SOLUTIONS

5

Different Types of Tastes

Solutions for *Microeconomics: An Intuitive Approach with Calculus*

Apart from end-of-chapter exercises provided in the student *Study Guide*, these solutions are provided for use by instructors. (End-of-Chapter exercises with solutions in the student *Study Guide* are so marked in the textbook.)

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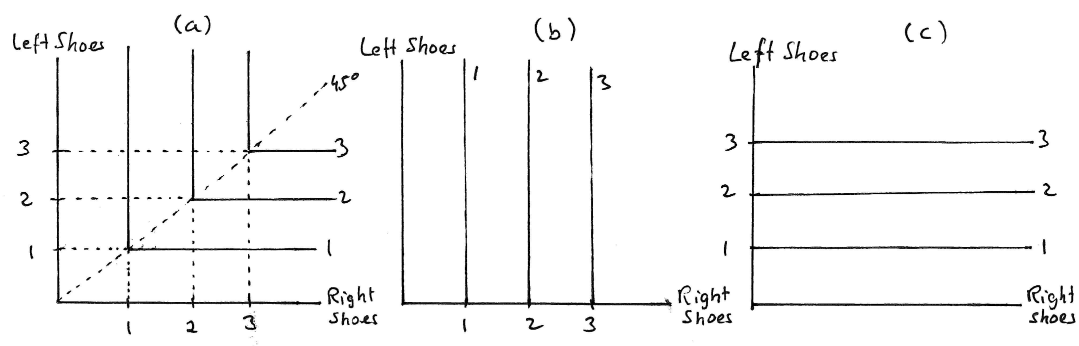
- Each end-of-chapter exercise begins on a new page. This is to facilitate maximum flexibility for instructors who may wish to share answers to some but not all exercises with their students.
- If you are assigning only the A-parts of exercises in *Microeconomics: An Intuitive Approach with Calculus*, you may wish to instead use the solution set created for the companion book *Microeconomics: An Intuitive Approach*.
- *Solutions to Within-Chapter Exercises are provided in the student* Study Guide.

5.1 *Consider your tastes for right and left shoes.*

A: *Suppose you, like most of us, are the kind of person that is rather picky about having the shoes you wear on your right foot be designed for right feet and the shoes you wear on your left foot be designed for left feet. In fact you are so picky that you would never wear a left shoe on your right foot or a right shoe on your left foot — nor would you ever choose (if you can help it) not to wear shoes on one of your feet.*

(a) *In a graph with the number of right shoes on the horizontal axis and the number of left shoes on the vertical, illustrate three indifference curves that are part of your indifference map.*

Answer: Panel (a) of Graph 5.1 illustrates the three indifference curves corresponding to the utility you get from 1 pair of shoes, 2 pair of shoes and 3 pair of shoes. Right and left shoes are perfect complements.



Graph 5.1: Right Shoes and Left Shoes

(b) *Now suppose you hurt your left leg and have to wear a cast (which means you cannot wear shoes on your left foot) for 6 months. Illustrate how the indifference curves you have drawn would change for this period. Can you think of why goods such as left shows in this case are called* neutral goods*?*

Answer: Panel (b) of Graph 5.1 illustrates such indifference curves. For any given number of right shoes, utility would not change as you get more left shoes since you have no use for left shoes. The only way to get to higher utility is to increase right shoes. Goods like left shoes in this example are sometimes called *neutral goods* because you do not care one way or another if you have any of them.

(c) *Suppose you hurt your right foot instead. How would this change your answer to part (b).*

Answer: This is illustrated in panel (c) of Graph 5.1. Now you can only become better off by getting more left shoes, but getting more right shoes (for any level of left shoes) does nothing to change your utility.

(d) *Are any of the tastes you have graphed homothetic? Are any quasilinear?*

Answer: All 3 are homothetic — the slopes (to the extent to which these are defined) of the indifference curves in all three maps are the same along any ray from the origin. The panel (a) perfect complements case is not quasilinear because, for any quantity of right shoes, the "slope" changes from perfectly horizontal to perfectly vertical at some level of left shoes. And for any quantity of left shoes, the "slope" changes from perfectly vertical to perfectly horizontal at some level of right shoes. But the tastes in panels (b) and (c) are quasilinear in both goods — along any horizontal and vertical line, the "slope" remains the same. You can view the latter two as the limit cases of perfect substitutes. For instance, in panel (c) we could add a slight negative slope to the indifference curves, and we would then have indifference curves with the same *MRS* everywhere. Put differently, we'd have perfect substitutes where we are willing to trade very small numbers of left shoes for many right shoes. Then imagine a sequence of such indifference maps, with each indifference curve in the sequence having a slope that is half the slope of the previous one. Every indifference map in that sequence is similarly one of perfect substitutes with constant MRS, and the limit of that sequence is the indifference map depicted in panel (c).

(e) *In the three different tastes that you graphed, are any of the goods ever "essential"? Are any not essential?*

Answer: A good is essential if there is no way to attain utility greater than what one would attain at the origin without consuming at least some of that good. In panel (a), both goods are therefore essential — because you have to consume the goods in pairs in order to get any utility from consuming either. In panel (b), right shoes are essential but left shoes are not, and in panel (c) left shoes are essential but right shoes are not.

B: *Continue with the description of your tastes given in part A above and let x_1 represent right shoes and let x_2 represent left shoes.*

(a) *Write down a utility function that represents your tastes as illustrated in A(a). Can you think of a second utility function that also represents these tastes?*

Answer: This is just a case of perfect complements — so the utility function $u(x_1,x_2) = \min\{x_1,x_2\}$ would be one that works for representing these tastes. So would a function $v(x_1,x_2) = \alpha \min\{x_1,x_2\}$ for any $\alpha > 0$, or $w(x_1,x_2) = (\min\{x_1,x_2\})^\beta$ for any $\beta > 0$, or any number of other transformations that don't alter the ordering of indifference curves.

(b) *Write down a utility function that represents your tastes as graphed in A(b).*

Answer: Since only right shoes (x_1) matter, utility cannot vary with the number of left shoes (x_2). A function like $u(x_1,x_2) = x_1$ would therefore suffice.

(c) *Write down a utility function that represents your tastes as drawn in A(c).*

Answer: Since only left shoes (x_2) matter, utility cannot vary with the number of right shoes (x_1). A function like $u(x_1,x_2) = x_2$ would therefore suffice.

(d) *Can any of the tastes you have graphed in part A be represented by a utility function that is homogeneous of degree 1? If so, can they also be represented by a utility function that is not homogeneous?*

Answer: A function $u(x_1,x_2)$ is homogeneous of degree 1 if $u(tx_1,tx_2) = tu(x_1,x_2)$. The utility function $u(x_1,x_2) = \min\{x_1,x_2\}$ in our answer to part B(a), for instance, is homogeneous of degree 1 because

$$u(tx_1,tx_2) = \min\{tx_1,tx_2\} = t\min\{x_1,x_2\} = tu(x_1,x_2). \tag{5.1}$$

Similarly, the functions $u(x_1,x_2) = x_1$ from part B(b) and $u(x_1,x_2) = x_2$ from part B(c) are homogeneous of degree 1. Each of these three functions can be turned into a function that is not homogeneous by simply adding a constant. Adding such a constant does not change the underlying shape of indifference curves — and so it does not alter the kinds of tastes that we are modeling. But, for instance, $f(x_1,x_2) = \alpha + \min\{x_1,x_2\}$ is such that

$$f(tx_1,tx_2) = \alpha + \min\{tx_1,tx_2\} \neq t^k\alpha + t^k\min\{x_1,x_2\} = t^k f(x_1,x_2) \tag{5.2}$$

for any $k > 0$.

(e) *Refer to end-of-chapter exercise 4.13 where the concepts of "strong monotonicity," "weak monotonicity" and "local non-satiation" were defined. Which of these are satisfied by the tastes you have graphed in this exercise?*

Answer: All satisfy local non-satiation because for any bundle, there is always another bundle close by that is more preferred. All satisfy weak monotonicity — because for any bundle, adding more of one of the goods is at least as good as the original bundle. But they don't satisfy strong monotonicity — because in each case there is a way to add more of one good to a bundle without making the individual strictly better off.

(f) *Refer again to end-of-chapter exercise 4.13 where the concepts of "strong convexity" and "weak convexity" were defined. Which of these are satisfied by the tastes you have graphed in this exercise?*

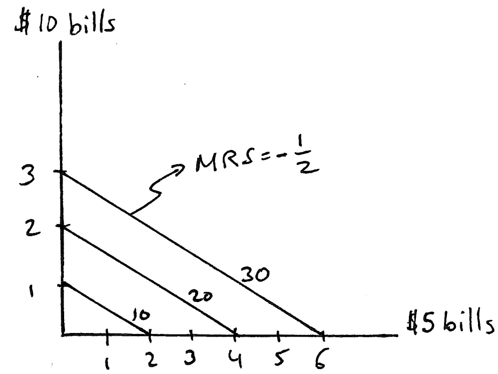
Answer: All satisfy weak convexity because, for any two bundles on a given indifference curve, any weighted average of the bundles (which lies on a line connecting the two bundles) is at least as good as the more extreme bundles. They do not satisfy strong convexity because in each case we can find two bundles that lie on a line segment of the indifference curves — and for those bundles, weighted averages are not strictly better than the extremes.

