SOLUTIONS

8

Wealth and Substitution Effects in Labor and Capital Markets

Solutions for *Microeconomics: An Intuitive Approach with Calculus*

Apart from end-of-chapter exercises provided in the student *Study Guide*, these solutions are provided for use by instructors. (End-of-Chapter exercises with solutions in the student *Study Guide* are so marked in the textbook.)

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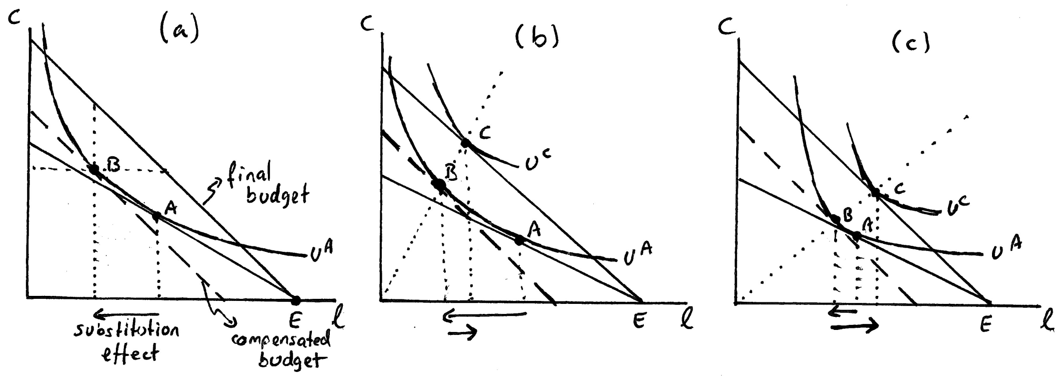
- Each end-of-chapter exercise begins on a new page. This is to facilitate maximum flexibility for instructors who may wish to share answers to some but not all exercises with their students.
- If you are assigning only the A-parts of exercises in *Microeconomics: An Intuitive Approach with Calculus*, you may wish to instead use the solution set created for the companion book *Microeconomics: An Intuitive Approach*.
- *Solutions to Within-Chapter Exercises are provided in the student* Study Guide.

8.1 *As we have suggested in the chapter, it is often important to know whether workers will work more or less as their wage increases.*

A: *In each of the following cases, can you tell whether a worker will work more or less as his wage increases?*

(a) *The worker's tastes over consumption and leisure are quasilinear in leisure.*

Answer: Panel (a) of Graph 8.1 illustrates the substitution effect for a wage increase. This effect depends only on the shape of the indifference curve that goes through the original bundle A — the more substitutable consumption and leisure are, the greater the substitution from leisure (and thus toward more labor) to consumption. If tastes are then quasilinear in leisure, we know that, as we move from the compensated to the final budget, there is no wealth effect on leisure and thus no further change in leisure (beyond the substitution effect). Thus, the worker will unambiguously work more.



Graph 8.1: Wage Increases with Different Tastes

(b) *The worker's tastes over consumption and leisure are homothetic.*

Answer: Panels (b) and (c) of Graph 8.1 illustrate that it is ambiguous in this case whether the worker will work more or less with an increase in the wage — it depends on the size of the substitution effect. In panel (b), the indifference curve u^A is relatively flat around A — indicating a great deal of willingness on the part of the worker to substitute leisure and consumption. This gives rise to a large substitution effect. B is tangent to the (dashed) compensated budget — which is parallel to the final budget. Homotheticity then implies that, if B is optimal on the compensated budget, the optimal final bundle C lies on a ray from the origin through B. Because of the willingness to substitute between consumption and leisure, the resulting wealth effect only outweighs part of the substitution effect — leaving us with less leisure (and more labor) at the higher wage than at the original lower wage (at A). In panel (c), on the other hand, consumption and leisure are not as substitutable around A — leading to a relatively small substitution effect that is more than outweighed by a wealth effect in the opposite direction. Thus, when consumption and leisure are relatively complementary, an increase in the wage causes an increase in leisure and thus a decrease in work hours.

(c) *Leisure is a luxury good.*

Answer: We can use the same graphs as in panels (b) and (c) to again show that the answer is ambiguous. If leisure is a luxury good, then as the budget shifts out parallel, the new optimal bundle will lie to the right of the ray from the origin through the original optimum (because consumption of leisure increases faster than under homotheticity). In panel (b), that would mean C lies to the right of where it is indicated in the graph — but that still makes it plausible that the wealth effect is smaller than the substitution effect leaving us with less leisure than at A (and thus more work). In panel (c), C will again lie to the right of where it is indicated in the graph — but that implies that the wealth effect is even larger and will still outweigh the substitution effect. This will again leave us with more leisure and thus less work.

(d) *Leisure is a necessity.*

Answer: For reasons analogous to those just cited for luxury goods, the answer is still ambiguous and depends on the size of the substitution effect. This time, C will lie to the left of where it is marked in panels (b) and (c) of the graph — but that still leaves room for the ambiguity.

(e) *The worker's tastes over consumption and leisure are quasilinear in consumption.*

Answer: Going back to panel (a) of the graph, if consumption is the quasilinear good, then it will remain unchanged from the optimal bundle B on the compensated budget to the final budget. This creates a wealth effect on leisure that is opposite to the substitution effect. As drawn in panel (a), it looks like that returns us to a bundle C that will lie right above A — thus returning us to the same leisure consumption (and thus the same amount of work) as before the wage increase. But had we drawn a smaller substitution effect, the horizontal line through B would take us to the right of A on the final budget — thus causing an increase in leisure (and a decrease in work). If, on the other hand, we had made the indifference curve u^A flatter and thus had produced a larger substitution effect, the horizontal line through B would take us to the left of A on the final budget — thus causing a decrease in leisure (and thus an increase in work) from the original optimum A. As is usually the case when we have competing substitution and wealth effects, the answer is therefore again ambiguous.

B: *Suppose that tastes take the form* $u(c,\ell) = (0.5c^{-\rho} + 0.5\ell^{-\rho})^{-1/\rho}$.

(a) *Set up the worker's optimization problem assuming his leisure endowment is L and his wage is w.*

Answer: The problem is

$$\max_{c,\ell} \left(0.5c^{-\rho} + 0.5\ell^{-\rho}\right)^{-1/\rho} \text{ subject to } w(L-\ell) = c. \tag{8.1}$$

(b) *Set up the Lagrange function corresponding to your maximization problem.*

Answer: The Lagrange function is

$$\mathcal{L}(c,\ell,\lambda) = \left(0.5c^{-\rho} + 0.5\ell^{-\rho}\right)^{-1/\rho} + \lambda(wL - w\ell - c). \tag{8.2}$$

(c) *Solve for the optimal amount of leisure.*

Answer: The first two first order conditions are

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial c} &= 0.5c^{-(\rho+1)}\left(0.5c^{-\rho} + 0.5\ell^{-\rho}\right)^{-(\rho+1)/\rho} - \lambda = 0,\\ \frac{\partial \mathcal{L}}{\partial \ell} &= 0.5\ell^{-(\rho+1)}\left(0.5c^{-\rho} + 0.5\ell^{-\rho}\right)^{-(\rho+1)/\rho} - \lambda w = 0.\end{aligned} \tag{8.3}$$

The problem simplifies quite a bit if we simply take the λ terms to the other side of each equation and then divide the second equation by the first — which gives

$$\left(\frac{c}{\ell}\right)^{(\rho+1)} = w. \tag{8.4}$$

If you remember the expression of the MRS for a CES utility function from Chapter 5, you could have just skipped to this equation — which simply says the MRS is equal to the slope of the budget. The equation can then be written in terms of just $c = \ell w^{1/(\rho+1)}$. When plugged into the budget constraint $w(L-\ell) = c$, we can solve for

$$\ell = \frac{L}{1 + w^{-\rho/(\rho+1)}}. \tag{8.5}$$

(d) *Does leisure consumption increase or decrease as w increases? What does your answer depend on?*

Answer: We can see whether leisure increases or decreases with the wage rate by checking whether the first derivative of the equation for optimal leisure consumption from above is positive or negative. This derivative is (after a little algebra)

$$\frac{\partial \ell}{\partial w} = \rho \left[\frac{L\left(1 + w^{-\rho/(\rho+1)}\right)^{-2}}{(\rho+1)\, w^{(2\rho+1)/(\rho+1)}} \right] \tag{8.6}$$

Note L and w are positive and, since ρ lies between -1 and ∞, $(\rho+1)$ is also positive. This implies that the entire term in the square brackets must be positive regardless of what value ρ takes. (The negatives in the exponents of course only affect whether the term appears in the numerator or denominator — not whether it is positive or not.) Since the bracketed term is positive, the sign of the derivative depends entirely on whether ρ is positive or negative.

If $\rho = 0$, the tastes are Cobb-Douglas with elasticity of substitution $1/(1-\rho) = 1$. In that case, $\partial\ell/\partial w = 0$ and the wage therefore does not affect leisure consumption (or labor supply). For $\rho < 0$ the elasticity of substitution is greater than 1 — and $\partial\ell/\partial w < 0$. Thus, as the elasticity of substitution rises above 1, leisure consumption declines with an increase in the wage — and work hours increase. For $\rho > 0$, on the other hand, the elasticity of substitution is less than 1 — and $\partial\ell/\partial w > 0$. Thus, as the elasticity of substitution falls below 1, leisure consumption increases with the wage — and work hours fall. (Note that the elasticity of substitution is $\sigma = 1/(1+\rho)$.)

