand, enjoy a good breakfast of Coco Puffs Cereal.*

A: *In each of the following, put boxes of grits on the horizontal axis and boxes of cereal on the vertical. Then graph three indifference curves and number them.*

(a) *Assume that my tastes satisfy the convexity and continuity assumptions and otherwise satisfy the description above.*

Answer: Panel (a) of Graph 4.1 graphs an example of such tastes. In the top right corner, arrows indicate the directions in which I become better off. As you will see in this exercise, convexity always implies that indifference curves bend toward the origin that is created by arrows such as these that indicate the directions in which a consumer becomes better off. In the graph, *A* and *B* appear on the same indifference curve — and the line connecting them lies "above" the curve in the sense that it contains only bundles to the northwest that are more preferred.



Graph 4.1: Grits and Cereal

(b) *How would your answer change if my tastes were "non-convex" — i.e. if averages were worse than extremes.*

Answer: Panel (b) graphs an example of such tastes. I still become better off moving toward the northwest where there are fewer grits and more cereal. But now the indifference curves bend in the other direction. *A* and *B* again lie on the same indifference curve, but the line connecting them now lies "below" the indifference curve in the sense that all bundles on that line segment lie to the southeast where I become worse off.

(c) *How would your answer to (a) change if I hated both Coco Puffs and grits but we again assumed my tastes satisfy the convexity assumption.*

Answer: An example of such tastes is graphed in panel (c) of Graph 4.1. Now the arrows at the top right of the graph point down and left, with better points lying to the southwest as we move toward the origin of the graph. Convexity again implies that indifference curves bend toward the origin that is created by the arrows that indicate which direction makes us better off. Bundles *A* and *B* again lie on the same indifference curve, and the line connecting them lies "above" the indifference curve in the sense that it lies to the southwest where I become better off.

(d) *What if I hated both goods and my tastes were non-convex?*

Answer: An example of such tastes is graphed in panel (d) of the graph. As in panel (c), the consumer becomes better off moving toward the southwest. But because tastes are non-convex, the indifference curves now bend in the other direction (and away from the origin that is created by the arrows in the top right corner of the graph). *A* and *B* are once again on the same indifference curve, but the line connecting them now lies "below" the indifference curve in the sense that it lies to the northeast where the consumer becomes worse off.

B: *Now suppose you like both grits and Coco Puffs, that your tastes satisfy our five basic assumptions and that they can be represented by the utility function* $u(x_1, x_2) = x_1 x_2$.

(a) *Consider two bundles, A=(1,20) and B=(10,2). Which one do you prefer?*

Answer: You would be indifferent between the two because, when you plug these into the utility function, you get the same utility value; i.e. $u(1,20) = 1(20) = 20$ and $u(10,2) = 10(2) = 20$.

(b) *Use bundles A and B to illustrate that these tastes are in fact convex.*

Answer: Suppose I construct a new bundle C that is the average of A and B — i.e. take half of A and mix it with half of B. This would give 5.5 boxes of grits and 11 boxes of cereal; i.e. C=(5.5,11). Plugging this into the utility function, we get $u(5.5,11) = 5.5(11) = 60.5$. Thus, utility of the average is higher than utility of the extremes.

(c) *What is the MRS at bundle A? What is it at bundle B?*

Answer: The MRS for this utility functions is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{x_2}{x_1}. \tag{4.1}$$

Plugging in the values for x_1 and x_2 at A and B, we then get $MRS^A = -20$ and $MRS^B = -2/10 = -1/5$.

(d) *What is the simplest possible transformation of this function that would represent tastes consistent with those described in A(d)?*

Answer: The simplest possible transformation would be to multiply the function by a negative 1. This would leave the shape of the indifference curves unchanged because the MRS would be the same. (The negative would cancel in the calculation of MRS.) But the ordering of the numbers accompanying the indifference curves would change because each number would now be multiplied by minus 1. This means that, rather than numbers going up as we move toward the northeast of the graph, numbers will go up as we go to the southwest of the graph. The indifference map would therefore look like the one we graphed in panel (d) of Graph 4.1.

(e) *Now consider tastes that are instead defined by the function $u(x_1,x_2) = x_1^2 + x_2^2$. What is the MRS of this function?*

Answer: The MRS is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{2x_1}{2x_2} = -\frac{x_1}{x_2}. \tag{4.2}$$

(f) *Do these tastes have diminishing marginal rates of substitution? Are they convex?*

Answer: Notice that the MRS is the inverse of what we calculated for the Cobb-Douglas utility function x_1x_2. Consider, for instance, the bundles (1,5) and (5,1) which both lie on the same indifference curve (that gets utility 26). At (1,5), $MRS = -1/5$ while at (5,1), $MRS = -5/1 = -5$. Thus, the MRS is shallow toward the left of the indifference curve and gets steeper toward the right — we have increasing marginal rates of substitution rather than diminishing marginal rates of substitution. Put differently, these indifference curves bend away from rather than toward the origin. Since more is better, this implies that tastes are not convex.

(g) *How could you most easily turn this utility function into one that represents tastes like those described in A(c)?*

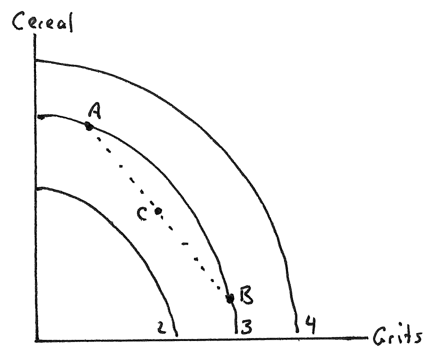
Answer: In A(c), the two goods are "bads" and tastes are convex. The tastes represented by the utility function $u(x_1,x_2) = x_1^2 + x_2^2$ in the previous part give rise to indifference curve with the shape needed for those in A(c) — but the direction of the labeling is one that assigns higher labels to bundles that contain more rather than fewer goods. By simply multiplying the function by -1, however, we reverse the labels and thus have indifference curves with the right shapes and labels increasing in the right direction. Thus, $v(x_1,x_2) = -x_1^2 - x_2^2$ would represent tastes such as those in A(c).

4.2 *Consider my wife's tastes for grits and cereal.*

A: *Unlike me, my wife likes both grits and cereal, but for her, averages (between equally preferred bundles) are worse than extremes.*

(a) *On a graph with boxes of grits on the horizontal and boxes of cereal on the vertical, illustrate three indifference curves that would be consistent with my description of my wife's tastes.*

Answer: This is illustrated in Graph 4.2. The shapes of these indifference curves arise from the following observation: Consider A and B that lie on the same indifference curve. Bundle C is the average of A and B — and since averages are worse than extremes, C must lie below the indifference curve that contains A and B.



Graph 4.2: Grits and Cereal Tastes

(b) *Suppose we ignored labels on indifference curves and simply looked at shapes of the curves that make up our indifference map. Could my indifference map look the same as my wife's if I hate both cereal and grits? If so, would my tastes be convex?*

Answer: Yes, I would simply become better off as I move in the direction of the origin in the graph while my wife would become better off moving in the opposite direction. And my tastes would indeed be convex in this case — because C would now lie "above" the indifference curve that contains A and B in the sense that I become better off as I move to the southwest in the graph.

B: *Consider the utility function* $u(x_1, x_2) = x_1^2 + 4x_2^2$.

(a) *Could this utility function represent the tastes you graphed in part A(a) above?*

Answer: Yes. To see this, we can calculate the marginal rate of substitution as

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{2x_1}{8x_2} = -\frac{x_1}{4x_2}. \tag{4.3}$$

When x_1 is low and x_2 is high, this implies that the MRS is small in absolute value — and when x_1 is high and x_2 is low, it implies that the MRS is large in absolute value. Put differently, for bundles that lie close to the vertical axis, the slope of the indifference curve is shallow, and for bundles that lie close to the horizontal axis, the slope of the indifference curve is steep. Thus, indifference curves exhibit increasing MRS as we move from left to right — giving the shape in Graph 4.2.

(b) *How could you transform this utility function to be consistent with my tastes as described in A(b)?*

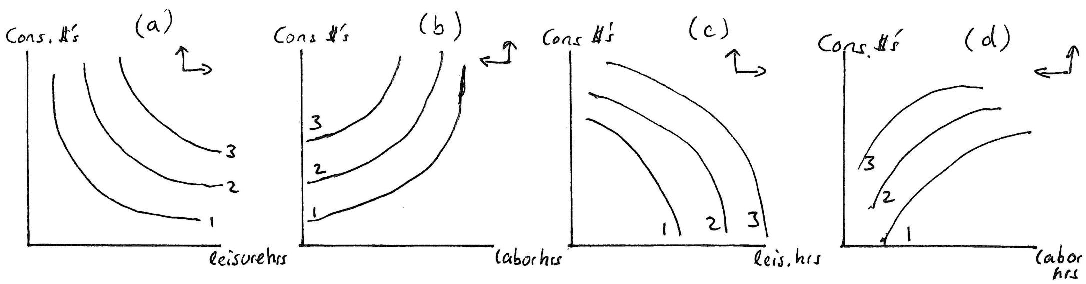
Answer: My tastes as described in A(b) give rise to the same indifference curves as my wife's, but I become better off as I get less of each good while she gets better off as she gets more. The function $u(x_1, x_2) = x_1^2 + 4x_2^2$ is increasing in both x_1 and x_2 — so it represents my wife's tastes. If I simply multiply it by negative 1, I get $v(x_1, x_2) = -x_1^2 - 4x_2^2$, a function that is decreasing in both x_1 and x_2. And the *MRS* for the function v is identical to the *MRS* of the function u — which implies the indifference curves maintain the same shape. We have thus transformed u to v where u can represent my wife's tastes and v can represent mine — with both of us having the same shapes of indifference curves.

4.3 *Consider my tastes for consumption and leisure.*

A: *Begin by assuming that my tastes over consumption and leisure satisfy our 5 basic assumptions.*

(a) *On a graph with leisure hours per week on the horizontal axis and consumption dollars per week on the vertical, give an example of three indifference curves (with associated utility numbers) from an indifference map that satisfies our assumptions.*

Answer: Panel (a) of Graph 4.3 graphs an example of three indifference curves that satisfy these assumptions.



Graph 4.3: Tastes over Consumption and Labor/Leisure

(b) *Now redefine the good on the horizontal axis as "labor hours" rather than "leisure hours". How would the same tastes look in this graph?*

Answer: Panel (b) illustrates these indifference curves with the good on the horizontal axis redefined. I now become better off going to the northwest in the graph since I prefer less labor.

(c) *How would both of your graphs change if tastes over leisure and consumption were non-convex — i.e. if averages were worse than extremes.*

Answer: Panels (c) and (d) illustrate examples of indifference curves with leisure (panel (c)) and labor (panel (d)) when tastes are non-convex. The line connecting any two points on any of these indifference curves contains only bundles that lie in the "worse" region — implying that averages are worse than extremes.

B: *Suppose your tastes over consumption and leisure could be described by the utility function* $u(\ell, c) = \ell^{1/2}c^{1/2}$.

(a) *Do these tastes satisfy our 5 basic assumptions?*

Answer: Yes. The utility function is one that has been used a number of times in the chapter. It is clearly a continuous function that assigns higher value to bundles that have more consumption and leisure (i.e. it represents monotonic tastes). The MRS for this utility function is given by $MRS = -\ell/c$. When ℓ is low and c is high (i.e. to the left in our graph), the MRS is therefore large in absolute value, and when ℓ is low and c is high (i.e. to the right in our graph), the MRS is small in absolute value. Thus, we have indifference curves that have the property of diminishing marginal rates of substitution — which is the case only when convexity is satisfied. Thus, continuity, monotonicity and convexity are all satisfied. (And the function clearly assigns utility values to all bundles — thus representing complete tastes — and any mathematical function automatically satisfies transitivity.)