5.2 *Consider your tastes for five dollar bills and ten dollar bills (and suppose that you could have partial \$10 and \$5 bills).*

A: *Suppose that all you care about is how much money you have, but you don't care whether a particular amount comes in more or fewer bills.*

(a) *With the number of five dollar bills on the horizontal axis and the number of ten dollar bills on the vertical, illustrate three indifference curves from your indifference map.*

Answer: Three such indifference curves are graphed in Graph 5.2. The first of these represents different ways of having \$10, the second represents different ways of having \$20 and the third represents different ways of having \$30 in your wallet.



Graph 5.2: \$5 and \$10 bills

(b) *What is your marginal rate of substitution of ten dollar bills for five dollar bills?*

Answer: The MRS is $-1/2$ — because you are willing to trade half a \$10 bill for one \$5 bill.

(c) *What is the marginal rate of substitution of five dollar bills for ten dollar bills?*

Answer: You are willing to trade 2 five dollar bills for 1 ten dollar bill — so your marginal rate of substitution of \$5 bills for \$10 bills is -2 — the inverse of the marginal rate of substitution of \$10 bills for \$5 bills.

(d) *Are averages strictly better than extremes? How does this relate to whether your tastes exhibit diminishing marginal rates of substitution?*

Answer: No, averages are just as good as extremes in this case. Diminishing marginal rates of substitution arise when averages are strictly better than extremes — and when averages are just as good as extremes, the MRS is constant rather than diminishing.

(e) *Are these tastes homothetic? Are they quasilinear?*

Answer: The MRS is the same everywhere. This implies the MRS is the same along any ray from the origin as well as any vertical or horizontal line. Thus, these tastes are homothetic as well as quasilinear (in both goods).

(f) *Are either of the goods on your axes "essential"?*

Answer: Neither of the goods are essential because you do not require any \$5 bills to attain utility higher than you would at the origin (as long as you get some \$10 bills), nor do you require any \$10 bills (so long as you get some \$5 bills).

B: *Continue with the assumption that you care only about the total amount of money in your wallet, and let five dollar bills be denoted x_1 and ten dollar bills be denoted x_2.*

(a) *Write down a utility function that represents the tastes you graphed in A(a). Can you think of a second utility function that also represents these tastes?*

Answer: These are perfect substitutes for which two of x_1 is needed to make up for one of x_2. Thus, the function $u(x_1, x_2) = x_1 + 2x_2$ would work because it places twice as much value on \$10 bills (x_2) as on \$5 bills (x_1). Any transformation of this function that simply relabels the indifference curve would also work, such as $v(x_1, x_2) = x_1 + 2x_2 + \alpha$ for any α or $w(x_1, x_2) = \beta(x_1 + 2x_2)$ for any $\beta > 0$.

(b) *Calculate the marginal rate of substitution from the utility functions you wrote down in B(a) and compare it to your intuitive answer in A(b).*

Answer: The MRS's for the 3 functions given in the previous part are given by

$$MRS^u = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{1}{2}; \tag{5.3}$$

$$MRS^v = -\frac{\partial v/\partial x_1}{\partial v/\partial x_2} = -\frac{1}{2}; \tag{5.4}$$

and

$$MRS^w = -\frac{\partial w/\partial x_1}{\partial w/\partial x_2} = -\frac{\beta}{2\beta} = -\frac{1}{2}. \tag{5.5}$$

This is precisely what we concluded intuitively in part A.

(c) *Can these tastes be represented by a utility function that is homogeneous of degree 1? If so, can they also be represented by a utility function that is not homogeneous?*

Answer: The function $u(x_1, x_2) = x_1 + 2x_2$ is homogeneous of degree 1 because

$$u(tx_1, tx_2) = tx_1 + 2tx_2 = t(x_1 + 2x_2) = tu(x_1, x_2). \tag{5.6}$$

The equation $v(x_1, x_2) = x_1 + 2x_2 + \alpha$, on the other hand, is not homogeneous because

$$v(tx_1, tx_2) = tx_1 + 2tx_2 + \alpha \neq t^k(x_1 + 2x_2 + \alpha) = t^k v(x_1, x_2) \tag{5.7}$$

for any $k > 0$.

(d) *Refer to end-of-chapter exercise 4.13 where the concepts of "strong monotonicity," "weak monotonicity" and "local non-satiation" were defined. Which of these are satisfied by the tastes you have graphed in this exercise?*

Answer: They satisfy local non-satiation because for any bundle, there is always another bundle close by that is more preferred. They also satisfy weak monotonicity — because for any bundle, adding more of one of the goods is at least as good as the original bundle. And they satisfy strong monotonicity — because adding some positive amount of either good to any bundle results in a bundle that is in fact strictly preferred to the original.

(e) *Refer again to end-of-chapter exercise 4.13 where the concepts of "strong convexity" and "weak convexity" were defined. Which of these are satisfied by the tastes you have graphed in this exercise?*

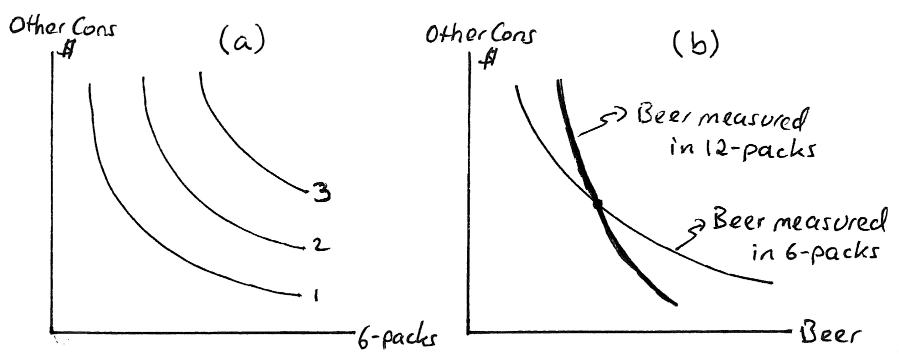
Answer: Weak convexity is satisfied because averages are at least as good as extremes, but strong convexity is not satisfied because averages are not strictly better than extremes.

5.3 *Beer comes in six and twelve-packs. In this exercise we will see how your model of tastes for beer and other consumption might be affected by the units in which we measure beer.*

A: *Suppose initially that your favorite beer is only sold in six-packs.*

(a) *On a graph with beer on the horizontal axis and other consumption (in dollars) on the vertical, depict three indifference curves that satisfy our usual five assumptions assuming that the units in which beer is measured is six-packs.*

Answer: An example of 3 such indifference curves is depicted in panel (a) of Graph 5.3



Graph 5.3: Six and 12-packs of Beer

(b) *Now suppose the beer company eliminates six-packs and sells all its beer in twelve-packs instead. What happens to the MRS at each bundle in your graph if 1 unit of beer now represents a twelve-pack instead of a six-pack.*

Answer: At every bundle, you would now be willing to give up twice as many dollars of other consumption for one more unit of beer than you were before — because one more unit of beer is twice as much beer as it was before. Thus, the *MRS* has to be twice as large in absolute value at every consumption bundle.

(c) *In a second graph, illustrate one of the indifference curves you drew in part (a). Pick a bundle on that indifference curve and then draw the indifference curve through that bundle assuming we are measuring beer in twelve-packs instead. Which indifference curve would you rather be on?*

Answer: In panel (b) of Graph 5.3, this is illustrated — with the indifference curve that measures beer in 12-packs having twice the slope in absolute value as the indifference curve that measures beer in 6-packs. You would of course rather be on the indifference curve with beer measured in 12 packs.

(d) *Does the fact that these indifference curves cross imply that tastes for beer change when the beer company switches from 6-packs to 12-packs?*

Answer: No. The shape of indifference curves on any indifference map is determined in part by the units used to measure quantities of the goods. The two indifference maps from which the indifference curves in panel (b) arise represent the same tastes if they have the same *MRS* adjusted for the units used to measure beer — i.e. if beer is measured in units twice as large, the *MRS* at every bundle has to be twice as large in absolute value.

B: *Let x_1 represent beer and let x_2 represent dollars of other consumption. Suppose that, when x_1 is measured in units of six-packs, your tastes are captured by the utility function $u(x_1, x_2) = x_1 x_2$.*

(a) *What is the MRS of other goods for beer?*

Answer: The *MRS* is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{x_2}{x_1} \tag{5.8}$$

(b) *What does the MRS have to be if x_1 is measured in units of 12-packs?*

Answer: As we argued above, the MRS has to be twice as large in absolute value since now you would be willing to pay twice as much for one more unit of x_1 since it is measured in units twice as large.

(c) *Give a utility function that represents your tastes when x_1 is measured in 12-packs and check to make sure it has the MRS you concluded it must have.*

Answer: The utility function $v(x_1, x_2) = x_1^2 x_2$ would be one function that could represent tastes over 12-packs of beer. The MRS of this function is

$$MRS = -\frac{\partial v/\partial x_1}{\partial v/\partial x_2} = -\frac{2x_1 x_2}{x_1^2} = -2\frac{x_2}{x_1}, \tag{5.9}$$

which is twice as large in absolute value as the MRS of the original utility function u.