(e) *Relate this to what you know about substitution and wealth effects in this type of problem.*

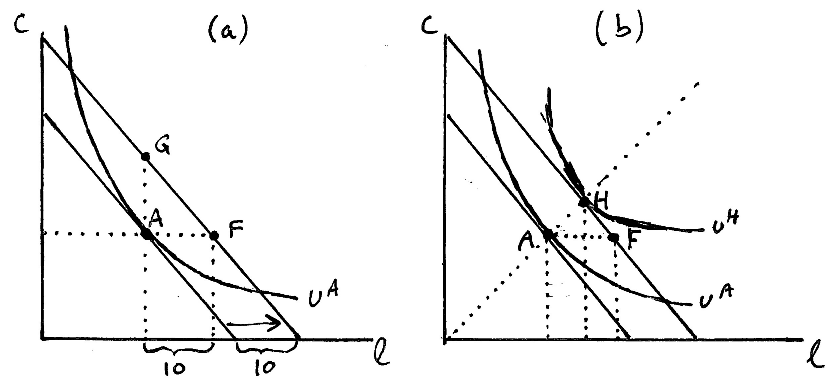
Answer: We have seen in part A of the question that the substitution effect points to less leisure (and more work) as wage increases — and, so long as leisure is a normal good, the wealth effect points in the opposite direction. For homothetic tastes (which CES tastes are), we showed that the overall effect of a wage increase on leisure consumption then depends on the substitutability of consumption and leisure. The greater the substitutability, the larger is the substitution effect — and the larger the substitution effect, the less likely it is that the wealth effect can fully offset it. We now see that for CES utility functions, the direction of the effect of a wage increase on leisure consumption depends entirely on ρ which determines the elasticity of substitution or the degree of substitutability between consumption and leisure. Elasticities below 1 make indifference curves look more like those in panel (c) of Graph 8.1 — with the wealth effect outweighing the substitution effect. Elasticities above 1, on the other hand, make the indifference curve look more like those in panel (b) where the substitution effect outweighs the wealth effect.

8.2 *Suppose that an invention has just resulted in everyone being able to cut their sleep requirement by 10 hours per week — thus providing an increase in their weekly leisure endowment.*

A: *For each of the cases below, can you tell whether a worker will work more or less?*

(a) *The worker's tastes over consumption and leisure are quasilinear in leisure.*

Answer: The increase in the leisure endowment without a change in wage shifts out the budget constraint without changing its slope. This is illustrated in panel (a) of Graph 8.2. Bundle A indicates the original optimal bundle. If F were the new optimal bundle on the new budget, then leisure consumption would have increased by exactly 10 hours — which means that labor hours would have remained exactly the same (since leisure endowment went up by 10 hours). Thus, in order for work hours to *decrease*, it would have to be the case that the new bundle lies to the right of F on the new budget. When tastes are quasilinear in leisure, the new optimal bundle lies at G — because there is no wealth effect on leisure. Thus, in this case the worker would work 10 hours more — devoting all additional leisure time to work.



Graph 8.2: Wage Increases with Different Tastes

(b) *The worker's tastes over consumption and leisure are homothetic.*

Answer: Panel (b) illustrates homothetic tastes. A is again the original optimal bundle. Under homotheticity, the MRS remains unchanged along any ray from the origin. This means that, since the slope of the budget has not changed, the new optimum will lie at H on the ray from the origin through A. This is to the left of F — and thus implies that work hours increase. In this case, the worker splits the additional 10 hours of leisure time — consuming part of it as leisure and spending part of it working.

(c) *Leisure is a luxury good.*

Answer: If leisure is a luxury good, then its consumption increases more than it would under homotheticity — which implies that the new optimal bundle will lie to the right of the ray from the origin through A. This could mean it will lie between H and F in panel (b) of the graph, or it could mean it will lie to the right of F. In the former case the worker still increases work hours; in the latter case he decreases work hours. (So long as consumption is a normal good, however, we know that the new optimum falls between H and F if leisure is a luxury good. It will lie to the right of F only if consumption is inferior.) Just knowing that leisure is a luxury good therefore does not allow us to definitively conclude whether the worker will work more or less with an increase in leisure endowment — it depends on whether consumption of other goods is normal or inferior.

(d) *Leisure is a necessity.*

Answer: If leisure is a necessity, than the new optimum will lie to the left of the ray from the origin through A. Thus, the new optimum lies to the left of H which means that the worker will definitely work more hours.

(e) *The worker's tastes over consumption and leisure are quasilinear in consumption.*

Answer: Returning to panel (a), quasilinearity in consumption implies that there are no wealth effects for consumption. Thus, the new optimum will lie at F — which we identified as the bundle at which work hours remain exactly as they were before (because the worker simply takes all the additional leisure endowment as leisure consumption).

(f) *Do any of your answers have anything to do with how substitutable consumption and leisure are? Why or why not?*

Answer: They do not — because substitution effects arise only when opportunity costs change — and here no opportunity cost has changed (as seen by the fact that the slope of the budget constraint does not change.)

B: *Suppose that a worker's tastes for consumption c and leisure ℓ can be represented by the utility function $u(c,\ell) = c^{\alpha}\ell^{(1-\alpha)}$.*

(a) *Write down the worker's constrained optimization problem and the Lagrange function used to solve it, using w to denote the wage and L to denote the leisure endowment.*

Answer: The problem is

$$\max_{c,\ell} c^{\alpha}\ell^{(1-\alpha)} \text{ subject to } w(L-\ell) = c. \tag{8.7}$$

The corresponding Lagrange function is

$$\mathscr{L}(c,\ell,\lambda) = c^{\alpha}\ell^{(1-\alpha)} + \lambda(wL - w\ell - c). \tag{8.8}$$

(b) *Solve the problem to determine leisure consumption as a function of w, α and L. Will an increase in L result in more or less Leisure consumption?*

Answer: The first two first order conditions are

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial c} &= \alpha c^{(\alpha-1)}\ell^{(1-\alpha)} - \lambda = 0, \\ \frac{\partial \mathscr{L}}{\partial \ell} &= (1-\alpha)c^{\alpha}\ell^{-\alpha} - \lambda w = 0. \end{aligned} \tag{8.9}$$

Taking the λ terms to the other side and dividing the equations by one another, we can solve for $c = (\alpha/(1-\alpha))w\ell$. Plugging this into the constraint, we can then solve for $\ell = (1-\alpha)L$. Since L clearly enters positively into this equation, an increase in L will increase leisure consumption. (You can show this more formally by simply taking the derivative of ℓ with respect to L — which is $(1-\alpha) > 0$.)

(c) *Can you determine whether an increase in leisure will cause the worker to work more?*

Answer: Suppose the leisure endowment increases from L to $L+H$. Then leisure consumption increases from $(1-\alpha)L$ to $(1-\alpha)(L+H)$. The number of hours spent working, however, is the leisure endowment minus the leisure consumption. Thus, initially the worker works for $L-(1-\alpha)L = \alpha L$ hours, and after the leisure increase, he works $(L+H)-(1-\alpha)(L+H) = \alpha(L+H)$. Thus, the worker works more after the increase in leisure — and spends a fraction α of his additional leisure on work.

(d) *Repeat the above parts using the utility function $u(c,\ell) = c + \alpha \ln \ell$ instead.*

Answer: The problem now becomes

$$\max_{c,\ell} u(c,\ell) = c + \alpha \ln \ell \text{ subject to } w(L-\ell) = c, \tag{8.10}$$

with corresponding Lagrange function

$$\mathscr{L}(c,\ell,\lambda) = c + \alpha \ln \ell + \lambda(wL - w\ell - c). \tag{8.11}$$

The first two first order conditions are

$$\frac{\partial \mathscr{L}}{\partial c} = 1 - \lambda = 0,$$
$$\frac{\partial \mathscr{L}}{\partial \ell} = \frac{\alpha}{\ell} - \lambda w = 0. \tag{8.12}$$

These solve straightforwardly to $\ell = \alpha/w$. Thus, leisure consumption is independent of leisure endowment (assuming we are not at a corner solution) — which means an increase in the leisure endowment translates directly into an increase in work hours by the same amount. This makes sense in light of our answer to A(a) given that the utility function here treats leisure as a quasilinear good.

(e) *Can you show that, if tastes can be represented by the CES utility function $u(c,\ell) = (\alpha c^{-\rho}(1-\alpha)\ell^{-\rho})^{-1/\rho}$, the worker will choose to consume more leisure as well as work more when there is an increase in the leisure endowment L? (Warning: The algebra gets a little messy. You can occasionally check your answers by substituting $\rho = 0$ and checking that this matches what you know to be true for the Cobb-Douglas function $u(c,\ell) = c^{0.5}\ell^{0.5}$.)*

Answer: Setting up the usual maximization problem and solving the first two first order conditions (or alternatively, recalling the CES formula for MRS and setting it equal to the slope of the budget line) gives us

$$c = \left(\frac{\alpha w}{(1-\alpha)}\right)^{1/(\rho+1)} \ell. \tag{8.13}$$

Substituting this into the constraint $w(L-\ell) = c$ and solving for ℓ we get

$$\ell = \frac{L}{1 + (\alpha w^{-\rho}/(1-\alpha))^{1/(\rho+1)}}. \tag{8.14}$$

(You can check that this is correct by substituting $\rho = 0$ to see if it gives the Cobb-Douglas answer $\ell = (1-\alpha)L$ — which it does.) The derivative of this function with respect to L is positive (since all the terms in the denominator are positive given $0 < \alpha < 1$ and $w > 0$. Thus, leisure consumption increases with leisure endowment regardless of the value of ρ — and thus regardless of the elasticity of substitution.