(b) *Can you find a utility function that would describe the same tastes when the second good is defined as labor hours instead of leisure hours? (*Hint*: Suppose your weekly endowment of leisure time is 60 hours. How do labor hours relate to leisure hours?)*

Answer: Let l represent labor hours and assume that I have a total of 60 hours per week in possible leisure time. Then, since $\ell = 60 - l$ (because the leisure hours we actually consume are just those during which we do not work), we can write the utility function in terms of l instead of ℓ by replacing ℓ with $(60 - l)$. Our new function is then $v(c, l) = c^{1/2}(60 - l)^{1/2}$.

(c) *What is the marginal rate of substitution for the function you just derived? How does that relate to the sign of the slopes of indifference curves you graphed in part A(b)?*

Answer: The marginal rate of substitution is

$$MRS = -\frac{\partial u/\partial l}{\partial u/\partial c} = -\frac{(-1/2)c^{1/2}(60-l)^{-1/2}}{(1/2)c^{-1/2}(60-l)^{1/2}} = \frac{c}{60-l}. \quad (4.4)$$

Note the minus sign that appears in the denominator (because of the Chain Rule), which cancels the minus sign in front of the fraction to give a positive MRS. This is exactly what we graphed in panel (b) of Graph 4.3 where the slope of indifference curves is positive. (The expression above also implies that slopes start shallow and become steeper as they do in our graph — see the answer to the next part for an explanation to this.)

(d) *Do the tastes represented by the utility function in part (b) satisfy our 5 basic assumptions?*

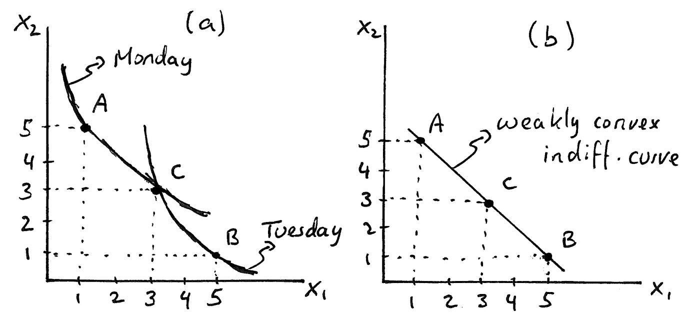
Answer: They do not because l enters negatively — which implies more l reduces utility. Thus, monotonicity is violated because of the way we have redefined the goods. The other assumptions, however, still hold. We still have a continuous function that assigns values to all bundles (i.e. we have continuity, completeness and transitivity). Also, when l and c are both low (to the left of the graph), the denominator of our MRS is large while the numerator is small — leading to a small positive number. When l and c are both high (to the right of the graph), on the other hand, the denominator becomes small while the numerator is large — leading to a large positive number. Thus, the slope of indifference curves starts small (i.e. shallow) and becomes large (i.e. steep) — precisely as depicted in panel (b) of Graph 4.3 that mapped out indifference curves under the assumption of convexity.

4.4 *Basket A contains 1 unit of x_1 and 5 units of x_2. Basket B contains 5 units of x_1 and 1 unit of x_2. Basket C contains 3 units of x_1 and 3 units of x_2. Assume throughout that tastes are monotonic.*

A: *On Monday you are offered a choice between basket A and C, and you choose A. On Tuesday you are offered a choice between basket B and C, and you choose B.*

(a) *Graph these baskets on a graph with x_1 on the horizontal and x_2 on the vertical axis.*

Answer: The three baskets are graphed in panels (a) and (b) of Graph 4.4.



Graph 4.4: Tastes on Monday and Tuesday

(b) *If I know your tastes on any given day satisfy a strict convexity assumption — by which I mean that averages are strictly better than extremes, can I conclude that your tastes have changed from Monday to Tuesday?*

Answer: Bundles *A*, *B* and *C* all lie in a straight line — with *C* the average between *A* and *B*. If you choose *A* over *C* on Monday, it must be that the indifference curve that goes through *A* lies *above* *C* (or at most also contains *C*). If your tastes are also strictly convex, this gives rise to an indifference curve through *A* that looks something like what is graphed in panel (a) of Graph 4.4. If you then choose *B* over *C* on Tuesday, it must be that a strictly convex indifference curve passes through *B* and lies above *C* (or goes through *C*). As is obvious from the graph, this implies that the indifference curve governing Monday's choice crosses the indifference curve that governs Tuesday's choice. Indifference curves from the same indifference map that represents rational tastes cannot cross. Thus, Monday's indifference curve must come from a different indifference map than Tuesday's — which implies tastes must have changed from one day to the next.

(c) *Suppose I only know that your tastes satisfy a weak convexity assumption — by which I mean that averages are at least as good as extremes. Suppose also that I know your tastes have not changed from Monday to Tuesday. Can I conclude anything about the precise shape of one of your indifference curves?*

Answer: We know from part (b) that it cannot be that the indifference curves have any curvature to them — otherwise we get crossing indifference curves. Thus, the only new option that we can exlore from knowing that tastes now satisfy only the weak convexity assumption is to think of the case where averages are just as good as extremes. If *A* and *B* lie on the same indifference curve, this assumption (averages being just as good as extremes) implies that *C* (which is the average between *A* and *B*) is just as good as *A* and *B*. In that case, it would be rational for you to choose *A* over *C* one day and *B* over *C* the next day without your tastes having changed. It also implies that the indifference curve through *A* is a straight line with slope -1 (that then also goes through *B* and *C*).

B: *Continue to assume that tastes satisfy the monotonicity assumption.*

(a) *State formally the assumption of "strict convexity" as defined in part A(b).*

Answer: Tastes represented by the relation $\succsim$ are strictly convex if and only if for all bundles A and B such that $A \sim B$,

$$\alpha A + (1-\alpha)B \succ A \text{ where } 0 < \alpha < 1. \tag{4.5}$$

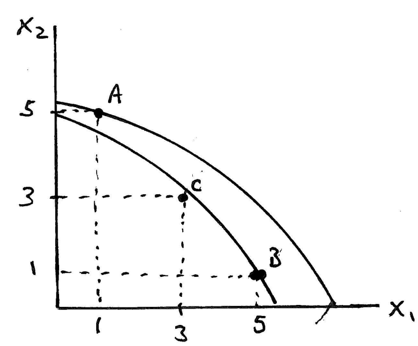
Note that by transitivity this also implies that the convex combination of A and B is strictly preferred to B.

(b) *Suppose your tastes over x_1 and x_2 were strictly non-convex — averages are strictly worse than extremes. State this assumption formally. Under this condition, would your answer to part A(b) change?*

Answer: Tastes represented by the relation $\succsim$ are strictly non-convex if and only if for all bundles A and B such that $A \sim B$,

$$A \succ \alpha A + (1-\alpha)B \text{ where } 0 < \alpha < 1. \tag{4.6}$$

If tastes were in fact non-convex, the answer to A(b) would indeed change. Graph 4.5 illustrates non-convex indifference curves through A and B that would make it rational to choose A over C and B over C. You could similarly have a single non-convex indifference curve passing through A and B and above C — which would also lead to the observed Monday and Tuesday behaviors.



Graph 4.5: Tastes on Monday and Tuesday: Part 2

(c) *Consider the utility function $u(x_1, x_2) = x_1 + x_2$. Demonstrate that this captures tastes that give rise to your conclusion about the shape of one of the indifference curves in part A(c).*

Answer: The MRS for this utility function is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{1}{1} = -1. \tag{4.7}$$

Thus, this utility function represents indifference curves that have slope of -1 everywhere — including through point A. This is precisely what is graphed in panel (b) of Graph 4.5.

4.5 *In this exercise, we explore the concept of marginal rates of substitution (and, in part B, its relation to utility functions) further.*

A: *Suppose I own 3 bananas and 6 apples, and you own 5 bananas and 10 apples.*

(a) *With bananas on the horizontal axis and apples on the vertical, the slope of my indifference curve at my current bundle is* -2*, and the slope of your indifference curve through your current bundle is* -1*. Assume that our tastes satisfy our usual five assumptions. Can you suggest a trade to me that would make both of us better off? (Feel free to assume we can trade fractions of apples and bananas).*

Answer: The slope of my indifference curve at my bundle tells us that I am willing to trade as many as 2 apples to get one more banana. The slope of your indifference curve at your bundle tells us that you are willing to trade apples and bananas one for one. If you offer me 1 banana in exchange for 1.5 apples, you would be better off because you would have been willing to accept as little as 1 apple for 1 banana. I would also be better off because I would be willing to give you as many as 2 apples for 1 banana — only having to give you 1.5 apples is better than that. (If you are uncomfortable with fractions of apples being traded, you could also propose giving me 2 bananas for 3 apples.)

This is only one possible example of a trade that would make us both better off. You could propose to give me 1 banana for x apples, where x can lie between 1 and 2. Since I am willing to give up as many as 2 apples for one banana, any such trade would make me better off, and since you are willing to trade them one for one, the same would be true for you.

(b) *After we engage in the trade you suggested, will our MRS's have gone up or down (in absolute value)?*

Answer: Any trade that makes both of us better off moves me in the direction of more bananas and fewer apples — which, given diminishing marginal rates of substitution, should decrease the absolute value of my MRS; i.e. as I get more bananas and fewer apples, I will be willing to trade fewer apples to get one more banana than I was willing to originally. You, on the other hand, are giving up bananas and getting apples, which moves you in the opposite direction toward fewer bananas and more apples. Thus, you will become less willing to trade 1 banana for 1 apple and will in future trades demand more bananas in exchange for 1 apple. Thus, in absolute value, your MRS will get larger.

(c) *If the values for our MRS's at our current consumption bundles were reversed, how would your answers to (a) and (b) change?*

Answer: The trades would simply go in the other direction; i.e. I would be willing to trade 1 banana for x apples so long as x is at least 1, and you would be willing to accept such a trade so long as x is no more than 2. Thus, x again lies between 1 and 2 if both of us are to be better off from the trade, only now I am giving you bananas in exchange for apples rather than the other way around.

(d) *What would have to be true about our MRS's at our current bundles in order for you not to be able to come up with a mutually beneficial trade?*

Answer: In order for us not to be able to trade in a mutually beneficial way, your MRS at your current bundle would have to be identical to my MRS at my current bundle.

(e) True or False*: If we have different tastes, then we will always be able to trade with both of us benefitting.*

Answer: This statement is generally false. What matters is not that we have different tastes (i.e. different maps of indifference curves). What matters instead is that, at our current consumption bundle, we value goods differently — that at our current bundle, our MRS's are different. It is quite possible for us to have different tastes (i.e. different maps of indifference curves) but to also be at bundles where our MRS is the same. In that case, we would have the same tastes *at the margin* even though we have different tastes overall (i.e. different indifference maps.)

(f) True or False*: If we have the same tastes, then we will never be able to trade with both of us benefitting.*

Answer: False. People with the same tastes but different bundles of goods may well have different marginal rates of substitution at their current bundles — and this opens the possibility of trading with benefits for both sides.

B: *Consider the following five utility functions and assume that α and β are positive real numbers:*

$$\begin{aligned}
&1.\ u^A(x_1,x_2) = x_1^{\alpha} x_2^{\beta}\\
&2.\ u^B(x_1,x_2) = \alpha x_1 + \beta x_2\\
&3.\ u^C(x_1,x_2) = \alpha x_1 + \beta \ln x_2\\
&4.\ u^D(x_1,x_2) = \left(\frac{\alpha}{\beta}\right)\ln\ x_1 + \ln x_2\\
&5.\ u^E(x_1,x_2) = -\alpha \ln x_1 - \beta \ln x_2
\end{aligned} \tag{4.8}$$

(a) *Calculate the formula for MRS for each of these utility functions.*

Answer: These would be

$$\begin{aligned}
&1.\ MRS^A = -\frac{\alpha x_1^{\alpha-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta-1}} = -\frac{\alpha x_2}{\beta x_1}\\
&2.\ MRS^B = -\frac{\alpha}{\beta}\\
&3.\ MRS^C = -\frac{\alpha}{\beta/x_2} = -\frac{\alpha x_2}{\beta}\\
&4.\ MRS^D = -\frac{\alpha/(\beta x_1)}{1/x_2} = -\frac{\alpha x_2}{\beta x_1}\\
&5.\ MRS^E = -\frac{-\alpha/x_1}{-\beta/x_2} = -\frac{\alpha x_2}{\beta x_1}
\end{aligned} \tag{4.9}$$

(b) *Which utility functions represent tastes that have linear indifference curves?*

Answer: Linear indifference curves are indifference curves that have the same slope everywhere — i.e. indifference curves with constant rather than diminishing MRS. Thus, the MRS cannot depend on x_1 or x_2 for the indifference curve to be linear — which is the case only for $u^B(x_1,x_2)$.