(d) *Can you use this example to explain why it is useful to measure the substitutability between different goods using percentage terms (as in the equation for the elasticity of substitution) rather than basing it simply on the absolute value of slopes at different bundles?*

Answer: The units used to measure goods affect the way that indifference curves look, but they don't affect the underlying tastes represented by those indifference curves. If a measure of substitutability were to use the absolute value of slopes at different bundles, the choice of units would partly determine the value of our measure of substitutability. But by using percentage changes instead of absolute changed in the formula for the elasticity of substitution, the units cancel — and our measure becomes independent of the units. For instance, the utility function we derived in the previous part for the case where we measure beer in 12-packs is Cobb-Douglas just as the utility function we used to measure those same tastes when beer was measured in 6-packs. We know that all Cobb-Douglas utility functions have elasticity of substitution of 1 — and so we know we have not changed the elasticity of substitution when we altered the units used to measure one of the goods. Thus, we have defined in the elasticity of substitution a measure of substitutability that is immune to the units chosen to measure the goods on each axis.

5.4 *Suppose two people want to see if they could benefit from trading with one another in a 2-good world.*

A: *In each of the following cases, determine whether trade might benefit the two individuals:*

(a) *As soon as they start talking with one another, they find that they own exactly the same amount of each good as the other does.*

Answer: This should in general not keep them from being able to gain from trading with one another as long as their tastes differ at the margin at the bundle that they own. What matters for gains from trade is whether there are differences in the two individual's MRS at the bundle they currently own.

(b) *They discover that they are long-lost twins who have identical tastes.*

Answer: Again, that should not generally keep them from being able to trade with one another, at least as long as they don't currently own the same bundle. People with the same map of indifference curves will typically have different MRS's when they own different bundles — and it is this difference in tastes at the margin that may arise even if people have the same map of indifference curves.

(c) *The two goods are perfect substitutes for each of them — with the same MRS within and across their indifference maps.*

Answer: In this case, there is no way to gain from trade — because no matter what bundle each of the individuals currently owns, their MRS is the same across the two individuals.

(d) *They have the same tastes, own different bundles of goods but are currently located on the same indifference curve.*

Answer: As long as averages are better than extremes, they will be able to trade toward a more "average" bundle and thus will both benefit from trade.

B: *Suppose that the two individuals have CES utility functions, with individual 1's utility given by* $u(x_1, x_2) = \left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-1/\rho}$ *and individual 2's by* $v(x_1, x_2) = \left(\beta x_1^{-\rho} + (1-\beta)x_2^{-\rho}\right)^{-1/\rho}$.

(a) *For what values of α, β and ρ is it the case that owning the same bundle will always imply that there are no gains from trade for the two individuals.*

Answer: Owning the same bundle implies identical MRS's for the two individuals only if tastes are the same. This implies that $\alpha = \beta$ (since both utility functions already share the same ρ.)

(b) *Suppose $\alpha = \beta$ and the two individuals therefore share the same preferences. For what values of $\alpha = \beta$ and ρ is it the case that the two individuals are not able to gain from trade regardless of what current bundles they own?*

Answer: When individuals have identical tastes but different current bundles of goods, the only way we know that they cannot trade for sure is if the two goods are in fact perfect substitutes for them (because then their MRS is in fact the same regardless of what bundles they own). This occurs when $\rho = -1$.

(c) *Suppose that person 1 owns twice as much of all goods as person 2. What has to be true about α, β and ρ for them not to be able to trade?*

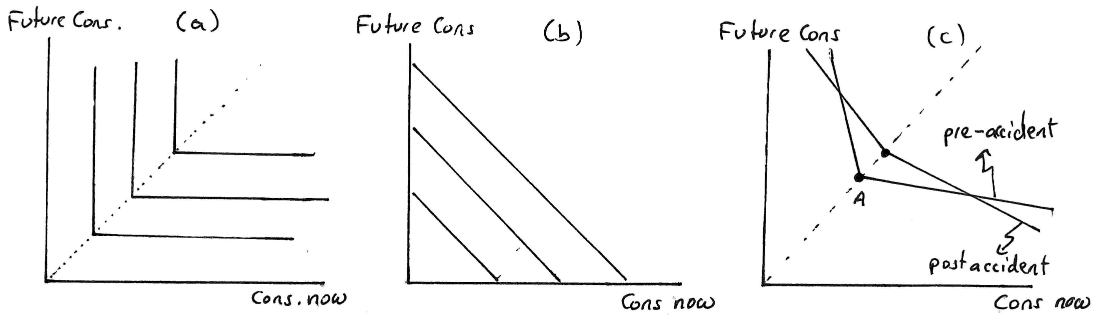
Answer: The tastes are homothetic — which means that the MRS is the same along any ray from the origin within a single indifference map. If the two indifference maps are furthermore identical, then the same ray from the origin will be associated with the same MRS across the two individuals. If person 1 owns twice as much of everything as person 2, then their current bundles lie on a single ray from the origin — which implies that if the two indifference maps are identical, the two individuals will not be able to trade. This is true if $\alpha = \beta$ for any ρ between -1 and infinity.

5.5 Everyday Application: *Personality and Tastes for Current and Future Consumption*: *Consider two brothers, Eddy and Larry, who, despite growing up in the same household, have grown quite different personalities.*

A: *Eddy is known to his friends as "steady Eddy" — he likes predictability and wants to know that he'll have what he has now again in the future. Larry, known to his friends as "crazy Larry", adapts easily to changing circumstances. One year he consumes everything around him like a drunken sailor, the next he retreats to a Buddhist monestary and finds contentment in experiencing poverty.*

(a) *Take the characterization of Eddy and Larry to its extreme (within the assumptions about tastes that we introduced in Chapter 4) and draw two indifference maps with "current consumption" on the horizontal axis and "future consumption" on the vertical — one for steady Eddy and one for crazy Larry.*

Answer: The description indicates that Eddy does not trade off consumption across time very easily while Larry does. In the extreme, that would mean that consumption now and consumption in the future are perfect complements for Eddy and perfect substitutes for Larry. (A less extreme version would have consumption now and consumption in the future be closer to perfect complements for Eddy than for Larry.) The extreme indifference maps for Eddy and Larry are drawn in panels (a) and (b) (respectively) of Graph 5.4.



Graph 5.4: Steady Eddy, Crazy Larry and Unstable Daryl

(b) *Eddy and Larry have another brother named Daryl who everyone thinks is a weighted average between his brothers' extremes. Suppose he is a lot more like steady Eddy than he is like crazy Larry — i.e. he is a weighted average between the two but with more weight placed on the Eddy part of his personality. Pick a bundle A on the 45 degree line and draw a plausible indifference curve for Daryl through A. (If you take the above literally in a certain way, you would get a kink in Daryl's indifference curve.) Could his tastes be homothetic?*

Answer: His indifference curves would be flatter than Eddy's but not as flat as Larry's, and since he is more like Eddy, they would look more like Eddy's. One plausible such indifference curve — labeled "pre-accident" — through a bundle A on the 45 degree line is drawn in panel (c) of the graph. The indifference curve has a kink at A because at A it is unclear what it would mean to "average" the indifference maps. A less literal interpretation of the problem might not have a kink at that point — but would have a somewhat smoother version of an indifference curve like the one graphed here. Both Eddy's and Larry's indifference maps are homothetic — and an average between their indifference maps should also be homothetic. In panel (c), the other indifference curves would contain parallel line segments emanating from the 45 degree line — and the MRS would therefore be the same along any ray from the origin. The same can easily be true of indifference maps without the sharp kink on the 45 degree line. This illustrates that homotheticity of tastes can allow for many different degrees of substitutability.

(c) *One day Daryl suffers a blow to his head — and suddenly it appears that he is more like crazy Larry than like steady Eddy; i.e. the weights in his weighted average personality have flipped. Can his tastes still be homothetic?*

Answer: Yes, they would simply have indifference curves with line segments flatter than the indifference curve through bundle A — indifference curves like the one labeled "post-accident" in panel (c) of the graph. This would continue to satisfy the homotheticity condition. This would also hold for smoother versions of the indifference curves — i.e. versions that don't have a kink point on the 45 degree line.

(d) *In end-of-chapter exercise 4.9, we defined what it means for two indifference maps to satisfy a "single crossing property". Would you expect that Daryl's pre-accident and post-accident indifference maps satisfy that property?*

Answer: No, they would not. This is easily seen in panel (c) of the graph where the pre- and post-accident indifference curve cross twice. (Note that this conclusion also is not dependent on the kink in the indifference curves.)

(e) *If you were told that either Eddy or Larry saves every month for retirement and the other smokes a lot, which brother is doing what?*

Answer: I would guess that a person who views consumption across time as not very substitutable would make sure to save so that he can consume at the same levels when he stops earning income. At the same time, someone who views consumption now and in the future substitutable might be willing to enjoy a lot of smoking now even if it decreases the quality of life later.