Labor hours l are just given by L minus leisure consumption; i.e.

$$l = L - \frac{L}{1 + (\alpha w^{-\rho}/(1-\alpha))^{1/(\rho+1)}} = \frac{L\left(\alpha w^{-\rho}/(1-\alpha)\right)^{1/(\rho+1)}}{1 + (\alpha w^{-\rho}/(1-\alpha))^{1/(\rho+1)}}. \tag{8.15}$$

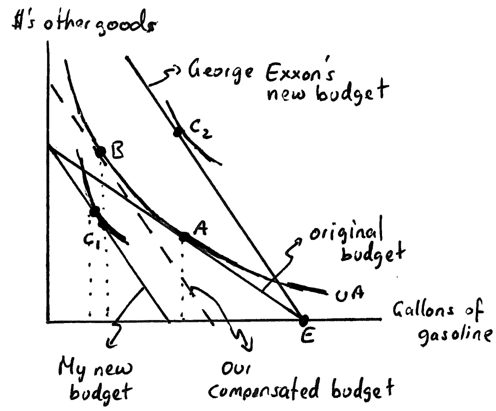
(You can again check that this is correct by substituting $\rho = 0$ to see if it gives the Cobb-Douglas answer $l = \alpha L$ — which it does.) The derivative of this with respect to L is similarly positive regardless of ρ. Thus, regardless of the elasticity of substitution, work hours increase as leisure endowment rises.

8.3 *In this chapter, we began by considering the impact of an increase in the price of gasoline on George Exxon who owns a lot of gasoline. In this exercise, assume that George and I have exactly the same tastes and that gasoline and other goods are both normal goods for us.*

A: *Unlike George Exxon, however, I do not own gasoline but simply survive on an exogenous income provided to me by my generous wife.*

(a) *With gallons of gasoline on the horizontal and dollars of other goods on the vertical, graph the income and substitution effects from an increase in the price of gasoline.*

Answer: Graph 8.3 illustrates the original budget as the budget line tangent at the original optimum A that lies on the indifference curve u^A. Then the compensated budget line is illustrated as the dashed line, with B on u^A tangent to it. The move from A to B is the substitution effect. My final budget then lies parallel below the compensated budget — with a bundle like C_1 representing my new optimum and the move from B to C_1 representing the income effect.



Graph 8.3: George Exxon and Me

(b) *Suppose George (who derives all his income from his gasoline endowment) had exactly the same budget before the price increase that I did. On the same graph, illustrate how his budget changes as a result of the price increase.*

Answer: Rather than rotating inward with a fixed point on the vertical axis, the budget rotates outward around his fixed endowment point. The new budget is indicated in the graph. It has the same slope as my new budget because we both face the same prices, but George's lies further out because he owns gasoline and thus saw his wealth increase as a result of the increase in the price of gasoline (while I saw mine decrease.)

(c) *Given that we have the same tastes, can you say whether the substitution effect is larger or smaller for George than it is for me?*

Answer: The substitution effect is exactly the same for me and George. We both start at A on the indifference curve u^A — and we both face the same compensated budget because we both face the same price increase. Thus, B is the same for George as it is for me.

(d) *Why do we call the change in behavior that is not due to the substitution effect an* income effect *in my case but a* wealth effect *in George Exxon's case?*

Answer: In my case, I simply have an exogenous income level that is not tied to any price in the economy. As a result, any price increase will make me worse off because my income can buy less. I change consumption partly because of the change in opportunity costs (giving rise to the substitution effect) but also because my "real income" has changed — thus the term income effect. The story for George is a bit different — he owns gasoline, and the value of his wealth from which he can draw income therefore depends on the price of gasoline. While an increase in the price of gasoline makes me worse off and decreases my "real income", the same increase in price actually makes George better off because it increases how much income he can draw from what he owns. Put differently, George's wealth has increased because something he owns has become more valuable — and this will impact his consumption behavior (in addition to the impact of the change in opportunity costs).

B: *In Section 8B.1, we assumed the utility function $u(x_1, x_2) = x_1^{0.1} x_2^{0.9}$ for George Exxon as well as an endowment of gasoline of 1000 gallons. We then calculated substitution and wealth effects when the price of gasoline goes up from \$2 to \$4 per gallon.*

(a) *Now consider me with my exogenous income $I = 2000$ instead. Using the same utility function we used for George in the text, derive my optimal consumption of gasoline as a function of p_1 (the price of gasoline) and p_2 (the price of other goods).*

Answer: Solving

$$\max_{x_1,x_2} x_1^{0.1} x_2^{0.9} \text{ subject to } p_1 x_1 + p_2 x_2 = 2000, \tag{8.16}$$

we can derive $x_1 = 200/p_1$ and $x_2 = 1800/p_2$.

(b) *Do I consume the same as George Exxon prior to the price increase? What about after the price increase?*

Answer: Prior to the price increase, $p_1 = 2$ — thus $x_1 = 200/2 = 100$ which is the same as we calculated in the text for George Exxon. After the price increase, I consume 200/4=50 gallons of gasoline — less than we calculated for George.

(c) *Calculate the substitution effect from this price change and compare it to what we calculated in the text for George Exxon.*

Answer: To calculate the substitution effect, we first have to know how much utility I get before the price increase. We already calculated that $x_1 = 100$, and we can similarly calculate that $x_2 = 1800/1 = 1800$ (since $p_2 = 1$). My original bundle is therefore $A = (100, 1800)$ — which gives utility $u^A = u(100, 1800) = 100^{0.1} 1800^{0.9} \approx 1348$ — same as for George Exxon. Then we ask what the least is that we could spend and reach this utility level again after the price increase; i.e. we solve

$$\min_{x_1,x_2} 4x_1 + x_2 \text{ subject to } x_1^{0.1} x_2^{0.9} = 1348. \tag{8.17}$$

Note that this is exactly the same problem we wrote down to determine the substitution effect for George Exxon in the text — because we are asking exactly the same question. Thus we get the same answer — $x_1 = 53.59$ and $x_2 = 1929.19$.

(d) *Suppose instead that the price of "other goods" fell from \$1 to 50 cents while the price of gasoline stayed the same at \$2. What is the change in my consumption of gasoline due to the substitution effect? Compare this to the substitution effect you calculated for the gasoline price increase above.*

Answer: We would now solve

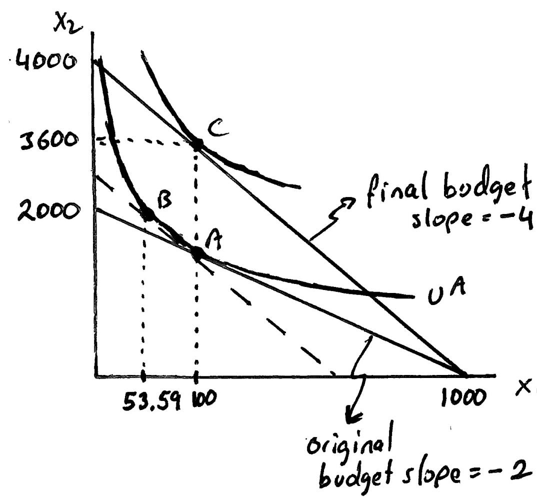
$$\min_{x_1,x_2} 2x_1 + 0.5x_2 \text{ subject to } x_1^{0.1} x_2^{0.9} = 1348. \tag{8.18}$$

Note that, while this problem looks different from the problem described above in (8.17), it will necessarily give the same answer because the *ratio* of the prices is exactly the same, as is the indifference curve we are trying to fit the compensated budget to. Thus, the solution will be $x_1 = 53.59$ and $x_2 = 1929.19$ as above — i.e. the substitution effect is the same.

(e) *How much gasoline do I end up consuming? Why is this identical to the change in consumption we derived in the text for George when the price of gasoline increases? Explain intuitively using a graph.*

Answer: In B(a) we derived my optimal consumption to be $x_1 = 200/p_1$ and $x_2 = 1800/p_2$. When $p_1 = 2$ and $p_2 = 0.5$, this implies $x_1 = 100$ and $x_2 = 3600$.

The price decrease of x_2 implies that my budget will rotate outward around the horizontal intercept of my original budget (at 1000). The price increase of x_1 will cause George's budget to similarly rotate around the horizontal intercept of his original budget — which, in his case, is his endowment bundle. Thus, a decrease in p_2 caused a qualitatively similar change in my budget as an increase in p_1 does in George's budget. The two are quantitatively the same — resulting in the same final budget — if the ratio p_1/p_2 ends up being the same. Starting at $p_1 = 2$ and $p_2 = 1$, a decrease in p_2 to 0.5 causes that ratio to change from 2/1 to 2/0.5 = 4. Similarly, an increase in p_1 to 4 will cause the ratio to change to 4. Thus, George and I experience the same change in our economic circumstances under the two scenarios — which is why the change in behavior is the same (assuming we have the same tastes). This is illustrated in Graph 8.4.