(c) *Which of these utility functions represent the same underlying tastes?*

Answer: Two conditions have to be met for utility functions to represent the same tastes: (1) the indifference curves they give rise to must have the same shapes, and (2) the numbering on the indifference curves needs to have the same order (though not the same magnitude.) To check that indifference curves from two utility functions have the same shape, we have to check that the MRS for those utility functions are the same. This is true for u^A, u^D and u^E. To check that the ordering of the numbers associated with indifference curves goes in the same direction, we need to go back to the utility functions. In u^A, for instance, more of x_1 and/or x_2 means higher utility values. The same is true for u^D. Thus u^A and u^D represent the same underlying tastes because they give rise to the same shapes for all the indifference curves and both have increasing numbers associated with indifference curves as we move northeast in the graph of the indifference curves. But u^E is different: While it gives rise to indifference curves with the same shapes as u^A and u^D, the utility values associated with the indifference curves become increasingly negative — i.e. they decline — as we increase x_1 and/or x_2. Thus, higher numerical labels for indifference curves happen to the southwest rather than the northeast — indicating that less is better than more. So the only two utility functions in this problem that represent the same tastes are u^A and u^D.

(d) *Which of these utility functions represent tastes that do not satisfy the monotonicity assumption?*

Answer: As just discussed in the answer to B(c), u^E represents tastes for which less is better than more — because the labeling on the indifference curves gets increasingly negative as we move to the northeast (more of everything) and increasingly less negative as we move toward the origin. In all other cases, more x_1 and/or more x_2 creates greater utility as measured by the utility functions.

(e) *Which of these utility functions represent tastes that do not satisfy the convexity assumption?*

Answer: As we move to the right on an indifference curve, x_1 increases and x_2 decreases. We can then look at the formulae for MRS that we derived for each utility function to see what happens to the MRS as x_1 increases while x_2 decreases. In MRS^A, for instance, this would result in a decrease in the numerator and an increase in the denominator — i.e. we are dividing a smaller number by a larger number as x_1 increases and x_2 decreases. Thus, in absolute value, the MRS declines as we move to the right in our graph — which implies we have diminishing MRS and the usual shape for the indifference curves. Since they share the same MRS, the same holds for u^D and u^E. For u^C, it is similarly true that an increase in x_1 accompanied by a decrease in x_2 (i.e. a movement along the indifference curve toward the right in the graph) causes the MRS to fall — only this time x_1 plays no role and the drop is entirely due to the reduction in the numerator. For u^B, the MRS is constant — implying no change in the MRS as we move along an indifference curve to the right in the graph.

We can then conclude the following: u^B satisfies the convexity assumption but barely so — averages are the same as extremes (but not better). Furthermore, u^A, u^C and u^D all represent monotonic tastes with diminishing marginal rates of substitution along indifference curves. Thus, averages between extremes that lie on the same indifference curve will be preferred to the extremes because the averages lie to the northeast of some bundles on the indifference curves on which the extremes lie, and, since more is better, this implies the averages are better than the extremes. So u^A, u^C and u^D all satisfy the convexity assumption. That leaves only u^E which we concluded before does not satisfy the monotonicity assumption but its indifference curves look exactly like they do for u^A and u^D. If you pick any two bundles on an indifference curve, it will therefore again be true that the average of those bundles lies to the northeast of some of the bundles on that indifference curve — but now a movement to the northeast makes the individual worse off, not better off. Thus, averages are worse than extremes for the tastes represented by u^E — which implies that u^E represents tastes that are neither convex nor monotonic.

(f) *Which of these utility functions represent tastes that are not rational (i.e. that do not satisfy the completeness and transitivity assumptions)?*

Answer: Each of these is a function that satisfies the mathematical properties of functions. In each case, you can plug in any bundle (x_1, x_2) and the function will assign a utility value. Thus, any two bundles can be compared — and completeness is satisfied. Furthermore, it is mathematically not possible for a function to assign a value to bundle A that is higher than the value it assigns to a different bundle B which in turn is higher than the value assigned to a third bundle C — without it also being true that the value assigned to C is lower than the value assigned to A. Thus, transitivity is satisfied.

(g) *Which of these utility functions represent tastes that are not continuous?*

Answer: All the functions are continuous without sudden jumps — and therefore represent tastes that are similarly continuous.

(h) *Consider the following statement: "Benefits from trade emerge because we have different tastes. If individuals had the same tastes, they would not be able to benefit from trading with one another." Is this statement ever true, and if so, are there any tastes represented by the utility functions in this problem for which the statement is true?*

Answer: What we found in our answers in part A is that, in order for individuals to be able to benefit from trading, it must be the case that their indifference curves through their current consumption bundle have different slopes. It does not matter whether their indifference maps are identical. So long as they are at different current bundles that have different MRS's, mutually beneficial trades are possible. You and I, for instance, might have identical tastes over apples and bananas, but I might have mostly bananas and you might have mostly apples. Then you would probably be willing to trade lots of apples for more bananas, and I'd be willing to let go of bananas pretty easily to get more apples. The only way we cannot benefit from trading with one another is if our MRS's through our current bundle are the same. This might be true for some bundles when we have identical tastes (such as when we currently own the same bundle), but it is not generally true just because we have the same tastes. The only utility function from this problem for which the statement generally holds is therefore u^B, the utility function that represents tastes with the same MRS at all bundles. If you and I shared those tastes, then we would have the same *MRS* regardless of which bundles we currently owned — and this makes it impossible for us to become better off through trade.

The statement in this problem could be re-phrased in a way that would make it universally true for all tastes: “Benefits from trade emerge because we have different tastes *at the margin*” — that is, when we have the same willingness to trade goods off for one another around the bundle we currently consume, then we have the same *MRS* and can’t trade.

4.6 Everyday Application: *Rating Movies on a Numerical Scale*: *My wife and I often go to movies and afterwards assign a rating ranging from 0 to 10 to the movie we saw.*

A: *Suppose we go to see a double feature, first "Terminator 2" with the great actor Arnold Schwarzenegger and then the adaptation of Jane Austin's boring novel "Emma". Afterwards, you hear me say that I rated "Terminator 2" as an 8 and "Emma" as a 2, and you hear my wife comment that she rated "Terminator 2" a 5 and "Emma" a 4.*

(a) *Do my wife and I agree on which movie is better?*

Answer: Yes, we both give a higher rating to "Terminator 2" and thus both believe "Terminator 2" is better than "Emma".

(b) *How would your answer change if my wife's ratings had been reversed?*

Answer: If my wife gave a 4 to "Terminator" and a 5 to "Emma" instead, then she would believe that "Emma" was the better movie. In that case, we would disagree on which movie is better.

(c) *Can you tell for sure whether I liked "Terminator 2" more than my wife did?*

Answer: No, you cannot. You know I like "Terminator 2" better than "Emma", but we have no way of knowing what an 8 rating or a 2 rating really mean to me relative to what a 5 rating and a 4 rating mean to my wife. The ordering of my ratings matters because it conveys real information — but the precise numbers don't mean much because we don't have an objective scale that everyone uses and that then allows for interpersonal comparisons.

(d) *Often, my wife and I then argue about our rankings.* True or False*: It makes little sense for us to argue if we both rank one movie higher than the other even if we assign very different numbers.*

Answer: True. It makes little sense for us to argue because we don't know what rating numbers actually mean to each of us — all we know is that higher numbers are better. Of course if we keep doing this for every movie we see, we might well start arguing because, even though we agree on the relative ranking of "Terminator 2" and "Emma", we may disagree about the rating we gave to one of these movies relative to some other movie that we saw last week. That would make for a good argument because then we can argue about two movies and which was better. But so long as we assign the higher number to one of the movies in a pair of movies, we agree on which is better and should stop fighting.

B: *Suppose that the only thing I really care about in evaluating movies is the fraction of "action" time (as opposed to thoughtful conversation) and let the fraction of screen time devoted to action be denoted x_1. Suppose that the only thing my wife cares about when evaluating movies is the fraction of time strong women appear on screen, and let that fraction be denoted x_2. "Terminator 2" has $x_1 = 0.8$ and $x_2 = 0.5$ while Emma has $x_1 = 0.2$ and $x_2 = 0.4$.*

(a) *Consider the functions $u(x_1) = 10x_1$ and $v(x_2) = 10x_2$ and suppose that I use the function u to determine my movie rating and my wife uses the function v. What ratings do we give to the two movies?*

Answer: I would rate "Terminator 2" an 8 and "Emma" a 2, while my wife would rate "Terminator 2" a 5 and "Emma" a 4. Thus, these functions replicate the ratings we used in part A.

(b) *One day I decide that I will assign ratings differently, using the function $\overline{u}(x_1) = 5.25x_1^{1/6}$. Will I rank any pair of movies differently using this function rather than my previous function u? What approximate values do I now assign to "Terminator 2" and "Emma"?*

Answer: Changing x_1 to $x_1^{1/6}$ does not alter the ranking of any two values of x_1 — it merely changes the magnitude of the numbers that are assigned. Similarly, multiplying x_1 by 5.25 does not alter the ranking of any two values of x_1. Thus, the new function $\overline{u}$ would rank any pair of movies the same way as the original function u. The approximate values now assigned by me to "Terminator 2" and "Emma" are 5 and 4. Note that these are exactly the values my wife assigns to the two movies — so, by changing the scaling of my ranking I now assign numbers identical to hers even though I care only about x_1 and she cares only about x_2.

(c) *My wife also decides to change her way of assigning ratings to movies. She will now use the function $\overline{v}(x_2) = 590x_2^{6.2}$. Will her rankings of any two movies change as a result? What approximate values does she now assign to the two movies?*

<u>Answer</u>: Again, neither of the two transformations — multiplying by 590 or taking x_1 to the power 6.2 — alters the ranking assigned to any two values for x_1. Thus, the ranking of any pair of movies would be the same for my wife under this new function. Using this new function, my wife would assign approximately an 8 rating to "Terminator 2" and a 2 ranking to "Emma". Thus, by changing the scaling of her ranking in a way that does not alter the relative ranking of any pair of movies, we now have a rule for my wife that has her assign exactly the ratings I originally assigned — despite the fact that I only care about x_1 and she only cares about x_2.

(d) *Suppose my wife had instead chosen the function* $\underline{v}(x_2) = 10(1-x_2)$. *Will she now rank movies differently?*

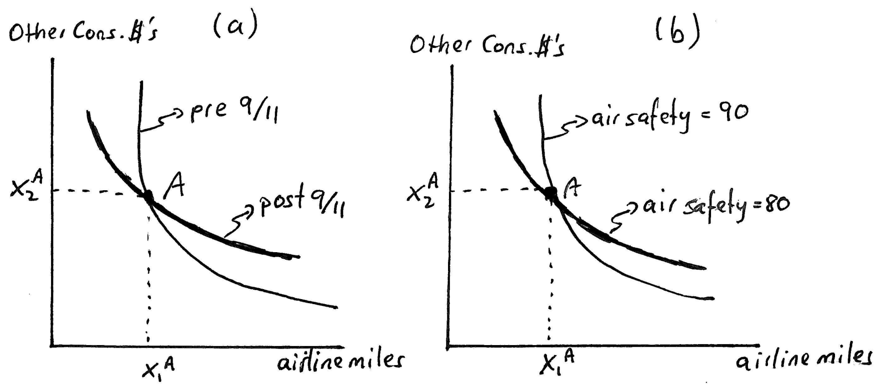
<u>Answer</u>: In that case, her rankings would change because x_2 now enters negatively rather than positively. In other words, changing the function in this way, while still producing ratings between 0 an 10, now places value on the fraction of screen time without strong women $(1-x_2)$ as opposed to the fraction of screen time with strong women (x_2). For instance, using the function $\underline{v}$, my wife would continue to assign 5 to "Terminator 2" but would change her rating of "Emma" from 4 to 6. This would imply that she now rates "Emma" higher than "Terminator."

