

SOLUTIONS

6

Doing the “Best” We Can

Solutions for *Microeconomics: An Intuitive Approach with Calculus*

Apart from end-of-chapter exercises provided in the student *Study Guide*, these solutions are provided for use by instructors. (End-of-Chapter exercises with solutions in the student *Study Guide* are so marked in the textbook.)

The solutions may be shared by an instructor with his or her students at the instructor’s discretion.

They may not be made publicly available.

If posted on a course web-site, the site must be password protected and for use only by the students in the course.

Reproduction and/or distribution of the solutions beyond classroom use is strictly prohibited.

In most colleges, it is a violation of the student honor code for a student to share solutions to problems with peers that take the same class at a later date.

- Each end-of-chapter exercise begins on a new page. This is to facilitate maximum flexibility for instructors who may wish to share answers to some but not all exercises with their students.
- If you are assigning only the A-parts of exercises in *Microeconomics: An Intuitive Approach with Calculus*, you may wish to instead use the solution set created for the companion book *Microeconomics: An Intuitive Approach*.
- *Solutions to Within-Chapter Exercises are provided in the student Study Guide.*

6.1 I have two 5-year old girls — Ellie and Jenny — at home. Suppose I begin the day by giving each girl 10 toy cars and 10 princess toys. I then ask them to plot their indifference curves that contain these endowment bundles on a graph with cars on the horizontal and princess toys on the vertical axis.

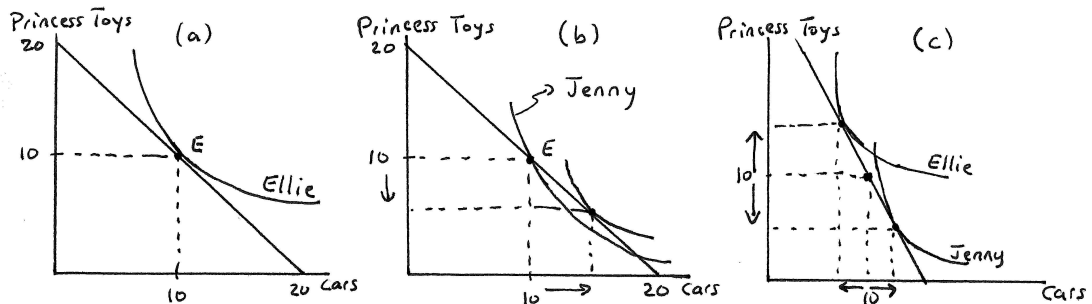
A: Ellie's indifference curve appears to have a marginal rate of substitution of -1 at her endowment bundle, while Jenny's appears to have a marginal rate of substitution of -2 at the same bundle.

(a) Can you propose a trade that would make both girls better off?

Answer: Any trade under which Jenny would give up x princess toys for 1 car and Ellie would accept x princess toys in exchange for giving up 1 car would work so long as $1 < x < 2$. This is because Jenny would be willing to give up as many as 2 princess toys for 1 car — so the trade will make her better off because she has to give up less; and Ellie would be willing to accept as little as 1 princess toy to give up 1 car — so the trade will make her better off because she gets more without giving up more.

(b) Suppose the girls cannot figure out a trade on their own. So I open a store where they can buy and sell any toy for \$1. Illustrate the budget constraint for each girl.

Answer: The budget constraints would be the same for the two girls — because they both have the same endowment point (10,10) and both face the same prices (that result in a slope of -1). These constraints are illustrated in panels (a) and (b) of Graph 6.1, with the endowment point labeled E .



Graph 6.1: Toy Cars and Princess Toys

(c) Will either of the girls shop at my store? If so, what will they buy?

Answer: We can then add Ellie's indifference curve through her endowment point in panel (a) and Jenny's indifference curve through her endowment point in panel (b). We know that Ellie's is tangent to her budget constraint because the budget constraint has a slope of -1 and the MRS described in A is also -1 at the endowment bundle E . So Ellie does not want to buy or sell anything at my store at these prices. Jenny's indifference curve at E , on the other hand, has slope -2 — and thus we know her indifference curve cuts her budget constraint at E from above. This implies that Jenny will have better points available in her choice set — with all better points lying to the right of E . Jenny will therefore want to sell princess toys and buy toy cars at my store.

(d) Suppose I do not actually have any toys in my store and simply want my store to help the girls make trades among themselves. Suppose I fix the price at which princess toys are bought and sold to \$1. Without being specific about what the price of toy cars would have to be, illustrate, using final indifference curves for both girls on the same graph, a situation where the prices in my store result in an efficient allocation of toys.

Answer: It would have to be that the girls have the same tastes at the margin when they leave my store. Thus, they would have to be at indifference curves that are tangent to the same budget line (because their budget goes through the same endowment bundle and has

the same slope). Since Jenny likes cars more than Ellie does at their endowment points, this implies that Jenny will end up selling princess toys and buying cars while Ellie will sell car toys and buy princess toys. For the allocation of toys to be efficient, the price of cars will have to be set so that the number of cars Ellie wants to sell is exactly equal to the number of cars that Jenny wants to buy, and the number of princess toys Ellie wants to buy is exactly equal to the number of princess toys Jenny wants to sell. Thus, the arrows on each axis in panel (c) of the graph have to be the same size.

- (e) What values might the price for toy cars take to achieve the efficient trades you described in your answer to (d)?

Answer: We concluded in (a) that mutually beneficial trades had to have terms of trades under which x princess toys are traded for 1 car, with x falling between 1 and 2. The price of toy cars must therefore be between 1 and 2 times the price of princess toys, allowing consumers to buy between 1 and 2 times as many princess toys as toy cars with any given dollar amount. Since the price of princess toys is fixed at \$1, this implies that the price of cars must lie between \$1 and \$2. You can see from panel (b) that the price of cars can't possibly be lower than \$1 because at a price of \$1 Jenny wants to buy cars and sell princess toys but Ellie is willing to do neither. Thus, the price of cars has to go up in order to induce Ellie to be willing to sell cars and to induce Jenny to demand fewer cars. At the same time, we could similarly show that the price can't be higher than \$2 — because at a price of \$2, Jenny would no longer want to trade but Ellie would definitely want to sell cars for princess toys. Depending on exactly what the indifference maps look like, some price between \$1 and \$2 will therefore be just right.

B: Now suppose that my girls' tastes could be described by the utility function $u(x_1, x_2) = x_1^\alpha x_2^{1-\alpha}$, where x_1 represents toy cars, x_2 represents princess toys and $0 < \alpha < 1$.

- (a) What must be the value of α for Ellie (given the information in part A)? What must the value be for Jenny?

Answer: The MRS for this utility function is

$$MRS = -\frac{\partial u / \partial x_1}{\partial u / \partial x_2} = -\frac{\alpha x_1^{\alpha-1} x_2^{1-\alpha}}{(1-\alpha) x_1^\alpha x_2^{-\alpha}} = -\frac{\alpha x_2}{(1-\alpha) x_1}. \quad (6.1)$$

At the bundle (10,10), Ellie's MRS is -1 — which implies that $\alpha = 0.5$ for Ellie. Similarly, for Jenny the MRS is -2 at the bundle (10,10) — which implies that $\alpha / (1 - \alpha) = 2$ or $\alpha = 2/3$ for Jenny.

- (b) When I set all toy prices to \$1, what exactly will Ellie do? What will Jenny do?

Answer: We can solve the general optimization problem in terms of α by writing the Lagrange function as

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^\alpha x_2^{1-\alpha} + \lambda(20 - x_1 - x_2), \quad (6.2)$$

where the 20 in the parentheses following λ is simply the value of the endowment of 10 car toys and 10 princess toys when the price of each is set to 1. The first two first order conditions of this problem are

$$\begin{aligned} \alpha x_1^{\alpha-1} x_2^{1-\alpha} &= \lambda \\ (1-\alpha) x_1^\alpha x_2^{-\alpha} &= \lambda. \end{aligned} \quad (6.3)$$

Since the right hand side of each of these is equal to λ , we can just set the left hand sides equal to each other and solve for x_2 to get

$$x_2 = \frac{(1-\alpha)}{\alpha} x_1. \quad (6.4)$$

Plugging this into the budget constraint $20 = x_1 + x_2$, we can solve for x_1 to get $x_1 = 20\alpha$. Plugging this back into equation (6.4), we can also get $x_2 = 20(1-\alpha)$.

Since $\alpha = 0.5$ for Ellie, this implies Ellie's optimal bundle is $(x_1, x_2) = (10, 10)$ — i.e. Ellie will not trade. Since $\alpha = 2/3$ for Jenny, it means Jenny's optimal bundle is $(x_1, x_2) = (13.33, 6.67)$. Jenny will therefore want to trade 3.33 princess toys for 3.33 toy cars.

- (c) *Given that I am fixing the price of princess toys at \$1, do I have to raise or lower the price of car toys in order for me to operate a store in which I don't keep inventory but simply facilitate trades between the girls?*

Answer: As we already concluded in part A(e), I will have to raise the price of cars to somewhere between \$1 and \$2.

- (d) *Suppose I raise the price of car toys to \$1.40, and assume that it is possible to sell fractions of toys. Have I found a set of prices that allow me to keep no inventory?*

Answer: The Lagrange function written in terms of α is then

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^\alpha x_2^{(1-\alpha)} + \lambda(24 - 1.4x_1 - x_2), \quad (6.5)$$

where 24 is the value of the endowment (10,10). The first two first order conditions are

$$\begin{aligned} \alpha x_1^{\alpha-1} x_2^{1-\alpha} &= 1.4\lambda \\ (1-\alpha)x_1^\alpha x_2^{-\alpha} &= \lambda. \end{aligned} \quad (6.6)$$

Dividing the first by the second (and thus canceling λ), we can solve for x_2 in terms of x_1 to get

$$x_2 = \frac{1.4(1-\alpha)}{\alpha} x_1. \quad (6.7)$$

Substituting this into the budget constraint and solving for x_1 , we get $x_1 = (24/1.4)\alpha = 17.143\alpha$. Plugging this back into equation (6.7) and solving for x_2 , we get $x_2 = 24(1-\alpha)$.

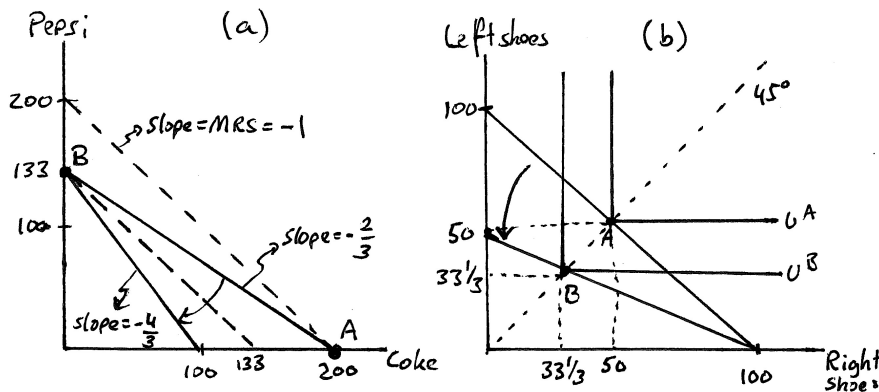
For Ellie, $\alpha = 0.5$ — which implies her optimal bundle will be (8.571,12). Thus, she wants to give up 1.429 of x_1 in exchange for receiving 2 of x_2 . For Jenny, $\alpha = 2/3$ — which implies her optimal bundle will be (11.429,8). Jenny therefore wants to get 1.429 of x_1 in exchange for giving up 2 of x_2 . The trades exactly offset each other — thus I have to keep no inventory at these prices. I am simply facilitating efficient trade between Ellie and Jenny by setting the price of cars equal to \$1.40 (while setting the price of princess toys to \$1.00).

6.2 Suppose Coke and Pepsi are perfect substitutes for me, and right and left shoes are perfect complements.

A: Suppose my income allocated to Coke/Pepsi consumption is \$100 per month, and my income allocated to Right/Left shoe consumption is similarly \$100 per month.

- (a) Suppose Coke currently costs 50 cents per can and Pepsi costs 75 cents per can. Then the price of Coke goes up to \$1 per can. Illustrate my original and my new optimal bundle with Coke on the horizontal and Pepsi on the vertical axis.

Answer: This is illustrated in panel (a) of Graph 6.2. Budget lines are drawn solid while indifference curves are drawn with dashed lines. Since Coke and Pepsi are perfect substitutes that I am willing to trade one for one, the MRS is -1 everywhere. The original budget has slope of $-133/200 = -2/3$, while the new budget constraint after the price increase has slope $-133/100 = -4/3$. Thus, my original budget had slope shallower than my indifference curves whereas my final budget has a slope that is steeper than my indifference curves. The highest indifference curve that contains a point in the original choice set is then the outer indifference curve containing A , whereas the highest indifference curve that contains a point in the new choice set is the lower indifference curve containing B . Before the price increase I spend my entire budget on Coke, but after the price increase I spend all of it on Pepsi. This should make intuitive sense — when Coke is cheaper than Pepsi, I should spend all my budget on Coke, but when Coke becomes more expensive than Pepsi, I should switch to spending my entire budget on Pepsi.



Graph 6.2: Coke and Pepsi, Right Shoes and Left Shoes

- (b) Suppose right and left shoes are sold separately. If right and left shoes are originally both priced at \$1, illustrate (on a graph with right shoes on the horizontal and left shoes on the vertical) my original and my new optimal bundle when the price of left shoes increases to \$2.

Answer: The change in budget constraints is illustrated in panel (b) of the graph, with an initial budget that runs from 100 left shoes to 100 right shoes when both cost \$1 each and the final budget running from 50 left shoes to 100 right shoes when the price of left shoes increases to \$2. The optimal indifference curves are then illustrated on each budget constraint. Since right and left shoes are perfect complements, it must be that they are consumed as pairs at any consumer optimum. This implies an initial bundle A of 50 pair of shoes and a final bundle B of 33.33 pair of shoes.

- (c) True or False: Perfect complements represent a unique special case of homothetic tastes in the following sense: Whether income goes up or whether the price of one of the goods falls, the optimal bundle will always lie on the same ray emerging from the origin.

Answer: This is true. Since perfect complements are always consumed in pairs, the optimal consumption level always lies on a ray from the origin. If the complementarity is such that the goods are consumed in pairs of 1 each, then that ray is the 45-degree line (as in panel (b)). Thus optimal bundles lie on the 45 degree line for the right shoe/left shoe case. (If the complementarity were such that 2 of good 1 is always paired with 1 of good 2, then that ray is the 30-degree line. In principle, the relevant ray can have any angle from the origin depending on the rate at which the two goods are paired — but in our example here they are paired 1 for 1.)

B: Continue with the assumptions about tastes from above.

- (a) Write down two utility functions: one representing my tastes over Coke and Pepsi, another representing my tastes over right and left shoes.

Answer: For Coke and Pepsi, the function $u(x_1, x_2) = x_1 + x_2$ would work. For right and left shoes, the function $u(x_1, x_2) = \min\{x_1, x_2\}$ would work. These are the simplest possible utility functions to describe the two sets of preferences. (Of course you could also choose a variety of transformations of these, transformations like $u(x_1, x_2) = 2x_1 + 2x_2$ for perfect substitutes or $u(x_1, x_2) = 2 \min\{x_1, x_2\}$ for perfect complements.)

- (b) Using the appropriate equation derived above, label the two indifference curves you drew in A(a).

Answer: The higher indifference curve is such that $x_1 + x_2 = 200$ — thus, the utility function $u(x_1, x_2) = x_1 + x_2$ assigns it a label of 200 utils. The lower indifference curve, on the other hand, is such that $x_1 + x_2 = 133.33$ — which means it has a label of 133.33 utils.

- (c) Using the appropriate equation derived in B(a), label the two indifference curves you drew in A(b).

Answer: The labels would be $u^A = 50$ and $u^B = 33.33$.

- (d) Consider two different equations representing indifference curves for perfect complements: $u^1(x_1, x_2) = \min\{x_1, x_2\}$ and $u^2(x_1, x_2) = \min\{x_1, 2x_2\}$. By inspecting two of the indifference curves for each of these utility functions, determine the equation for the ray along which all optimal bundles will lie for individuals whose tastes can be represented by these equations.

Answer: In u^1 , the two goods are always used in pairs of 1 each. Thus, the optimal bundles lie on the 45 degree line. In u^2 , on the other hand, you always use pairs of 2 of x_1 with 1 of x_2 . The optimal bundles therefore lie on the 30 degree ray from the origin — i.e. a ray with slope 1/2.

- (e) Explain why the Lagrange method does not seem to work for calculating the optimal consumption bundle when the goods are perfect substitutes.

Answer: This is because the optimal bundles will almost certainly be corner solutions (unless the ratio of prices happens to give the same slope of the budget line as the slope of all the indifference curves). The Lagrange method finds points of tangency — but there are no such points in the case of perfect substitutes.

- (f) Explain why the Lagrange method cannot be applied to calculate the optimal bundle when the goods are perfect complements.

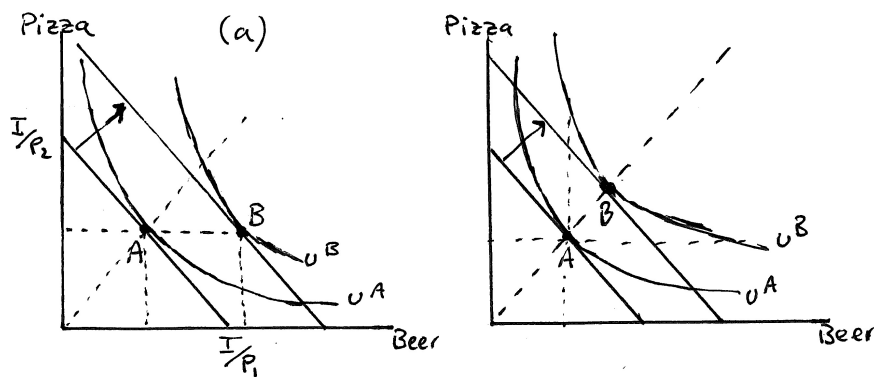
Answer: The Lagrange method implicitly assumes differentiability. But the slope of the indifference curve at the kink points is undefined — so the Lagrange method cannot find tangencies. Thus, the Lagrange method requires smoothness of the indifference curves.