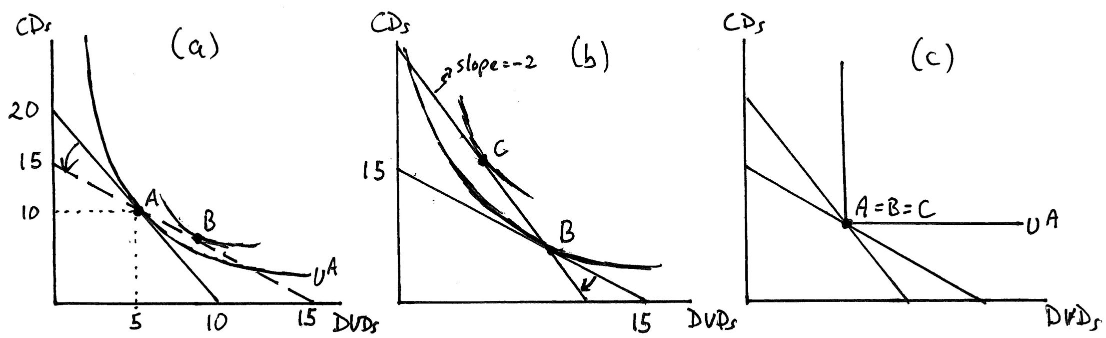
Graph 8.4: George Exxon and Me: Part II

8.4 Business Application: *Merchandise Exchange Policies*: *Suppose you have $200 in discretionary income that you would like to spend on ABBA CDs and Arnold Schwarzenegger DVDs.*

A: *On the way to work, you take your $200 to the Wal-Mart and buy 10 CDs and 5 DVDs at CD prices of $10 and DVD prices of $20.*

(a) *On a graph with DVDs on the horizontal and CDs on the vertical, illustrate your budget constraint and your optimal bundle A.*

Answer: This is done in panel (a) of Graph 8.5 where the indifference curve u^A is tangent to the original budget line at A.



Graph 8.5: Store Credit at Wal-Mart

(b) *On the way home, you drive by the the same Wal-Mart and see a big sign: "All DVDs half-price — only $10!" You also know that Wal-Mart has a policy of either refunding returned items for the price at which they were bought if you provide them with a Wal-Mart receipt — or, alternatively, giving store credit in the amount that those items are currently priced in the store if you have "lost" your receipt.*[1] *What is the most in store credit that you could get?*

Answer: You could get $10 for each of the items in your shopping bag — your 10 CDs and 5 DVDs. Thus, the most store credit you can get is $150.

(c) *Given that you have no more cash and only a bag full of DVDs and CDs, will you go back into Wal-Mart and shop?*

Answer: Yes. This is also illustrated in panel (a) of the graph where the new (dashed) budget is drawn. With $150 in store credit, you could buy back bundle A, or you could buy any of the other bundles on the new dashed budget. (A is like an endowment point for this problem — no matter how the prices change, you can always still consume A.) But now there is a set of new bundles to the right of A that lie on the new budget as well as above the indifference curve u^A. Thus, you will buy more DVDs and fewer CDs (at a bundle like B) as you use your store credit — and you will be better off as a result.

(d) *On the way to work the next day, you again drive by Wal-Mart — and you notice that the sale sign is gone. You assume that the price of DVDs is back to $20 (with the price of CDs still unchanged), and you notice you forgot to take your bag of CDs and DVDs out of the car last night and have it sitting right there next to you. Will you go back into the Wal-Mart store (assuming you still have an empty wallet)?*

Answer: Yes. This is illustrated in panel (b) where the new budget goes through B (because B is now your new endowment bundle) but has steeper slope. This makes more preferred bundles to the left of B possible — causing you to switch to a bundle like C that contains fewer DVDs and more CDs.

[1] "Store credit" means that you get a card to which you can charge the amount of credit for anything you buy in the store.

(e) *Finally, you pass Wal-Mart again on the way home — and this time see a sign: "Big Sale — All CDs only $5, All DVDs only $10!" With your bag of merchandise still sitting next to you and your wallet still empty, will you go back into Wal-Mart?*

Answer: No, you will not. The slope of the last budget constraint you faced at Wal-Mart was −2 — and resulted in you leaving with bundle C in panel (b) of the graph. C is now your new endowment bundle. When prices of both CDs and DVDs fall by the same proportion, the ratio of the price of DVDs to CDs is still 2 (i.e. 10/5=2) — which implies the slope of the budget is not changing. If you went back into Wal-Mart, you could therefore get store credit in an amount that would allow you to buy C again — or any bundle on the budget that goes through C with slope −2. But that is the same budget you had in the morning — which would mean C, the bundle that is in your bag already, would still be the optimal bundle.

(f) *If you are the manager of a Wal-Mart with this "store credit" policy, would you tend to favor — all else being equal — across the board price changes or sales on selective items?*

Answer: All else being equal, you would favor across the board price cuts. These would keep the ratio of prices the same and would make it impossible for people to game the store credit policy. Selective price changes, as we have seen — whether they are increases or decreases — make it possible for people to game the system.

(g) True or False*: If it were not for substitution effects, stores would not have to worry about people gaming their "store credit" policies as you did in this example.*

Answer: This is true. To see this, we can draw an indifference curve without any substitutability at the original optimal bundle A. This is done in panel (c) of Graph 8.5. The steeper slope is the original budget. When the price of DVDs falls, the new budget goes through A but has half the slope. Without any substitutability between the goods, this would imply that A is still the optimal bundle — i.e. A=B. Then the price of DVDs increases again — causing the budget constraint to revert back to the original. A is still optimal — i.e. A=C.

B: *Suppose your tastes for DVDs (x_1) and CDs (x_2) can be characterized by the utility function* $u(x_1,x_2)=x_1^{0.5}x_2^{0.5}$. *Throughout, assume that it is possible to buy fractions of CDs and DVDs.*

(a) *Calculate the bundle you initially buy on your first trip to Wal-Mart.*

Answer: By now you probably probably realize that, when tastes are Cobb-Douglas and the exponents sum to 1, the optimal quantity of each good is the exponent on that good times income divided by the good's price. (You can of course derive this by solving the usual maximization problem.) This gives us

$$x_1=\frac{0.5I}{p_1}=\frac{0.5(200)}{20}=5 \text{ and } x_2=\frac{0.5I}{p_2}=\frac{0.5(200)}{10}=10. \tag{8.19}$$

Your original bundle A is therefore (5,10) (as in part A of the question).

(b) *Calculate the bundle you buy on your way home from work on the first day (when p_1 falls to 10).*

Answer: The value of your goods when exchanged for store credit when $p_1=p_2=10$ is $150. When dealing with endowments, this value of the endowment takes the place of the exogenous income I in our equations. Thus, using the same insight about the equations for optimal consumption under Cobb-Douglas tastes as we used in B(a), we get

$$x_1=\frac{0.5(150)}{10}=7.5 \text{ and } x_2=\frac{0.5(150)}{10}=7.5. \tag{8.20}$$

In essence, you are trading 2.5 CDs for 2.5 DVDs because the opportunity cost of DVD's has fallen.

(c) *If you had to pay the store some fixed fee for letting you get store credit, what's the most you would be willing to pay on that trip?*

Answer: The most you'd be willing to pay is an amount that will make you as well off after using your store credit as you were before you went back into Wal-Mart. Before you went back in, you had 5 DVDs and 10 CDs — which gave you utility $u^A=5^{0.5}10^{0.5}\approx 7.071$. To calculate the bundle on that same indifference curve that you would buy if you paid the maximum bribe fee you are willing to pay, we have to solve the problem

$$\min_{x_1,x_2} 10x_1 + 10x_2 \text{ subject to } x_1^{0.5}x_2^{0.5} = 7.071. \tag{8.21}$$

Setting up the Lagrange function and solving the first two first order conditions, we get $x_1 = x_2$. Plugging this back into the constraint and solving, we get $x_1 = 7.071 = x_2$. This bundle costs $10(7.071) + 10(7.071) \approx 141.42$. Thus, you would be willing to pay up to about \$8.58 to be able to get store credit at Wal-Mart.