B: *Suppose that one of the brothers' tastes can be captured by the function* $u(x_1,x_2) = \min\{x_1,x_2\}$ *where* x_1 *represents dollars of current consumption and* x_2 *represents dollars of future consumption.*

(a) *Which brother is it?*

Answer: It's steady Eddy — since he is not willing to trade consumption across time periods and thus has indifference curves that treat consumption now and consumption in the future as perfect complements (or something close to it).

(b) *Suppose that when people say that Daryl is the weighted average of his brothers, what they mean is that his elasticity of substitution of current for future consumption lies in between those of his borthers. If Larry and Daryl have tastes that could be characterized by one (or more) of the utility functions from end-of-chapter exercise 4.5, which functions would apply to whom?*

Answer: Crazy Larry's would be perfect substitutes — which are given by utility function (2) in problem 4.5. By looking at MRS's and the ordering of indifference curves, we concluded in the answer to problem 4.5 that the utility functions (1) and (4) represented the same Cobb-Douglas tastes. Cobb-Douglas tastes are members of the family of CES tastes — which have perfect complements and perfect substitutes at the extremes. Thus, one way of thinking about Daryl being the average of his brothers would be to think of his elasticity of substitution being in some sense in between those of his brothers'. In that case, Cobb-Douglas tastes seem plausible tastes for Daryl. Note that is a different notion of what it might mean for Daryl to be the weighted average of his brothers from that which resulted in a kink point in the indifference curves in our graph.

(c) *Which of the functions in end-of-chapter exercise 4.5 are homothetic? Which are quasilinear (and in which good)?*

Answer: When the MRS depends only on the ratio of x_2 to x_1, then this means that it is the same for bundle A as it is for any bundle that multiplies the goods in A by the same constant — i.e. it is the same along any ray from the origin. Utility functions (1), (4) and (5) all have this feature and thus all represent homothetic tastes. Furthermore, utility function (2) gives rise to the same MRS everywhere — so it, too, represents tastes that are homothetic. Utility function (3), on the other hand, has MRS that depends only on x_2 — which means that, if me multiply both goods in a bundle A, we end up getting a different MRS. Thus, utility function (3) is not homothetic.

For a utility function to represent tastes that are quasilinear, the MRS has to be independent of one of the two goods. This is true for utility function (3) where the MRS depends only on

x_2. Thus, for any level of x_2, the MRS is unchanged regardless of how much x_1 is in the bundle. Put differently, we can draw a horizontal line in our graph with x_2 on the vertical axis and know that the MRS along that line is constant. Thus, utility function (3) represents tastes quasilinear in x_2. Finally, utility function (2) has the same MRS everywhere and is thus quasilinear in both goods. The other utility functions have MRS varying with both goods and are therefore not quasilinear.

(d) *Despite being so different, is it possible that both steady Eddy and crazy Larry have tastes that can be represented by Cobb Douglas utility functions?*

Answer: No, that does not seem plausible since the description of steady Eddy and crazy Larry clearly indicates very different elasticities of substitution between current and future consumption — and all Cobb-Douglas preferences have elasticity of substitution of 1.

(e) *Is it possible that all their tastes could be represented by CES utility functions? Explain.*

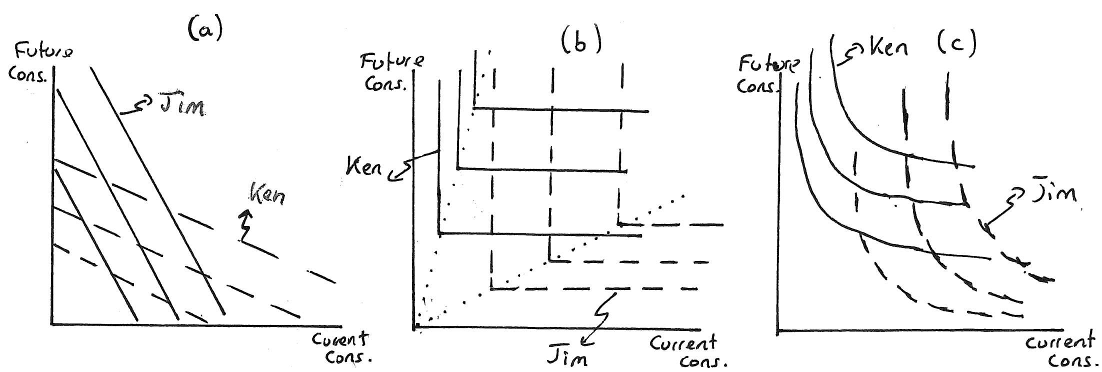
Answer: Yes, this is possible since CES utility functions encompass functions ranging from elasticity of substitution of 0 (perfect complements) to ∞ (perfect substitutes). The essential difference between the three brothers is their elasticity of substitution between current and future consumption — and the CES family of utility functions gives the flexibility to allow that to vary completely across individuals.

5.6 Everyday Application: *Thinking About Old Age: Consider two individuals who take a very different view of life — and consider how this shapes their tastes over intertemporal tradeoffs.*

A: *Jim is a 25 year-old athlete who derives most of his pleasure in life from expensive and physically intense activities — mountain climbing in the Himalayas, kayaking in the Amazon, bungee jumping in New Zealand, Lion safaris in Africa and skiing in the Alps. He does not look forward to old age when he can no longer do this and plans on getting as much fun in early on as he can. Ken is quite different — he shuns physical activity but enjoys reading in comfortable surroundings. The more he reads, the more he wants to read and the more he wants to retreat to luxurious libraries in the comfort of his home. He looks forward to quiet years of retirement when he can do what he loves most.*

(a) *Suppose both Jim and Ken are willing to perfectly substitute current for future consumption — but at different rates. Given the descriptions of them, draw two different indifference maps and indicate which is more likely to be Jim's and which is more likely to be Ken's.*

Answer: Panel (a) of Graph 5.5 illustrates two indifference maps in one graph — with Jim's indifference curves in solid lines and Ken's in dashed lines. Since Jim is more interested in focusing his consumption now, his MRS is larger in absolute value — i.e. he is willing give up more future consumption for current consumption.



Graph 5.5: Jim and Ken's Intertemporal Tastes

(b) *Now suppose neither Jim nor Ken are willing to substitute at all across time periods. How would their indifference maps differ now given the descriptions of them above?*

Answer: Panel (b) illustrates the case where they are not willing to substitute across time — with Jim's indifference curves again dashed and Ken's solid. Even though they are not willing to substitute across time, knowing that Jim wants to consume more now while Ken wants to postpone tells us where the corners of the indifference curves are relative to one another.

(c) *Finally, suppose they both allowed for some substitutability across time periods but not as extreme as what you considered in part (a). Again, draw two indifference maps and indicate which refers to Jim and which to Ken.*

Answer: These are now illustrated in panel (c) — with indifference curve similar to those in (b) except that we add some curvature to allow for some (though not complete) substitutability.

(d) *Which of the indifference maps you have drawn could be homothetic?*

Answer: The indifference maps in (a) are definitely homothetic (since they have the same MRS within each map. The others can certainly be homothetic. In panel (b), they are in fact clearly drawn as homothetic since the corners of the indifference curves are drawn along rays from the origin. The same could be true for panel (c).

(e) *Can you say for sure if the indifference maps of Jim and Ken in part (c) satisfy the single crossing property (as defined in end-of-chapter exercise 4.9)?*

Answer: You can't say for sure. The way they are drawn in panel (c), it certainly seems like the single crossing property might hold. If the indifference maps are close to those of perfect complements, the single crossing property will, in fact hold. But you can also imagine starting with the indifference maps in panel (a) and bending them slightly — thus creating two points at which indifference curves from the two maps cross one another.

B: *Continue with the descriptions of Jim and Ken as given in part A and let c_1 represent consumption now and let c_2 represent consumption in retirement.*

(a) *Suppose that Jim's and Ken's tastes can be represented by $u^J(c_1,c_2) = \alpha c_1 + c_2$ and $u^K(c_1,c_2) = \beta c_1 + c_2$ respectively. How does α compare to β — i.e. which is larger?*

Answer: The marginal rates of substitution are

$$MRS^J = -\frac{\partial u^J/\partial c_1}{\partial u^J/\partial c_2} = -\alpha \text{ and } MRS^K = -\frac{\partial u^K/\partial c_1}{\partial u^K/\partial c_2} = -\beta. \tag{5.10}$$

Thus, it must be that $\alpha > \beta$ for us to get indifference maps such as those in panel (a) of Graph 5.5.

(b) *How would you similarly differentiate, using a constant α for Jim and β for Ken, two utility functions that give rise to tastes as described in A(b)?*

Answer: Such tastes can be represented by

$$u^J(c_1,c_2) = \min\{\alpha c_1, c_2\} \text{ and } u^K(c_1,c_2) = \min\{\beta c_1, c_2\}, \tag{5.11}$$

where $\alpha < 1 < \beta$. Consider, $u^J(c_1,c_2) = \min\{\alpha c_1, c_2\}$. Suppose, for instance, that $\alpha = 1/2$. This means that Jim's utility for the bundle $(c_1,c_2) = (2x,x)$ would be equal to $u^J(2x,x) = \min\{(1/2)2x, x\} = \min\{x,x\}$. Thus, $(2x,x)$ for different values of x represent the location of all the corners of the indifference curves for Jim — or, put differently, those corners lie on a ray from the origin with slope (1/2). More generally, they will lie on a ray from the origin with slope α for Jim and with slope β for Ken — and from panel (b) in Graph 5.5, we know that Ken's ray has slope greater than 1 (because his corners lie above the 45-degree line) while Jim's ray has slope less than 1 (because his corners lie below the 45-degree line.)