6.3 Pizza and Beer: Sometimes we can infer something about tastes from observing only two choices under two different economic circumstances.

A: Suppose we consume only beer and pizza (sold at prices p_1 and p_2 respectively) with an exogenously set income I .

- (a) With the number of beers on the horizontal axis and the number of pizzas on the vertical, illustrate a budget constraint (clearly labeling intercepts and the slope) and some initial optimal (interior) bundle A .

Answer: Panel (a) of Graph 6.3 illustrates the original budget line containing the optimal bundle A .



Graph 6.3: Beer and Pizza

- (b) When your income goes up, I notice that you consume more beer and the same amount of pizza. Can you tell whether my tastes might be homothetic? Can you tell whether they might be quasilinear in either pizza or beer?

Answer: The shift in income is also indicated in panel (a), with the new optimal bundle B containing more beer but the same amount of pizza. Since the two indifference curves have the same MRS along the horizontal line that holds pizza fixed at its original quantity, the tastes might indeed be quasilinear in pizza. But the tastes could not be homothetic — because, on the ray that passes through A from the origin, the MRS is greater in absolute value along the higher indifference curve than along the lower. The only way this would not be the case is if pizza and beer were perfect substitutes *and* the price of pizza is the same as the price of beer. In that case, all points on both budgets are optimal — including A initially and B after the income change. This would be the one case where tastes are both quasilinear and homothetic.

- (c) How would your answers change if I had observed you decreasing your beer consumption when income goes up?

Answer: If I simply would have observed a decrease in your beer consumption, I could say for sure that your tastes are not quasilinear in beer (unless beer and pizza are perfect substitutes and prices happen to be such that the slopes of the budget constraints are equal to the MRS everywhere). I could similarly conclude that your tastes are not quasilinear in pizza — because, if you consume less beer with an increase in income, you must be consuming more pizza (if pizza and beer is all you consume). Finally, I could also say for sure that your tastes are not homothetic — because under homothetic tastes, consumption of all goods goes up with increases in income. Again, the one exception is the case where pizza and beer are perfect substitutes with MRS equal to the slopes of the budgets. In that case, we would again have tastes that are both quasilinear and homothetic.

- (d) How would your answers change if both beer and pizza consumption increased by the same proportion as income?

Answer: This case is graphed in the second panel of Graph 6.3. The original bundle A and the new optimal bundle B lie on the same ray from the origin — with the indifference curves at both bundles tangent to the same slope. Thus, along this ray, the two indifference curves we know about have the same slope — which is consistent with tastes being homothetic. But the vertical and horizontal lines through A will contain bundles along u^B where the MRS differs from that at A — which implies that the tastes are not quasilinear, at least so long as we rule out the special case that the goods are perfect substitutes and the ratio of prices happens to be such that the budget lines have the same slope as the indifference curves everywhere.

B: Suppose your tastes over beer (x_1) and pizza (x_2) can be summarized by the utility function $u(x_1, x_2) = x_1^2 x_2$ and that $p_1=2$, $p_2=10$ and weekly income $I=180$.

- (a) Calculate your optimal bundle A of weekly beer and pizza consumption by simply using the fact that, at any interior solution, $MRS = -p_1/p_2$.

Answer: Using the fact that we know $MRS = -p_1/p_2 = -2/10 = -1/5$ at the optimum, we can write

$$MRS = -\frac{\partial u / \partial x_1}{\partial u / \partial x_2} = -\frac{2x_1 x_2}{x_1^2} = -\frac{2x_2}{x_1} = -\frac{1}{5}, \quad (6.8)$$

and the last equality can be written as $x_2 = x_1/10$. Plugging this into the budget constraint $180 = 2x_1 + 10x_2$, we get

$$180 = 2x_1 + 10 \frac{x_1}{10} = 3x_1, \quad (6.9)$$

which solves to $x_1 = 60$. Plugging this back into $x_2 = x_1/10$, we also get $x_2 = 6$.

- (b) What numerical label does this utility function assign to the indifference curve that contains your optimal bundle?

Answer: $u(60, 6) = (60^2)(6) = 21,600$.

- (c) Set up the more general optimization problem where, instead of using the prices and income given above, you simply use p_1 , p_2 and I . Then, derive your optimal consumption of x_1 and x_2 as a function of p_1 , p_2 and I .

Answer: The more general optimization problem is

$$\max_{x_1, x_2} u(x_1, x_2) = x_1^2 x_2 \text{ subject to } p_1 x_1 + p_2 x_2 = I, \quad (6.10)$$

with corresponding Lagrange function

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^2 x_2 + \lambda(I - p_1 x_1 - p_2 x_2). \quad (6.11)$$

The first two first order conditions are then

$$\begin{aligned} 2x_1 x_2 &= \lambda p_1 \\ x_1^2 &= \lambda p_2. \end{aligned} \quad (6.12)$$

Dividing the first by the second equation, we get $2x_2/x_1 = p_1/p_2$ which can be solved for x_2 to get $x_2 = (p_1 x_1)/(2p_2)$. Substituting this into the budget constraint $I = p_1 x_1 + p_2 x_2$ (which is also the third first order condition), we get

$$I = p_1 x_1 + p_2 \frac{p_1 x_1}{2p_2} = \frac{2p_1 x_1}{2} + \frac{p_1 x_1}{2} = \frac{3p_1 x_1}{2}, \quad (6.13)$$

and this can be solved for x_1 as $x_1 = 2I/(3p_1)$. Plugging this back into the expression $2x_2/x_1 = p_1/p_2$, we can then solve for x_2 as $x_2 = I/(3p_2)$.

- (d) Plug the values $p_1=2$, $p_2=10$ and $I=180$ into your answer to B(c) and verify that you get the same result you originally calculated in B(a).

Answer: Our solution above was $x_1 = 2I/(3p_1)$ and $x_2 = I/(3p_2)$. Plugging in the specific values for prices and income, we therefore get $x_1 = 2(180)/(3(2)) = 360/6 = 60$ and $x_2 = 180/(3(10)) = 180/30 = 6$ — 60 beers and 6 pizzas just as we concluded in B(a).

- (e) Using your answer to part B(c), verify that your tastes are homothetic.

Answer: You can tell how consumption of each good changes with income by taking the derivative of $x_1 = 2I/(3p_1)$ and $x_2 = I/(3p_2)$ with respect to I . This gives

$$\frac{\partial x_1}{\partial I} = \frac{2}{3p_1} \quad \text{and} \quad \frac{\partial x_2}{\partial I} = \frac{1}{3p_2}. \quad (6.14)$$

Thus, as income increases, consumption of both goods increases linearly. Put differently, as income doubles, consumption of both goods doubles. This is true only for homothetic tastes where the MRS is the same along rays from the origin — which implies that optimal bundles lie on rays from the origin as income changes.

- (f) Which of the scenarios in A(b) through (d) could be generated by the utility function $u(x_1, x_2) = x_1^2 x_2$?

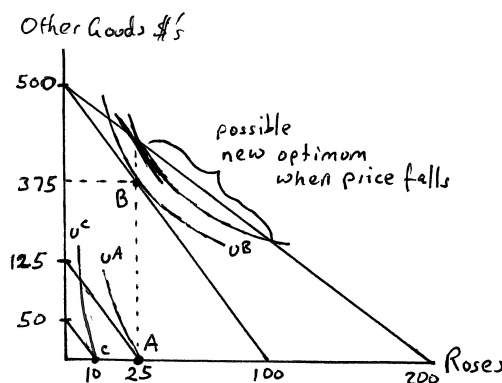
Answer: Only the last scenario in A(d) could be generated by this utility function since we know it represents homothetic tastes. The scenario in A(b) has tastes that are quasilinear in pizza, while the scenario in A(c) has beer consumption decreasing with an increase in income (which is inconsistent with what we derived above).

6.4 Inferring Tastes for Roses (and Love) from Behavior. *I express my undying love for my wife through weekly purchases of roses that cost \$5 each.*

A: *Suppose you have known me for a long time and you have seen my economic circumstances change with time. For instance, you knew me in graduate school when I managed to have \$125 per week in disposable income that I could choose to allocate between purchases of roses and “other consumption” denominated in dollars. Every week I brought 25 roses home to my wife.*

- (a) *Illustrate my budget as a graduate student — with roses on the horizontal and “dollars of other consumption” on the vertical axis. Indicate my optimal bundle on that budget as A. Can you conclude whether either good is not “essential”?*

Answer: Graph 6.4 illustrates a number of different budget constraints for this problem, including the one described in this part — which starts at \$125 on the vertical axis and ends at 25 roses on the horizontal axis. Thus, I am spending all my income on roses — which implies other goods are not essential for me.



Graph 6.4: Love and Roses

- (b) *When I became an assistant professor, my disposable income rose to \$500 per week, and the roses I bought for my wife continued to sell for \$5 each. You observed that I still bought 25 roses each week. Illustrate my new budget constraint and optimal bundle B on your graph. From this information, can you conclude whether my tastes might be quasilinear in roses? Might they not be quasilinear?*

Answer: The new budget constraint is the one starting at \$500 on the vertical axis and ending at 100 roses on the horizontal. The new optimal B lies exactly above the original optimal A. These tastes could be quasilinear — it is possible that the *MRS* at A is exactly equal to the *MRS* at B. But tastes might also not be quasilinear because the *MRS* at A could in fact be larger in absolute value (i.e. the slope could be steeper) at A than at B — which would still make the corner solution at A optimal.

- (c) *Suppose for the rest of the problem that my tastes in fact are quasilinear in roses. One day while I was an assistant professor, the price of roses suddenly dropped to \$2.50. Can you predict whether I then purchased more or fewer roses?*

Answer: The new budget line is the one beginning at \$500 on the vertical axis and ending at 200 roses on the horizontal. The bundle that lies on this budget line and directly above B must have an *MRS* that is the same as the *MRS* that goes through B if tastes are indeed quasilinear. But that implies that the indifference curve through that point cuts the budget line from above — making bundles to the right more preferred. Thus, I can conclude I will consume more roses when the price of roses falls.

- (d) Suppose I had not gotten tenure — and the best I could do was rely on a weekly allowance of \$50 from my wife. Suppose further that the price of roses goes back up to \$5. How many roses will I buy for my wife per week?

Answer: This budget constraint begins at \$50 on the vertical axis and ends at 10 roses on the horizontal. If tastes are indeed quasilinear, the *MRS* at the corner bundle *C* is larger in absolute value (i.e. the slope is steeper) than it is at *A* or *B*. Thus, if *A* was an optimum under the higher budget, *C* must be an optimum under the lower income. I will therefore buy 10 roses per week.

- (e) True or False: Consumption of quasilinear goods always stays the same as income changes.

Answer: This is almost true but not quite. As we have shown, once we reach the corner solution where we are only consuming the quasilinear good, we will reduce our consumption of that good as income falls (because we just don't have enough income to keep buying the same amount).

- (f) True or False: Over the range of prices and incomes where corner solutions are not involved, a decrease in price will result in increased consumption of quasilinear goods but an increase in income will not.

Answer: This is true. We have demonstrated in part (c) that decreases in prices will lead to increased consumption of the quasilinear good. We also know that, for quasilinear goods, the *MRS* stays constant along any consumption level of the other good — which implies that the tangency of the budget line and the optimal indifference curve remains at the same level of the quasilinear good as income increases (because increases in income do not change the slope of budget constraints and thus don't change the slope of the optimal indifference curve at the optimum so long as we are not at corner solutions).

B: Suppose my tastes for roses (x_1) and other goods (x_2) can be represented by utility function $u(x_1, x_2) = \beta x_1^\alpha + x_2$.

- (a) Letting the price of roses be denoted by p_1 , the price of other goods by 1, and my weekly income by I , determine my optimal weekly consumption of roses and other goods as a function of p_1 and I .

Answer: The Lagrange function for this optimization problem is

$$\mathcal{L}(x_1, x_2, \lambda) = \beta x_1^\alpha + x_2 + \lambda(I - p_1 x_1 - x_2). \quad (6.15)$$

The first two first order conditions are

$$\begin{aligned} \alpha \beta x_1^{\alpha-1} &= \lambda p_1 \\ 1 &= \lambda. \end{aligned} \quad (6.16)$$

Replacing λ with 1 in the first equation and solving for x_1 , we get

$$x_1 = \left(\frac{p_1}{\alpha \beta} \right)^{1/(\alpha-1)} = \left(\frac{\alpha \beta}{p_1} \right)^{1/(1-\alpha)} \quad (6.17)$$

and substituting this into the budget constraint and solving for x_2 , we get

$$x_2 = I - p_1 \left(\frac{\alpha \beta}{p_1} \right)^{1/(1-\alpha)} = I - \frac{\alpha \beta^{1/(1-\alpha)}}{p_1^\alpha}. \quad (6.18)$$

Note that x_1 is not a function of I — the optimal level of the quasilinear good is independent of I (so long as the Lagrange method applies — i.e. so long as we are not at a corner solution).

- (b) Suppose $\beta = 50$ and $\alpha = 0.5$. How many roses do I purchase when $I = 125$ and $p_1 = 5$? What if my income rises to \$500?

Answer: Substituting $\beta = 50$, $\alpha = 0.5$, $I = 125$ and $p_1 = 5$ into equations (6.17) and (6.18), we get $x_1 = 25$ and $x_2 = 0$, exactly like point *A* in our graph. When we replace income by \$500, we get $x_1 = 25$ and $x_2 = 375$, again exactly as in our graph.

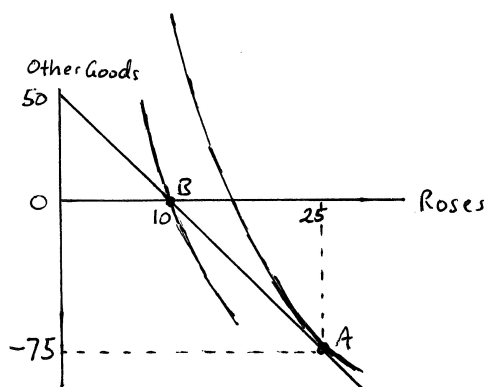
- (c) Comparing your answers to your graph from part A, could the actions observed in part A(b) be rationalized by tastes represented by the utility function $u(x_1, x_2)$? Give an example of another utility function that can rationalize the behavior described in part A(b).

Answer: Yes, as we just showed, the utility function gives us the same optimal consumption levels as those graphed in Graph 6.4. Any order preserving transformation of the utility function will similarly rationalize this behavior — as, for instance, $v(x_1, x_2) = (\beta x_1^\alpha + x_2)^2$.

- (d) What happens when the price of roses falls to \$2.50? Is this consistent with your answer to part A(c)?

Substituting $\beta = 50$, $\alpha = 0.5$, $I = 500$ and $p_1 = 2.5$ into equations (6.17) and (6.18), we get $x_1 = 100$ and $x_2 = 250$ — which is consistent with the answer we gave in A(c), i.e. the answer that I will buy more roses when the price falls.

- (e) What happens when my income falls to \$50 and the price of roses increases back to \$5? Is this consistent with your answer to part A(d)? Can you illustrate in a graph how the math is giving an answer that is incorrect?



Graph 6.5: Love and Roses: Part 2

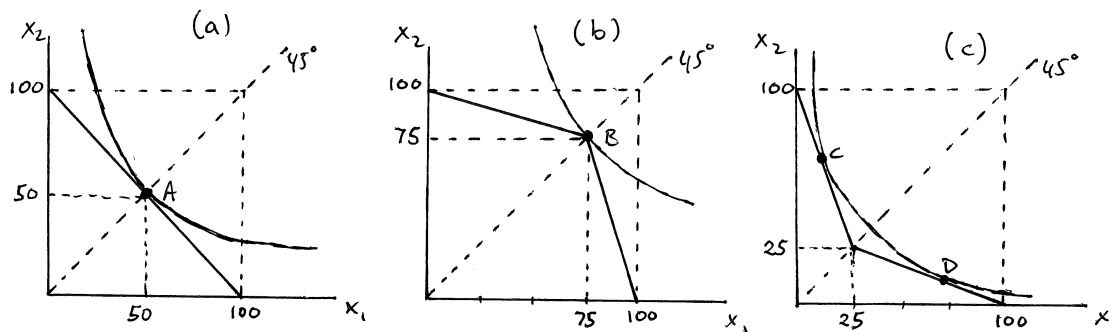
Answer: Substituting $\beta = 50$, $\alpha = 0.5$, $I = 50$ and $p_1 = 5$ into equations (6.17) and (6.18), we get $x_1 = 25$ and $x_2 = -75$. This can't be a true optimum because it would involve a negative consumption level for other goods. Graph 6.5 illustrates what is happening — that math picks out bundle A in the graph because that is where an indifference curve extended into the negative “other goods” quadrant is tangent to a budget line that is similarly extended. The negative value for “other goods” suggests that there is a corner solution that is missed by the math because there is no tangency at that solution. This solution, we know from our intuition, is the bundle $B=(10,0)$. The indifference curve through that bundle has a steeper slope than the budget constraint as we can see by calculating the MRS for this utility function. Applying our formula for MRS , we get $MRS = -\alpha\beta x_1^{\alpha-1}$ which, when $\alpha = 0.5$, $\beta = 50$ and $x_1 = 10$ is substituted into it, implies that the MRS at $x_1 = 10$ is -7.91 while we know the budget constraint has a slope of just -5 . Thus, the answer given by the math is just wrong because there is a corner solution, but the corner solution it points us to is exactly the one we arrived at in part A(d).

6.5 Suppose you have an income of \$100 to spend on goods x_1 and x_2 .

A: Suppose that you have homothetic tastes that happen to have the special property that indifference curves on one side of the 45 degree line are mirror images of indifference curves on the other side of the 45 degree line.

(a) Illustrate your optimal consumption bundle graphically when $p_1 = 1 = p_2$.