4.7 Everyday Application: *Did 9/11 Change Tastes?: In another textbook, the argument is made that consumer tastes over "airline miles traveled" and "other goods" changed as a result of the tragic events of September 11, 2001.*

A: *Below we will see how you might think of that argument as true or false depending on how you model tastes.*

(a) *To see the reasoning behind the argument that tastes changed, draw a graph with "airline miles traveled" on the horizontal axis and "other goods" (denominated in dollars) on the vertical. Draw one indifference curve from the map of indifference curves that represent a typical consumer's tastes (and that satisfy our usual assumptions.)*

Answer: This is illustrated in panel (a) of Graph 4.6 with the indifference curve labeled "pre-9/11".



Graph 4.6: Tastes before and after 9/11

(b) *Pick a bundle on the indifference curve on your graph and denote it A. Given the perception of increased risk, what do you think happened to the typical consumer's MRS at this point after September 11, 2001?*

Answer: The *MRS* tells us how much in "dollars of other goods" a consumer is willing to give up to travel one more mile by air. After 9/11, it would stand to reason that the typical consumer would give up fewer dollars for additional air travel than before. Thus, the slope of the indifference curve at *A* should become shallower — which implies that the *MRS* is falling in absolute value.

(c) *For a consumer who perceives a greater risk of air travel after September 11, 2001, what is likely to be the relationship of the indifference curves from the old indifference map to the indifference curves from the new indifference map at every bundle?*

Answer: The reasoning from (b) holds not just at *A* but at all bundles. Thus, we would expect the new indifference map to have indifference curves with shallower slopes at every bundle.

(d) *Within the context of the model we have developed so far, does this imply that the typical consumer's tastes for air-travel have changed?*

Answer: Rationality (as we have defined it) rules out the possibility for indifference curves to cross. Thus, within the context of this model, it certainly seems that tastes must have changed.

(e) *Now suppose that we thought more comprehensively about the tastes of our consumer. In particular, suppose we add a third good that consumers care about — "air safety". Imagine a 3-dimensional graph, with "air miles traveled" on the horizontal axis and "other goods" on the vertical (as before) — and with "air safety" on the third axis coming out at you. Suppose "air safety" can be expressed as a value between 0 and 100, with 0 meaning certain death when* *one steps on an airplane and 100 meaning no risk at all. Suppose that before 9/11, consumers thought that air safety stood at 90. On the slice of your 3-dimensional graph that holds air safety constant at 90, illustrate the pre-9/11 indifference curve that passes through (x_1^A, x_2^A), the level of air miles traveled (x_1^A) and other goods consumed (x_2^A) before 9/11.*

Answer: This is illustrated in panel (b) of Graph 4.6 as the indifference curve labeled "air safety = 90".

(f) *Suppose the events of 9/11 cause air safety to fall to 80. Illustrate your post-9/11 indifference curve through (x_1^A, x_2^A) on the slice that holds air safety constant at 80 but draw that slice on top of the one you just drew in (e).*

Answer: This is also done in panel (b) of the graph.

(g) *Explain that, while you could argue that our tastes changed in our original model, in a bigger sense you could also argue that our tastes did not change after 9/11, only our circumstances did.*

Answer: When we explicitly include air safety as something we value as consumers, we get indifference surfaces that lie in 3 dimensions. But since we don't get to choose the level of air safety, we effectively operate on a 2-dimensional slice of that 3-dimensional indifference surface — the slice that corresponds to the current level of air safety. That slice looks just like any ordinary indifference curve in a 2-good model even though it comes from a 3-good model. When 9/11 changes the perceptions of air safety, outside circumstances are shifting us to a different portion of our 3-dimensional indifference surface — with that slice once again giving rise to indifference curves that look like the ones we ordinarily graph in a 2-good model. But when viewed from this perspective, the fact that the indifference curve that corresponds to more air safety crosses the indifference curve that corresponds to less air safety merely arises because we are graphing two different slices of a 3-dimensional surface in the same 2-dimensional space. While both curves then contain the bundle (x_1^A, x_2^A), they occur at different levels of x_3. The pre-9/11 indifference curve really goes through bundle $(x_1^A, x_2^A, 90)$ while the post-9/11 indifference curve really goes through bundle $(x_1^A, x_2^A, 80)$ — and the two therefore do not cross. Thus, when viewed from this larger perspective, tastes have not changed, only circumstances have.

B: *Suppose an average traveler's tastes can be described by the utility function $u(x_1, x_2, x_3) = x_1 x_3 + x_2$, where x_1 is miles traveled by air, x_2 is "other consumption" and x_3 is an index of air safety that ranges from 0 to 100.*

(a) *Calculate the MRS of other goods for airline miles — i.e. the MRS that represents the slope of the indifference curves when x_1 is on the horizontal and x_2 is on the vertical axis.*

Answer: The MRS is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{x_3}{1} = -x_3. \tag{4.10}$$

(b) *What happens to the MRS when air safety (x_3) falls from 90 to 80?*

Answer: It changes from −90 to −80.

(c) *Is this consistent with your conclusions from part A? In the context of this model, have tastes changed?*

Answer: The change in the MRS as air safety falls is a decrease in absolute value — i.e. the slope of the indifference curve over x_1 and x_2 becomes shallower just as we concluded in part A. But we are representing tastes with exactly the same utility function as before — so tastes cannot have changed.

(d) *Suppose that $u(x_1, x_2, x_3) = x_1 x_2 x_3$ instead. Does the MRS of other consumption for air miles traveled still change as air safety changes? Is this likely to be a good model of tastes for analyzing what happened to consumer demand after 9/11?*

Answer: The MRS now is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{x_2 x_3}{x_1 x_3} = -\frac{x_2}{x_1}. \tag{4.11}$$

Thus, the MRS for tastes represented by this utility function is unaffected by x_3 — the level of air safety. This would imply that the two indifference curves in panel (b) of Graph 4.6 would lie on top of one another. If we think consumers felt differently about air travel after 9/11 than before, then this utility function would not be a good one to choose for analyzing changes in consumer behavior.

(e) *What if* $u(x_1, x_2, x_3) = x_2x_3 + x_1$*?*

Answer: In this case, the MRS is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{1}{x_3}. \tag{4.12}$$

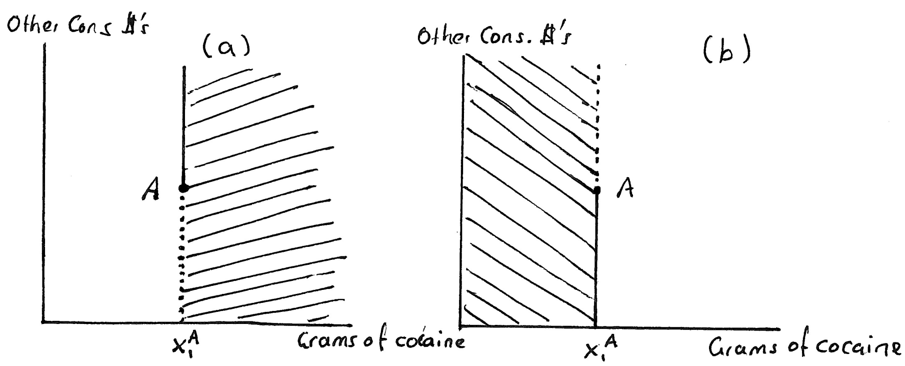
This would imply that as x_3 — air safety — falls, the MRS increases in absolute value; i.e. it would mean that a decrease in air safety would make us willing to spend more on additional air travel than what we were willing to spend before. It would thus result in a steeper rather than a shallower slope for indifference curves post-9/11. It seems unlikely that a typical consumer would respond in this way to changes in air safety.

4.8 Everyday Application: *Tastes of a Cocaine Addict: Fred is addicted to cocaine. Suppose we want to model his tastes over cocaine and other goods.*

A: *I propose to model his tastes in the following way: For any two bundles A and B of "grams of cocaine" and "dollars of other consumption," I will assume that Fred always prefers bundle A if it contains more grams of cocaine than bundle B. If bundles A and B contain the same amount of cocaine, then I will assume he prefers A to B if and only if A contains more other consumption than B.*

(a) *On a graph with "grams of cocaine" on the horizontal axis and "other consumption" (denominated in dollars) on the vertical, denote one arbitrary bundle as A. Then indicate all the bundles that are strictly preferred to A.*

Answer: Panel (a) of Graph 4.7 illustrates the set of bundles that are strictly preferred to *A*. This set includes all bundles to the right of *A* since all those bundles include more cocaine than *A* — and Fred's tastes are such that he always prefers bundles with more cocaine to bundles with less regardless of how much other consumption is in the bundles. Among bundles that contain the same amount of cocaine, Fred prefers bundles with more other consumption — so, in addition to the bundles to the right of *A*, the set of bundles that are strictly preferred to *A* also includes all the bundles that contain x_1^A of cocaine and more other consumption. These are represented by the solid line above *A*.



Graph 4.7: An Addict's Tastes for Cocaine

(b) *On a separate graph, indicate all bundles that are strictly less preferred than A.*

Answer: Panel (b) of Graph 4.7 illustrates the set of bundles that are strictly less preferred than *A*. This set includes all bundles that contain less cocaine than *A* — i.e. all bundles to the left of *A*. It also includes all bundles that have the same amount x_1^A of cocaine as bundle *A* but less other goods consumption. These bundles are the ones that fall on the solid line below *A*.

(c) *Looking over your two graphs, is there any bundle that Fred would say gives him exactly as much happiness as A? Are there any two bundles (not necessarily involving bundle A) that Fred is indifferent between?*

Answer: Panels (a) and (b) graphed two sets — the set of bundles strictly preferred to *A* (panel (a)) and the set of bundles that *A* is strictly preferred to (panel (b)). Together, these two sets include every bundle other than bundle *A*. Thus, there is no bundle other than *A* that will give Fred exactly as much happiness as *A* does. Since we picked *A* arbitrarily to start with, the same logic applies to every other bundle. Thus, there exist no two bundles that Fred is indifferent between.

(d) *In order for this to be a useful model for studying Fred's behavior, how severe would Fred's addiction have to be?*

Answer: It would have to be pretty severe. It essentially implies that, so long as we restrict him to not buying cocaine with the money we give him, there is no amount of money that would get Fred to part with even a fraction of a gram of cocaine.

(e) *Are these tastes rational? In other words, are they complete and transitive?*

Answer: Yes, these tastes are rational in the way that we have defined the term rational. Remember that we do not mean rational in some philosophical sense — all we mean is that tastes are complete and transitive. We can in fact pick any two bundles and Fred will be able to tell us which he prefers. So tastes are complete — there are no two bundles that Fred would not be able to compare. If A is preferred to B and B is preferred to C because A has more cocaine than B and B has more cocaine than C, then A has more cocaine than C and is thus preferred to C. Similarly, if A is preferred to B and B to C because they all have the same amount of cocaine but A has more other consumption than B and B has more other consumption than C, then A has the same amount of cocaine as C but more other consumption and is thus preferred to C. Finally, if A is preferred to B because it has more cocaine and B is preferred to C because it has the same amount of cocaine but more other consumption, then A has more cocaine than C and is thus preferred to C. Thus, there are no three bundles such that A is preferred to B and B is preferred to C with C then preferred to A — transitivity holds as well.

(f) *Do these tastes satisfy the monotonicity property?*

Answer: Yes. Starting at any bundle, more cocaine (i.e. moving to the right) is better as is more other consumption (i.e. moving up).