(c) *Now consider the case described in A(c), with their tastes now described by the Cobb-Douglas utility functions $u^J(c_1,c_2) = c_1^{\alpha} c_2^{(1-\alpha)}$ and $u^J(c_1,c_2) = c_1^{\beta} c_2^{(1-\beta)}$. How would α and β in those functions be related to one another?*

Answer: The MRS for the two functions is

$$MRS^J = \frac{\alpha c_2}{(1-\alpha)c_1} \text{ and } MRS^K = \frac{\beta c_2}{(1-\beta)c_1}. \tag{5.12}$$

Recall that the MRS is equal to -1 along the 45 degree line (where $c_1 = c_2$) when $\alpha = 0.5$. For Ken, the indifference curves are shallower than this on the 45 degree line — implying a smaller MRS in absolute value. This can only happen if $\beta < 0.5$. The reverse is true for Jim — implying $\alpha > 0.5$. Thus, $\alpha > \beta$.

(d) *Are all the tastes described by the above utility functions homothetic? Are any of them quasilinear?*

Answer: Yes, they are all homothetic. The only one that is quasilinear is the one for perfect substitutes in B(a)

(e) *Can you show that the tastes in B(c) satisfy the single crossing property (as defined in end-of-chapter exercise 4.9)?*

Answer: Pick any arbitrary bundle (c_1,c_2). Given that $\alpha > 0.5 > \beta$ (from (c) above), the absolute value of the marginal rates of substitution at that bundle satisfy

$$\left|MRS^J\right| = \frac{\alpha c_2}{(1-\alpha)c_1} > \frac{\beta c_2}{(1-\beta)c_1} = \left|MRS^K\right|. \tag{5.13}$$

Thus, the slope of Jim's indifference curves at every bundle is steeper than that of Ken's indifference curve at that bundle — which means that indifference curves can only be crossing once.

(f) *Are all the functions in B(a)-(c) members of the family of CES utility functions?*

Answer: Yes. As shown in the text, CES utility functions range from perfect substitutes to perfect complements and include Cobb-Douglas tastes.

5.7 Everyday Application: *Tastes for Paperclips*: *Consider my tastes for paperclips and "all other goods" (denominated in dollar units).*

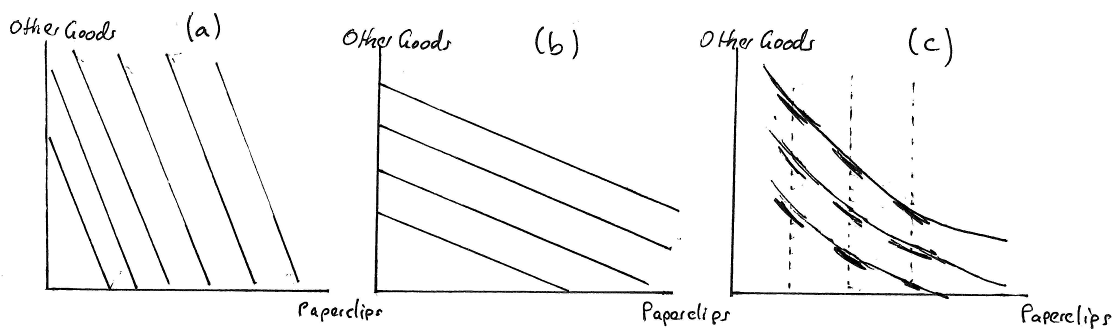
A: *Suppose that my willingness to trade paper clips for other goods does* not *depend on how many other goods I am also currently consuming.*

(a) *Does this imply that "other goods" are "essential" for me?*

Answer: This means that, plotting paperclips on the horizontal axis, the *MRS* is unchanged along any vertical line we draw — which in turn means that the indifference curves must cross the paperclip axis. When an indifference curve crosses the paperclip axis, it means that I am able to get utility greater than at the origin without consuming any "other good". Thus, other goods are not essential for me.

(b) *Suppose that, in addition, my willingness to trade paperclips for other goods does not depend on how many paperclips I am currently consuming. On two graphs, each with paperclips on the horizontal axis and "dollars of other goods" on the vertical, give two examples of what my indifference curves might look like.*

Answer: If my *MRS* does not depend on either my level of paperclip consumption or my level of other good consumption, it means that my *MRS* is the same everywhere. Thus, paperclips and other goods are perfect substitutes — and the only thing we are not sure about is at what rate I am willing to substitute one for the other. Thus, we can get different maps of indifference curves where those maps differ in terms of the *MRS* that holds everywhere within each map. Two such indifference maps are illustrated in panels (a) and (b) of Graph 5.6.



Graph 5.6: Paperclips and other Goods

(c) *How much can the MRS vary* within *an indifference map that satisfies the conditions in part (b)? How much can it vary* between two *indifference maps that both satisfy the conditions in part (b)?*

Answer: As we just concluded, the *MRS* cannot vary *within* an indifference map under these conditions — but it can vary *across* indifference maps that satisfy these conditions.

(d) *Now suppose that the statement in (a) holds for my tastes but the statement in part (b) does not. Illustrate an indifference map that is consistent with this.*

Answer: Tastes are now simply quasilinear in paperclips — and if they are not also quasilinear in other goods, we have ruled out the case of perfect substitutes. An example of such an indifference map is illustrated in panel (c) of Graph 5.6.

(e) *How much can the MRS vary* within *an indifference map that satisfies the conditions of part (d)?*

Answer: The *MRS* can range from zero to minus infinity within the same map.

(f) *Which condition do you think is more likely to be satisfied in someone's tastes — that the willingness to trade paperclips for other goods is independent of the level of paperclip consumption or that it is independent of the level of other goods consumption?*

Answer: It seems more realistic to assume that our MRS is independent of how much in other goods we are consuming. Paperclips are a small part of our overall consumption bundle — and as we consume more of other goods (because, for instance, our income is increasing), it seems unlikely that we will change how we feel about paperclips on the margin. We have only so much need for paperclips, though — so as we get more paperclips in our consumption bundle, we are probably less willing to pay the same amount for more paperclips as we were willing to pay for the original paperclips that made it into the consumption bundle. So it seems unlikely that our MRS is independent of the level of paperclip consumption.

(g) *Are any of the indifference maps above homothetic? Are any of them quasilinear?*

Answer: The maps in panels (a) and (b) are homothetic and quasilinear, while the map in panel (c) is only quasilinear (in paperclips).

B: *Let paperclips be denoted by x_1 and other goods by x_2.*

(a) *Write down two utility functions, one for each of the indifference maps from which you graphed indifference curves in A(b).*

Answer: Consider the utility function $u(x_1,x_2) = \alpha x_1 + x_2$. When $\alpha = 1$, we have the case of perfect substitutes where the consumer is willing to always trade the goods one for one. More generally, the consumer is willing to trade α of x_2 for one x_1. Thus, when $\alpha < 1$, the consumer is willing to trade less than one unit of x_2 for one unit of x_1 — implying shallower indifference curves as in panel (b); and when $\alpha > 1$, the consumer is willing to trade more than one unit of x_2 for one unit of x_1 — implying steeper indifference curves as in panel (a). You can also see this by simply deriving the MRS as $(-\alpha)$.

(b) *Are the utility functions you wrote down homogeneous? If the answer is no, could you find utility functions that represent those same tastes and are homogeneous? If the answer is yes, could you find utility functions that are not homogeneous but still represent the same tastes?*

Answer: Yes, the utility function $u(x_1,x_2) = \alpha x_1 + x_2$ is homogeneous. In fact, it is homogeneous of degree 1 since

$$u(tx_1,tx_2) = \alpha(tx_1) + tx_2 = t(\alpha x_1 + x_2) = tu(x_1,x_2). \tag{5.14}$$

You can transform this into a non-homogeneous utility function by simply adding a constant — say 10 — to get $v(x_1,x_2) = \alpha x_1 + x_2 + 10$. Then

$$v(tx_1,tx_2) = \alpha(tx_1) + tx_2 + 10 \neq t(\alpha x_1 + x_2) + 10t = t^k v(x_1,x_2) \tag{5.15}$$

for all $k > 0$.

(c) *Are the functions you wrote down homogeneous of degree 1? If the answer is no, could you find utility functions that are homogeneous of degree 1 and represent the same tastes? If the answer is yes, could you find utility functions that are not homogeneous of degree k and still represent the same tastes?*

Answer: Equation (5.14) demonstrates that the function $u(x_1,x_2) = \alpha x_1 + x_2$ is homogeneous of degree 1. You can turn this into a function that is homogeneous of degree k by taking it to the kth power; i.e. $w(x_1,x_2) = (u(x_1,x_2))^k = (\alpha x_1 + x_2)^k$. Then

$$w(tx_1,tx_2) = (\alpha(tx_1) + tx_2)^k = (t(\alpha x_1) + x_2))^k = t^k(\alpha(x_1) + x_2)^k = t^k w(x_1,x_2). \tag{5.16}$$

(d) *Is there any indifference map you could have drawn when answering A(d) which can be represented by a utility function that is homogeneous? Why or why not?*

Answer: No. Homogeneous functions have the property that they give rise to homothetic indifference maps — with the MRS constant along any ray from the origin. The indifference map in A(d) is quasilinear — and the only way that it can be both quasilinear and homothetic is for the goods to be perfect substitutes. But A(d) specifically ruled out linear indifference curves.