Answer: Panel (a) of Graph 6.6 illustrates the budget line in this case. Symmetry around the 45-degree line implies that the slope of indifference curves on the 45 degree line must be -1 . Since the budget constraint in this case also has slope -1 , the optimum must occur on the 45 degree line. This is indicated as point A in the graph.



Graph 6.6: Homothetic Tastes and Optimization

(b) Now suppose the price of the first 75 units of x_1 you buy is $1/3$ while the price for any additional units beyond that is 3. The price of x_2 remains at 1 throughout. Illustrate your new budget and optimal bundle.

Answer: This implies that the first 75 units of x_1 cost \$25, leaving you with \$75 to spend on x_2 . The kink point therefore happens at the bundle (75,75). Since the price of x_1 is 3 from then on, you can buy at most 25 more units with the \$75 you have left after buying the first 75 units of x_1 . The budget constraint therefore looks as it does in panel (b) of the graph. The symmetry of the indifference curves then still implies that the optimum happens on the 45 degree line at the kink point B.

(c) Suppose instead that the price for the first 25 units of x_1 is 3 but then falls to $1/3$ for all units beyond 25 (with the price of x_2 still at 1). Illustrate this budget constraint and indicate what would be optimal.

Answer: After buying 25 units of x_1 at \$3 per unit, you have only \$25 left. Thus, the new kink point happens at (25,25). Since the resulting budget line (graphed in panel (c)) is symmetric around the 45 degree line, the symmetry of the indifference curves implies that there will be two optimal bundles (indicated by C and D). These may happen anywhere along the budget line depending on how substitutable the two goods are for one another. If the indifference curves themselves are kinked at the 45-degree line, it may even be the case that $C = D$ so long as the kink is more severe than the kink of the budget constraint (as would be the case for perfect complements).

(d) If the homothetic tastes did not have the symmetry property, which of your answers might not change?

Answer: Without the symmetry property, the optimal bundle in (a) would be to the left or right of the 45 degree line, and there would not be two optimal bundles at symmetric distances from the 45 degree line in panel (c). (There might still be two optimal bundles, or there might only be one.) But in panel (b), the optimum might well still occur at the kink point because many different marginal rates of substitution can be “tangent” at that kink.

B: Suppose that your tastes can be summarized by the Cobb-Douglas utility function $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$.

(a) Does this utility function represent tastes that have the symmetry property described in A?

Answer: Yes. The MRS for this utility function is $-x_2/x_1$ — which is equal to -1 when $x_1 = x_2$ on the 45 degree line. We can furthermore see that the symmetry holds — if we place x_1 on the vertical instead of the horizontal axis, the MRS simply switches to $-x_1/x_2$ and thus retains the same shape as before.

(b) Calculate the optimal consumption bundle when $p_1 = 1 = p_2$.

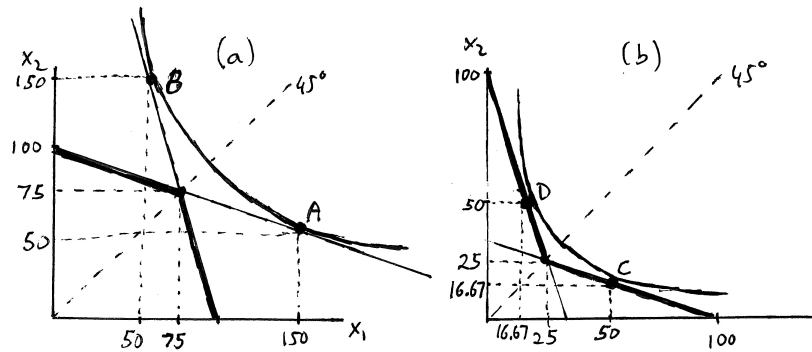
Answer: The optimum occurs where $MRS = -p_1/p_2$ which is $-x_2/x_1 = -1$. Solving for x_2 we get $x_2 = x_1$, and plugging this into the budget constraint, we get $x_1 + x_2 = x_1 + x_1 = 2x_1 = 100$ or $x_1 = 50$ (which then also implies $x_2 = 50$).

(c) Derive the two equations that make up the budget constraint you drew in part A(b) and use the method described in the appendix to this chapter to calculate the optimal bundle under that budget constraint.

Answer: The first segment of the budget constraint is $x_2 = 100 - (1/3)x_1$ and the second line segment is $x_2 = 300 - 3x_1$. Optimal tangencies occur where $MRS = -x_2/x_1 = -p_1/p_2$, which implies $x_2 = (p_1 x_1)/p_2$ or $x_2 = p_1 x_1$ since $p_2 = 1$.

Along the first line segment, $p_1 = 1/3$. Substituting $x_2 = p_1 x_1 = (1/3)x_1$ into $x_2 = 100 - (1/3)x_1$, we get $(1/3)x_1 = 100 - (1/3)x_1$ or $(2/3)x_1 = 100$. Solving for x_1 , we get $x_1 = 150$ which lies on the portion of the budget line that is not truly part of the kinked budget. This is illustrated as bundle A in panel (a) of Graph 6.7.

Along the second line segment, $p_1 = 3$. Substituting $x_2 = p_1 x_1 = 3x_1$ into $x_2 = 300 - 3x_1$, we get $3x_1 = 300 - 3x_1$ or $6x_1 = 300$. Solving for x_1 , we get $x_1 = 50$ which also lies on the portion of the budget line that is not truly part of the kinked budget. This is illustrated as bundle B in panel (a) of Graph 6.7. Note that, due to the symmetry of the indifference curves, bundles A and B lie on the same indifference curve.



Graph 6.7: Homothetic Tastes and Optimization: Part 2

Since both optimization problems — i.e. the problems using both of the extended line segments as budgets — result in solutions outside the actual kinked budget, the actual optimum lies at the kink point.

(d) Repeat for the budget constraint you drew in A(c).

Answer: The first segment of the budget constraint is now $x_2 = 100 - 3x_1$ and the second line segment is $x_2 = 33.33 - (1/3)x_1$. Optimal tangencies occur again where $MRS = -x_2/x_1 = -p_1/p_2$, which implies $x_2 = (p_1 x_1)/p_2$ or $x_2 = p_1 x_1$ since $p_2 = 1$.

Along the first line segment, $p_1 = 3$. Substituting $x_2 = p_1 x_1 = 3x_1$ into $x_2 = 100 - 3x_1$, we get $3x_1 = 100 - 3x_1$ or $6x_1 = 100$. Solving for x_1 , we get $x_1 = (100/6) = 16.67$ which lies on

the portion of the budget line that is in fact part of the kinked budget. This is illustrated as bundle *C* in panel (b) of Graph 6.7.

Along the second line segment, $p_1 = (1/3)$. Substituting $x_2 = p_1 x_1 = (1/3)x_1$ into $x_2 = 33.33 - (1/3)x_1$, we get $(1/3)x_1 = 33.33 - (1/3)x_1$ or $(2/3)x_1 = 33.33$. Solving for x_1 , we get $x_1 = 50$ which also lies on the portion of the budget line that is in fact part of the kinked budget. This is illustrated as bundle *D* in panel (b) of Graph 6.7. Note again that, due to the symmetry of the indifference curves, bundles *C* and *D* lie on the same indifference curve. Both of these bundles are therefore optimal.

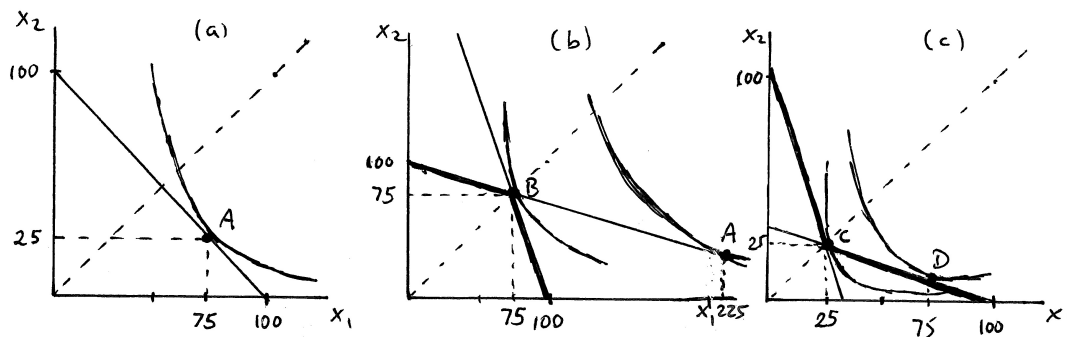
- (e) Repeat (b) through (d) assuming instead $u(x_1, x_2) = x_1^{3/4} x_2^{1/4}$ and illustrate your answers in graphs.

Answer: The MRS for this function is $MRS = -3x_2/x_1$. Thus, optimal solutions occur at $MRS = -3x_2/x_1 = -p_1/p_2$ or, equivalently, where $x_2 = p_1 x_1 / 3p_2$ which can furthermore be simplified to $x_2 = p_1 x_1 / 3$ since $p_2 = 1$.

When $p_1 = p_2 = 1$ as in part (b), our optimality condition reduces to $x_2 = x_1 / 3$. Putting this into the budget constraint, we get $x_1 + x_2 = x_1 + (x_1 / 3) = 100$ or $(4/3)x_1 = 100$. Solving for x_1 we then get $x_1 = 75$ which implies $x_2 = 25$. This is graphed as *A* in panel (a) of Graph 6.8.

In the scenario of part (c), the first segment of the budget constraint is $x_2 = 100 - (1/3)x_1$ and the second line segment is $x_2 = 300 - 3x_1$. Substituting our optimality condition $x_2 = p_1 x_1 / 3$ into the first equation and letting $p_1 = 1/3$, we get $x_2 = (1/3)(x_1 / 3) = 100 - (1/3)x_1$ or $(1/9)x_1 = 100 - (1/3)x_1$ which solves to $x_1 = 225$ which is clearly outside the actual kinked budget and is illustrated as *A* in panel (b) of Graph 6.8. Similarly, substituting our optimality condition $x_2 = p_1 x_1 / 3$ into the second equation and letting $p_1 = 3$, we get $x_2 = 3x_1 / 3 = 300 - 3x_1$ or $x_1 = 300 - 3x_1$. Solving for x_1 , we get $x_1 = 75$. This is exactly the kink point — and is therefore the optimal solution, illustrated as *B* in panel (b) of Graph 6.8.

In the scenario of part (d), the first segment of the budget constraint is $x_2 = 100 - 3x_1$ and the second line segment is $x_2 = 33.33 - (1/3)x_1$. Substituting our optimality condition $x_2 = p_1 x_1 / 3$ into the first equation and letting $p_1 = 3$, we get $x_2 = 3(x_1 / 3) = 100 - 3x_1$ or $x_1 = 100 - 3x_1$ which solves to $x_1 = 25$. This happens right at the kink point — which means it could not possibly be an optimum since the indifference curve cuts the other part of the budget constraint. This is illustrated in panel (c) of Graph 6.8 where the kink point is denoted *C*. Substituting our optimality condition $x_2 = p_1 x_1 / 3$ into the second equation and letting $p_1 = 1/3$, we get $x_2 = (1/3)x_1 / 3 = 33.33 - (1/3)x_1$ or $(1/9)x_1 = 33.33 - (1/3)x_1$. Solving for x_1 , we get $x_1 = 75$. This, illustrated as *D* in panel (c) of Graph 6.8, is in fact on the actual kinked budget and is therefore the optimal bundle.



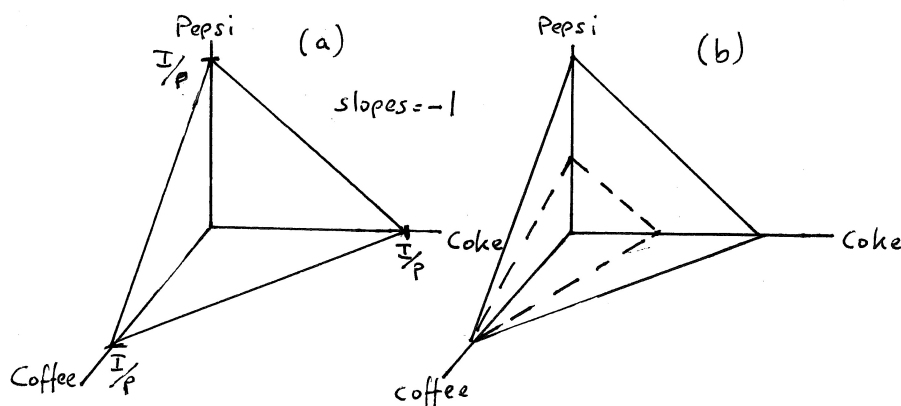
Graph 6.8: Homothetic Tastes and Optimization: Part 3

6.6 Coffee, Coke and Pepsi: Suppose there are three different goods: cans of Coke (x_1), cups of coffee (x_2) and cans of Pepsi (x_3).

A: Suppose each of these goods costs the same price p and you have an exogenous income I .

(a) Illustrate your budget constraint in three dimensions and carefully label all intercepts and slopes.

Answer: Panel (a) of Graph 6.9 illustrates the budget constraint. Each intercept is simply I divided by the price p which is the same for all three goods. All slopes are -1 .



Graph 6.9: Coffee, Coke and Pepsi

(b) Suppose each of the three drinks has the same caffeine content, and suppose caffeine is the only characteristic of a drink you care about. What do “indifference curves” look like?

Answer: This implies that all three goods are perfect substitutes for one another — which means the MRS between any two of them (holding the third fixed) is always exactly -1 . Thus, the indifference curves would be planes that look very much like the budget constraint graphed in panel (a). In fact, there is one indifference curve that lies exactly on this budget constraint, with all other indifference curves parallel to that plane.

(c) What bundles on your budget constraint would be optimal?

Answer: Since, when all three goods have the same prices, the slopes of the budget constraint are exactly identical to the slopes of the indifference planes, all bundles on the budget constraint are optimal. In other words, there is an indifference plane that lies exactly on top of the budget plane. This should make sense — if the three goods are perfect substitutes and they all cost the same, then it does not matter which combination you buy because they are all the same to you.

(d) Suppose that Coke and Pepsi become more expensive. How does your answer change? Are you now better or worse off than you were before the price change?

Answer: Panel (b) illustrates the new budget with the plane created by the dashed lines. The new budget shares one point in common with the original budget — the point at which all income is spent on coffee (since the price of coffee has not changed). Since the indifference planes look like the budgets before the price change, the plane formed by the solid lines is both the original budget (before the price change) and the highest possible indifference plane you can get to and still have it contain a point in your new (dashed) budget. That optimal point contains only coffee — and it gives you the same utility as you had before the price change (since it appears on the same indifference curve). Again, that should make sense — when coffee is cheaper than Coke and Pepsi but is perfectly substitutable for Coke and Pepsi, you should spend all your income on coffee. And when all three are initially

equally priced and then two become more expensive, you do not become worse off because you can still consume the same quantity of a good that is perfectly substitutable for the ones whose price has gone up.

B: Assume again that the three goods cost the same price p .

(a) Write down the equation of the budget constraint you drew in part A(a).

Answer: $px_1 + px_2 + px_3 = I$.

(b) Write down a utility function that represents the tastes described in A(b).

Answer: $u(x_1, x_2, x_3) = x_1 + x_2 + x_3$.

(c) Can you extend our notion of homotheticity to tastes over three goods? Are the tastes represented by the utility function you derived in (b) homothetic?

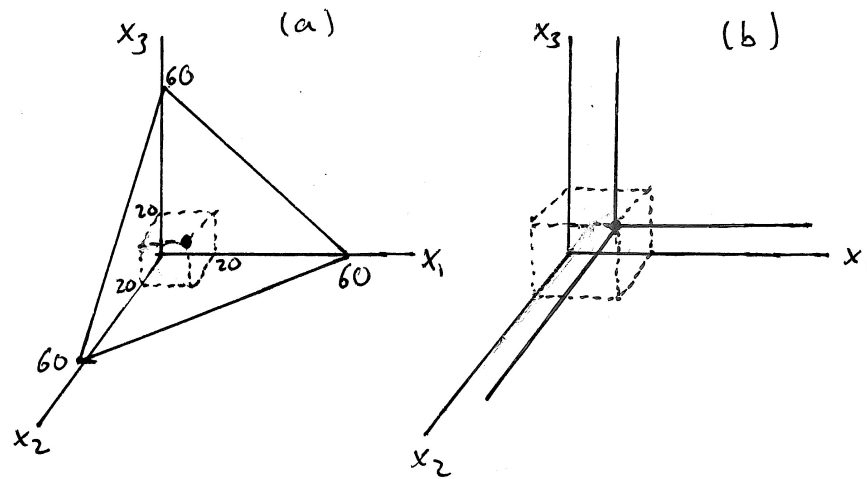
Answer: Homotheticity would now mean that, along any ray from the origin (where that ray can be along any plane where one of the goods is held at zero or it can vary all three goods at the same time), the MRS 's in all direction (i.e. always holding one of the goods fixed) are the same along that ray. This clearly holds for perfect substitutes where all MRS 's between any two goods at all bundles is always identical. The tastes represented by our utility function in (b) are therefore homothetic.

6.7 Coffee, Milk and Sugar. Suppose there are three different goods: cups of coffee (x_1), ounces of milk (x_2) and packets of sugar (x_3).

A: Suppose each of these goods costs 25 cents and you have an exogenous income of \$15.

(a) Illustrate your budget constraint in three dimensions and carefully label all intercepts.

Answer: At 25 cents a piece, I can buy as much as 60 of any one of the goods with \$15 assuming I don't buy anything else. Thus, panel (a) in Graph 6.10 has intercept of 60 on each axis (which makes all the slopes equal to -1 .)



Graph 6.10: Coffee, Milk and Sugar

(b) Suppose that the only way you get enjoyment from a cup of coffee is to have at least one ounce of milk and one packet of sugar in the coffee, the only way you get enjoyment from an ounce of milk is to have at least one cup of coffee and one packet of sugar, and the only way you get enjoyment from a packet of sugar is to have at least one cup of coffee and one ounce of milk. What is the optimal consumption bundle on your budget constraint.