(d) *What bundle will you eventually end up with if you follow all the steps in part A?*

Answer: Eventually you go back and exchange your $x_1 = x_2 = 7.071$ bundle for store credit when $p_1 = 20$ and $p_2 = 10$. This would give you store credit of $20(7.071) + 10(7.071) \approx$ \$212.13. With that, you would buy

$$x_1 = \frac{0.5(212.13)}{20} \approx 5.30 \text{ and } x_2 = \frac{0.5(212.13)}{10} \approx 10.61. \tag{8.22}$$

(e) *Suppose that your tastes were instead characterized by the function* $u(x_1, x_2) = (0.5x_1^{-\rho} + 0.5x_2^{-\rho})^{-1/\rho}$. *Can you show that your ability to game the store credit policy diminishes as the elasticity of substitution goes to zero (i.e. as ρ goes to ∞)?*

Answer: If we solve

$$\max_{x_1,x_2} \left(0.5x_1^{-\rho} + 0.5x_2^{-\rho}\right)^{-1/\rho} \text{ subject to } p_1x_1 + p_2x_2 = I \tag{8.23}$$

in the usual way, we get

$$x_1 = \frac{I}{p_1 + p_1^{1/(\rho+1)}p_2^{\rho/(\rho+1)}} \text{ and } x_2 = \frac{I}{p_2 + p_2^{1/(\rho+1)}p_1^{\rho/(\rho+1)}}. \tag{8.24}$$

Note that the limit of $1/(\rho+1)$ as ρ goes to ∞ is 0, and the limit of $\rho/(\rho+1)$ as ρ goes to ∞ is 1. Thus, the limit of the solutions for x_1 and x_2 as ρ goes to ∞ is

$$x_1 = \frac{I}{p_1 + p_1^0p_2^1} = \frac{I}{p_1 + p_2} \text{ and } x_2 = \frac{I}{p_2 + p_1^1p_2^0} = \frac{I}{p_1 + p_2}. \tag{8.25}$$

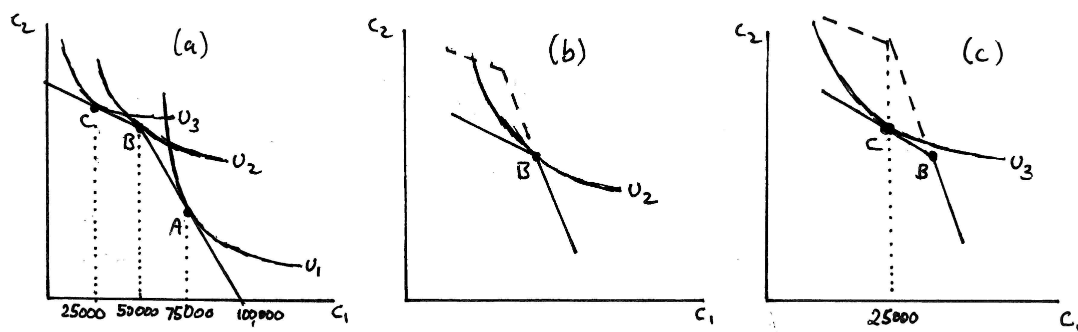
Initially, $I = 200$, $p_1 = 20$ and $p_2 = 10$. Thus, $x_1 = 200/(20 + 10) = 6.67 = x_2$. Thus, the initial optimum is A=(6.67,6.67). Then p_1 falls to 10. The value of the endowment A is the $10(6.67) + 10(6.67) = 133.33$. Plugging this into our equations for x_1 and x_2 — and using the new prices $p_1 = p_2 = 10$ — we get $x_1 = 133.33/(10 + 10) = 6.67 = x_2$. Thus, our new optimal bundle if we use store credit is B=(6.67,6.67) which is equal to A. Nothing was gained by using the store credit. When p_1 then goes back to 20, store credit for this bundle is back to \$200 — which implies the optimal bundle is once again A.

8.5 Policy Application: *Savings Behavior and Tax Policy: Suppose you consider the savings decisions of three households - households 1, 2 and 3. Each household plans for this year's consumption and next year's consumption, and each household anticipates earning $100,000 this year and nothing next year. The real interest rate is 10%. Assume throughout that consumption is always a normal good.*

A: *Suppose the government does not impose any tax on interest income below $5,000 but taxes any interest income above $5,000 at 50%.*

(a) *On a graph with "Consumption this period" (c_1) on the horizontal axis and "Consumption next period" (c_2) on the vertical, illustrate the choice set faced by each of the three households.*

Answer: Panel (a) of Graph 8.6 illustrates the shape of the budget constraint which has a kink at $50,000 of consumption now (c_1) because, when consumption now is $50,000, then savings is also $50,000 — which, at a 10% interest rate, results in $5,000 of interest income. This first $5,000 of interest income is exempt — which means the slope of the lower part of the budget constraint is simply $-(1+r) = -(1+0.1) = -1.1$. At current consumption below $50,000, however, savings are above $50,000 — which means interest income is above $5,000. Thus, as interest income goes above $5,000 at $50,000 of savings, the slope of the budget constraint becomes shallower because the government now taxes the additional interest income at 50%. To be specific, the slope goes to $-(1+0.5r) = -1.05$.



Graph 8.6: Savings of 3 Households

(b) *Suppose you observe that household 1 saves $25,000, household 2 saves $50,000 and household 3 saves $75,000. Illustrate indifference curves for each household that would make these rational choices.*

Answer: Panel (a) of the graph also indicates three indifference curves that make the choices of the 3 households optimal ones — with each indifference curve labeled by the relevant household. (For instance, u_1 refers to the optimal indifference curve from household 1's indifference map, where household 1 is the household that saves $25,000 and thus consumes $75,000 now.)

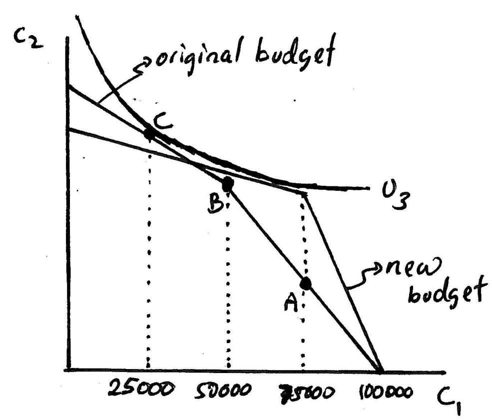
(c) *Now suppose the government changes the tax system by exempting the first $7,500 rather than the first $5,000 from taxation. Thus, under the new tax, the first $7,500 in interest income is not taxed, but any interest income above $7,500 is taxed at 50%. Given what you know about each household's savings decisions before the tax change, can you tell whether each of these households will now save more?* (Note*: It is extremely difficult to draw the scenarios in this question to scale — and when not drawn to scale, the graphs can become confusing. It is easiest to simply worry about the general shapes of the budget constraints around the relevant decision points of the households that are described.)*

Answer: This policy change would extend the steep portion of the budget from $50,000 in current consumption to $25,000 in current consumption (where savings hits $75,000 and thus interest income hits $7,500). Household 1 would be unaffected by this change since the indifference curve u_1 that is tangent at A lies above any new bundle that becomes available as a result of the policy change. Thus, household 1's savings would not change.

Household 2's savings, on the other hand, would almost certainly increase. In order for B to be optimal before the policy change, this household has an indifference curve that "hangs" on the kink of the original budget constraint. That means the MRS could lie between -1.1 (which is the slope of the steep portion of the budget) and -1.05 (which is the slope of the shallower portion). If the $MRS = -1.1$ at B, then the indifference curve u_2 is tangent to the extended steep budget that runs through B after the policy change — and thus B would continue to be optimal. However, if the MRS falls anywhere from -1.05 to -1.1 at B, then the new (dashed) budget constraint will cut the indifference curve u_2 as illustrated in panel (b) of Graph 8.6 — thus enabling the household to choose from a set of new bundles that lie above the original indifference curve. All of these bundles are such that consumption now (c_1) falls — i.e. savings increases.

Household 3, however, will definitely not save more. Panel (c) of the graph illustrates the change for this household. The new kink point now happens right above C. If the household were to choose a bundle on the flat portion of the new (dashed) budget line, then c_1 would be an inferior good and we have assumed that consumption is always normal. (It would be inferior because, when faced with a parallel outward shift in the budget, the household would be choosing to consume less.) Thus we know that the household will choose either the kink point (and keep savings the same) or a point on the steeper portion of the new (dashed) budget — with more c_1 and thus less savings.

(d) *Instead of the tax change in part (c), suppose the government had proposed to subsidize interest income at 100% for the first $2,500 in interest income while raising the tax on any interest income above $2,500 to 80%. (Thus, if someone earns $2,500 in interest, she would receive an additional $2,500 in cash from the government. If someone earns $3,500, on the other hand, she would receive the same $2,500 cash subsidy but would also have to pay $800 in a tax.) One of the three households is overheard saying:"I actually don't care whether the old policy (i.e. the policy described in part A) or this new policy goes into effect." Which of the three households could have said this, and will that household save more or less (than under the old policy) if this new policy goes into effect?*



Graph 8.7: Savings of 3 Households: Part II

Answer: By subsidizing savings initially, the government in effect raises the interest rate from 10% to 20% for the first $25,000 in savings. Thus, beginning at the $100,000 intercept on the c_1 axis, the budget constraint is twice as steep. From that point on, however, the government is in effect reducing the interest rate from 10% to 2% because of the 80% tax on interest income. Thus, beginning at $75,000 of current ($c_1$) consumption and moving leftward, the budget constraint becomes shallower than it was before. It seems clear that the two budget constraints will cross at some point — the question is where. We can check, for instance, which budget gives higher consumption next period (c_2) at $50,000 of savings where the original kink occurred. Under the original policy, you make $5,000 in interest when you save $50,000 — giving you $c_2 = \$55,000$. Under the new policy, you get $5,000 of interest (including the subsidy) for the first $25,000 you save, you earn another $2,500 of interest for the next $25,000 in savings — but that is taxed at 80% to leave you with only $500 of after-tax interest income. Thus, your total interest income (including the subsidy and subtracting out the tax) is $5,500 — leaving you with $55,500 in c_2. This is $500 more than under the original policy. If you save an additional $25,000 (for a total of $75,000), you would earn an additional $2,500 in interest. Under the original policy, half of that would be taxed away, leaving you with $1,250. Under the new policy, 80% is taxed away — leaving you with only $500 more. Thus, at $75,000 of savings, the old policy results in greater c_2 than the new policy — $250 more, to be exact. The old and the new budgets therefore intersect between $75,000 and $50,000 in savings.