5.8 Everyday Application: *Inferring Tastes for "Mozartkugeln": I love the Austrian candy "Mozartkugeln". They are a small part of my budget, and the only factor determining my willingness to pay for additional Mozartkugeln is how many I already have.*

A: *Suppose you know that I am willing to give up $1 of "other consumption" to get one more Mozartkugel when I consume bundle A — 100 Mozartkugeln and $500 in other goods per month.*

(a) *What is my MRS when my Mozartkugeln consumption remains unchanged from bundle A but I only consume $200 per month in other goods?*

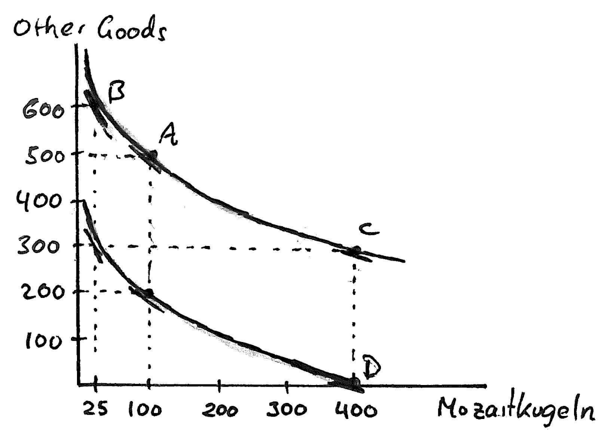
Answer: Since the only factor determining my willingness to pay for additional Mozartkugeln is how many I already have, the *MRS* is unchanged when other consumption goes to $200. I would therefore still be willing to trade $1 in other consumption for 1 more Mozartkugel.

(b) *Are my tastes quasilinear? Could they be homothetic?*

Answer: My tastes as defined are quasilinear in Mozartkugeln. They could be homothetic if my *MRS* is also independent of Mozartkugel consumption — in which case Mozartkugeln and other consumption would be perfect substitutes.

(c) *You notice that this month I am consuming bundle B — $600 in other goods and only 25 Mozartkugeln. When questioning me about my change in behavior (from bundle A), I tell you that I am just as happy as I was before. The following month you observe that I consume bundle C — 400 Mozartkugeln and $300 in other goods, and I once gain tell you my happiness remains unchanged. Does the new information about B and C change your answer in (b)?*

Answer: Graph 5.7 illustrates bundles *A*, *B* and *C* and an indifference curve running through all three bundles (since I am equally happy at each bundle). Knowing about these three bundles lying on the same indifference curve tells me that Mozartkugeln and other goods could not be perfect substitutes. Thus, my tastes cannot be homothetic (since we know they are quasilinear).



Graph 5.7: Mozartkugeln and other goods

(d) *Is consumption (other than of Mozartkugeln) essential for me?*

Answer: Since we know my tastes are quasilinear in Mozartkugeln, the *MRS* is the same along any vertical line in our graph. Thus, I know that there is some indifference curve like the second one drawn in the graph that intersects the horizontal axis at a point like *D*. In fact, all indifference curves must intersect the horizontal axis at some point. Thus, it is possible for me to have utility greater than at the origin without consuming any other goods. Other goods are therefore not essential for me.

B: *Suppose my tastes could be modeled with the utility function* $u(x_1, x_2) = 20x_1^{0.5} + x_2$, *where* x_1 *refers to Mozartkugeln and* x_2 *refers to other consumption.*

(a) *Calculate the MRS for these tastes and use your answer to prove that my tastes are quasilinear in* x_1.

Answer: The MRS is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{0.5(20)x_1^{-0.5}}{1} = -\frac{10}{x_1^{0.5}}. \tag{5.17}$$

Since the MRS is independent of the level of x_2, we know that tastes are quasilinear in x_1.

(b) *Consider the bundles A, B and C as defined in part A. Verify that they lie on one indifference curve when tastes are described by the utility function defined above.*

Answer: Plugging in the values for the three bundles, we get

$$\begin{aligned} u(100,500) &= 200(100^{0.5}) + 500 = 200(10) + 500 = 700 \\ u(25,600) &= 200(25^{0.5}) + 600 = 200(5) + 600 = 700 \\ u(400,300) &= 200(400^{0.5}) + 300 = 200(20) + 300 = 700. \end{aligned} \tag{5.18}$$

(c) *Verify that the* MRS *at bundle A is as described in part A and derive the* MRS *at bundles B and C.*

Answer: Equation (5.17) gives us that $MRS = -10/(x_1^{0.5})$. Thus, at bundle A where $x_1 = 100$, $MRS - 10/(100^{0.5}) = -1$ as described in part A. At B and C, the MRS is -2 and $-1/2$ respectively.

(d) *Verify that the* MRS *at the bundle (100,200) corresponds to your answer to A(a).*

Answer: Using equation (5.17), we again get $MRS = 10/(100^{0.5}) = -1$.

(e) *How much "other goods" consumption occurs on the indifference curve that contains (100,200) when my Mozartkugeln consumption falls to 25 per month? What about when it rises to 400 per month?*

Answer: My utility at (100,200) is $u(100,200) = 20(100^{0.5}) + 200 = 20(10) + 200 = 400$. When Mozartkugel consumption falls to 25, the utility I get from the first term in the utility function falls to $20(25^{0.5}) = 100$. Thus, in order to get the same utility as I do at (100,200), I must have 300 in other consumption. When Mozartkugel consumption is 400, on the other hand, my utility from the first portion of the utility function is $20(400^{0.5}) = 20(20) = 400$. Thus, to stay on the same indifference curve, I must have zero in other consumption.

(f) *Are Mozartkugeln essential for me?*

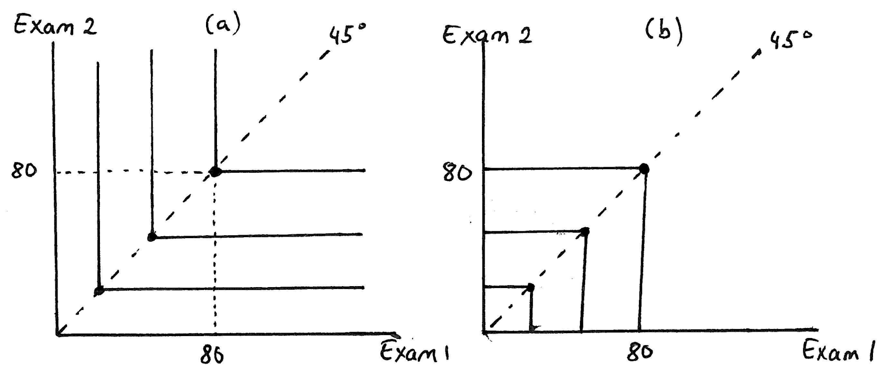
Answer: No. Suppose I have $700 in other good consumption. Then my utility is 700 — just as it is at A, B and C. So I can get more utility than I would at the origin without consuming Mozartkugeln.

5.9 Everday Application: *Syllabi-Induced Tastes over Exam Grades: Suppose you are taking two classes, economics and physics. In both classes, only two exams are given during the semester.*

A: *Since economists are nice people, your economics professor drops the lower exam grade and bases your entire grade on the higher of the two grades. Physicists are another story. Your physics professor will do the opposite — he will drop your highest grade and base your entire class grade on your lower score.*

(a) *With the first exam grade (ranging from 0 to 100) on the horizontal axis and the second exam grade (also ranging from 0 to 100) on the vertical, illustrate your indifference curves for your physics class.*

Answer: These are illustrated in panel (a) of Graph 5.8. Since the physics professor will only count the lower score, only the lower score matters for generating utility. Thus, the two exam scores are perfect complements — the only way to get more utility is for the minimum score to increase.



Graph 5.8: Physics and Economics Exams

(b) *Repeat this for your economics class.*

Answer: Since the economics professor is only counting the higher score, only the higher score matters for utility. Thus, if you have an 80, for instance, on your first exam, your utility will remain the same if your second exam is an 80 or less. Similarly, if your second exam is an 80, your first exam does not matter if it is less than or equal to 80. One of your indifference curves therefore is the point (80,80) as well as the bundles (x,80) and (80,x) for all $x \leq 80$. This forms one of the indifference curves in panel (b) of Graph 5.8, with the remaining indifference curves in the graph similarly derived.