(g) *Do they satisfy the convexity property?*

Answer: Yes. We can answer this in two different ways. First, we have defined convexity as follows: Take any two bundles that the consumer is indifferent between and then create an average of these indifferent bundles. Convexity holds so long as the average bundle is at least as good as the extremes. In this case, every indifference curve is just a single bundle — so we would have to take the average of two identical bundles to apply the definition — which in turn would give us the same bundle which is in fact at least as good as the original bundles. A second way to look at this is as follows: The reason we call the "averages are better than extremes" assumption "convexity" is because, when tastes satisfy this property, the set of bundles that is preferred to any bundle A is a convex set. A convex set is a set that has the property that you can take any two points in the set, connect them and find that all points on the connecting line also lie in the set. Panel (a) of Graph 4.7 illustrates the set of bundles that are preferred to A. This is in fact a convex set.

B: *The tastes defined above are called* lexicographic. *Formally, we can define them as follows: For any* $A, B \in \mathbb{R}^2_+$, $A \succ B$ *if either "*$x_1^A > x_1^B$*" or "*$x_1^A = x_1^B$ *and* $x_2^A > x_2^B$*".*

(a) *In this formal definition, which good is cocaine,* x_1 *or* x_2*?*

Answer: Since in the formal definition you always prefer bundles with more x_1, x_1 is cocaine in the example.

(b) *On a graph with* x_1 *on the horizontal axis and* x_2 *on the vertical, pick an arbitrary bundle* $A = (x_1^A, x_2^A)$*. Then pick a second bundle* $D = (x_1^D, x_2^D)$ *such that* $x_1^A = x_1^D$ *and* $x_2^A > x_2^D$*.*

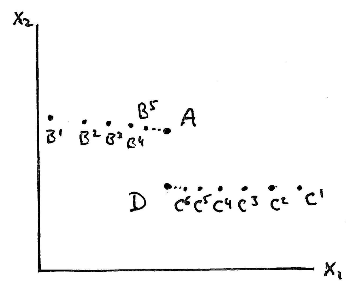
Answer: A and D are illustrated in Graph 4.8.

(c) *On your graph, illustrate an infinite sequence of bundles* $(B^1, B^2, B^3...)$ *that converges to* A *from the left. Then illustrate an infinite sequence of bundles* $(C^1, C^2, C^3...)$ *that converges to* D *from the right.*

Answer: These are also illustrated in Graph 4.8.

(d) True or False*: Every bundle in the* C*-sequence is strictly preferred to every bundle in the* B*-sequence.*

Answer: This is true. You can easily see this by referring back to panel (a) of Graph 4.7 where we illustrated the bundles strictly preferred to A and to panel (b) where we illustrated the bundles strictly less preferred than A. All the B-bundles lie in the less preferred region while all the C-bundles lie in the more preferred region. This also makes sense given our definition of the cocaine addiction under which Fred will immediately prefer a bundle so long as it has slightly more cocaine than another bundle. This implies bundles to the right in the graph are always preferred to bundles to the left, and all B-bundles lie to the left of all C-bundles.



Graph 4.8: Violation of Continuity for Cocaine Addict

(e) True or False*: Bundle A is strictly preferred to bundle D*.

Answer: True — *A* and *D* have the same amount of cocaine, but *A* has more other consumption than *D*.

(f) *Based on the answers you just gave to the previous two True/False questions, do lexicographic tastes satisfy the continuity property?*

Answer: No. Every element of the *C*-sequence is strictly preferred to every element of the *B*-sequence, but the limit bundle of the *B*-sequence (*A*) is strictly preferred to the limit bundle of the *C*-sequence (*D*). There is a sudden jump at the limit of the sequences that reverses the preference ordering in the limit — and that is a straightforward violation of the continuity assumption.

(g) *Can these tastes be represented by a utility function?*

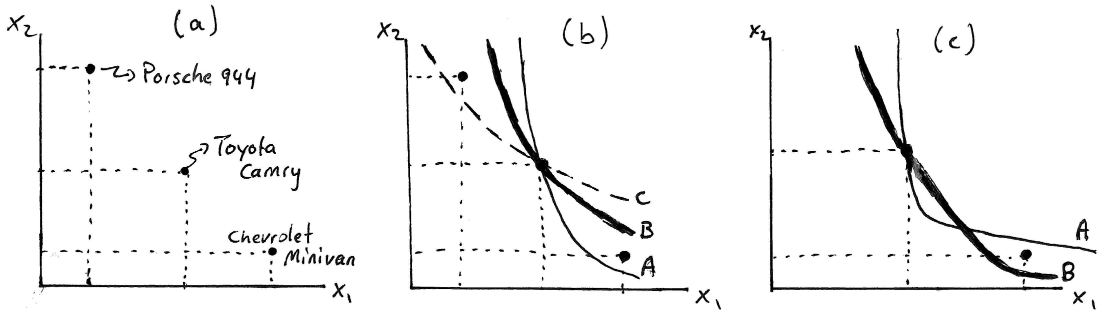
Answer: No, they cannot. While lexicographic tastes satisfy completeness and transitivity (which is necessary for tastes to be representable by utility functions), they do not satisfy the continuity assumption that is also required.

4.9 Business Application: *Tastes for Cars and Product Characteristics*: *People buy all sorts of different cars depending on their income levels as well as their tastes. Industrial organization economists who study product characteristic choices (and advise firms like car manufacturers) often model consumer tastes as tastes over product characteristics (rather than as tastes over different types of products). We explore this concept below.*

A: *Suppose people cared about two different aspects of cars: the size of the interior passenger cabin and the quality of handling of the car on the road.*

(a) *Putting* x_1 = *"cubic feet of interior space" on the horizontal axis and* x_2 = *"speed at which the car can handle a curved mountain road" on the vertical, where would you generally locate the following types of cars assuming that they will fall on one line in your graph: a Chevrolet Minivan, a Porsche 944, and a Toyota Camry.*

Answer: Panel (a) of Graph 4.9 illustrates where the product characteristics of these cars would place them in a graph with interior space on the horizontal axis and speed on the vertical. Porsche's do not have much space in the interior but they handle well at high speeds. Minivans have tons of interior space but don't handle that well at high speeds. And Toyota Camrys are somewhere in between — with more space than Prosche's but not as much as minivans, and with better handling at high speeds than minivans but not as good as Porsches.



Graph 4.9: Porsche, Toyota and Chevy

(b) *Suppose we considered three different individuals whose tastes satisfy our 5 basic assumptions, and suppose each person owns one of the three types of cars. Suppose further that each indifference curve from one person's indifference map crosses any indifference curve from another person's indifference map at most once. (When two indifference maps satisfy this condition, we often say that they satisfy the* single crossing property.*) Now suppose you know person A's MRS at the Toyota Camry is larger (in absolute value) than person B's, and person B's MRS at the Toyota Camry is larger (in absolute value) than person C's. Who owns which car?*

Answer: The indifference curves (through the Toyota Camry) for the 3 individuals are depicted in panel (b) of Graph 4.9. In order for one of these cars to be the most preferred for one and only one of the individuals, it must be that the Porsche lies above one person's indifference curve through the Camry and the minivan lies above another person's indifference curve through the Camry. If indifference curves from different indifference maps cross only once, it logically has to follow that the steepest indifference curve through the Camry lies below the minivan and the shallowest indifference curve through the Camry falls below the Porsche. Since person A's *MRS* is largest in absolute value, person A's indifference curve through the Camry has the steepest slope. By the same reasoning, person C has the shallowest slope going through the Camry. Thus, person A owns the minivan, person B owns the Camry and person C owns the Porsche.

(c) *Suppose we had not assumed the "single crossing property" in part (a). Would you have been able to answer the question "Who owns which car" assuming everything else remained the same?*

Answer: No, you would not have been able to answer the question. The ambiguity that arises when indifference curves from different indifference maps can cross more than once is depicted in panel (c) of Graph 4.9. Here, person B's (bold) indifference curve is shallower at the Camry than person A's just as described in the problem. However, person A's indifference curve takes a sharp turn at some point to the right of the Camry while person B's continues at roughly the same slope. Thus, B's indifference curve ends up below the minivan (making the minivan better for B than the Camry) while person A's indifference curve ends up above the minivan (making the Camry better for him than the minivan). Thus, once we allow multiple crossing of indifference curves from different indifference maps, it becomes ambiguous who is driving which car.

(d) *Suppose you are currently person B and you just found out that your uncle has passed away and bequeathed to you his 3 children, aged 4, 6 and 8 (and nothing else). This results in a change in how you value space and maneuverability. Is your new MRS at the Toyota Camry now larger or smaller (in absolute value)?*

Answer: You would now be wiling to sacrifice more speed and maneuverability for an increase in interior cabin space — which means the slope of your indifference curve at the Camry should get steeper. Thus, the MRS will increase in absolute value.

(e) *What are some other features of cars that might matter to consumers but that you could not fit easily into a 2-dimensional graphical model?*

Answer: You could think of many other car features: the quality of the upholstery, the shape of the seats, the color of the exterior and interior, whether there is a sun-roof, the quality of the speakers on the stereo system, the degree to which each passenger can control air temperature, the size of the engine, etc.

B: *Let x_1 denote cubic feet of interior space and let x_2 denote maneuverability as defined in part A. Suppose that the tastes of persons A, B and C can be represented by the utility functions $u^A(x_1,x_2) = x_1^{\alpha}x_2$, $u^B(x_1,x_2) = x_1^{\beta}x_2$ and $u^C(x_1,x_2) = x_1^{\gamma}x_2$ respectively.*

(a) *Calculate the MRS for each person.*

Answer: The MRS for person A is

$$MRS^A = -\frac{\partial u^A/\partial x_1}{\partial u^A/\partial x_2} = -\frac{\alpha x_1^{\alpha-1}x_2}{x_1^{\alpha}} = -\alpha\frac{x_2}{x_1}. \tag{4.13}$$

Similarly, $MRS^B = -\beta x_2/x_1$ and $MRS^C = -\gamma x_2/x_1$.

(b) *Assuming α, β and γ take on different values, is the "single crossing property" defined in part A(b) satisfied?*

Answer: Pick any product characteristic bundle $(\overline{x}_1,\overline{x}_2)$. Consider individual A and individual B and how their MRS's are related to one another at that bundle by dividing one MRS by the other; i.e.

$$\frac{MRS^A}{MRS^B} = \frac{-\alpha\overline{x}_2/\overline{x}_1}{-\beta\overline{x}_2/\overline{x}_1} = \frac{\alpha}{\beta}. \tag{4.14}$$

Now, it does not matter what bundle $(\overline{x}_1,\overline{x}_2)$ I use, the above equation tells me that the MRS^A is always equal to α/β times the MRS^B. Thus, any indifference curve from A's indifference map can cross any indifference curve from B's indifference map only once. If that were not the case, (as in panel (c) of the graph), the relationship between the slopes of the indifference curves would have to be different at the second crossing — but we have just concluded that this relationship is the same everywhere. The same of course holds for any other pair of individuals from our group of persons A, B and C.

(c) *Given the description of the three persons in part A(b), what is the relationship between α, β and γ?*

Answer: Since A's indifference curve at any product characteristic bundle is steeper than B's and B's is steeper than C's, it must be that $\alpha > \beta > \gamma$.

(d) *How could you turn your graphical model into a mathematical model that includes factors you raised in part A(e)?*

Answer: All that's required is that the utility function includes more product characteristics. So, if we identify n different product characteristics that matter to consumers, we would model their tastes as represented by a utility function $u(x_1, x_2, ..., x_n)$ where x_i is the ith product characteristic.