The general relationship between the original and the new budget constraints is graphed in Graph 8.7 (previous page). Households 1 and 2 must prefer the new policy since it opens up new bundles to the northeast of their original optimal bundles. Household 3, however, might be indifferent — as illustrated with the indifference curve u_3. Under the new policy, household 3 would then consume more now — and save less — if indeed it is indifferent between the policies.

B: *Now suppose that our 3 households had tastes that can be represented by the utility function* $u(c_1, c_2) = c_1^{\alpha} c_2^{(1-\alpha)}$*, where* c_1 *is consumption now and* c_2 *is consumption a year from now.*

(a) *Suppose there were no tax on savings income. Write down the intertemporal budget constraint with the real interest rate denoted* r *and current income denoted* I *(and assume that consumer anticipate no income next period).*

Answer: The intertemporal budget constraint is

$$(1+r)c_1 + c_2 = (1+r)I. \tag{8.26}$$

(b) *Write down the constrained optimization problem and the accompanying Lagrange function. Then solve for* c_1*, current consumption, as a function of* α*, and solve for the implied level of savings as a function of* α*,* I *and* r*. Does savings depend on the interest rate?*

Answer: The maximization problem is

$$\max_{c_1, c_2} c_1^{\alpha} c_2^{(1-\alpha)} \text{ subject to } (1+r)c_1 + c_2 = (1+r)I. \tag{8.27}$$

The Lagrange function for this problem is

$$\mathscr{L}(c_1, c_2, \lambda) = c_1^{\alpha} c_2^{(1-\alpha)} + \lambda((1+r)I - (1+r)c_1 - c_2). \tag{8.28}$$

The first two first order conditions can be solved to yield

$$c_2 = \frac{(1-\alpha)(1+r)}{\alpha} c_1. \tag{8.29}$$

Plugging this into the constraint $(1+r)c_1 + c_2 = (1+r)I$, we can solve for $c_1 = \alpha I$. Savings s is then simply c_1 subtracted from current income; i.e.

$$s = I - \alpha I = (1-\alpha)I. \tag{8.30}$$

Savings therefore does not depend on the interest rate.

(c) *Determine the* α *value for consumer 1 as described in part A.*

Answer: Consumer 1 saves 25% of her income on the portion of the budget where there is no tax — thus, it must be that $(1-\alpha) = 0.25$ or $\alpha = 0.75$.

(d) *Now suppose the initial 50% tax described in part A is introduced. Write down the budget constraint (assuming current income I and before-tax interest rate r) that is now relevant for consumers who end up saving more than $50,000. (*Note*: Don't write down the equation for the kinked budget — write down the equation for the linear budget on which such a consumer would optimize.)*

Answer: To write down this budget, we need to know an intercept and a slope. The slope is simply $-(1+0.5r)$ since the government is taxing interest income at 50%. We can determine the c_2 intercept by calculating the total interest a consumer would earn if she saved all her income I assuming $I > 50,000$. For the first $50,000, she would save at the untaxed interest rate of r — thus accumulating $(1+r)50000$ for next period. She would then have $(I-50000)$ left to save — on which she would earn $0.5r$ interest. In addition to accumulating $(1+r)50000$ for the first $50,000 in savings, she would therefore accumulate $(1+0.5r)(I-50000)$ if she saved all her income. Her total possible c_2 consumption is therefore

$$(1+r)50000+(1+0.5r)(I-50000) = (1+0.5r)I + 25000r. \tag{8.31}$$

This, then, is the c_2 intercept. Given we already determined the slope to be $-(1+0.5r)$, the budget constraint is $c_2 = (1+0.5r)I + 25000r - (1+0.5)c_1$ or

$$(1+0.5r)c_1 + c_2 = (1+0.5r)I + 25000r. \tag{8.32}$$

(e) *Use this budget constraint to write down the constrained optimization problem that can be solved for the optimal choice given that households save more than $50,000. Solve for c_1 and for the implied level of savings as a function of α, I and r.*

Answer: The maximization problem is

$$\max_{c_1,c_2} c_1^{\alpha} c_2^{(1-\alpha)} \text{ subject to } (1+0.5r)c_1 + c_2 = (1+0.5r)I + 25000r. \tag{8.33}$$

The Lagrange function for this problem is

$$\mathcal{L}(c_1,c_2,\lambda) = c_1^{\alpha} c_2^{(1-\alpha)} + \lambda((1+0.5r)I + 25000r - (1+0.5r)c_1 - c_2). \tag{8.34}$$

Solving this in the same way as before, we then get

$$c_1 = \alpha I + \frac{25000\alpha r}{(1+0.5r)} \tag{8.35}$$

and an implied savings s of

$$s = (1-\alpha)I - \frac{25000\alpha r}{(1+0.5r)}. \tag{8.36}$$

(f) *What value must α take for household 3 as described in part A?*

Answer: Household 3 saves $75,000 with income of $100,000 and before-tax interest rate $r = 0.1$. Thus

$$75000 = (1-\alpha)100000 - \frac{25000\alpha(0.1)}{(1+0.5(0.1))} \tag{8.37}$$

which solves to $\alpha \approx 0.244$.

(g) *With the values of α that you have determined for households 1 and 3, determine the impact that the tax reform described in (c) of part A would have?*

Answer: Using panels (b) and (c) of Graph 8.6, we concluded in part A that both households will choose to locate on the steeper portion of the budget under the new policy — i.e. on the portion defined by the constraint $(1+r)c_1 + c_2 = (1+r)I$ where $I = 100,000$ and $r = 0.1$. In B(b), we determined that savings in this case is given by $s = (1-\alpha)I$. Thus, household 1 for whom $\alpha = 0.75$ would save $(1-0.25)100,000 = 25,000$ as before. Household 3, for whom $\alpha \approx 0.244$, will save approximately $(1-0.244)100,000 = 75,600$ — but that level of savings lies to the left of the kink point of the dashed budget in panel (c). Thus, household 3 optimizes at the kink point, implying unchanged savings at $75,000.

(h) *What range of values can α take for household 2 as described in part A?*

Answer: There are several ways you could use to figure this out. One way is to note that

$$MRS = -\frac{\alpha c_2}{(1-\alpha)c_1} \tag{8.38}$$

and that this must, for household 2, lie between -1.05 and -1.10 at $(c_1, c_2) = (50000, 55000)$ in order for that kink point in the budget to be optimal. Substituting these values for c_1 and c_2 into the expression for MRS and setting it equal to these two endpoint values, we can solve

$$-\frac{55000\alpha}{50000(1-\alpha)} = -1.05 \text{ and } -\frac{55000\alpha}{50000(1-\alpha)} = -1.10 \tag{8.39}$$

to conclude that $0.488 \leq \alpha \leq 0.5$.

Another way to solve for this is to use our results from the previous parts. Household 2 might have a tangency with the steep portion of the budget at $(c_1, c_2) = (50000, 55000)$. We concluded in B(b) (equation (8.30)) that in this case, savings s is $s = (1-\alpha)I$. Thus, for household 2 to choose \$50,000 in savings under the steeper portion of the budget, $50000 = (1-\alpha)100000$ which implies $\alpha = 0.5$.

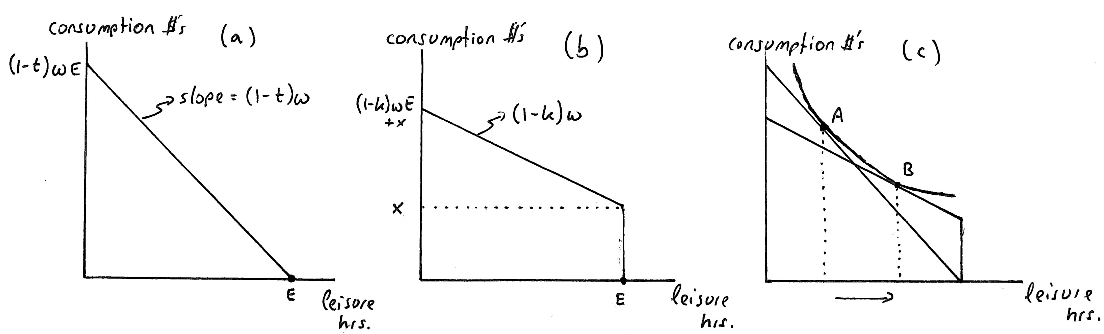
Alternatively, household 2 could have a tangency with the shallow portion of the budget at $(c_1, c_2) = (50000, 55000)$. We concluded in B(3) that savings then satisfies equation (8.36). Substituting $s = 50,000$, $I = 100,000$ and $r = 0.1$ into that equation, we can solve for $\alpha \approx 0.488$. Thus, again we get that $0.488 \leq \alpha \leq 0.5$.