(c) *Suppose all you care about is your final grade in a class and you otherwise value all classes equally. Consider a pair of exam scores* (x_1, x_2) *and suppose you knew before registering for a class what that pair will be — and that it will be the same for the economics and the physics class. What must be true about this pair in order for you to be indifferent between registering for economics and registering for physics?*

Answer: The fact that you care only about your final grade and you value all classes equally means that getting an 80 in your economics class means exactly as much to you as getting an 80 in your physics class. The highest indifference curve in panel (a) of the graph illustrates all the different ways of getting an 80 in your physics class. Similarly, the highest indifference curve in panel (b) of the graph illustrates all the different ways of getting an 80 in the economics class. Thus, any of the pairs of grades on the relevant indifference curve in panel (a) is just as good as any of the pairs of grades on the relevant indifference curve in panel (b).

But those indifference curves only share one pair of grades in common — the pair (80,80). The same reasoning holds for any other pair of exam grades that might make you indifferent between the two classes. Thus, it must be that $x_1 = x_2$.

B: *Consider the same scenario as the one described in part A.*

(a) *Give a utility function that could be used to represent your tastes as you described them with the indifference curves you plotted in A(a)?*

Answer: The simples example of such a utility function is $u(x_1, x_2) = \min\{x_1, x_2\}$.

(b) *Repeat for the tastes as you described them with the indifference curves you plotted in A(b).*

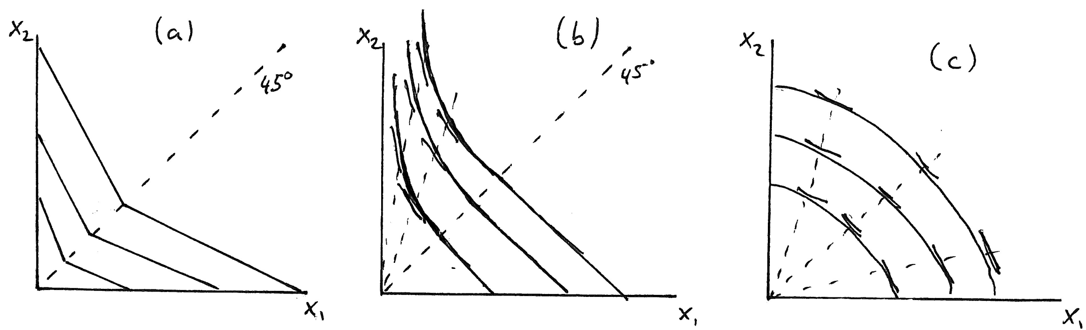
Answer: Now it is the maximum, not the minimum, grade that matters. So the corresponding utility function could take the form $u(x_1, x_2) = \max\{x_1, x_2\}$.

5.10 *Consider again the family of homothetic tastes.*

A: *Recall that essential goods are goods that have to be present in positive quantities in a consumption bundle in order for the individual to get utility above what he would get by not consuming anything at all.*

(a) *Aside from the case of perfect substitutes, is it possible for neither good to be essential but tastes nevertheless to be homothetic? If so, can you give an example?*

Answer: Yes, this is possible. The vertical and horizontal axes are rays from the origin just as all other possible rays (with angles between 0 and 90 degrees) are — and there is nothing in principle to keep indifference curves from having the same slopes as they cross these axes. One simple example is illustrated in panel (a) of Graph 5.9.



Graph 5.9: Homothetic Tastes

(b) *Can there be homothetic tastes where one of the two goods is essential and the other is not? If so, give an example.*

Answer: Yes. An example is illustrated in panel (b) where the indifference curves converge to the vertical axis but not the horizontal.

(c) *Is it possible for tastes to be non-monotonic (less is better than more) but still homothetic?*

Answer: Yes. Monotonicity just has to do with the labeling of the indifference curves. So you can take any homothetic tastes that are monotonic and multiply the labels of indifference curves by -1 — you then still have a homothetic indifference map but you have turned goods from being "goods" into being "bads".

(d) *Is it possible for tastes to be monotonic (more is better), homothetic but strictly non-convex (i.e. averages are worse than extremes)?*

Answer: Yes. An example is graphed in panel (c) of Graph 5.9. (The labels should increase as we move toward the northeast in the graph — since monotonicity holds.)

B: *Now relate the homotheticity property of indifference maps to utility functions.*

(a) *Aside from the case of perfect substitutes, are there any CES utility functions that represent tastes for goods that are not essential?*

Answer: No. CES utility functions give rise to indifference maps with all indifference curves converging to the axes (but not crossing them) so long as the goods are not perfect substitutes. In the text, we calculated the MRS for CES utility functions to be

$$MRS = -\left(\frac{\alpha}{\beta}\right)\left(\frac{x_2}{x_1}\right)^{\rho+1}. \tag{5.19}$$

From this it is easy to see, for instance, that the MRS is undefined for $x_1 = 0$ (i.e. along the vertical axis).

(b) *All CES utility functions represent tastes that are homothetic. Is it also true that all homothetic indifference maps can be represented by a CES utility function? (*Hint*: Consider your answer to A(a) and ask yourselves, in light of your answer to B(a), if it can be represented by a CES function.)*

Answer: No. The tastes graphed in panel (a) of Graph 5.9, for instance, cannot be represented by a CES utility function. The same is true of any set of tastes (other than perfect substitutes) that are homothetic and for whom one (or more) of the goods is not essential — because, as we have just concluded above, CES functions represent tastes over goods that are essential.

(c) True or False*: The elasticity of substitution can be the same at all bundles only if the underlying tastes are homothetic.*

Answer: This is true — such tastes can be represented by CES utility functions, and all tastes representable by the CES utility function are in fact homothetic. Again, this is easily seen from the formula for the MRS for CES utility functions — which depends on the ratio of the goods only. Since the ratio of the goods is constant along any ray from the origin, the MRS for indifference curves represented by CES utility functions is constant along rays from the origin.

(d) True or False*: If tastes are homothetic, then the elasticity of substitution is the same at all bundles.*

Answer: False. Panel (b) of Graph 5.9 represents tastes that are perhaps Cobb-Douglas above the 45 degree line but give rise to linear indifference curves below the 45 degree line. This would imply an elasticity of substitution of 1 above the 45 degree line and infinity below.

(e) *What is the simplest possible transformation of the CES utility function that can generate tastes which are homothetic but non-monotonic?*

Answer: Multiply the CES utility function by -1.

(f) *Are the tastes represented by this transformed CES utility function convex?*

Answer: No. While the MRS is still diminishing (because the shapes of indifference curves are not the same), bundles that lie to the norhteast of an indifference curve (and thus contain "averages") are now worse than those on the indifference curve (because monotonicity does not hold and more is worse). Thus, averages are worse than extremes that lie on an indifference curves — implying that convexity does not hold.

(g) *So far, we have always assumed that the parameter ρ in the CES utility function falls between -1 and ∞. Can you determine what indifference curves would look like when ρ is less than -1?*

Answer: Consider again the MRS for the CES utility function:

$$MRS = -\left(\frac{\alpha}{\beta}\right)\left(\frac{x_2}{x_1}\right)^{\rho+1}. \tag{5.20}$$

We argued in B(f) that we can see in this formula that we have diminishing MRS for CES tastes. That argument, however, is based on the presumption that $\rho > -1$ which implies that the exponent $(\rho+1)$ is positive and the MRS increases in absolute value with the ratio x_2/x_1. When $\rho < -1$, however, the exponent in the MRS formula becomes negative, which implies that the MRS *decreases* in absolute value with the ratio x_2/x_1. Rather than having diminishing MRS, we therefore have increasing MRS as we move to the right in the graph. Thus, indifference curves look like those in panel (c) of Graph 5.9.

(h) *Are such tastes convex? Are they monotonic?*

Answer: They are not convex but they are monotonic.

(i) *What is the simplest possible transformation of this utility function that would change both your answers to the previous question?*

Answer: Multiply the function by -1. This reverses the labeling — thus making the tastes non-monotonic, and in the process the set of bundles that are better than those on an indifference curve now fall below rather than above the indifference curve. And that set is now convex.

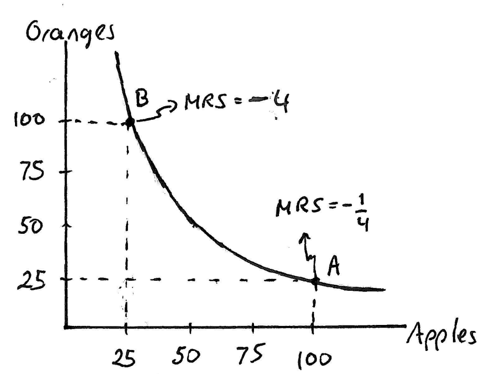
5.11 *In this exercise, we are working with the concept of an elasticity of substitution. This concept was introduced in part B of the Chapter. Thus, this entire question relates to material from part B, but the A-part of the question can be done simply by knowing the formula for an elasticity of substitution while the B-part of the question requires further material from part B of the Chapter. In Section 5B.1, we defined the elasticity of substitution as*

$$\sigma = \left| \frac{\%\Delta(x_2/x_1)}{\%\Delta MRS} \right|. \tag{5.21}$$

A: *Suppose you consume only apples and oranges. Last month, you consumed bundle A=(100,25) — 100 apples and 25 oranges, and you were willing to trade at most 4 apples for every orange. Two months ago, oranges were in season and you consumed B=(25,100) and were willing to trade at most 4 oranges for 1 apple. Suppose your happiness was unchanged over the past two months.*

(a) *On a graph with apples on the horizontal axis and oranges on the vertical, illustrate the indifference curve on which you have been operating these past two months and label the MRS where you know it.*

Answer: This is illustrated in Graph 5.10.