4.10 Business Application: *Investor Tastes over Risk and Return: Suppose you are considering where to invest money for the future.*

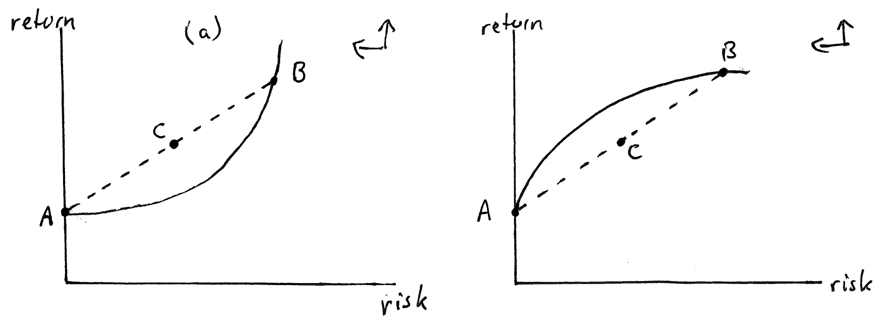
A: *Like most investors, you care about the expected return on your investment as well as the risk associated with the investment. But different investors are willing to make different kinds of tradeoffs relative to risk and return.*

(a) *On a graph, put risk on the horizontal axis and expected return on the vertical. (For purposes of this exercise, don't worry about the precise units in which these are expressed.) Where in your graph would you locate "safe" investments like inflation indexed government bonds — investments for which you can predict the rate of return with certainty?*

Answer: Such investments would appear on the vertical axis since risk is represented on the horizontal axis. All such investments have zero risk.

(b) *Pick one of these "safe" investment bundles of risk and return and label it A. Then pick a riskier investment bundle B that an investor could plausibly find equally attractive (given that risk is bad in the eyes of investors while expected returns are good).*

Answer: Panel (a) of Graph 4.10 depicts a safe investment A on the vertical axis. An investment bundle with risk must then have a higher return since investors like greater returns and less risk. Put differently, investors become better off moving up and to the left (as indicated by the arrows in the top left of panel (a)), which means that bundles indifferent to A must lie to the northeast.



Graph 4.10: Tastes over Risk and Return

(c) *If your tastes are convex and you only have investments A and B to choose from, would you prefer diversifying your investment portfolio by putting half of your investment in A and half in B?*

Answer: Such diversification would result in bundle C in panel (a) of the graph. Convexity of tastes implies the illustrated shape of the indifference curve through A and B — such that the set of bundles better than A (and B) is a convex set. This causes C to lie to the northwest of some of the bundles that are indifferent to A and B, which means C must be preferred to A and B. So, yes, you would choose to diversify.

(d) *If your tastes are non-convex, would you find such diversification attractive?*

Answer: If tastes are not convex, then C lies below the indifference curve through A and B as illustrated in panel (b) of the graph. Thus, you would not choose to diversify.

B: *Suppose an investor has utility function $u(x_1, x_2) = (R - x_1)x_2$ where x_1 represents the risk associated with an investment, x_2 is the expected return and R is a constant.*

(a) *What is the MRS of risk for return for this investor.*

Answer: The MRS is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{-x_2}{R-x_1} = \frac{x_2}{R-x_1}, \tag{4.15}$$

which is positive for sufficiently high R.

(b) *Suppose A is a risk free investment, with $x_1^A = 0$, and suppose that B is risky but our investor is indifferent between A and B. What must the return x_2^A on the risk-free investment be in terms of x_1^B and x_2^B?*

Answer: The utility of investment B is $(R-x_1^B)x_2^B$. The utility of investment A is Rx_2^A. For the investor to be indifferent, these utilities have to be equal to one another; i.e. $(R-x_1^B)x_2^B = Rx_2^A$. Solving this for x_2^A, we get

$$x_2^A = \frac{(R-x_1^B)x_2^B}{R}. \tag{4.16}$$

(c) *Do this investor's tastes satisfy convexity? Illustrate by considering whether this investor would be willing to switch from A or B in part (b) to putting half his investment in A and half in B.*

Answer: The answer is yes, the tastes satisfy convexity — and yes, the person would be willing to diversify by mixing A and B. To show this, let's calculate the utility of investment C that mixes A and B. The risk characteristic of this investment would be

$$x_1^C = \frac{1}{2}x_1^A + \frac{1}{2}x_1^B = \frac{x_1^B}{2} \tag{4.17}$$

since $x_1^A = 0$. The expected return of investment C would be

$$x_2^C = \frac{1}{2}x_2^A + \frac{1}{2}x_2^B = \frac{1}{2}\left(\frac{(R-x_1^B)x_2^B}{R}\right) + \frac{1}{2}x_2^B = \frac{2Rx_2^B - x_1^B x_2^B}{2R}. \tag{4.18}$$

The utility from C is then

$$u(x_1^C, x_2^C) = (R-x_1^C)x_2^C = \left(R - \frac{x_1^B}{2}\right)\left(\frac{2Rx_2^B - x_1^B x_2^B}{2R}\right) = \frac{4R^2 x_2^B - 4Rx_1^B x_2^B + (x_1^B)^2 x_2^B}{4R} \tag{4.19}$$

which reduces to

$$\begin{aligned} u(x_1^C, x_2^C) = {} & Rx_2^B - x_1^B x_2^B + \frac{(x_1^B)^2 x_2^B}{4R} = \\ & = \left(R - x_1^B\right)x_2^B + \frac{(x_1^B)^2 x_2^B}{4R} = u(x_1^B, x_2^B) + \frac{(x_1^B)^2 x_2^B}{4R}. \end{aligned} \tag{4.20}$$

The utility of C is therefore equal to the utility of B plus a positive amount; i.e. the utility of C is strictly greater than the utility of B. Put differently, the average of A and B is strictly preferred.

(d) *Suppose $R = 10$ for our investor. Imagine he is offered the following 3 investment portfolios: (1) a no-risk portfolio of government bonds with expected return of 2 and 0 risk; (2) a high risk portfolio of volatile stocks with expected return of 10 and risk of 8; or a portfolio that consists half of government bonds and half of volatile stocks, with expected return of 6 and risk of 4. Which would he choose?*

Answer: By plugging in these risk and return values into the utility function $u(x_1, x_2) = (10-x_1)x_2$, we get utility of 20 for the no-risk portfolio, utility of 20 for the high risk portfolio and utility 36 for the mixed portfolio. He would choose the mixed portfolio.

(e) *Suppose a second investor is offered the same three choices. This investor is identical to the first in every way, except that R in his utility function is equal to 20 instead of 10. Which portfolio will he choose?*

Answer: Plugging the risk and return numbers into $u(x_1, x_2) = (20-x_1)x_2$, we now get utility of 40 for the risk free portfolio, utility of 120 for the high risk portfolio and utility of 96 for the mixed portfolio. He will choose the high risk portfolio.

(f) True or False*: The first investor's tastes are convex while the second one's are not.*

Answer: False. It's easy to see that the first investor's tastes are indeed convex — the no risk and high risk portfolios have the same utility value, and the average between them has a higher utility value. Averages are better than extremes for the first investor. For the second investor, the average is not better than the extremes — but the extremes are not on the same indifference curve. Convexity only says that the average of indifferent bundles is better than the extremes — not the average between any two bundles. And we showed in part B(c), that tastes represented by the utility function $u(x_1, x_2) = (R - x_1)x_2$ are convex regardless of the value of R — so the second investor's tastes are also convex.

(g) *What value of R would make the investor choose the no-risk portfolio?*

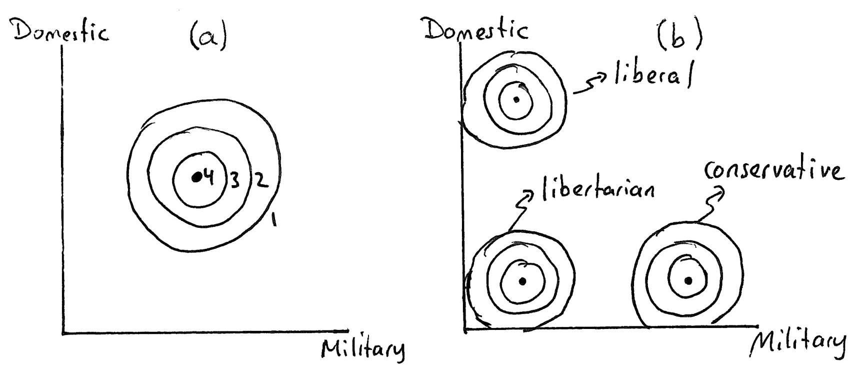
Answer There are many values of R that would do this. For instance, if $R = 5$, the utility of the risk free portfolio is 10 while the utility of the risky portfolio is −30 and the utility of the mixed portfolio is 6. For $R = 6$, the utility of the no risk portfolio is 12, as is the utility of the mixed portfolio (with the risky portfolio getting utility −20). Thus, $R = 6$ is the highest value of R that can justify a choice of the risk-free portfolio.

4.11 Policy Application: *Ideology and Preferences of Politicians*: *Political scientists often assume that politicians have tastes that can be thought of in the following way: Suppose that the two issues a politician cares about are domestic spending and military spending. Put military spending on the horizontal axis and domestic spending on the vertical axis. Then each politician has some "ideal point" — some combination of military and domestic spending that makes him/her happiest.*

A: *Suppose that a politician cares only about how far the actual policy bundle is from his ideal point, not the direction in which it deviates from his ideal point.*

(a) *On a graph, pick any arbitrary "ideal point" and illustrate what 3 indifference "curves" would look like for such a politician. Put numerical labels on these to indicate which represent more preferred policy bundles.*

Answer: The first panel in Graph 4.11 illustrates an example of such indifference curves. The ideal point is at the center of concentric circles, with circles farther way from the ideal point representing policy bundles with less and less utility. Since distance from the ideal point is all that matters, the indifference "curves" should be circles with the ideal point at their center.



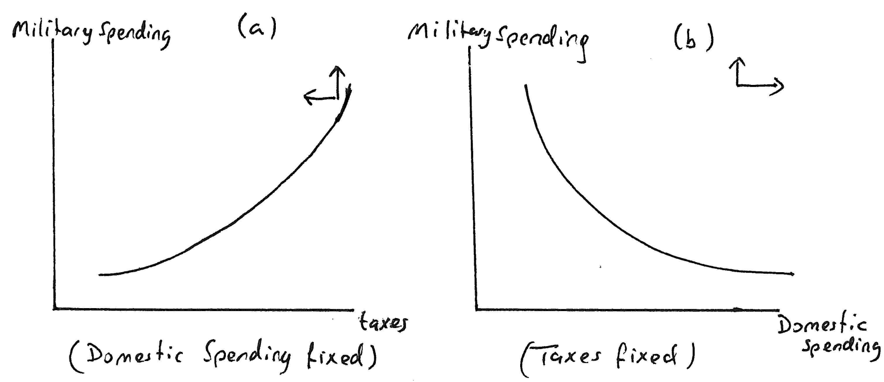
Graph 4.11: Ideology and Political Tastes

(b) *On a separate graph, illustrate how tastes would be different for a political conservative (who likes a lot of military spending but is not as keen on domestic spending), a liberal (who likes domestic spending but is not as interested in military spending) and a libertarian (who does not like government spending in any direction to get very large).*

Answer: This is illustrated in the second panel of Graph 4.11. The politician's ideology determines the location of his ideal point, with ideal points lining up as described in the problem. Indifference "curves" will then again be concentric circles with each ideal point at the center of the circular indifference curves.

(c) *This way of graphing political preferences is a short-cut because it incorporates directly into tastes the fact that there are taxes that have to pay for government spending. Most politicians would love to spend increasingly more on everything, but they don't because of the increasing political cost of having to raise taxes to fund spending. Thus, there are really 3 goods we could be modeling: military spending, domestic spending and taxes, where a politician's tastes are monotone in the first two goods but not in the last. First, think of this as three goods over which tastes satisfy all our usual assumptions* — including monotonicity *and convexity — where we define the goods as spending on military, spending on domestic goods and the "relative absence of taxes". What would indifference "curves" for a politician look like in a 3-dimensional graph? Since it is difficult to draw this, can you describe it in words and show what a 2-dimensional slice looks like if it holds one of the goods fixed?*

Answer: The indifference "curves" in this 3-dimensional graph would be bowl-shaped, with the tip of the bowl facing the origin. Along any slice that holds one of the goods fixed, the shape would be the usual shape of an indifference curve in 2 dimensions as, for example, that depicted in panel (b) of Graph 4.12.