Answer: The three goods are therefore perfect complements. This would mean that you would want to consume equal amounts of all three goods — which, given the prices and income, you would do when $x_1 = x_2 = x_3 = 20$. Put differently, you would want to consume 20 perfectly balanced cups of coffee (with an ounce of milk and a packet of sugar in each).

(c) What does your optimal indifference curve look like?

Answer: The indifference curve has a corner along the ray from the origin on which all goods are represented in identical quantities. The rest of the indifference “curve” is composed of planes parallel to each of the planes formed by the axes in the graph but ending at the corner. Panel (b) of Graph 6.10 is an attempt to graph this. Essentially, the indifference “curve” is like three sides of a box with the corner of the box pointing toward the origin and located along the ray that holds all goods equal to one another.

(d) If your income falls to \$10 — what will be your optimal consumption bundle?

Answer: You would still want to consume the three goods in equal amounts — which means now you could consume $2/3$ of what you did before. Before, you were able to consume 20 cups of coffee (with milk and sugar). Now you can only consume $40/3 = 13.33$ cups (with milk and sugar).

(e) If instead of a drop in income the price of coffee goes to 50 cents, how does your optimal bundle change?

Answer: Because the goods are perfect complements, it would still need to be the case that you buy the same quantity of each of the goods. Thus, $0.5x_1 + 0.25x_2 + 0.25x_3 = 15$ but $x_1 = x_2 = x_3$ at any optimum. Thus, letting x denote the quantity of each of the goods, $0.5x + 0.25x + 0.25x = 15$ or $x = 15$. Thus, you would drink 15 cups of coffee with milk and sugar.

- (f) Suppose your tastes are less extreme and you are willing to substitute some coffee for milk, some milk for sugar and some sugar for coffee. Suppose that the optimal consumption bundle you identified in (b) is still optimal under these less extreme tastes. Can you picture what the optimal indifference curve might look like in your picture of the budget constraint?

Answer: The indifference “curve” would still point toward the origin but would now be more “bowl-shaped” rather than “box-shaped” since the corner on the indifference curve would become smooth.

- (g) If tastes are still homothetic (but of the less extreme variety discussed in (f)), would your answers to (d) or (e) change?

Answer: If 20 cups of coffee with 20 ounces of milk and 20 packets of sugar is optimal under the original income of \$15, and if tastes are homothetic, then the ratio of the goods will remain the same if income changes. Thus, the answer to (d) does not change — you would consume 13.33 cups of coffee with as many sugars and ounces of milk when income falls to \$10. But when opportunity costs change — as in (e) where the price of a cup of coffee doubles, you will now substitute away from coffee and toward milk and sugar. Thus, you would drink fewer cups of coffee than we concluded in (e), but the coffee would be lighter (because of more milk) and sweeter (because of more sugar).

B: Continue with the assumption of an income of \$15 and prices for coffee, milk and sugar of 25 cents each.

- (a) Write down the budget constraint.

Answer: $0.25x_1 + 0.25x_2 + 0.25x_3 = 15$.

- (b) Write down a utility function that represents the tastes described in A(b).

Answer: $u(x_1, x_2, x_3) = \min\{x_1, x_2, x_3\}$.

- (c) Suppose that instead your tastes are less extreme and can be represented by the utility function $u(x_1, x_2, x_3) = x_1^\alpha x_2^\beta x_3$. Calculate your optimal consumption of x_1 , x_2 and x_3 when your economic circumstances are described by the prices p_1 , p_2 and p_3 and income is given by I .

Answer: It becomes notationally a bit easier to just take the log of the utility function before doing this problem. Thus, we can use the function $v(x_1, x_2, x_3) = \alpha \ln x_1 + \beta \ln x_2 + \ln x_3$. This gives us an optimization problem that can be written as

$$\max_{x_1, x_2, x_3} \alpha \ln x_1 + \beta \ln x_2 + \ln x_3 \quad \text{subject to} \quad p_1 x_1 + p_2 x_2 + p_3 x_3 = I. \quad (6.19)$$

The Lagrange function for this problem is

$$\mathcal{L}(x_1, x_2, x_3, \lambda) = \alpha \ln x_1 + \beta \ln x_2 + \ln x_3 + \lambda(I - p_1 x_1 + p_2 x_2 + p_3 x_3), \quad (6.20)$$

which gives us first order conditions of

$$\begin{aligned} \frac{\alpha}{x_1} &= \lambda p_1 \\ \frac{\beta}{x_2} &= \lambda p_2 \\ \frac{1}{x_3} &= \lambda p_3 \\ p_1 x_1 + p_2 x_2 + p_3 x_3 &= I. \end{aligned} \quad (6.21)$$

Solving the third equation for λ and substituting this into the first and second equations, we can solve for x_1 and x_2 to get

$$x_1 = \frac{\alpha p_3 x_3}{p_1} \text{ and } x_2 = \frac{\beta p_3 x_3}{p_2}. \quad (6.22)$$

We can then substitute these into the final first order condition (which is equal to the budget constraint) to get

$$p_1 x_1 + p_2 x_2 + p_3 x_3 = p_1 \frac{\alpha p_3 x_3}{p_1} + p_2 \frac{\beta p_3 x_3}{p_2} + p_3 x_3 = (\alpha + \beta + 1) p_3 x_3 = I. \quad (6.23)$$

Solving for x_3 , and then using this to plug into equations (6.22), gives

$$x_1 = \frac{\alpha I}{(\alpha + \beta + 1) p_1}, x_2 = \frac{\beta I}{(\alpha + \beta + 1) p_2} \text{ and } x_3 = \frac{I}{(\alpha + \beta + 1) p_3}. \quad (6.24)$$

- (d) What values must α and β take in order for the optimum you identified in A(b) to remain the optimum under these less extreme tastes?

Answer: In A(b), $p_1 = p_2 = p_3 = 0.25$ and $I = 15$. Thus, the solutions in (6.24) become

$$x_1 = \frac{60\alpha}{(\alpha + \beta + 1)}, x_2 = \frac{60\beta}{(\alpha + \beta + 1)} \text{ and } x_3 = \frac{60}{(\alpha + \beta + 1)}. \quad (6.25)$$

In order for the solution to be $x_1 = x_2 = x_3 = 20$ as in A(b), this implies that $\alpha = \beta = 1$.

- (e) Suppose α and β are as you concluded in part B(d). How does your optimal consumption bundle under these less extreme tastes change if income falls to \$10 or if the price of coffee increases to 50 cents? Compare your answers to your answer for the more extreme tastes in A(d) and (e).

Answer: Using $\alpha = \beta = 1$ as we have just concluded, the expressions become $x_1 = I/(3p_1)$, $x_2 = I/(3p_2)$ and $x_3 = I/(3p_3)$. Substituting $p_1 = p_2 = p_3 = 0.25$ and $I = 10$, we get $x_1 = x_2 = x_3 = 13.33$ which is identical to what we concluded in A(d) under the more extreme tastes. Substituting $p_1 = 0.50$, $p_2 = p_3 = 0.25$ and $I = 15$, on the other hand, we get $x_1 = 10$, $x_2 = 20$ and $x_3 = 20$. This differs from the answer in A(e) where no substitutability between the goods was permitted — now you end up drinking less coffee but with more milk and sugar in each cup.

- (f) True or False: Just as the usual shapes of indifference curves represent two dimensional “slices” of a 3-dimensional utility function, 3-dimensional “indifference bowls” emerge when there are three goods — and these “bowls” represent slices of a 4-dimensional utility function.

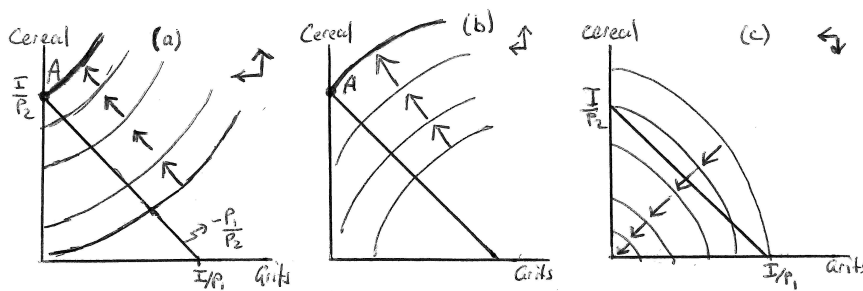
Answer: This is true. The utility function with three goods can be plotted in 4 dimensions — one for each good and one to indicate the utility level of each bundle — but the indifference “curves” hold utility fixed and can therefore be represented in 3 dimensions. This is analogous to slicing a 3 dimensional utility function with two goods to get two dimensional indifference curves.

6.8 Grits and Cereal: In end-of-chapter exercise 4.1, I described my dislike for grits and my fondness for Coco Puffs Cereal.

A: In part A of exercise 4.1, you were asked to assume that my tastes satisfy convexity and continuity and then to illustrate indifference curves on a graph with grits on the horizontal axis and cereal on the vertical.

- (a) Now add a budget constraint (with some positive prices for grits and cereal and some exogenous income I for me). Illustrate my optimal choice given my tastes.

Answer: This is depicted in panel (a) of Graph 6.11 where higher indifference curves occur to the northwest with fewer grits and more cereal. The highest indifference curve that still contains a bundle within the budget set is then a corner solution at point A. This should make sense — if I don't like grits but do like cereal, I should spend no money on grits.



Graph 6.11: Grits and Cereal

- (b) Does your answer change if my tastes are non-convex (as in part (b) of exercise 4.1A)?

Answer: No — the same corner solution would arise since I still don't like grits and therefore won't buy any at positive prices. This is illustrated in panel (b) of the graph.

- (c) In part (c) of exercise 4.1A, you were asked to imagine that I hate cereal as well and that my tastes are again convex. Illustrate my optimal choice under this assumption.

Answer: Panel (c) of the graph illustrates optimization in this case where higher indifference curves lie further to the southwest with fewer grits and less cereal. The optimum would be at $(0,0)$ — I would not spend any money since I dislike consuming both cereal and grits.

- (d) Does your answer change when my tastes are not convex (as in part (d) of exercise 4.1A)?

Answer: No, it would not change — moving southwest would still be optimal until one gets to $(0,0)$ since I still dislike both cereal and grits.

B: In part B of exercise 4.1 you derived a utility function that was consistent with my dislike for grits.

- (a) Can you explain why the Lagrange method will not work if you used it to try to solve the optimization problem using this utility function?

Answer: The Lagrange method identifies points at which indifference curves are tangent to budget lines. But the solution in this case is a corner solution where there is no tangency. Thus, the Lagrange method cannot be employed to solve this problem.

- (b) What would the Lagrange method offer as the optimal solution if you used a utility function that captured a dislike for both grits and cereal when tastes are non-convex? Illustrate your answer using $u(x_1, x_2) = -x_1 x_2$ and graph your insights.

Answer: The Lagrange function for this problem is

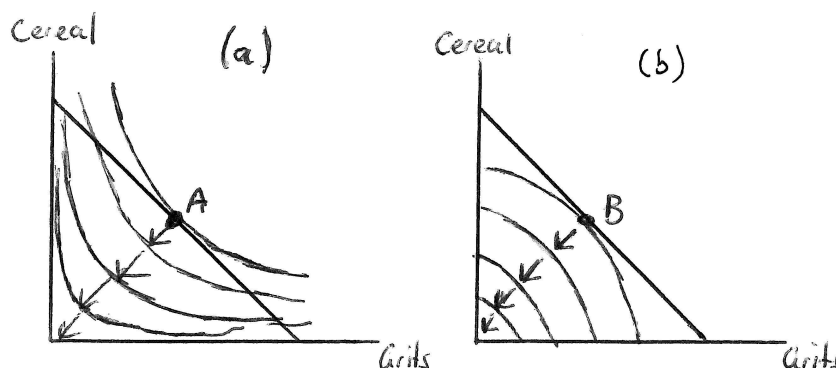
$$\mathcal{L}(x_1, x_2, \lambda) = -x_1 x_2 + \lambda(I - p_1 x_1 - p_2 x_2), \quad (6.26)$$

which gives as its first two first order conditions

$$-x_2 = \lambda p_1 \text{ and } -x_1 = \lambda p_2. \quad (6.27)$$

Dividing these by each other, we get $x_2/x_1 = p_1/p_2$ or $x_2 = (p_1/p_2)x_1$. Substituting this into the budget constraint, we get $p_1x_1 + p_2x_2 = p_1x_1 + p_2(p_1/p_2)x_1 = 2p_1x_1 = I$ which solves to $x_1 = I/(2p_1)$. Substituting back into $x_2 = (p_1/p_2)x_1$, we also get $x_2 = I/(2p_2)$. This is exactly the same solution that we would get if we had maximized the utility function $u(x_1, x_2) = x_1x_2$ where both goods are “goods” rather than “bads”. That is because the Lagrange method simply looks for tangencies — and we have not changed the map of indifference curves by multiplying the utility function by -1 — all we have done is change the ordering.

Put differently, the Lagrange method finds a bundle like A in panel (a) of graph 6.12 — but that is not an optimum because I can jump to higher indifference curves by moving south-west in the graph toward the origin. In fact, the Lagrange method has found the lowest possible utility I can get while still being in the choice set.



Graph 6.12: Grits and Cereal: Part 2

- (c) What would the Lagrange method offer as a solution if a utility function that captures a dislike for both grits and cereal represented convex tastes? Illustrate your answer using the function $u(x_1, x_2) = -x_1^2 - x_2^2$ and show what happens graphically.

Answer: The method would again identify tangencies — and thus would find a bundle like B in panel (b) of Graph 6.12. Of course that is again not an optimum because better bundles lie to the southwest. For the utility function $u(x_1, x_2) = -x_1^2 - x_2^2$, the Lagrange function would be

$$\mathcal{L}(x_1, x_2, \lambda) = -x_1^2 - x_2^2 + \lambda(I - p_1x_1 - p_2x_2) \quad (6.28)$$

with the first two first order conditions giving

$$-2x_1 = \lambda p_1 \text{ and } -2x_2 = \lambda p_2. \quad (6.29)$$

Dividing these by each other, we get $x_1/x_2 = p_1/p_2$ or $x_1 = p_1x_2/p_2$. Substituting into the budget constraint and solving for x_2 (and then substituting back into $x_1 = p_1x_2/p_2$) we get

$$x_2 = \frac{Ip_2}{p_1^2 + p_2^2} \text{ and } x_1 = \frac{Ip_1}{p_1^2 + p_2^2}. \quad (6.30)$$

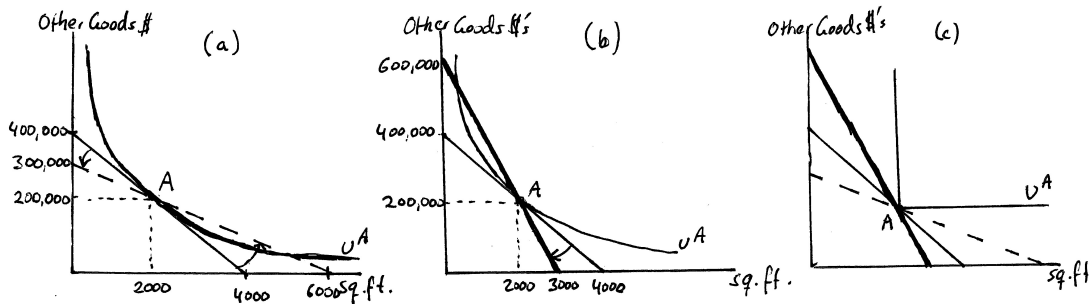
Thus, the math gives us an interior solution on the budget constraint where an indifference curve is tangent — but the real solution is to consume nothing (which gives utility of zero as opposed to negative utility from positive consumption.)

6.9 Everyday Application: *Price Fluctuations in the Housing Market:* Suppose you have \$400,000 to spend on a house and “other goods” (denominated in dollars).

A: The price of 1 square foot of housing is \$100 and you choose to purchase your optimally sized house at 2000 square feet. Assume throughout that you spend money on housing solely for its consumption value (and not as part of your investment strategy).

- (a) On a graph with “square feet of housing” on the horizontal axis and “other goods” on the vertical, illustrate your budget constraint and your optimal bundle A.

Answer: The budget constraint would have vertical intercept of \$400,000 (since this is how much other goods you can consume if you buy no housing) and horizontal intercept of 4,000 square feet of housing (since that is how much you can afford at \$100 per square foot if you spend all your money on housing.) The slope of this budget is -100 . The budget is depicted as the solid line in panel (a) of Graph 6.13.



Graph 6.13: Housing Price Fluctuations

- (b) After you bought the house, the price of housing falls to \$50 per square foot. Given that you can sell your house from bundle A if you want to, are you better or worse off?

Answer: The (dashed) new budget line is also drawn in panel (a) of the graph. Note that it has to go through A because A is your endowment point once you have bought the 2000 square foot house. Thus, you can always choose to consume that bundle regardless of what happens to prices. But you can also sell your 2000 square foot house for \$100,000 — which would give you \$300,000 in consumption, your new vertical intercept. Or you can take that \$300,000 and spend it on a new house and thereby buy as much as a 6,000 square foot house since housing now only costs \$50 per square foot. Since your indifference curve at A is tangent to your original budget line, the new (shallower) budget line cuts that indifference curve from below at bundle A. All the new bundles that are now affordable and that lie above the original indifference curve u^A therefore lie to the right of A. You are better off at any of those bundles on the dashed line that lie above the indifference curve u^A .

- (c) Assuming you can easily buy and sell houses, will you now buy a different house? If so, is your new house smaller or larger than your initial house?

Answer: You will buy a larger house — since all the better bundles on the dashed line in panel (a) are to the right of A and therefore include a house larger than 2000 square feet.

- (d) Does your answer to (c) differ depending on whether you assume tastes are quasilinear in housing or homothetic?

Answer: No — in both cases you would end up better off consuming a larger house.

- (e) How does your answer to (c) change if the price of housing went up to \$200 per square foot rather than down to \$50.