Graph 5.10: Elasticity of Subsitution

(b) *Using the formula for elasticity of substitution, estimate your elasticity of substitution of apples for oranges.*

Answer: This is

$$\sigma = \left| \frac{((100/25)-(25/100))/(100/25)}{(-4-(-1/4))/(-4)} \right| = \left| \frac{(15/4)/4}{(15/4)/4} \right| = 1. \tag{5.22}$$

(c) *Suppose we know that the elasticity of substitution is in fact the same at every bundle for you and is equal to what you calculated in (b). Suppose the bundle C=(50,50) is another bundle that makes you just as happy as bundles A and B. What is the MRS at bundle C?*

Answer: Using B and C in the elasticity of substitution formula, setting σ equal to 1 and letting the MRS at C be denoted by x, we get

$$\left| \frac{((100/25)-(50/50))/(100/25)}{(-4-x)/(-4)} \right| = \left| \frac{3/4}{(4+x)/4} \right| = 1, \tag{5.23}$$

and solving this for x, we get $x = -1$ — i.e. the MRS at C is equal to -1.

(d) *Consider a bundle $D = (25,25)$. If your tastes are homothetic, what is the MRS at bundle D?*

Answer: Since it, like bundle C, lies on the 45 degree line, homotheticity implies the MRS is again -1.

(e) *Suppose you are consuming 50 apples, you are willing to trade 4 apples for one orange and you are just as happy as you were when you consumed at bundle D. How many oranges are you consuming (assuming the same elasticity of substitution)?*

Answer: Let the number of oranges be denoted y. Using the bundle $(50, y)$ and $D = (25,25)$ in the elasticity formula and setting it to 1, we get

$$\left|\frac{((50/y)-(25/25))/(50/y)}{(-4-(-1))/(-4)}\right| = \left|\frac{((50/y)-1)/(50/y)}{(3/4)}\right| = 1. \tag{5.24}$$

Solving this for y, we get $y = 12.5$.

(f) *Call the bundle you derived in part (e) E. If the elasticity is as it was before, at what bundle would you be just as happy as at E but would be willing to trade 4 oranges for 1 apple?*

Answer: If the elasticity is 1 from D to E and is again supposed to be 1 from D to this new bundle, there must be symmetry around the 45 degree line (as there was between A and B). At $E = (50,12.5)$, the MRS is $-1/4$, and the necessary symmetry then means that $MRS = -4$ at (12.5,50).

B: *Suppose your tastes can be summarized by the utility function* $u(x_1,x_2) = \left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-1/\rho}$.

(a) *In order for these tastes to contain an indifference curve such as the one containing bundle A that you graphed in A(a), what must be the value of ρ? What about α?*

Answer: The elasticity of substitution for the CES utility function can be written as $\sigma = 1/(1+\rho)$. Above, we determined that the elasticity of substitution in this problem is 1. Thus, $1 = 1/(1+\rho)$ which implies $\rho = 0$. Since our graph is symmetric around the 45 degree line, it must furthermore be true that $\alpha = 0.5$ — i.e. x_1 and x_2 enter symmetrically into the utility function.

(b) *Suppose you were told that the same tastes can be represented by $u(x_1,x_2) = x_1^{\gamma} x_2^{\delta}$. In light of your answer above, is this possible? If so, what has to be true about γ and δ given the symmetry of the indifference curves on the two sides of the 45 degree line?*

Answer: Yes — it is possible because we determined that the elasticity of substitution is 1 everywhere, which is true for Cobb-Douglas utility functions of the form $u(x_1,x_2) = x_1^{\gamma} x_2^{\delta}$. The symmetry implies $\gamma = \delta$.

(c) *What exact value(s) do the exponents γ and δ take if the label on the indifference curve containing bundle A is 50? What if that label is 2,500? What if the label is 6,250,000?*

Answer: If the utility at A is 50, it means $50^{\gamma}50^{\delta} = 50$. Since we just concluded in (a) that $\gamma = \delta$, this implies that $\gamma = \delta = 0.5$. If the utility is 2,500, then $\gamma = \delta = 1$, and if the utility is 6,250,000, $\gamma = \delta = 2$.

(d) *Verify that bundles A, B and C (as defined in part A) indeed lie on the same indifference curve when tastes are represented by the three different utility functions you implicitly derived in B(c). Which of these utility functions is homogeneous of degree 1? Which is homogeneous of degree 2? Is the third utility function also homogeneous?*

Answer: The bundles are A=(100,25), B=(25,100) and C=(50,50). The following equations hold, verifying that these must be on the same indifference curve for each of the three utility functions: $u(x_1,x_2) = x_1^{0.5}x_2^{0.5}$, $v(x_1,x_2) = x_1x_2$ and $w(x_1,x_2) = x_1^2x_2^2$:

$$\begin{aligned} u(100,25) &= u(25,100) = u(50,50) = 50 \\ v(100,25) &= v(25,100) = v(50,50) = 2,500 \\ w(100,25) &= w(25,100) = w(50,50) = 6,250,000. \end{aligned} \tag{5.25}$$

The following illustrate the homogeneity properties of the three functions:

$$\begin{aligned} u(tx_1,tx_2) &= (tx_1)^{0.5}(tx_2)^{0.5} = tx_1^{0.5}x_2^{0.5} = tu(x_1,x_2) \\ v(tx_1,tx_2) &= (tx_1)(tx_2) = t^2x_1x_2 = t^2v(x_1,x_2) \\ w(tx_1,tx_2) &= (tx_1)^2(tx_2)^2 = t^4x_1^2x_2^2 = t^4w(x_1,x_2). \end{aligned} \tag{5.26}$$

Thus, u is homogeneous of degree 1, v is homogeneous of degree 2 and w is homogeneous of degree 4.

(e) *What values do each of these utility functions assign to the indifference curve that contains bundle D?*

Answer: Recall that $D = (25,25)$. Thus, the three utility functions assign values of $u(25,25) = 25^{0.5}25^{0.5} = 25$; $v(25,25) = 25(25) = 625$; and $w(25,25) = 25^2(25^2) = 390,625$.

(f) True or False*: Homogeneity of degree 1 implies that a doubling of goods in a consumption basket leads to "twice" the utility as measured by the homogeneous function, whereas homogeneity greater than 1 implies that a doubling of goods in a consumption bundle leads to more than "twice" the utility.*

Answer: This is true. Above, we showed an example of this. More generally, you can see this from the definition of a function that is homogeneous of degree k; i.e. $u(tx_1, tx_2) = t^k u(x_1, x_2)$. Substituting $k = 2$, $u(2x_1, 2x_2) = 2^k u(x_1, x_2)$. When $k = 1$ — i.e. when the utility function is homogeneous of degree 1, this implies $u(2x_1, 2x_2) = 2u(x_1, x_2)$ — a doubling of goods leads to a doubling of utility assigned to the bundle. More generally, a doubling of goods leads to 2^k times as much utility assigned to the new bundle — and 2^k is greater than 2 when $k > 1$ (and less than 2 when $k < 1$.)

(g) *Demonstrate that the MRS is unchanged regardless of which of the three utility functions derived in B(c) is used.*

Answer: The MRS of a Cobb-Douglas utility function $u(x_1, x_2) = x_1^{\gamma} x_2^{\delta}$ is $MRS = -(\gamma x_2)/(\delta x_1)$ which reduces to $-x_2/x_1$ when $\gamma = \delta$ which is the case for all three of the utility functions above. Thus, the MRS is the same for the three functions.

(h) *Can you think of representing these tastes with a utiliy function that assigns the value of 100 to the indifference curve containing bundle A and 75 to the indifference curve containing bundle D? Is the utility function you derived homogeneous?*

Answer: The function $u(x_1, x_2) = x_1^{0.5} x_2^{0.5} + 50$ would work. This function is not homogeneous (but it is homothetic).

(i) True or False: *Homothetic tastes can always be represented by functions that are homogeneous of degree k (where k is greater than zero), but even functions that are not homogeneous can represent tastes that are homothetic.*

Answer: This is true. We showed in the text that $MRS(tx_1, tx_2) = MRS(x_1, x_2)$ for homogeneous functions — thus, for homogeneous functions, the MRS is constant along any ray from the origin, the definition of homothetic tastes. At the same time, we just saw in the answer to the previous part an example of a non-homogeneous function that still represents homothetic tastes.

(j) True or False*: The marginal rate of substitution is homogeneous of degree 0 if and only if the underlying tastes are homothetic.*

Answer: For any set of homothetic tastes, the MRS is constant along rays from the origin; i.e. $MRS(tx_1, tx_2) = MRS(x_1, x_2)$. Thus, for homothetic tastes, the MRS is indeed homogeneous of degree 0. But $MRS(tx_1, tx_2) = MRS(x_1, x_2)$ defines homotheticity — so non-homothetic tastes will not have this property, which implies their MRS is not homogeneous of degree zero. The statement is therefore true.