Graph 4.12: Ideology and Political Tastes: Part 2

(d) *Now suppose you model the same tastes, but this time you let the third good be defined as "level of taxation" rather than "relative absence of taxes". Now monotonicity no longer holds in one dimension. Can you now graph what a slice of this 3-dimensional indifference surface would look like if it holds domestic spending fixed and has taxes on the horizontal and military spending on the vertical axis? What would a slice look like that holds taxes fixed and has domestic spending on the horizontal and military spending on the vertical axis?*

Answer: The indifference surface would still be bowl shaped but would now point toward the far end of the tax axis. The slice with military spending on the vertical and taxes on the horizontal is graphed in panel (a) of Graph 4.12 where the politician becomes better off with less taxes and more military spending. The slice with military spending on the vertical and domestic spending on the horizontal axis is illustrated in panel (b) — and looks like an ordinary indifference curve since taxes are fixed along the slice.

(e) *Pick a point on the slice that holds taxes fixed. How does the MRS at that point differ for a conservative from that of a liberal?*

Answer: The slope at that point would be shallower for a conservative than for a liberal because a conservative is willing to give up less military spending to get one more dollar of domestic spending. So, in absolute value, the conservative's MRS is smaller than the liberal's.

(f) *Pick a point on the slice that holds domestic spending fixed. How would the MRS at that point differ for a libertarian compared to a conservative?*

Answer: Libertarians would need to get a lot more military spending to justify one more unit of taxation while conservatives would need less. Thus, libertarians would have a steeper slope — i.e. a higher MRS (in absolute value).

B: *Consider the following equation* $u(x_1, x_2) = P - \left((x_1 - a)^2 + (x_2 - b)^2\right)$.

(a) *Can you verify that this equation represents tastes such as those described in this problem (and graphed in part A(a))?*

Answer: Along any indifference curve, the utility level is constant. Consider one such indifference curve with utility constant at $\overline{u}$. This can then be written as

$$P - \overline{u} = (x_1 - a)^2 + (x_2 - b)^2. \tag{4.21}$$

which is the equation of a circle with center (a, b) and radius $(P - \overline{u})^{1/2}$. At the ideal point $(x_1, x_2) = (a, b)$, utility is at its peak P. As x_1 deviates in either direction (with $x_2 = b$), utility declines by $(x_1 - a)^2$. For instance, if x_1 deviates in either direction by 1, utility declines to $(P - 1)$, and if x_1 deviates by 2 in either direction, utility declines to $(P - 4)$. The same is true for deviations of x_2 in either direction (holding $x_1 = a$). And the same holds for any deviation from (a, b) in directions that involve changes in both x_1 and x_2. Thus, utility declines from its peak in relation to a policy bundle's distance from the ideal point (a, b).

(b) *What would change in this equation as you model conservative, liberal and libertarian politicians?*

Answer: Conservatives would have $a > b$ and liberals $b > a$. Libertarians would have low values of a relative to those of conservatives and low levels of b relative to liberals.

(c) *Do these tastes satisfy the convexity property?*

Answer: Yes, they do. To see this, take any two points on an indifference circle. The line connecting those two points lies in the region of policy bundles that are better than those on the indifference circle. Thus, averages of policy bundles that the politician is indifferent between are better than extremes.

(d) *Can you think of a way to write a utility function that represents the tastes you were asked to envision in A(c) and A(d)? Let t represent the tax rate with an upper bound of 1.*

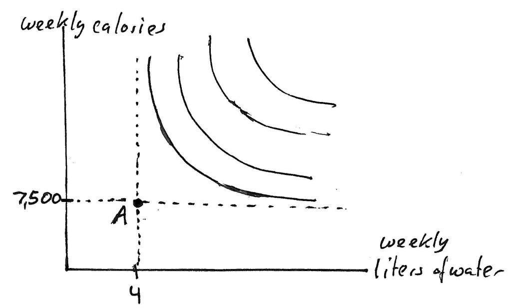
Answer: To turn the tax from a "bad" to a "good", we can define it as the "relative absence of a tax" by writing it as $(1 - t)$. We can then treat $(1 - t)$ just like any other good, writing the utility function, for instance, as $u(x_1, x_2, t) = x_1^{\alpha} x_2^{\beta} (1 - t)^{\gamma}$ where α, β and γ are just numbers on the real line.

4.12 Policy Application: *Subsistence Levels of Consumption*: *Suppose you are interested in modeling a policy issue involving poor households in an under-developed country.*

A: *The households we are trying to model are primarily worried about survival, with a minimum quantity of certain goods (like food and water) necessary for survival. Suppose that one cannot live without at least 4 liters of water per week and at least 7,500 calories of food per week. These quantities of water and food are then* subsistence levels *of water and food.*

(a) *Suppose you graph weekly liters of water on the horizontal axis and weekly intake of calories on the vertical. Indicate the bundle required for subsistence.*

Answer: This is indicated as bundle A in Graph 4.13.



Graph 4.13: Tastes with Subsistence Levels

(b) *If life below the subsistence quantities is not sustainable, we might find it reasonable not to model tastes below the subsistence quantities. Illustrate a plausible map of indifference curves that takes this into account.*

Answer: Such a map is drawn in Graph 4.13. The indifference curves cannot fall below subsistence levels — thus effectively making bundle A the origin relative to the map of indifference curves.

(c) *Subsistence levels are a biological reality for all of us, not just for the poor in developing countries. Why might we nevertheless not worry about explicitly modeling subsistence levels for policy analysis in richer countries?*

Answer: While subsistence levels are indeed a reality for all human beings, they are not of economic significance in many contexts where incomes are such that everyone consumes substantially above subsistence levels. Economics fundamentally deals with trade-offs in a world of scarcity — and for the poor in developing countries the scarcity of food and clean water is quite real. In such circumstances, we might need to model the reality that individuals simply cannot trade off one for the other below certain levels of food and water. But for the circumstances that most of us face, these are not the salient trade-offs that we make. While our tastes too will be undefined below subsistence levels, we are so far above such levels that we can simply abstract away from such considerations because they play no role in the subset of the bundles of goods from which we are actually choosing.

B: *The following utility function is known as the* Stone-Geary utility function*:* $u(x_1,x_2) = (x_1 - \overline{x}_1)^{\alpha}(x_2 - \overline{x}_2)^{(1-\alpha)}$*, where* $0 < \alpha < 1$.

(a) *When interpreted as a model of tastes such as those described in part A, what are the subsistence levels of* x_1 *and* x_2*?*

Answer: The subsistence bundle (analogous to A in our graph) would be $(\overline{x}_1, \overline{x}_2)$. This is because the quantities in the parentheses are above zero only if consumption is above these quantities.

(b) *How does this utility function treat tastes below subsistence levels?*

Answer: Bundles below the subsistence level have utility that is undefined. If $x_1 < \overline{x}_1$, for instance, then $(x_1 - \overline{x}_1) < 0$. Since the exponent α lies between 0 and 1, the quantity $(x_1 - \overline{x}_1)^{\alpha}$ is undefined.

(c) *What is the MRS when consumption is above subsistence levels?*

Answer: The MRS is

$$MRS = -\frac{\alpha(x_1 - \overline{x}_1)^{\alpha-1}(x_2 - \overline{x}_2)^{1-\alpha}}{(1-\alpha)(x_1 - \overline{x}_1)^{\alpha}(x_2 - \overline{x}_2)^{-\alpha}} = -\frac{\alpha(x_2 - \overline{x}_2)}{(1-\alpha)(x_1 - \overline{x}_1)} \qquad (4.22)$$

(d) *Suppose that instead of water and food for someone poor in the developing world, we modeled calories from food (x_1) and dollars spent on vacations (x_2) for someone in the developed world (taking for granted that he is consuming his desired quantity of water). How would you modify the Stone-Geary utility function assuming that you still want to recognize the absence of tastes for food levels below subsistence?*

Answer: Since there is no subsistence level for dollars spent on vacations, you would set $\overline{x}_2$ to zero. The function would then become

$$u(x_1, x_2) = (x_1 - \overline{x}_1)^{\alpha} x_2^{1-\alpha}. \qquad (4.23)$$

4.13 *In this exercise, we will explore some logical relationships between families of tastes that satisfy different assumptions.*

A: *Suppose we define a strong and a weak version of convexity as follows: Tastes are said to be* strongly convex *if, whenever a person with those tastes is indifferent between A and B, she strictly prefers the average of A and B (to A and B). Tastes are said to be* weakly convex *if, whenever a person with those tastes is indifferent between A and B, the average of A and B is at least as good as A and B for that person.*

(a) *Let the set of all tastes that satisfy strong convexity be denoted as SC and the set of all tastes that satisfy weak convexity as WC. Which set is contained in the other? (We would, for instance, say that "WC is contained in SC" if any taste that satisfies weak convexity also automatically satisfies strong convexity.)*

Answer: Suppose your tastes satisfy the strong convexity condition. Then you always strictly prefer averages to extremes (where the extremes are such that you are indifferent between them). That automatically means that the average between such extremes is *at least as good as* the extremes — which means that your tastes automatically satisfy weak convexity. Thus, the set SC must be fully contained within the set WC.

(b) *Consider the set of tastes that are contained in one and only one of the two sets defined above. What must be true about some indifference curves on any indifference map from this newly defined set of tastes?*

Answer: We already concluded above that all strongly convex tastes are also weakly convex. So tastes that are strongly convex cannot be in the newly defined set because they appear in both SC and WC — and we are defining our new set to contain tastes that are only in one of these sets. The newly defined set therefore contains only tastes that satisfy weak convexity but not strong convexity. The only difference between weak and strong convexity is that the former permits averages to be just as good as extremes while the latter insists that averages are strictly better than extremes. When an average is just as good as two extremes from the same indifference curve, it must be that the line connecting the extremes is all part of the same indifference curve. Thus, some indifference curves in a weakly convex indifference map that lies outside SC must have "flat spots" that are line segments.

(c) *Suppose you are told the following about 3 people: Person 1 strictly prefers bundle A to bundle B whenever A contains more of each and every good than bundle B. If only some goods are represented in greater quantity in A than in B while the remaining goods are represented in equal quantity, then A is at least as good as B for this person. Such tastes are often said to be* weakly monotonic. *Person 2 likes bundle A strictly better than B whenever at least some goods are represented in greater quantity in A than in B while others may be represented in equal quantity. Such tastes are said to be* strongly monotonic. *Finally, person 3's tastes are such that, for every bundle A, there always exists a bundle B very close to A that is strictly better than A. Such tastes are said to satisfy* local nonsatiation. *Call the set of tastes that satisfy strict monotonicity SM, the set of tastes that satisfy weak monotonicity WM, and the set of tastes that satisfy local non-satiation L. What is the relationship between these sets? Put differently, is any set contained in any other set?*

Answer: If your tastes satisfy strong monotonicity, it means that A is strictly preferred to B even if A and B are identical in every way except that A has more of one good than B. This means that your tastes would automatically satisfy weak monotonicity — because weak monotonicity only requires that A is at least as good under that condition and thus permits indifference between A and B unless all goods are more highly represented in A than in B. All strongly monotone tastes are weakly monotone, which means SM is fully contained in WM. Local non-satiation only requires that, for every bundle A, there exists some bundle B close to A such that B is preferred to A. If your tastes satisfy strong monotonicity, then we know such a bundle always exists: Begin at some A and then add a tiny bit of every good to A to form B. As long as we add a tiny bit to all goods, strong monotonicity says B is strictly better than A. The same works for weakly monotonic tastes. Thus, both SM and WM are fully contained in L. But there are also tastes in L such that these tastes are not in WM. Consider tastes where at some bundle A there are no bundles with more goods close to A that are preferred to A but there is a bundle with slightly fewer goods that is preferred to B. Then such tastes would satisfy local non-satiation but not weak (or strong) convexity.