8.6 Policy Application: *The Negative Income Tax: Suppose the current tax system is such that the government takes some fixed percentage t of any labor income that you make.*

A: *Some in Congress have proposed the following alternative type of tax system known as the* negative income tax*: You get a certain guaranteed income x even if you do not work at all. Then, for any income you earn in the labor market, the government takes a certain percentage k in taxes. In order to finance the guaranteed income x, the tax rate on labor income in this alternative system has to be higher than the tax rate under the current system (i.e. $t < k$).*[2]

(a) *On a graph with leisure on the horizontal axis and consumption on the vertical, illustrate what your budget constraint under the current tax system looks like — and indicate what the intercepts and slopes are assuming a leisure endowment of E and before tax wage w.*

Answer: This is illustrated in panel (a) of Graph 8.8.



Graph 8.8: Work and the Negative Income Tax

(b) *On a similar graph, illustrate what your budget constraint looks like under the alternative system.*

Answer: This is done in panel (b) where, even if the person does not work at all, he is still able to consume the guaranteed income level x.

(c) *You hear me say:"You know what — after looking at the details of the tax proposal, I can honestly say I don't care whether we keep the current system or switch to the proposed one." Without knowing what kind of goods leisure and consumption are for me, can you tell whether I would work more or less under the negative income tax? Explain.*

Answer: If you are indifferent, it must be that the optimum on each of the budgets lies on the same indifference curve as illustrated in panel (c) of the graph. The difference in behavior between the two budgets is therefore a pure substitution effect — with more leisure (i.e. less work) under the negative income tax. We do not need to know precisely what kinds of goods (i.e. normal, inferior, etc.) consumption and leisure are for this conclusion since the conclusion arises simply from the presence of substitutability between consumption and leisure.

(d) *What would your tastes have to look like in order for you to be equally happy under the two systems while also working exactly the same number of hours in each case?*

Answer: Leisure and consumption could be perfect complements — with the corner in the indifference curve lying at the intersection of the two budgets. (It is also possible for the indifference curve to have a sharp kink at that point — sharp enough for the intersection point to be optimal.)

[2]In some proposals, the requirement that $t < k$ actually does not hold because proponents of the negative income tax envision replacing a number of social welfare programs with the guaranteed income x.

(e) True or False*: The less substitutable consumption and leisure are, the less policy makers have to worry about changes in people's willingness to work as we switch from one system to the other.*

Answer This is true — the closer the tastes are to perfect complements, the closer will be A and B in panel (c) of the graph.

B: *Consider your weekly decision of how much to work, and suppose that you have 60 hours of available time to split between leisure and work. Suppose further that your tastes over consumption and leisure can be captured by the utility function* $u(c,\ell) = c\ell$ *and that your market wage is* $w = 20$ *per hour.*

(a) *Write down the budget constraint under the two different tax policies described above; i.e. write down the first budget constraint as a function of* c*,* ℓ *and* t *and the second as a function of* c*,* ℓ*,* k *and* x*.*

Answer: The initial budget constraint is

$$c = 20(1-t)(60-\ell). \tag{8.40}$$

The negative income tax budget constraint is

$$c = x + 20(1-k)(60-\ell). \tag{8.41}$$

(b) *Derive the optimal choice under the current tax system (as a function of* t*.) In the absence of anything else changing, do changes in wage taxes cause you to change how much you work? Can you relate your answer (intuitively) to wealth and substitution effects?*

Answer: To solve for the optimal choice, we solve the problem

$$\max_{c,\ell} c\ell \text{ subject to } c = 20(1-t)(60-\ell). \tag{8.42}$$

Setting up the Lagrangian and solving the first two first order conditions, we get $c = 20(1-t)\ell$. Plugging this into the constraint and solving for ℓ, we get $\ell = 60/2 = 30$. Substituting this back into $c = 20(1-t)\ell$, we get $c = 600(1-t)$. The optimal choice under the current tax system is therefore

$$c = 600(1-t) \text{ and } \ell = 30. \tag{8.43}$$

Since ℓ — the amount of leisure consumed — does not depend on t, work effort under these tastes does not depend on the tax rate. Thus, the substitution effect (that says we should work less under higher tax rates) is exactly offset by the wealth effect (which says we work more).

(c) *Now derive your optimal leisure choice under a negative income tax (as a function of* k *and* x*). How is your work decision now affected by an increase in* k *or an increase in* x*?*

Answer: We now solve the problem

$$\max_{c,\ell} c\ell \text{ subject to } c = 20(1-k)(60-\ell) + x. \tag{8.44}$$

Following the same steps as in the last part, we again get from the first two first order conditions that $c = 20(1-k)\ell$. Plugging this into the budget constraint $c = 20(1-k)(60-\ell) + x$ and solving for ℓ (and then plugging the solution back into $c = 20(1-k)\ell$), however, we now get the following optimal choices:

$$c = 600(1-k) + \frac{x}{2} \text{ and } \ell = 30 + \frac{x}{40(1-k)}. \tag{8.45}$$

An increase in k decreases the denominator in our expression for the optimal ℓ — which means it increases the value of the optimal ℓ. Thus, an increase in the tax rate k increases leisure consumption and thus decreases work effort. An increase in the guaranteed income x increases ℓ — and thus also decreases work effort. (You can also check the partial derivative of ℓ with respect to k and x — and you should find that both are positive — which implies that leisure consumption is positively related to both k and x and thus negatively to work effort.)

(d) *Suppose that* $t = 0.2$. *Using your utility function to measure happiness, what utility level do you attain under the current tax system?*

Answer: Under the current tax system, equation (8.43) tells us that, with $t = 0.2$, your consumption is $c = 480$ and your leisure consumption is $\ell = 30$. Thus, your utility is

$$u(c,\ell) = 480(30) = 14,400. \tag{8.46}$$

(e) *Now the government wants to set* $k = 0.3$. *Suppose you are the pivotal voter — if you approve of the switch to the negative income tax, then it will pass. What is the minimum level of guaranteed income* x *that the negative income tax proposal would have to include in order to win your support?*

Answer: From equation (8.45) we know that optimal consumption and leisure when $k = 0.3$ are

$$c = 420 + \frac{x}{2} \text{ and } \ell = 30 + \frac{x}{28}. \tag{8.47}$$

In order for you to be indifferent between the original tax (where you received utility of 14,400) and the negative income tax with $k = 0.3$, it must then be that your utility from consuming c and ℓ in equation (8.47) is equal to 14,400; i.e.

$$\left(420 + \frac{x}{2}\right)\left(30 + \frac{x}{28}\right) = 14,400. \tag{8.48}$$

Writing this as

$$\frac{1}{56}x^2 + 30x - 1800 = 0, \tag{8.49}$$

we can apply the quadratic formula to solve this for x. This gives us $x = \$58$.[3]

(f) *How much less will you work if this negative income tax is implemented (assuming* x *is the minimum necessary to get your support)?*

Answer: Plugging $x = 58$ into equation (8.47), we get $c = 449$ and $\ell = 32.07$. You will therefore work less by a little over two hours per week.

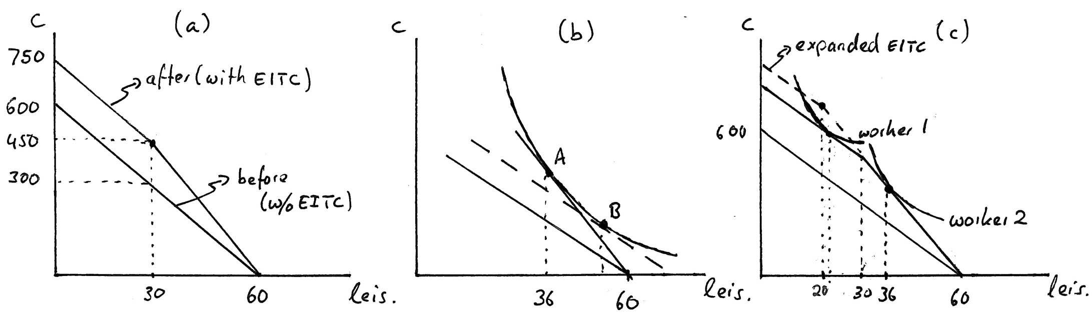
[3] The quadratic formula yields two solutions — with the other being $x = -1,738$. The latter is not economically meaningful — it is not possible to take an amount away from the consumer while also raising his tax and at the same time keep him equally happy.

8.7 Policy Application: *The Earned Income Tax Credit: Since the early 1970's, the U.S. government has had a program called the Earned Income Tax Credit (previously mentioned in end-of-chapter exercises in Chapter 3.) A simplified version of this program works as follows: The government subsidizes your wages by paying you 50% in addition to what your employer paid you but the subsidy applies only to the first $300 (per week) you receive from your employer. If you earn more than $300 per week, the government gives you only the subsidy for the first $300 you earned but nothing for anything additional you earn. For instance, if you earn $500 per week, the government would give you 50% of the first $300 you earned — or $150.*

A: *Suppose you consider workers 1 and 2. Both can work up to 60 hours per week at a wage of $10 per hour, and after the policy is put in place you observe that worker 1 works 39 hours per week while worker 2 works 24 hours per week. Assume throughout that Leisure is a normal good.*

(a) *Illustrate these workers' budget constraints with and without the program.*

Answer: These are illustrated in panel (a) of Graph 8.9. At a wage rate of $10 per hour, the earned income tax credit described in the problem raises the effective wage to $15 per hour for the first 30 hours of work.