Answer: Panel (b) of Graph 6.13 illustrates this change in prices. The original budget constraint (from \$400,000 on the vertical to 4,000 square feet on the horizontal axis) with bundle A is replicated from panel (a) and illustrates the budget when the price per square foot of

housing is \$100. The steeper bold line going through A illustrates the new budget line when A is the endowment point and the price of housing goes to \$200 per square foot. If you sell your 2000 square foot house at \$200 per square foot, you would get \$400,000 for it — which, added to the \$200,000 you have would give you as much as \$600,000 in consumption if you choose not to buy another house. If you do buy another house, the largest possible house at the new prices is now a 3000 square foot house. But you can always choose to stay at A — so A too is on the new budget line. The bundles on the new bold budget that also lie above the indifference curve u^A all lie to the left of A — indicating that the new house that you would purchase would be smaller than your original 2000 square foot house.

- (f) *What form would tastes have to take in order for you to not sell your \$2000 square foot house when the price per square foot goes up or down?*

Answer: The indifference curve through A would have to have a kink in it, as would be the case if housing and other goods are perfect complements. This is illustrated in panel (c) of Graph 6.13 where all three budget lines are drawn, as is an indifference curve u^A that treats the two goods as perfect complements. Technically, it could also be the case that the indifference curve through A has a less severe kink at A — one where the slope to the left of A is steeper than the bold budget line and the slope to the right of A is shallower than the slope of the dashed budget line. What is important is that there is a sufficiently severe kink — with no substitutability on the margin between the goods at the kink point. If there is no kink at A — i.e. if there is any substitutability at the margin between housing and other goods at A — then the bold and dashed indifference curves must necessarily cut the indifference curve at A in the ways (though not necessarily with the magnitudes) illustrated in (a) and (b).

- (g) True or False: *So long as housing and other consumption is at least somewhat substitutable, any change in the price per square foot of housing makes homeowners better off (assuming it is easy to buy and sell houses.)*

Answer: This is true, as just argued in the answer above.

- (h) True or False: *Renters are always better off when the rental price of housing goes down and worse off when it goes up.*

Answer: This is true. Renters do not have endowment points in this model as homeowners do. So changes in the rental price of housing rotate the budget line through the vertical intercept — which implies that a drop in housing prices unambiguously expands the budget set at every level of housing and an increase in housing prices unambiguously shrinks the choice set at every level of housing.

B: Suppose your tastes for “square feet of housing” (x_1) and “other goods” (x_2) can be represented by the utility function $u(x_1, x_2) = x_1 x_2$.

- (a) Calculate your optimal housing consumption as a function of the price of housing (p_1) and your exogenous income I (assuming of course that p_2 is by definition equal to 1.)

Answer: We want to solve the problem

$$\max_{x_1, x_2} u(x_1, x_2) = x_1 x_2 \quad \text{subject to} \quad p_1 x_1 + x_2 = I. \quad (6.31)$$

The Lagrange function for this problem is

$$\mathcal{L}(x_1, x_2, \lambda) = x_1 x_2 + \lambda(I - p_1 x_1 - x_2), \quad (6.32)$$

which give us first order conditions

$$\begin{aligned} x_2 &= \lambda p_1 \\ x_1 &= \lambda \\ p_1 x_1 + x_2 &= I. \end{aligned} \quad (6.33)$$

Substituting the second equation into the first, we get $x_2 = x_1 p_1$, and substituting this into the last equation, we get $p_1 x_1 + p_1 x_1 = I$ or $x_1 = I/(2p_1)$. Finally, plugging this back into $x_2 = x_1 p_1$, we get $x_2 = I/2$.

- (b) Using your answer, verify that you will purchase a 2000 square foot house when your income is \$400,000 and the price per square foot is \$100.

Answer: We just concluded that $x_1 = I/(2p_1)$. When $p_1 = 100$ and $I = 400,000$, this implies $x_1 = 400,000/(2(100)) = 2000$.

- (c) Now suppose the price of housing falls to \$50 per square foot and you choose to sell your 2000 square foot house. How big a house would you now buy?

Answer: By selling your 2000 square foot house at \$50 per square foot, you would make \$100,000. Added to the \$200,000 you had left over after you bought your original 2000 square foot house, this gives you a total income of \$300,000. Plugging $I=300,000$ and $p_1 = 50$ into our equation for the optimal housing quantity $x_1 = I/(2p_1)$, we get $x_1 = 300,000/(2(50)) = 3000$. Thus, you will buy a 3000 square foot house.

- (d) Calculate your utility (as measured by your utility function) at your initial 2000 square foot house and your new utility after you bought your new house? Did the price decline make you better off?

Answer: Your initial consumption bundle was (2000, 200000). That gives utility

$$u(2000, 200000) = 2000(200000) = 400,000,000. \quad (6.34)$$

When price fell, you end up at the bundle (3000, 150000) which gives utility

$$u(3000, 150000) = 3000(150000) = 450,000,000. \quad (6.35)$$

Since your utility after the price decline is higher than before, you are better off.

- (e) How would your answers to B(c) and B(d) change if, instead of falling, the price of housing had increased to \$200 per square foot?

Answer: Again, we have already calculated that $x_1 = I/(2p_1)$ and $x_2 = I/2$. When price increases to \$200 and you already own a 2000 square foot house, you can now sell your house for \$400,000 which, added to the \$200,000 you had left over after buying your original house, gives you up to \$600,000 to spend. Treating this as your new I and plugging in the new housing price $p_1 = 200$, we then get that your new optimal bundle has $x_1 = 600000/(2(200)) = 1500$ and $x_2 = 600000/2 = 300,000$. Thus you will buy a 1500 square foot house and consume \$300,000 in other goods. This gives you utility

$$u(1500, 300000) = 1500(300000) = 450,000,000, \quad (6.36)$$

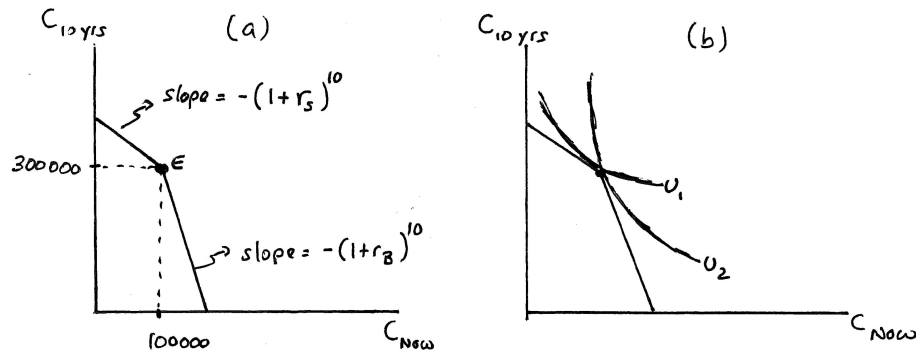
which is greater than the utility you had originally and equal to the utility you received from the price decrease above. Thus, a price increase to \$200 per square foot makes you better off, exactly as much as a drop in price to \$50 per square foot. You are therefore indifferent between the price increase and the price decrease.

6.10 Everyday Application: *Different Interest Rates for Borrowing and Lending.* You first analyzed intertemporal budget constraints with different interest rates for borrowing and saving (or lending) in end-of-chapter exercise 3.8.

A: Suppose that you have an income of \$100,000 now and you expect to have an income of \$300,000 10 years from now, and suppose that the interest rate for borrowing from the bank is twice as high as the interest rate the bank offers for savings.

- (a) Begin by drawing your budget constraint with “consumption now” and “consumption in 10 years” on the horizontal and vertical axes. (Assume for purposes of this problem that your consumption in the intervening years is covered and not part of the analysis.)

Answer: Suppose that the interest rate for savings is r_S while the interest rate for borrowing is r_B — with $r_S < r_B$. Panel (a) of Graph 6.14 then illustrates that this results in an outward kink of the intertemporal budget constraint at point E.



Graph 6.14: Different Interest Rates for Borrowing and Lending

- (b) Can you explain why, for a wide class of tastes, it is rational for someone in this position not to save or borrow?

Answer: The larger the difference between r_S and r_B , the sharper the kink at point E in panel (a) of the graph. And the sharper the kink, the greater the range of slopes of the indifference curves at E that would make it optimal to consume at E — which is equivalent to neither borrowing nor lending. Panel (b) of the graph illustrates two such indifference curves — with very different slopes — that are both consistent with this.

- (c) Now suppose that the interest rate for borrowing was half the interest rate for saving. Draw this new budget constraint.

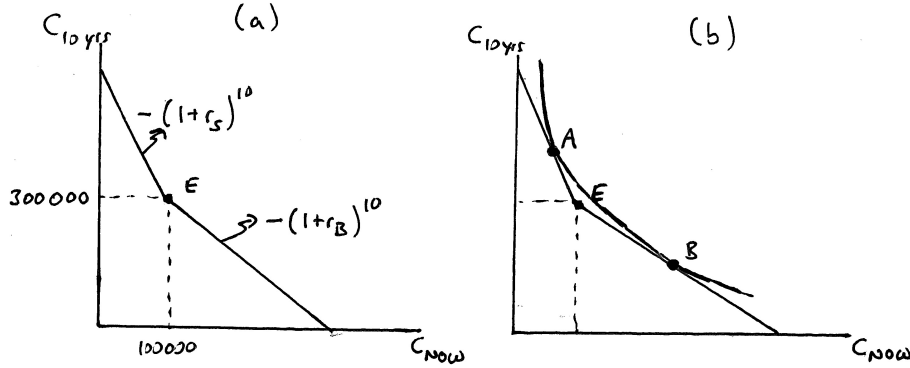
Answer: Panel (a) of Graph 6.15 (next page) illustrates that the kink at E would now point inward.

- (d) Illustrate a case where it might be rational for a consumer to flip a coin to determine whether to borrow a lot or to save a lot.

Answer: If the consumer flips a coin, she must be indifferent between borrowing and saving. Thus, she has two optimal bundles — as illustrated in panel (b) of Graph 6.15. At A, she saves a large fraction of her current income and consumes it in the future; at B she borrows a large fraction of future income and consumes it now.

B: Suppose that your incomes are as described in part A and that the annual interest rate for borrowing is 20% and the annual interest rate for saving is 10%. Also, suppose that your tastes over current consumption c_1 and consumption 10 years from now c_2 can be captured by the utility function $u(c_1, c_2) = c_1^\alpha c_2^{(1-\alpha)}$.

- (a) Assuming that interest compounds annually, what are the slopes of the different segments of the budget constraint that you drew in A(a)? What are the intercepts?



Graph 6.15: Different Interest Rates for Borrowing and Lending

Answer: The slope of the line segment above E is $-(1+0.1)^{10} = -2.594$ while the slope of the line segment below E is $-(1+0.2)^{10} = -6.192$. If you save \$100,000 for 10 years at an interest rate of 10% compounded annually, then you will have $100,000(1+0.1)^{10} = \$259,374$ in the bank 10 years from now. Added to the \$300,000 you will earn in 10 years, this makes for an intercept of \$559,374 on the c_2 axis. If you borrow on \$300,000 expected income 10 years from now at a 20% interest rate, on the other hand, you can only borrow $300,000/(1+0.2)^{10} = \$48,452$. Added to the \$100,000 in current income, that makes for a c_1 intercept of \$148,452.

(b) For what ranges of α is it rational to neither borrow nor save?

Answer: The marginal rate of substitution for these tastes is

$$MRS = -\frac{\alpha c_2}{(1-\alpha)c_1}. \quad (6.37)$$

At the kink point $E = (100000, 300000)$, this reduces to

$$MRS(100000, 300000) = -\frac{300000\alpha}{100000(1-\alpha)} = -\frac{3\alpha}{(1-\alpha)}. \quad (6.38)$$

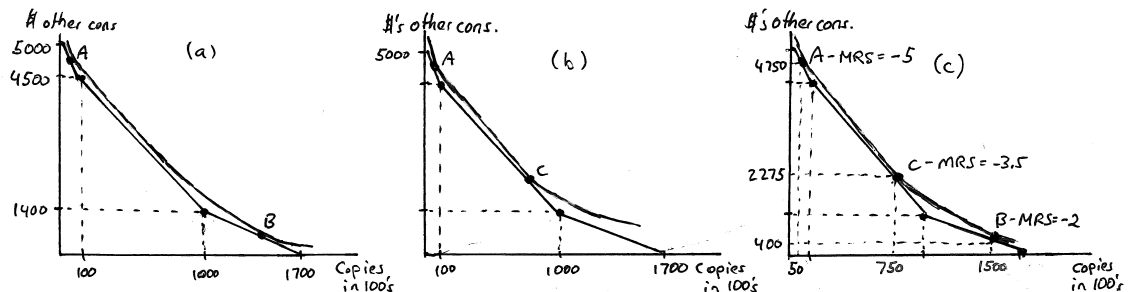
In order for E to be optimal, it must be that the MRS at E falls between the shallower slope of the budget constraint above E and the steeper slope below E . In part (a), we concluded that the former is -2.594 and the latter is -6.192 . The MRS at bundle E then falls between these values so long as $0.4637 < \alpha < 0.6736$.

6.11 Business Application: Quantity Discounts and Optimal Choices. In end-of-chapter exercise 2.9, you illustrated my department's budget constraint between “pages copied in units of 100” and “dollars spent on other goods” given the quantity discounts our local copy service gives the department. Assume the same budget constraint as the one described in 2.9A.

A: In this exercise, assume that my department's tastes do not change with time (or with who happens to be department chair). When we ask below whether someone is “respecting the department's tastes” we mean whether that person is using the department's tastes to make optimal decisions for the department given the circumstances faced by the department. Assume throughout that my department's tastes are convex.

- (a) True or False: If copies and other expenditures are very substitutable for my department, then you should observe either very little or a great deal of photocopying by our department at the local copy shop.

Answer: This is true. Panel (a) of Graph 6.16 illustrates one possibility of this with a single indifference curve tangent at low and high numbers of photocopies (bundles A and B). If the indifference curves have more curvature, then the tangency would lie on the middle portion of the budget constraint with a single optimal quantity that lies in between what one might consider as high and low. It is of course also possible that indifference curves with very little curvature are steeper than the steepest part of the budget — leading to an extreme corner solution on one end of the budget; or that they are very shallow leading to an extreme corner solution on the other end.



Graph 6.16: Discounts and Photocopies

- (b) Suppose that I was department chair last year and had approximately 5,000 copies per month made. This year, I am on leave and an interim chair has taken my place. He has chosen to make 150,000 copies per month. Given that our department's tastes are not changing over time, can you say that either I or the current interim chair is not respecting the department's tastes?

Answer: No, we cannot say that with any certainty. In fact, the indifference curve in panel (a) of Graph 6.16 illustrates the case where both chairs are respecting the department's tastes despite making very different decision. The reason for this is the non-convexity in the budget set created by the discount policy of the photocopy store.

- (c) Now the interim chair has decided to go on vacation for a month — and an interim interim chair has been named for that month. He has decided to purchase 75,000 copies per month. If I was respecting the department's tastes, is this interim interim chair necessarily violating them?

Answer: No, not necessarily. Panel (b) of the graph gives an example of an indifference curve that would make both choices, A and C, optimal from the department's perspective.

- (d) If both I and the initial interim chair were respecting the department's tastes, is the new interim interim chair necessarily violating them?

Answer: Again, not necessarily. This is illustrated in panel (c) of Graph 6.16.

B: Consider the decisions made by the 3 chairs as described above.

- (a) If I and the second interim chair (i.e. the interim interim chair) both respected the department's tastes, can you approximate the elasticity of substitution of the department's tastes?

Answer: The elasticity of substitution σ is given by

$$\sigma = \left| \frac{\% \Delta(x_2/x_1)}{\% \Delta MRS} \right| = \left| \left(\frac{(x_2^A/x_1^A) - (x_2^C/x_1^C)}{(x_2^A/x_1^A)} \right) \left(\frac{MRS^A}{MRS^A - MRS^C} \right) \right|, \quad (6.39)$$

where bundle A is (50,4750) and C is (750,2275) as depicted in panel (c) of the graph. We furthermore know that $MRS^A = -5$ and $MRS^C = -3.5$. Thus

$$\sigma = \left| \left(\frac{(4750/50) - (2275/750)}{4750/50} \right) \left(\frac{-5}{-5 - (-3.5)} \right) \right| = 3.23. \quad (6.40)$$

- (b) If the first and second interim chairs both respected the department's tastes, can you approximate the elasticity of substitution for the department?

Answer: Now the relevant bundles are C=(750,2275) and B=(1500,400) with $MRS^C = -3.5$ and $MRS^B = -2$, which implies

$$\sigma = \left| \left(\frac{(2275/750) - (400/1500)}{2275/750} \right) \left(\frac{-3.5}{-3.5 - (-2)} \right) \right| = 2.13. \quad (6.41)$$

- (c) Could the underlying tastes under which all three chairs respect the department's tastes be represented by a CES utility function?

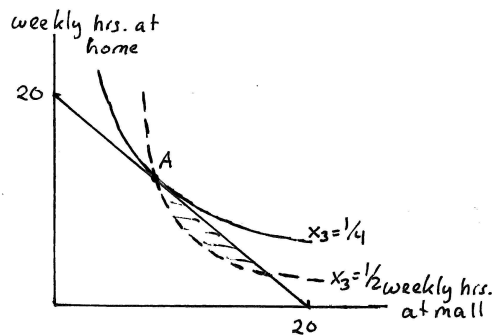
Answer: Since we get different elasticity of substitution estimates from the different pairs of choices, tastes that rationalize all three choices given the budget constraint cannot be represented by a *constant* elasticity of substitution utility function that has the same elasticity of substitution everywhere.

6.12 Business Application: Retail Industry Lobbying for Daylight Savings Time. In 2005, the U.S. Congress passed a bill to extend daylight savings time earlier into the spring and later into the fall (beginning in 2007). The change was made as part of an Energy Bill, with some claiming that daylight savings time reduces energy use by extending sunlight to later in the day (which requires fewer hours of artificial light). Among the biggest advocates for daylight savings time, however, was the retail and restaurant industry that believes consumers will spend more time shopping and eating in malls for reasons explored here.