(d) *Give an example of tastes that fall in one and only one of these three sets?*

Answer: Since we have just concluded that SM is contained in WM which is contained in L, such tastes must satisfy local non-satiation but not weak monotonicity. Consider tastes over labor and consumption. We would generally like to expend less labor and have more consumption. Such tastes are not strongly or weakly monotonic because A is strictly less preferred to B if A contains the same amount of consumption but more labor. But they do satisfy local non-satiation because for every A, we can make the person better off through less labor or more consumption (or both).

(e) *What is true of tastes that are in one and only one of the two sets SM and WM?*

Answer: Since SM is contained in WM, such tastes must be weakly monotonic. (If they were strongly monotonic, they would be contained in both sets). Consider bundles A and B that are identical in every way except that A has more of one of the goods than B. For tastes to be weakly monotonic but not strongly monotonic, it must be that there exists such an A and B and that a person with such tastes is indifferent between A and B. (If such a person strictly preferred all such A bundles to all such B bundles, her tastes would be strongly monotonic.) Thus, tastes that fall in WM but not SM must have some indifference curves with either horizontal or vertical "flat spots".

B: *Below we will consider the logical implications of convexity for utility functions. For the following definitions,* $0 \le \alpha \le 1$. *A function* $f : \mathbb{R}^2_+ \to \mathbb{R}^1$ *is defined to be* quasiconcave *if and only if the following is true: Whenever* $f(x_1^A, x_2^A) \le f(x_1^B, x_2^B)$*, then* $f(x_1^A, x_2^A) \le f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right)$. *The same type of function is defined to be* concave *if and only if* $\alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) \le f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right)$.

(a) True or False*: All concave functions are quasiconcave but not all quasiconcave functions are concave.*

Answer: True. Suppose we start with a concave function f. Then

$$\alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) \le f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right). \qquad (4.24)$$

Now suppose that $f(x_1^A, x_2^A) \le f(x_1^B, x_2^B)$. Then it must be true that

$$f(x_1^A, x_2^A) \le \alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B). \qquad (4.25)$$

But that implies that whenever $f(x_1^A, x_2^A) \le f(x_1^B, x_2^B)$, then

$$f(x_1^A, x_2^A) \le f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right) \qquad (4.26)$$

— which is the definition of a quasi-concave function. Thus, *concavity of a function implies quasi-concavity.*

But the reverse does not have to hold. Suppose that when $\alpha = 0.5$, $f(x_1^A, x_2^A) = 10$, $f(x_1^B, x_2^B) = 100$ and $f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right) = 20$. The condition for quasi-concavity is satisfied — so suppose f is in fact quasi-concave throughout. Notice, however, that $\alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) = 0.5(10) + (0.5)100 = 55$. Thus,

$$20 = f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right) < \alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) = 55, \qquad (4.27)$$

which directly violates concavity.

An example of a function that is quasi-concave but not concave is $u(x_1, x_2) = x_1^2 x_2^2$.

(b) *Demonstrate that, if u is a quasiconcave utility function, the tastes represented by u are convex.*

Answer: Tastes are convex if averages of bundles over which we are indifferent are better than those bundles. Suppose tastes are represented by u and u is quasiconcave. Pick $A = (x_1^A, x_2^A)$ and $B = (x_1^B, x_2^B)$ such that $u(x_1^A, x_2^A) = u(x_1^B, x_2^B)$. Let bundle C be some weighted average between A and B; i.e.

$$C = (x_1^C, x_2^C) = \left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right). \tag{4.28}$$

Then quasiconcavity of u implies that

$$u(x_1^A, x_2^A) \le u(x_1^C, x_2^C), \tag{4.29}$$

which tells us that the average bundle C is at least as good as the extreme bundles A and B (since $u(x_1^A, x_2^A) = u(x_1^B, x_2^B)$) that the individual is indifferent between. Thus, *quasiconcavity of the utility function implies convexity of underlying tastes represented by that utility function.*

(c) *Do your conclusions above imply that, if u is a concave utility function, the tastes represented by u are convex?*

Answer: Since we concluded in (a) that all concave functions are quasiconcave, and since we concluded in (b) that all quasiconcave utility functions represent tastes that satisfy convexity, it must be that all concave utility functions also represent tastes that are convex.

(d) *Demonstrate that, if tastes over two goods are convex, any utility functions that represents those tastes must be quasiconcave.*

Answer: Suppose we consider bundle $A = (x_1^A, x_2^A)$ and $B = u(x_1^B, x_2^B)$ over which an individual with convex tastes is indifferent. Any utility function that represents these tastes must therefore be such that $u(x_1^A, x_2^A) = u(x_1^B, x_2^B)$ which makes the statement

$$u(x_1^A, x_2^A) \le u(x_1^B, x_2^B) \tag{4.30}$$

also true (since the inequality is weak). Now define a weighted average C of bundles A and B; i.e.

$$C = (x_1^C, x_2^C) = \left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right). \tag{4.31}$$

Convexity of tastes implies that C is at least as good as A. Thus, any utility function that represents these tastes must be such that

$$u(x_1^A, x_2^A) \le u(x_1^C, x_2^C). \tag{4.32}$$

We have therefore concluded that the utility function representing convex tastes must be such that, whenever $u(x_1^A, x_2^A) \le u(x_1^B, x_2^B)$, then

$$u(x_1^A, x_2^A) \le \left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right), \tag{4.33}$$

which is the definition of a quasiconcave function. Thus, *convexity of tastes implies quasiconcavity of any utility function that represents those tastes.*[1]

(e) *Do your conclusions above imply that, if tastes over two goods are convex, any utility function that represents those tastes must be concave?*

Answer: No. We have concluded that convexity of tastes implies quasiconcavity of utility functions and we have shown in (a) that there are quasiconcave utility functions that are *not* concave. So the fact that convexity is represented by quasiconcave utility functions does not imply that convexity requires concave utility functions. In fact it does not — it only requires quasiconcavity.

(f) *Do the previous conclusions imply that utility functions which are not quasiconcave represent tastes that are not convex?*

Answer: Yes. In (d) we showed that convexity *necessarily* means that utility functions will be quasiconcave. Thus, when utility functions are *not* quasiconcave, they cannot represent convex tastes. They must therefore represent non-convex tastes.

[1] We actually showed that this statement holds when $u^A = u^B$ — but the same reasoning holds when $u^A < u^B$.

4.14 *In this exercise, you will prove that, as long as tastes satisfy rationality, continuity and monotonicity, there always exists a well-defined indifference map (and utility function) that can represent those tastes.*[2]

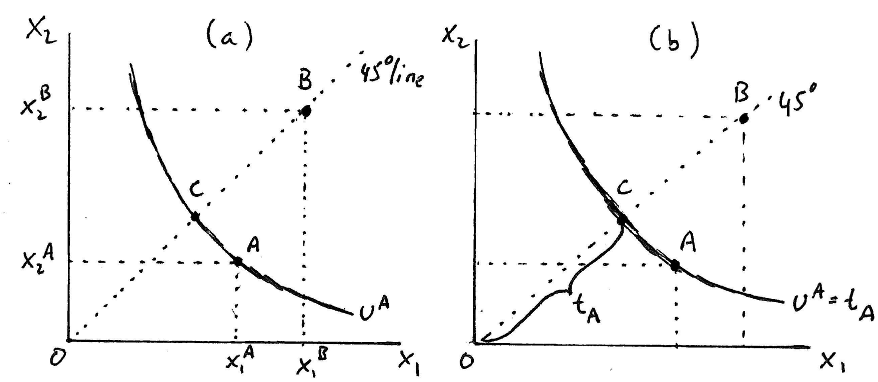
A: *Consider a 2-good world, with goods x_1 and x_2 represented on the two axes in any graphs you draw.*

(a) *Draw your two axes and pick some arbitrary bundle $A = (x_1^A, x_2^A)$ that contains at least some of each good.*

(b) *Draw the 45-degree line in your graph — this is a ray that represents all bundles that have equal amounts of x_1 and x_2 in them.*

(c) *Pick a second bundle $B = (x_1^B, x_2^B)$ such that $x_1^B = x_2^B$ and $x_1^B > \max\{x_1^A, x_2^A\}$. In other words, pick B such that it has equal amounts of x_1 and x_2 and such that it has more of x_1 and x_2 than A.*

Answer: This is all done in panel (a) of Graph 4.14.



Graph 4.14: Constructing Indifference Maps (and Utility Functions)

(d) *Is A more or less preferred than the bundle (0,0)? Is B more or less preferred than A?*

Answer: *A* is better than (0,0) and *B* is better than *A* by the monotonicity assumption.

(e) *Now imagine moving along the 45-degree line from (0,0) toward B. Can you use the continuity property of tastes we have assumed to conclude that there exists some bundle C between (0,0) and B such that the consumer is indifferent between A and C?*

Answer: We know from the monotonicity assumption that, as we move up the 45-degree line, the bundles become "better" because more of each good is added. Since (0,0) is, as we just concluded, worse than *A* and *B* is better than *A*, at some point along the 45-degree line between (0,0) and *B* we switch from having a bundle that is worse than *A* to one that is better than *A*. The continuity assumption says that there are no sudden jumps in tastes — which implies we can't just jump from a bundle that is worse to a bundle that is better — there must be a bundle in between that is just as good as *A*. That bundle is bundle *C*.

(f) *Does the same logic imply that there exists such an indifferent bundle along* any *ray from the origin and not just along the 45-degree line?*

Answer: Yes, the same logic holds along any ray from the origin, not just the 45 degree line.

[2] It can actually be demonstrated that this is true as long as tastes satisfy rationality and continuity only — but it is easier to demonstrate the intuition if we also assume monotonicity.

(g) *How does what you have just done demonstrate the existence of a well-defined indifference map?*

Answer: We began by finding a bundle that is indifferent to A and lies on the 45 degree line. Since the same logic holds for any other ray from the origin, we can find similar indifferent bundles along all other rays — which means we can construct an entire continuous indifference curve that contains bundle A. And since we picked A arbitrarily at the beginning, we can do the same thing for any other bundle — which implies we can construct indifference curves through every possible consumption bundle.

B: *Next we show that the same logic implies that there exists a utility function that represents these tastes.*

(a) *If you have not already done so, illustrate A(a)-(e).*

(b) *Denote the distance from (0,0) to C on the 45-degree line as* $t_A = t(x_1^A, x_2^A)$ *and assign the value* t_A *to the bundle A.*

Answer: This is done in panel (b) of Graph 4.14.

(c) *Imagine the same procedure for labeling each bundle in your graph — i.e. for each bundle, determine what bundle on the 45-degree line is indifferent and label the bundle with the distance on the 45-degree line from (0,0) to the indifferent bundle. The result is a function* $u(x_1, x_2)$ *that assigns to every bundle a number. Can you explain how this function meets our definition of a utility function?*

Answer: We said that a utility function assigns numbers to bundles in such a way that the same number is assigned to indifferent bundles and higher numbers are assigned to more preferred bundles. By always assigning the number that equals the distance from (0,0) to the bundle that is indifferent on the 45 degree line, we are assigning the same number to all bundles that are indifferent to the same bundle on the 45-degree line — i.e., we are assigning the same number to indifferent bundles. All bundles that are better lie above that indifference curve — and thus are indifferent to a bundle that lies higher up on the 45 degree line further away from (0,0). Thus, better bundles are assigned higher numbers.

(d) *Can you see how the same method of proof would work to prove the existence of a utility function when there are more than 2 goods (and when tastes satisfy rationality, continuity and monotonicity)?*

Answer: When there are more goods, we can still think of the analog of the 45 degree line. In the case of 3 goods, for instance, we can define this as the ray into 3-dimensional space that keeps all 3 bundles at the same quantities. When there are n goods, that same ray is defined by

$$\left\{(x_1, x_2, ..., x_n) \in \mathbb{R}^n_+ \mid x_1 = x_2 = ... = x_n\right\} \tag{4.34}$$

and the distance from the origin $(0, 0, ..., 0) \in \mathbb{R}^n_+$ to indifferent bundles in this set can be defined just as it was above.

(e) *Could we have picked a ray other than the 45-degree line to construct the utility values associated with each bundle?*

Answer: Yes, we simply used one of many possible ways of assigning numbers to bundles such that the resulting function is a utility functions.