A: Consider a consumer who returns home from work at 6PM and goes to sleep at 10PM. In the month of March, the sun sets by 7PM in the absence of daylight savings time, but with daylight savings time, the sun does not set until 8PM. When the consumer comes home from work, she can either spend time (1) at home eating food from her refrigerator while e-mailing friends and surfing/shopping on the internet or (2) at the local mall meeting friends for a bite to eat and strolling through stores to shop. Suppose this consumer gets utility from (1) and (2) (as defined here) but she also cares about x_3 which is defined as the fraction of daylight hours after work.

- (a) On a graph with “weekly hours at the mall” on the horizontal axis and “weekly hours at home” on the vertical, illustrate this consumer’s typical weekly after-work time constraint (with a total of 20 hours per week available — 4 hours on each of the 5 workdays). (For purposes of this problem, assume the consumer gets as much enjoyment from driving to the mall as she does being at the mall).

Answer: This is illustrated in Graph 6.17. The consumer can spend either 20 hours at home or 20 hours at the mall or some combination of 20 hours between the two places. Thus, the opportunity cost of spending 1 hour at the mall is not being able to spend that hour at home — leading to a slope of -1 .



Graph 6.17: Daylight Savings Time

- (b) Consider first the scenario of no daylight savings time in March. This implies only 1 hour of daylight in the 4 hours after work and before going to sleep; i.e. the fraction x_3 of daylight hours after work is $1/4$. Pick a bundle A on the budget constraint from (a) as the optimum for this consumer given this fraction of after-work daylight hours.

Answer: This is also depicted in the graph, with the (solid) indifference curve going through A labeled by $x_3 = 1/4$ and tangent to the budget constraint.

- (c) Now suppose daylight savings time is moved into March, thus raising the number of after-work daylight hours to 2 per day. Suppose this changes the MRS at every bundle. If the retail and restaurant industry is right, which way does it change the MRS?

Answer: If the industry is right, then more sunlight leads consumers to, at every bundle, be willing to give up more hours at home for an hour at the mall. Thus, the MRS becomes larger in absolute value — leading to indifference curves with steeper slopes.

- (d) Illustrate how, if the retail and restaurant industry is right, this results in more shopping and eating at malls every week.

Answer: This is illustrated with the second (dashed) indifference curve in the graph — with that indifference curve also passing through A but now labeled $x_3 = 1/2$. This indifference curve has steeper slope as we concluded it must have if the retail and restaurant industry is right. The shaded bundles between the new indifference curve and the budget line all make the consumer better off than bundle A when $x_3 = 1/2$ — and each of these bundles involves more time spent at the mall and restaurants.

- (e) *Explain the following statement: “While it appears in our 2-dimensional indifference maps that tastes have changed as a result of a change in daylight savings time, tastes really haven’t changed at all because we are simply graphing 2-dimensional slices of the same 3-dimensional indifference surfaces.”*

Answer: The consumer’s tastes have really not changed in any fundamental way — the consumer always cared about sunlight but simply does not have control over how much sunlight there is. Thus, for purposes of analyzing choices given the level of sunlight (i.e. given x_3), we only have to focus on the slice of the 3-dimensional indifference surfaces that illustrate combinations of mall hours, home hours and sunlight that the consumer is indifferent between. When daylight savings time goes into effect, sunlight changes and the consumer switches to a different portion of the 3-dimensional indifference surface — the portion that is now relevant for the new level of sunlight. When these two slices of the 3-dimensional surfaces are depicted on a single graph, it looks like indifference curves cross and tastes must have changed — but that is only because we are projecting two slices of the same tastes onto a single 2-dimensional graph.

- (f) *Businesses can lobby Congress to change the circumstances under which we make decisions, but Congress has no power to change our tastes. Explain how the change in daylight savings time illustrates this in light of your answer to (e).*

Answer: Congress did not need to change tastes in order to change behavior in line with what retailers and restaurant owners lobbied for — all it needed to do was change the circumstances consumers face — which in this case includes the number of hours of daylight after working hours. Often Congress changes individual circumstances through such policies as taxes or spending — but it also does so through regulations like when daylight savings time begins. As circumstances change, behavior changes even if tastes remain the same.

- (g) *Some have argued that consumers must be irrational for shopping more just because daylight savings is introduced. Do you agree?*

Answer: Our model suggests there is nothing irrational at all about shopping more under daylight savings time. By extending daylight hours at the end of the day, Congress has made it more desirable to go shopping because people like to shop while it is still daylight out. As a result, people shop more — they are merely responding to changed circumstances but are optimizing given their circumstances both before and after daylight savings time goes into effect.

- (h) *If we consider not just energy required to produce light but also energy required to power cars that take people to shopping malls, is it still clear that the change in daylight savings time is necessarily energy saving?*

Answer: No, it is not clear since there are offsetting effects that result from the change in people’s behavior. In fact, there are studies that claim to show that daylight savings time actually costs more energy because of such adjustments in individual behavior to changed circumstances.

B: Suppose a consumer’s tastes can be represented by the utility function $u(x_1, x_2, x_3) = 12x_3 \ln x_1 + x_2$, where x_1 represents weekly hours spent at the mall, x_2 represents weekly after-work hours spent at home (not sleeping), and x_3 represents the fraction of after-work (before-sleep) time that has daylight.

- (a) Calculate the MRS of x_2 for x_1 for this utility function and check to see whether it has the property that retail and restaurant owners hypothesize.

Answer: The MRS is

$$MRS = -\frac{\partial u / \partial x_1}{\partial u / \partial x_2} = -\frac{12x_3 / x_1}{1} = -\frac{12x_3}{x_1}. \quad (6.42)$$

We concluded in part A that retail and restaurant owners believe that, if there is more daylight, people will be willing to give up more hours at home for every hour at the mall and restaurants. This implies that the *MRS* should become larger in absolute value as x_3 — the fraction of evening time with daylight — increases. That is indeed the case — as x_3 increases, $12x_3/x_1$ increases as well — causing the indifference curves at every bundle to become steeper.

- (b) Which of the three things the consumer cares about — x_1, x_2 and x_3 — are choice variables for the consumer?

Answer: The consumer only get to choose x_1 and x_2 — the amount of time spent outside and inside the house. She does not get to decide how much daylight there is in the rest of the day.

- (c) Given the overall number of weekly after-work hours our consumer has (i.e. 20), calculate the number of hours per week this consumer will spend in malls and restaurants as a function of x_3 .

Answer: The problem we have to solve is

$$\max_{x_1, x_2} u(x_1, x_2, x_3) = 12x_3 \ln x_1 + x_2 \quad \text{subject to} \quad x_1 + x_2 = 20. \quad (6.43)$$

Note that, since x_3 is not a choice variable, it does not appear as part of the max notation in the specification of the problem. The Lagrange function for this problem is then

$$\mathcal{L}(x_1, x_2, \lambda) = 12x_3 \ln x_1 + x_2 + \lambda(20 - x_1 - x_2), \quad (6.44)$$

where the Lagrange function is not a function of x_3 because x_3 enters the function only as a parameter (exactly as the number “12” does), not as a variable. First order conditions are then taken only with respect to x_1 and x_2 (and λ), with the first two of these giving us

$$\begin{aligned} \frac{12x_3}{x_1} &= \lambda \\ 1 &= \lambda, \end{aligned} \quad (6.45)$$

where the second equation can just be substituted into the first to get $x_1 = 12x_3$. Notice that x_1 is only a functions of x_3 and not of x_2 — that’s because tastes are quasilinear in x_1 . (We can also derive the number of hours spent at home by simply putting $x_1 = 12x_3$ into the budget constraint $x_1 + x_2 = 20$ to get $12x_3 + x_2 = 20$ or $x_2 = 20 - 12x_3$.)

- (d) How much time per week will she spend in malls and restaurants in the absence of daily savings time? How does this change when daylight savings time is introduced?

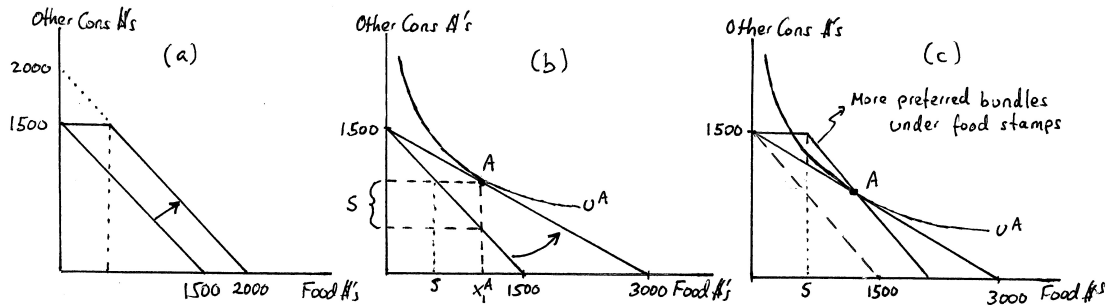
Answer: In the absence of daylight savings time, $x_3 = 1/4$ which implies $x_1 = 12x_3 = 3$. With daylight savings time, $x_3 = 1/2$ which implies $x_1 = 12x_3 = 6$. Thus, daylight savings time causes this consumer to go to the mall for 6 hours per week instead of 3 hours per week.

6.13 Policy Application: Food Stamps versus Food Subsidies: In exercise 2.13, you considered the food stamp programs in the US. Under this program, poor households receive a certain quantity of "food stamps" — stamps that contain a dollar value which is accepted like cash for food purchases at grocery stores.

A: Consider a household with monthly income of \$1,500 and suppose that this household qualifies for food stamps in the amount of \$500.

- (a) Illustrate this household's budget, both with and without the food stamp program, with "dollars spent on food" (on the horizontal axis) and "dollars spent on other goods" on the vertical. What has to be true for the household to be just as well off under this food stamp program as it would be if the government simply gave \$500 in cash to the household (instead of food stamps)?

Answer: Panel (a) of Graph 6.18 illustrates these two budgets. The budget under food stamps has a flat spot at the top because the first \$500 in food consumption can be paid for through the food stamps but non-food items cannot be paid for with those stamps. As long as the household would have purchased at least \$500 in food under a budget of \$2,000 per month, the food stamp program is exactly like a cash subsidy program for this household. Put differently, so long as the indifference curve tangent to the extended outer budget in panel (a) is tangent at food consumption levels greater than \$500, there is no difference between the two types of programs.



Graph 6.18: Food Stamps, Cash and Food Subsidies

- (b) Consider the following alternate policy: Instead of food stamps, the government tells this household that it will reimburse 50% of the household's food bills. On a separate graph, illustrate the household's budget (in the absence of food stamps) with and without this alternate program.

Answer: Panel (b) of the graph illustrates the initial budget (going from \$1,500 on the vertical axis to \$1,500 on the horizontal) and the new budget that has shallower slope because \$1 of food now only costs 50 cents.

- (c) Choose an optimal bundle A on the alternate program budget line and determine how much the government is paying to this household (as a vertical distance in your graph). Call this amount S.

Answer: This is also illustrated in panel (b) of the graph. At bundle A, the household is consuming x_1^A in food. We can then read off the vertical axis how much in other consumption the household was able to undertake at A and compare it to how much it would have been able to consume of other goods had it consumed x_1^A in food prior to the subsidy. The difference between these two amounts is S.

- (d) Now suppose the government decided to abolish the program and instead gives the same amount S in food stamps. How does this change the household's budget?

Answer: This change is illustrated in panel (c) of the graph. In both cases, the bundle A will be available to the consumer because the government is giving S under both programs. However, under the food stamp program, the subsidy amount remains the same regardless of how much food the household consumes, whereas under the price subsidy program the amount of government transfer decreases if the household consumes less food and increases if it consumes more food. Put differently, there is no change in opportunity costs under the cash subsidy, with the price of food going back up to \$1 for every \$1 of food.

- (e) Will this household be happy about the change from the first alternate program to the food stamp program?

Answer: The household prefers the food stamps to the price subsidy. You can see this in panel (c) where the indifference curve that makes A optimal under the price subsidy is tangent to the shallower (price subsidy) budget. But this means that the new food stamp budget cuts this indifference curve from above, making a set of new bundles that lie above the indifference curve u^A available to the household.

- (f) If some politicians want to increase food consumption by the poor and others just want to make the poor happier, will they differ on what policy is best?

Answer: Yes, they will differ. The food price subsidy causes the poor to consume more food whereas the equally costly food stamp program is more preferred by poor households (i.e. makes them happier).

- (g) True or False: The less substitutable food is for other goods, the greater the difference in food consumption between equally funded cash and food subsidy programs.

Answer: This is false. Imagine making u^A in panel (c) of our graph the shape that presumes food and other goods are perfect complements. In that case, the equally costly food stamp program, which still contains A , will no longer cut the indifference curve u^A — thus eliminating the “better” bundles on the food stamp budget that we identified in panel (c). The household would therefore consume the same amount of food under either program. Then imagine increasing the substitutability between food and other goods at point A — as you do so, more and more “better” bundles become available.

- (h) Consider a third possible alternative — giving cash instead of food stamps. True or False: As the food stamp program becomes more generous, the household will at some point prefer a pure cash transfer over an equally costly food stamp program.

Answer: This is true and relates to our answer to part (a). Since food stamps can only be spent on food, they are equivalent to cash so long as the household would choose to spend at least the value of food stamps on food even if the stamps were replaced by cash. But as the food stamp program becomes more generous, it will at some point be the case that the household would in fact use the food stamps to buy non-food items if it could — and it is at that point that the household would strictly prefer the cash program over the food stamp program.

B: Suppose this household's tastes for spending on food (x_1) and spending on other goods (x_2) can be characterized by the utility function $u(x_1, x_2) = \alpha \ln x_1 + \ln x_2$.

- (a) Calculate the level of food and other good purchases as a function of I and the price of food p_1 (leaving the price of dollars on other goods as just 1).

Answer: We are asked to solve the problem

$$\max_{x_1, x_2} u(x_1, x_2) = \alpha \ln x_1 + \ln x_2 \quad \text{subject to} \quad p_1 x_1 + x_2 = I. \quad (6.46)$$

The Lagrange function for this problem is

$$\mathcal{L}(x_1, x_2, \lambda) = \alpha \ln x_1 + \ln x_2 + \lambda(I - p_1 x_1 - x_2), \quad (6.47)$$

which gives rise to first order conditions, the first two of which are

$$\begin{aligned} \frac{\alpha}{x_1} &= \lambda p_1 \\ \frac{1}{x_2} &= \lambda \end{aligned} \quad (6.48)$$

Substituting the second equation into the first for λ , we get $x_2 = p_1 x_1 / \alpha$. And substituting this into the budget constraint (which is the third first order condition), we get $p_1 x_1 + x_2 = p_1 x_1 + p_1 x_1 / \alpha = I$ which we can solve for x_1 to get

$$x_1 = \frac{\alpha I}{(\alpha + 1)p_1}, \quad (6.49)$$

and substituting this back into $x_2 = p_1 x_1 / \alpha$,

$$x_2 = \frac{I}{(\alpha + 1)}. \quad (6.50)$$

- (b) For the household described in part A, what is the range of α that makes the \$500 food stamp program equivalent to a cash gift of \$500?

Answer: The food stamps are equivalent to a cash gift so long as the household would have spent at least the value of the food stamps on food were it to receive the cash gift instead. Our household has income $I = 1500$ and the price of food is $p_1 = \$1$ in the absence of a price subsidy. To determine the value of α at which the household would buy exactly \$500 of food with a cash gift of \$500, we need to substitute \$2,000 for I and \$1 for p_1 into our equation for x_1 , set it to \$500 and solve for α ; i.e.

$$\frac{2000\alpha}{\alpha + 1} = 500 \text{ implies } \alpha = \frac{1}{3}. \quad (6.51)$$

Thus, for $\alpha > 1/3$, the cash subsidy is equivalent to the food stamp program of \$500.

- (c) Suppose for the remainder of the problem that $\alpha = 0.5$. How much food will this household buy under the alternate policy described in A(b)?

Answer: Under this policy, p_1 drops to $1/2$ while I remains at 1500. The household will therefore buy

$$x_1 = \frac{\alpha I}{(\alpha + 1)p_1} = \frac{0.5(1500)}{1.5(0.5)} = 1000. \quad (6.52)$$

- (d) How much does this alternate policy cost the government for this household? Call this amount S .

Answer: If the household buys \$1,000 of food and the government reimburses half, then $S = 500$.

- (e) How much food will the household buy if the government gives S as a cash payment and abolishes the alternate food subsidy program?

Answer: In that case, $I = 1500 + S = 1500 + 500 = 2000$ and p_1 goes back to 1. Thus,

$$x_1 = \frac{\alpha I}{(\alpha + 1)p_1} = \frac{0.5(2000)}{1.5(1)} = 666.67. \quad (6.53)$$

- (f) Determine which policy — the price subsidy that leads to an amount S being given to the household, or the equally costly cash payment in part (e) — is preferred by the household.

Answer: Under the price subsidy policy, the household pays \$500 to get \$1000 of food, leaving it with \$1000 in other consumption. Thus, it consumes a bundle (1000,1000). This gives utility

$$u(1000, 1000) = 0.5 \ln(1000) + \ln(1000) = 10.362. \quad (6.54)$$

Under the cash subsidy policy, the household gets \$500 in cash to raise its total income to \$2000 of which it spends \$666.67 on food, leaving it with \$1333.33 in other spending; i.e. under cash subsidy, the household consumes bundle (666.67,1333.33). This gives utility

$$u(666.67, 1333.33) = 0.5 \ln(666.67) + \ln(1333.33) = 10.447. \quad (6.55)$$

The household is happier under the cash subsidy policy.

- (g) Now suppose the government considered subsidizing food more heavily. Calculate the utility that the household will receive from three equally funded policies: a 75% food price subsidy (i.e. a subsidy where the government pays 75% of food bills), a food stamp program and a cash gift program.

Answer: First, consider the price subsidy program that lowers the price p_1 from 1 to 0.25 while keeping I at \$1,500. This will result in food and other good consumption of

$$x_1 = \frac{\alpha I}{(\alpha + 1)p_1} = \frac{0.5(1500)}{(1.5)(0.25)} = 2000 \quad \text{and} \quad x_2 = \frac{I}{(\alpha + 1)} = \frac{1500}{1.5} = 1000. \quad (6.56)$$

The utility of this bundle is then

$$u(\text{price subsidy}) = u(2000, 1000) = 0.5 \ln(2000) + \ln(1000) = 10.708. \quad (6.57)$$

Since food consumption under the price subsidy is 2000, this implies $S = 0.75(2000) = 1500$. If $S = 1500$ is simply given as cash (where income then becomes \$3000 and p_1 goes back up to 1), this will result in food and other consumption of

$$x_1 = \frac{\alpha I}{(\alpha + 1)p_1} = \frac{0.5(3000)}{(1.5)(1)} = 1000 \quad \text{and} \quad x_2 = \frac{I}{(\alpha + 1)} = \frac{3000}{1.5} = 2000, \quad (6.58)$$

giving utility of

$$u(\text{cash}) = u(1000, 2000) = 0.5 \ln(1000) + \ln(2000) = 11.055. \quad (6.59)$$

Finally, under the food stamp program of size $S = 1500$, the household would be forced to consume \$1500 of food rather than \$1000 of food that it would have chosen had the money been given in terms of unrestricted cash. Thus, under food stamps, the consumer would buy the bundle (1500, 1500) giving utility

$$u(\text{food stamps}) = u(1500, 1500) = 0.5 \ln(1500) + \ln(1500) = 10.970. \quad (6.60)$$

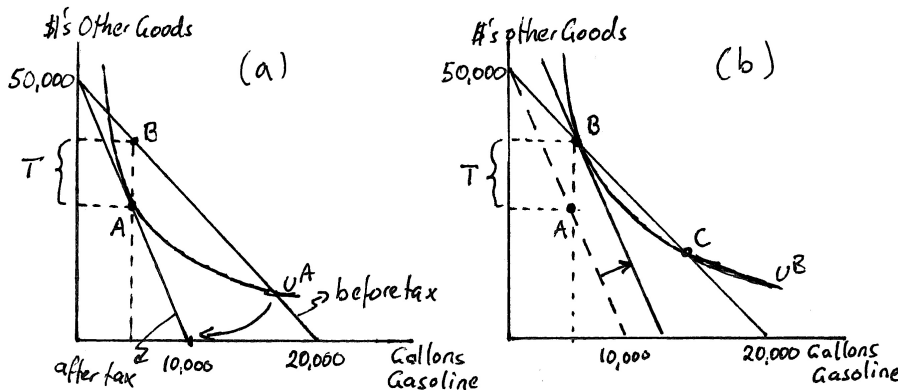
Here the food stamp program has gotten so large that it is no longer equivalent to getting cash — and so the consumer prefers the cash to the food stamps but still prefers the food stamps to the food price subsidy.

6.14 Policy Application: Gasoline Taxes and Tax Rebates: Given the concerns about environmental damage from car pollution, many have proposed increasing the tax on gasoline. We will consider the social benefits of such legislation later on in the text when we introduce externalities. For now, however, we can look at the impact on a single consumer.

A: Suppose a consumer has annual income of \$50,000 and suppose the price of a gallon of gasoline is currently \$2.50.

- (a) Illustrate the consumer's budget constraint with "gallons of gasoline" per year on the horizontal axis and "dollars spent on other goods" on the vertical. Then illustrate how this changes if the government imposes a tax on gasoline that raises the price per gallon to \$5.00.

Answer: In panel (a) of Graph 6.19, the original budget constraint runs from \$50,000 on the vertical axis to 20,000 gallons of gasoline on the horizontal — with slope -2.5 which is the pre-tax opportunity cost of gasoline. The tax increases that opportunity cost to 5 — thus rotating the budget constraint in, with the consumer being able to now purchase at most 10,000 gallons of gasoline per year.



Graph 6.19: Gasoline Taxes and Rebates

- (b) Pick some bundle A on the after tax budget constraint and assume that bundle is the optimal bundle for our consumer. Illustrate in your graph how much in gasoline taxes this consumer is paying and call this amount T.

Answer: This is also depicted in panel (a) of the graph. Once we have picked A, we simply ask how much other consumption is left when we consume this much gasoline after the tax — and compare it to how much other consumption would have been left had we consumed the same amount of gasoline before the tax (at B). The difference is T.

- (c) One of the concerns about using gasoline taxes to combat pollution is that it will impose hardship on consumers (and, perhaps more importantly, voters). Some have therefore suggested that the government simply rebate all revenues from a gasoline tax to taxpayers. Suppose that our consumer receives a rebate of exactly T. Illustrate how this alters the budget of our consumer.

Answer: In panel (b) of Graph 6.19 we show the after tax (before rebate) budget with a dashed line containing the optimal after tax (before rebate) bundle A and the tax payment T. A rebate of T would shift up this budget without changing prices — which implies the new budget that includes the tax and rebate goes through bundle B.

- (d) Suppose our consumer's tastes are quasilinear in gasoline. How much gasoline will she consume after getting the rebate?

Answer: If tastes are quasilinear in gasoline, then the MRS along any vertical line is the same. Thus, if point A is a tangency between an indifference curve and the after-tax (before-rebate) budget, then point B must also lie at a tangency between an indifference curve and the parallel after-tax, after-rebate budget. Thus, she will buy the same amount of gasoline after the rebate as she did before (as long as she faces the same tax-distorted price in both cases).

- (e) Can you tell whether the tax/rebate policy is successful at getting our consumer to consume less gasoline than she would were there neither the tax nor the rebate?

Answer: Yes. The indifference curve u^B cuts the original (before-tax, before-rebate) budget from above — causing bundles to the right of B and above the indifference curve u^B to become available. These are better than B — so the consumer would choose some bundle between B and C in the graph. All of those bundles have more gasoline — so the tax/rebate is effective at lowering gasoline consumption.

- (f) True or False: Since the government is giving back in the form of a rebate exactly the same amount as it collected in gasoline taxes from our consumer, the consumer is made no better or worse off from the tax/rebate policy.

Answer: False. While it is true that the consumer gets back the dollars she sent to the government as tax payment, she nevertheless ends up on a lower indifference curve than the pre-tax, pre-rebate indifference curve that is tangent to the original budget somewhere between B and C . Thus, the consumer is worse off as a result of the tax/rebate. (This ignores any potential benefit to the consumer from lower pollution that results from everyone consuming less gasoline.)

B: Suppose our consumer's tastes can be captured by the quasilinear utility function $u(x_1, x_2) = 200x_1^{0.5} + x_2$, where x_1 denotes gallons of gasoline and x_2 denotes dollars of other goods.

- (a) Calculate how much gasoline this consumer consumes as a function of the price of gasoline (p_1) and income I . Since other consumption is denominated in dollars, you can simply set its price (p_2) to 1.

Answer: We have to solve the problem

$$\max_{x_1, x_2} u(x_1, x_2) = 200x_1^{0.5} + x_2 \quad \text{subject to} \quad p_1 x_1 + x_2 = I. \quad (6.61)$$

The Lagrange function is then

$$\mathcal{L}(x_1, x_2, \lambda) = 200x_1^{0.5} + x_2 + \lambda(I - p_1 x_1 - x_2). \quad (6.62)$$

The first two first order conditions are

$$\begin{aligned} 0.5(200)x_1^{-0.5} &= \lambda p_1 \\ 1 &= \lambda, \end{aligned} \quad (6.63)$$

which implies $x_1^{0.5} = 100/p_1$ or $x_1 = 10000/(p_1^2)$. Note that gasoline consumption is therefore not a function of income — which is a consequence of the fact that utility is quasilinear in x_1 .

- (b) After the tax raises the price of gasoline to \$5, how much gasoline does our consumer purchase this year?

Answer: $x_1 = 10000/(p_1^2) = 10000/25 = 400$.

- (c) How much of a tax does she pay?

Answer: By purchasing 400 gallons at \$5 per gallon, she is spending \$2,000 on gasoline, half of which is due to the tax. Thus, she is paying $T = \$1000$.

- (d) Can you verify that her gasoline consumption will not change when the government sends her a rebate check equal to the tax payments she has made?

Answer: We determined that her gasoline consumption is $x_1 = 10000/(p_1^2)$ and is not a function of income. Since the rebate does not change the price p_1 , it does not impact x_1 — and therefore gasoline consumption remains the same after the rebate.

- (e) *How does annual gasoline consumption for our consumer differ under the tax/rebate program from what it would be in the absence of either a tax or rebate?*

Answer: In the absence of either program, the price of gasoline would be $p_1 = 2.5$. Thus $x_1 = 10000/(p_1^2) = 10000/(2.5^2) = 1600$. The tax/rebate program has therefore reduced this consumer's gasoline consumption from 1600 gallons per year to 400 gallons per year.

- (f) *Illustrate that our consumer would prefer no tax/rebate program but, if there is to be a tax on gasoline, she would prefer to have the rebate rather than no rebate.*

Answer: Without the tax/rebate program, the consumer buys 1600 gallons of gasoline. This costs $1600(\$2.50) = \$4,000$ which leaves \$46,000 for other consumption. The before tax/rebate utility is therefore

$$u(\text{no tax, no rebate}) = u(1600, 46000) = 200(1600^{0.5}) + 46000 = 54000. \quad (6.64)$$

With the tax and rebate, the consumer buys 400 gallons of gasoline at \$2000 but also gets a \$1000 rebate check — leaving her with \$49,000 in other consumption for utility of

$$u(\text{tax and rebate}) = u(400, 49000) = 200(400^{0.5}) + 49000 = 53000. \quad (6.65)$$

Finally, if only a tax were imposed without a rebate program, the consumer would still buy 400 gallons of gasoline for \$2000 — but now would only be left with \$48,000 in other consumption (because she does not get a rebate check). This leaves her with utility

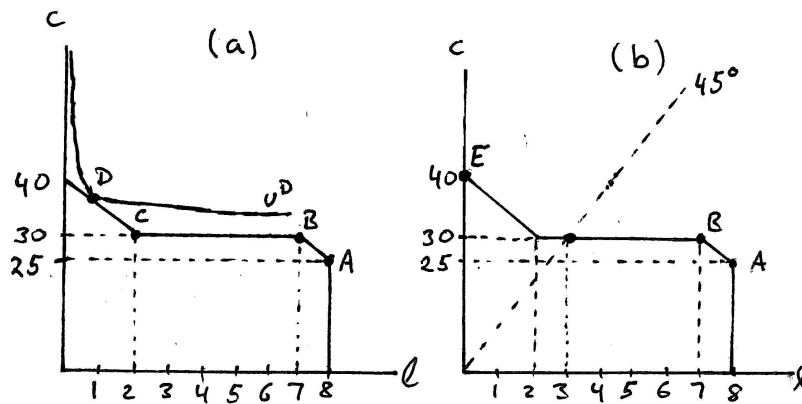
$$u(\text{tax and no rebate}) = u(400, 48000) = 200(400^{0.5}) + 48000 = 52000. \quad (6.66)$$

6.15 Policy Application: AFDC and Work Disincentives: Consider the AFDC program for an individual as described in end-of-chapter exercise 3.18.

A: Consider again an individual who can work up to 8 hours per day at a wage of \$5 per hour.

(a) Replicate the budget constraint you were asked to illustrate in 3.18A.

Answer: This is done in panel (a) of Graph 6.20, with leisure hours on the horizontal and consumption dollars on the vertical axis.



Graph 6.20: AFDC and Work Disincentives

(b) True or False: If this person's tastes are homothetic, then he/she will work no more than 1 hour per day.

Answer: This is false. Suppose, for instance, that leisure and consumption were perfect complements in the sense that this person wants to consume 1 hour of leisure with every \$35 of consumption. Indifference curves would then be L-shaped, with corners happening at bundles like (1,35) and (2,70). This would imply an optimal choice at (1,35) where the worker takes exactly 1 hour of leisure per day and works 7 hours per day. Such tastes are homothetic, as are less extreme tastes that allow for some (but not too much) substitutability between leisure and consumption. An example of an indifference curve u^D from a somewhat less extreme indifference map is illustrated in panel (a) of the graph — with tangency at D.

(c) For purposes of defining a 45-degree line for this part of the question, assume that you have drawn hours on the horizontal axis 10 times as large as dollars on the vertical. This implies that the 45-degree line contains bundles like (1,10), (2,20), etc. How much would this person work if his tastes are homothetic and symmetric across this 45-degree line? (By "symmetric across the 45-degree line" I mean that the portions of the indifference curves to one side of the 45 degree line are mirror images to the portions of the indifference curves to the other side of the 45 degree line.)

Answer: Panel (b) of the graph depicts this "45 degree line" where \$10 on the vertical axis is the same distance as 1 hour on the horizontal. In order for indifference curves to be symmetric around this line, it must be that the slope of the indifference curve for bundles on the 45 degree line is -1 . But since we are measuring \$10 as geometrically equivalent to 1 hour, a slope of -1 is really a slope, or MRS of -10 . If we were to draw a line from the point (0,40) to (3,30), this line would have a slope of $-10/3$. But any indifference curve has a slope of -10 on the 45 degree line — so we know that the indifference curve at (3,30) has a slope of

–10 at that point and gets steeper to the left. So all indifference curves going through (3,30) or above on the 45 degree line pass above the budget constraint to the left of the 45 degree line. Thus, such “symmetric” tastes will have an optimum to the right of the 45 degree line — most likely at *B* but plausibly between *B* and *A*.

- (d) Suppose you knew that the individual's indifference curves were linear but you did not know the MRS. Which bundles on the budget constraint could in principle be optimal and for what ranges of the MRS?

Answer: Bundles on the budget between *A* and *B* could be optimal, as could bundle *E*. In particular for MRS between 0 and –10/7, *E* would be optimal and the individual would work all the time and take no leisure. This is because indifference curves would be straight lines with sufficiently shallow slope to make the corner solution *E* optimal. For MRS between –10/7 and –5, *B* would be optimal. For MRS = –5, any bundle on the budget between *B* and *A* is optimal, with all these bundles lying on one indifference curve that is also the highest possible indifference curve for such an individual. Finally, for MRS less than –5, *A* becomes the optimal bundle.

- (e) Suppose you knew that, for a particular person facing this budget constraint, there are two optimal solutions. How much in AFDC payments does this person collect at each of these optimal bundles (assuming the person's tastes satisfy our usual assumptions)?

Answer: The only way there can be exactly two optimal solutions is if one of these is *B* and the other lies anywhere from *E* to *C*. The person collects no AFDC between *E* and *C* but the full \$25 daily benefit at *B*.

B: Suppose this worker's tastes can be summarized by the Cobb-Douglas utility function $u(\ell, c) = \ell^{1-\alpha} c^\alpha$ where ℓ stands for leisure and c for consumption.

- (a) Forget for a moment the AFDC program and suppose that the budget constraint for our worker could simply be written as $c = I - 5\ell$. Calculate the optimal amount of consumption and leisure as a function of α and I .

Answer: We need to solve the problem

$$\max_{\ell, c} u(\ell, c) = \ell^{1-\alpha} c^\alpha \text{ subject to } c = I - 5\ell. \quad (6.67)$$

Setting up the Lagrangian, taking first order conditions and solving for ℓ and c , we get

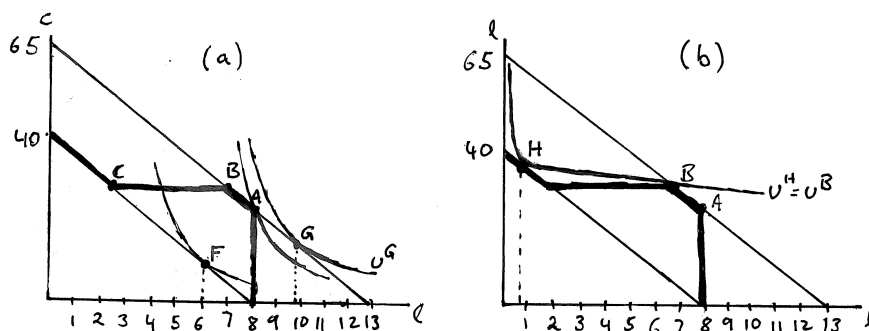
$$\ell = \frac{(1-\alpha)I}{5} \text{ and } c = \alpha I. \quad (6.68)$$

- (b) On your graph of the AFDC budget constraint for this worker, there are two line segments with slope –5 — one for 0-2 hours of leisure and another for 7-8 hours of leisure. Each of these lie on a line defined by $c = I - 5\ell$ except that I is different for the two equations that contain these line segments. What are the relevant I 's to identify the right equations on which these budget constraint segments lie?

Answer: It's easy to see from the graph that I is 40 for the lower line and 65 for the higher.

- (c) Suppose $\alpha = 0.25$. If this worker were to optimize using the two budget constraints you have identified with the two different I 's, how much leisure would he choose under each constraint? Can you illustrate what you find in a graph and tell from this where on the real AFDC budget constraint this worker will optimize?

Answer: When $I = 40$, he would optimize at $\ell = (1 - 0.25)40/5 = 6$ and when $I = 65$, he would optimize at $\ell = (1 - 0.25)65/5 = 9.75$. This is illustrated in panel (a) of Graph 6.21 where *F* with 6 hours of leisure occurs on the lower budget line and *G* with 9.75 hours of leisure occurs on the higher. *F* cannot be optimal inside the (bold) AFDC budget because it lies inside that budget. *G*, on the other hand, lies outside the (bold) AFDC budget and is therefore not feasible. But we do see that the indifference curve u^G is steeper than –5 on the ray connecting the origin to the kink point *A* — which implies the highest possible indifference curve on the bold AFDC budget goes through that kink point. Utility at $A = (8, 25)$, for instance, would be $u(8, 25) = 8^{0.75} 25^{0.25} = 10.63$ while utility at $B = (7, 30)$ is $u(7, 30) = 7^{0.75} 30^{0.25} = 10.07$. Thus, the real optimum when $\alpha = 0.25$ is bundle *A* with no work and all leisure.



Graph 6.21: AFDC and Work Disincentives: Part 2

- (d) As α increases, what happens to the MRS at each bundle?

Answer: The MRS for $u(\ell, c) = \ell^{1-\alpha} c^\alpha$ is

$$MRS = -\frac{\partial u / \partial \ell}{\partial u / \partial c} = -\frac{-(1-\alpha)\ell^{-\alpha} c^\alpha}{\alpha \ell^{1-\alpha} c^{\alpha-1}} = -\frac{(1-\alpha)c}{\alpha \ell}. \quad (6.69)$$

Thus, at any bundle (ℓ, c) , the MRS becomes larger in absolute value as α decreases and smaller in absolute value as α increases. Put differently, the slope of an indifference curve at any bundle becomes steeper as α gets smaller and shallower as α gets larger.

- (e) Repeat B(c) for $\alpha = 0.3846$ and for $\alpha = 0.4615$. What can you now say about this worker's choice for any $0 < \alpha < 0.3846$? What can you say about this worker's leisure choice if $0.3846 < \alpha < 0.4615$?

Answer: When $\alpha = 0.3846$, $\ell = (1 - 0.3846)40/5 = 4.92$ at the lower budget line and $\ell = (1 - 0.3846)65/5 = 8$ on the higher budget line. The solution on the lower budget line lies inside the AFDC budget and is therefore not optimal. The solution of 8 hours of leisure on the higher budget, on the other hand, is within the AFDC budget — it is bundle A. Thus, when $\alpha = 0.3846$, the highest possible indifference curve on the AFDC budget is just tangent to the extended budget line $c = 65 - 5\ell$ at A. Since lower α 's mean steeper indifference curves at every point, we can conclude from that that A will be optimal for all α 's that lie between 0 and 0.3846. When $\alpha = 0.4615$, $\ell = (1 - 0.4615)40/5 = 4.31$ at the lower budget line and $\ell = (1 - 0.4615)65/5 = 7$ on the higher budget line. The solution on the lower budget line is again inside the AFDC budget — so it cannot be optimal. The solution of 7 leisure hours on the higher budget, on the other hand, corresponds to B on the AFDC budget. Thus, when $\alpha = 0.4615$, the highest indifference curve on the AFDC budget is just tangent to the extended budget line $c = 65 - 5\ell$ at B. Since the slope of indifference curves becomes steeper as α falls, this implies that, for α between 0.3846 and 0.4615, the optimal leisure choice will lie in between A and B on the AFDC budget at $\ell = (1 - \alpha)65/5 = 13(1 - \alpha)$.

- (f) Repeat B(c) for $\alpha = 0.9214$ and calculate the utility associated with the resulting choice. Compare this to the utility of consuming at the kink point (7,30) and illustrate what you have found on a graph. What can you conclude about this worker's choice if $0.4615 < \alpha < 0.9214$?

Answer: When $\alpha = 0.9214$, $\ell = (1 - 0.9214)40/5 = 0.629$ giving consumption of $w(8 - \ell) = 5(8 - 0.629) = 36.856$. (On the higher budget line, $\ell = (1 - 0.9214)40/5 = 1.02$ which lies outside the AFDC budget). The bundle on the lower $c = 40 - 5\ell$ line, (0.629, 36.856), gives utility $u(0.629, 36.856) = 0.629^{(1-0.9214)} 36.856^{0.9214} = 26.76$. At B, the consumer would get utility $u(7, 30) = 7^{(1-0.9214)} 30^{0.9214} = 26.76$. Thus, the optimal bundle H on the budget line $c = 40 - 5\ell$ lies on the same indifference curve as B — as depicted in panel (b) of Graph 6.21. For $\alpha < 0.9214$, the indifference curve at H would be steeper and would therefore cut the

AFDC budget while passing below B — and thus B is optimal for α just below 0.9214. Thus B is the optimal bundle for $0.4615 < \alpha < 0.9214$.

- (g) *How much leisure will the worker take if $0.9214 < \alpha < 1$?*

Answer: Given that indifference curves become shallower at every bundle as α increases, we know that the indifference curve at H will be shallower for $\alpha > 0.9214$ than the one depicted in panel (b) of Graph 6.21. This implies that the optimal bundle for $\alpha > 0.9214$ lies to the left of H at $\ell = (1 - \alpha)40/5 = 8(1 - \alpha)$.

- (h) *Describe in words what this tells you about what it would take for a worker to overcome the work disincentives under the AFDC program.*

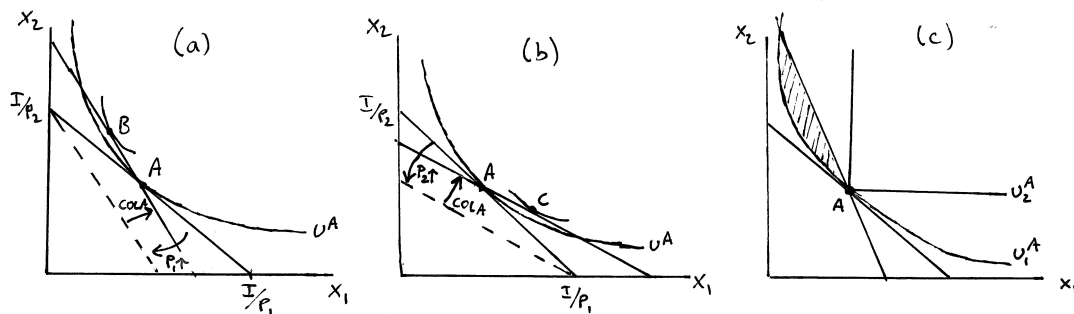
Answer: The exponent α tells us how much weight a person places in his tastes on consumption rather than leisure. When α is high, consumption is valued much more than leisure — so even a small increase in consumption can justify giving up a lot of leisure. Thus, for very high α , it is possible that someone with the AFDC budget constraint will in fact work close to full time despite the work disincentives. But that person's tastes would have to be pretty extreme — he would have to place virtually no value on leisure time. For anyone that places some non-trivial value on leisure time — which implies α isn't close to 1 or, to be more precise, $\alpha < 0.9214$ — the payoff from working close to full time is simply not high enough to sacrifice that much leisure. Thus, for most values of α , the person will choose to work less than 1 hour per day.

6.16 Policy Application: Cost of Living Adjustments of Social Security Benefits: Social Security payments to the elderly are adjusted every year in the following way: The government has in the past determined some average bundle of goods consumed by an average elderly person. Each year, the government then takes a look at changes in the prices of all the goods in that bundle and raises social security payments by the percentage required to allow the hypothetical elderly person to continue consuming that same bundle. This is referred to as a “cost of living adjustment” or COLA.

A: Consider the impact on an average senior’s budget constraint as cost of living adjustments are put in place. Analyze this in a 2-good model where the goods are simply x_1 and x_2 .

- (a) Begin by drawing such a budget constraint in a graph where you indicate the “average bundle” the government has identified as A and assume that initially this average bundle is indeed the one our average senior would have chosen from his budget.

Answer: This is done in panel (a) of Graph 6.22 with the initial budget line going from I/p_2 on the vertical to I/p_1 on the horizontal and with u^A tangent to that budget line at A.



Graph 6.22: Social Security COLAs

- (b) Suppose the prices of both goods went up by exactly the same proportion. After the government implements the COLA, has anything changed for the average senior? Is behavior likely to change?

Answer: If both prices go up by exactly the same proportion, the slope of the budget line $-p_1/p_2$ does not change. Rather, in the absence of a COLA adjustment, the budget would simply shift inward in a parallel way. But if the government will give enough additional money to the senior to allow him to again consume A after the increase in prices, then the government shifts the budget right back through A. Since the slope has not changed, the new budget lies exactly on top of the old. Thus, the senior will not change his behavior — A will still be optimal.

- (c) Now suppose that the price of x_1 went up but the price of x_2 stayed the same. Illustrate how the government will change the average senior’s budget constraint when it calculates and passes along the COLA. Will the senior alter his behavior? Is he better off, worse off or not affected?

Answer: This is also illustrated in panel (a) of the graph. The increase in p_1 causes the rotation in the budget to the dashed budget line, and the COLA adjustment shifts that steeper budget up to bundle A so that the person can still afford A. But in the process, the new budget cuts the original indifference curve in a way that makes new bundles above u^A available. These bundles are more preferred by the senior — so the senior will optimize at a new bundle like B which contains less of x_1 and more of x_2 . As a result, the senior moves to a higher indifference curve and is therefore better off.

- (d) How would your answers change if the price of x_2 increased and the price of x_1 stayed the same?

Answer: This is illustrated in panel (b) of Graph 6.22. Now the increase in p_2 causes the rotation of the budget to the dashed budget line, and the COLA adjustment then pushes this line outward until it once again goes through A . In the process, new bundles that lie above u^A become available to the right of A , causing the person to optimize at a new bundle like C . Thus, the senior will reduce consumption of x_2 and increase consumption of x_1 — and will end up on a higher indifference curve at C . Thus, the senior is better off as a result of the price increase and COLA adjustment.

- (e) Suppose the government's goal in paying COLAs to senior citizens is to insure that seniors become neither better nor worse off from price changes. Is the current policy successful if all price changes come in the form of general “inflation” — i.e. if all prices always change together by the same proportion? What if inflation hits some categories of goods more than others?

Answer: The policy is successful when inflation is of a general kind and affects all prices by the same proportion. But if prices change at different rates, the COLA adjustment is too large to keep seniors just as happy as before.

- (f) If you could “choose” your tastes under this system, would you choose tastes for which goods are highly substitutable or would you choose tastes for which goods are highly complementary?

Answer: Seniors benefit from COLA adjustments to the extent to which prices change differentially and the extent to which they are willing to substitute between goods. Consider, for instance, the impact of a higher price p_1 (like the one graphed in panel (a)) in the case of the two indifference curves u_1^A and u_2^A in panel (c) of the graph. The indifference curve u_2^A treats x_1 and x_2 as perfect complements — and as a result, the steeper new budget line with the COLA adjustment does not cut the indifference curve because the indifference curve has a sharp corner at A . Since the person with this indifference curve does not view x_1 and x_2 as in any way substitutable, he continues to buy A on the new budget. He therefore does not become any better off as a result of the COLA adjustment. Compare this to the individual with indifference curve u_1^A . This indifference curve is one that treats x_1 and x_2 as quite substitutable — and as a result, a large number of “better” bundles become available in the new COLA budget. These are indicated in the graph as the shaded area between u_1^A and the new budget constraint. Thus, individuals who view goods as more substitutable benefit more from the way the government adjusts social security checks in response to price changes.

B: Suppose the average senior has tastes that can be captured by the utility function $u(x_1, x_2) = (x_1^{-\rho} + x_2^{-\rho})^{-1/\rho}$.

- (a) Suppose the average senior has income from all sources equal to \$40,000 per year and suppose that prices are given by p_1 and p_2 . How much will our senior consume of x_1 and x_2 ? (Hint: It may be easiest to simply use what you know about the MRS of CES utility functions to solve this problem.)

Answer: For the general CES utility function $v(x_1, x_2) = (\alpha x_1^{-\rho} + \beta x_2^{-\rho})^{-1/\rho}$, the text derives the MRS to be $MRS = -(\alpha/\beta)(x_2/x_1)^{(\rho+1)}$. In the utility function u of this problem, α and β are implicitly set to 1. Thus, $MRS = -(x_2/x_1)^{(\rho+1)}$. At an optimum, $MRS = -p_1/p_2$, or

$$-\left(\frac{x_2}{x_1}\right)^{\rho+1} = -\frac{p_1}{p_2}. \quad (6.70)$$

Solving for x_2 , we get

$$x_2 = \left(\frac{p_1}{p_2}\right)^{1/(\rho+1)} x_1. \quad (6.71)$$

Substituting this into the budget constraint $p_1 x_1 + p_2 x_2 = 40000$ and solving for x_1 , we then get (after some tricky manipulation of exponents)

$$x_1 = \frac{40000}{p_1 + (p_2^\rho p_1)^{1/(\rho+1)}}. \quad (6.72)$$

Substituting this back into equation (6.71), we can then also solve for x_2 (after some more tricky exponents) as

$$x_2 = \frac{40000}{p_2 + (p_1^\rho p_2)^{1/(\rho+1)}}. \quad (6.73)$$

- (b) If $p_1 = p_2 = 1$ initially, how much of each good will the senior consume? Does your answer depend on the elasticity of substitution?

Answer: Substituting $p_1 = p_2 = 1$ into equations (6.72) and (6.73), we get

$$\begin{aligned} x_1 &= \frac{40000}{1 + (1^\rho 1)^{1/(\rho+1)}} = \frac{40000}{2} = 20,000 \text{ and} \\ x_2 &= \frac{40000}{1 + (1^\rho 1)^{1/(\rho+1)}} = \frac{40000}{2} = 20,000. \end{aligned} \quad (6.74)$$

Thus, the solution does not depend on ρ and therefore does not depend on the elasticity of substitution.

- (c) Now suppose that the price of x_1 increases to $p_1 = 1.25$. How much does the government have to increase the senior's social security payment in order for the senior to still be able to purchase the same bundle as he purchased prior to the price change?

Answer: Before the price change, the senior purchases (20000, 20000). To be able to afford this bundle after the price change, he must have income of $1.25(20000) + 1(20000) = 45000$, or \$5000 more than he had before. Thus, the COLA adjustment is \$5000.

- (d) Assuming the government adjusts the social security payment to allow the senior to continue to purchase the same bundle as before the price increase, how much x_1 and x_2 will the senior actually end up buying if $\rho = 0$?

Answer: The new income is now \$45000. Thus, when $\rho = 0$ and income is raised to \$45000, equations (6.72) and (6.73) reduce to

$$x_1 = \frac{45000}{p_1 + (p_2^0 p_1)^{1/(0+1)}} = \frac{45000}{2p_1} \text{ and } x_2 = \frac{45000}{p_2 + (p_1^0 p_2)^{1/(0+1)}} = \frac{45000}{2p_2}. \quad (6.75)$$

Thus, when $p_1 = 1.25$ and $p_2 = 1$, we get

$$x_1 = \frac{40000}{2(1.25)} = 18,000 \text{ and } x_2 = \frac{40000}{2(1)} = 22,500. \quad (6.76)$$

- (e) How does your answer change if $\rho = -0.5$ and if $\rho = -0.95$? What happens as ρ approaches -1 ?

Answer: When $\rho = -0.5$ and income again is set to \$45000, equations (6.72) and (6.73) give

$$x_1 = 16,000 \text{ and } x_2 = 25,000, \quad (6.77)$$

and when $\rho = -0.95$,

$$x_1 = 511 \text{ and } x_2 = 44,361. \quad (6.78)$$

Thus, the smaller ρ gets, the more the person will deviate from the original bundle. And as ρ approaches -1 , x_1 approaches 0 while x_2 approaches 45,000.

- (f) How does your answer change when $\rho = 1$ and when $\rho = 10$? What happens as ρ approaches infinity?

Answer: Again using equations (6.72) and (6.73), when $\rho = 1$,

$$x_1 = 19,003 \text{ and } x_2 = 21,246. \quad (6.79)$$

and when $\rho = 10$,

$$x_1 = 19,819 \text{ and } x_2 = 20,225. \quad (6.80)$$

Thus, the larger ρ gets, the less the person will deviate from his original bundle. And as ρ approaches infinity, x_1 goes to 20,000, as does x_2 ; i.e. as ρ approaches infinity, the change in behavior disappears.

- (g) *Can you come to a conclusion about the relationship between how much a senior benefits from the way the government calculates COLAs and the elasticity of substitution that the senior's tastes exhibit? Can you explain intuitively how this makes sense, particularly in light of your answer to A(f)?*

Answer: Above we noticed that as ρ gets smaller, the person deviates more from his original consumption bundle when prices change and COLAs are implemented. When p_1 increases from 1 to 1.25, he does not change his consumption when ρ is very large but reduces his consumption from 20,000 to 19,003 when $\rho = 1$, to 18,000 when $\rho = 0$, to 16,000 when $\rho = -0.5$, to 511 when $\rho = -0.95$ and finally to 0 as ρ approaches zero. The elasticity of substitution is $\sigma = 1/(1 + \rho)$ — which implies that, as ρ gets smaller, the elasticity of substitution increases. Thus, our results above say that, as the elasticity of substitution gets larger, the person deviates more from his original consumption bundle. This is exactly what our intuition told us in panel (c) of Graph 6.22 in our answer to A(f).

- (h) *Finally, show how COLAs affect consumption decisions by seniors under general inflation that raises all prices simultaneously and in proportion to one another as, for instance, when both p_1 and p_2 increase from 1 to 1.25 simultaneously.*

Answer: When both p_1 and p_2 increase to 1.25, the COLA has to be sufficient for the senior to once again be able to afford (20000,20000) at the new prices. This implies the senior needs an income of $1.25(20000) + 1.25(20000) = 50000$, which is 10000 more than the 40000 he has before the inflation. Thus, the COLA has to be \$10,000. Replacing 40000 with 50000 in equations (6.72) and (6.73) and replacing p_1 with $1.25p_1$ and p_2 with $1.25p_2$, we get

$$\begin{aligned} x_1 &= \frac{50000}{1.25p_1 + ((1.25p_2)^\rho (1.25p_1))^{1/(\rho+1)}} \\ &= \frac{50000}{1.25 \left(p_1 + (p_2^\rho p_1)^{1/(\rho+1)} \right)} = \frac{40000}{p_1 + (p_2^\rho p_1)^{1/(\rho+1)}}. \end{aligned} \quad (6.81)$$

and

$$\begin{aligned} x_2 &= \frac{50000}{1.25p_2 + ((1.25p_1)^\rho (1.25p_2))^{1/(\rho+1)}} \\ &= \frac{50000}{1.25 \left(p_2 + (p_1^\rho p_2)^{1/(\rho+1)} \right)} = \frac{40000}{p_2 + (p_1^\rho p_2)^{1/(\rho+1)}}, \end{aligned} \quad (6.82)$$

both of which are identical to what we had before the inflation and COLA increase.