Graph 8.9: Earned Income Tax Credit

(b) *Can you tell whether the program has increased the amount that worker 1 works? Explain.*

Answer: Worker 1 works 39 hours — which means he takes 21 hours of leisure after the EITC is implemented. Removing the EITC would therefore be like a parallel shift in of the budget for this worker — and would thus produce a pure wealth (or income) effect, no substitution effect. If leisure is a normal good, that means that removing the EITC would cause the worker to reduce consumption of leisure — i.e. he would work more. It must therefore be the case that the introduction of the EITC did the opposite — it increased the worker's consumption of leisure, thus causing him to work less than he did before.

(c) *Can you tell whether worker 2 works more or less after the program than he did before? Explain.*

Answer: Worker 2 works for 24 hours per week after the introduction of the EITC — implying a leisure consumption of 36 hours per week. Thus, this worker (unlike worker 1 in the previous part) locates to the right of the kink point in the EITC budget. This implies that removing the EITC would imply an inward rotation of the budget — thus causing both a substitution and a wealth effect. This is pictured in panel (b) of the graph — where the substitution effect is the change from the bundle A to B. Removing the EITC would make consuming leisure less expensive (since now the worker would only give up $10 for every hour of leisure rather than $15), which is why the substitution effect says the worker will take more leisure when the EITC is removed and work less. However, from the dashed compensated budget in the graph to the no-EITC budget below, there is a decrease in wealth, and if leisure is a normal good, a decrease in wealth implies less consumption of leisure. Thus, there is a wealth effect that points in the opposite direction from the substitution effect — leaving the overall effect ambiguous. The more substitutable leisure and consumption are, the more likely it is that the removal of the EITC would cause the worker to work less. The introduction of the EITC is of course the mirror image — the more substitutable leisure and consumption are, the more likely it is that the introduction of the EITC will cause the worker to work more.

(d) *Now suppose the government expands the program by raising the cut-off from $300 to $400. In other words, now the government applies the subsidy to earnings up to $400 per week. Can you tell whether worker 1 will now work more or less? What about worker 2?*

Answer: Panel (c) of the graph illustrates the initial without-EITC budget and the $300 EITC budget as in panel (a). In addition, the dashed extension of the EITC budget represents the expanded $400 EITC budget. This extension of the steeper EITC slope has no impact on worker 2 — worker 2 originally optimized at 36 hours of leisure, and no better bundles are made available by the expanded budget. Thus, worker 2 would do nothing differently. Worker 1, on the other hand, is affected by the change in the EITC. He initially takes 21 hours of leisure — which means the new EITC budget affects him both because it has a different slope and because it is further out. Since leisure is a normal good, we know the worker will not choose to optimize on the part of the new budget that lies to the left of the new kink point (at 20 hours of leisure) — because that would be equivalent to reducing the amount of leisure when wealth increases. So worker 1 will end up somewhere on the steeper portion of the new EITC budget — somewhere between 20 hours of leisure and 30 hours of leisure. We can't tell exactly where — there are once again offsetting wealth and substitution effects. The substitution effect says that worker 1 should now consume less leisure (i.e. work more) because leisure has become more expensive ($15 rather than $10 per hour). The wealth effect, on the other hand, says the worker is richer and therefore should consume more leisure (i.e. work less). Either effect may dominate. The more substitutable leisure and consumption are for the worker, the more likely it is that the worker will work more under the expanded EITC. The most he will work more, however, is 1 hour.

B: *Suppose that workers have tastes over consumption c and leisure ℓ that can be represented by the function $u(c,\ell) = c^{\alpha}\ell^{(1-\alpha)}$.*

(a) *Given you know which portion of the budget constraint worker 2 ends up on, can you write down the optimization problem that solves for his optimal choice? Solve the problem and determine what value α must take for worker 2 in order for him to have chosen to work 24 hours under the EITC program.*

Answer: The optimization problem for worker 2 is

$$\max_{c,\ell} c^{\alpha}\ell^{(1-\alpha)} \text{ subject to } c = 15(60-\ell). \tag{8.50}$$

Setting up the Lagrange function and solving the first two first order conditions, we get $c = (15\alpha\ell)/(1-\alpha)$. Plugging this into the budget constraint and solving for ℓ, we get $\ell = 60(1-\alpha)$, and plugging this into $c = (15\alpha\ell)/(1-\alpha)$, we get $c = 900\alpha$.

In order for the worker to choose 24 hours of work and thus 36 hours of leisure, it must then be that $\ell = 36 = 60(1-\alpha)$. Solving for α, we get $\alpha = 24/60 = 0.4$.

(b) *Repeat the same for worker 1 — but be sure you specify the budget constraint correctly given that you know the worker is on a different portion of the EITC budget. (*Hint*: If you extend the relevant portion of the budget constraint to the leisure axis, you should find that it intersects at 75 leisure hours.)*

Answer: The optimization problem now would be

$$\max_{c,\ell} c^{\alpha}\ell^{(1-\alpha)} \text{ subject to } c = 750-10\ell. \tag{8.51}$$

Going through the same steps as above, we then get $\ell = 75(1-\alpha)$ and $c = 750\alpha$. In order for this worker to choose 39 hours of work or 21 hours of leisure, it therefore has to be the case that $\ell = 21 = 75(1-\alpha)$ or $\alpha = 0.72$.

(c) *Having identified the relevant α parameters for workers 1 and 2, determine whether either of them works more or less than he would have in the absence of the program.*

Answer: In the absence of the EITC program, the workers would solve

$$\max_{c,\ell} c^{\alpha}\ell^{(1-\alpha)} \text{ subject to } c = 10(60-\ell) \tag{8.52}$$

which gives $\ell = 60(1-\alpha)$ and $c = 600\alpha$. Worker 1 has $\alpha = 0.72$ — which means he takes $60(1-0.72) = 16.8$ hours of leisure without EITC and 21 hours of leisure with the EITC. Thus, worker 1 works 4.2 hours less under EITC. This is consistent with our intuitive graphs — where we concluded that the EITC has a pure wealth effect for worker 1 — causing him to work less. Worker 2 has $\alpha = 0.4$ — which means he takes $60(1-0.6) = 36$ hours of leisure before EITC and 36 hours of leisure after EITC. Thus, worker 2 does not change his work hours as a result of the EITC. This is also consistent with our graphical analysis where we found competing wealth and substitution effects for worker 2 — effects that exactly offset each other when the worker has the tastes modeled here.

(d) *Determine how each worker would respond to an increase in the EITC cut-off from \$300 to \$400.*

Answer: We already know from our intuitive analysis that nothing changes for worker 2 — he continues to operate on the steeper portion of the budget defined by the equation $c = 15(60-\ell)$ which we used in problem (8.50). Since the problem remains unchanged, the solution remains unchanged. For worker 1, however, the relevant budget constraint now is $c = 900 - 15\ell$ (rather than $750 - 10\ell$ as in problem (8.51)). Thus, since the relevant constraint has changed, we need to solve the problem with the new constraint — which gives us $\ell = 60(1-\alpha) = 60(1-0.72) = 16.8$ and $c = 900\alpha = 900(0.72) = 648$. But this would put him on the steep budget to the right of the kink — which implies the true optimum is at the kink where $\ell = 20$. Thus, he will work 1 hour more.

(e) *For what ranges of α would a worker choose the kink-point in the original EITC budget you drew (i.e. the one with a \$300 cutoff)?*

Answer: To figure out this range, we need to determine the values of α for which 30 hours of leisure is optimal for the problems written out in equations (8.50) and (8.51). In other words, for each of the two budget line segments, what are the values of α for which a worker would optimize at precisely the kink point. Any α between the values we get from these two exercises will be such that the kink point is optimal.

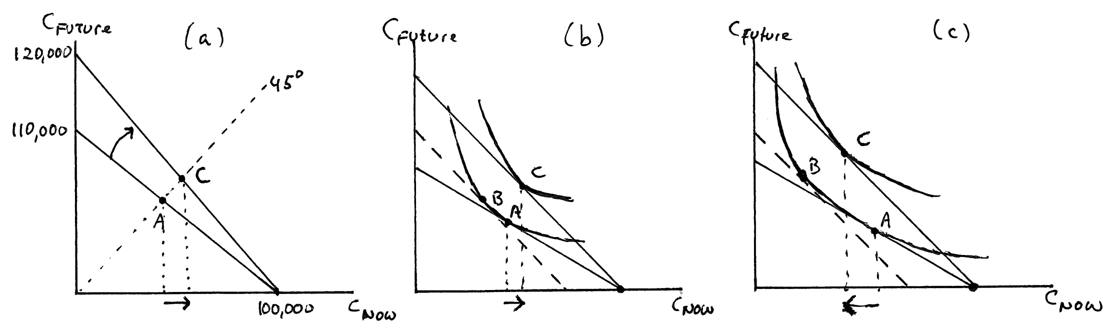
For the problem in (8.51), we calculated $\ell = 60(1-\alpha)$. Setting ℓ equal to 30, we can solve for $\alpha = 0.5$. For the problem in (8.50), we calculated $\ell = 75(1-\alpha)$. Setting $\ell = 30$, we can solve for $\alpha = 0.6$. Thus, for $0.5 \le \alpha \le 0.6$, the kink point where the worker works for 30 hours a week is optimal.

8.8 Policy Application: *Advising Congress on Savings Subsidies and Substitution Effects: Suppose you are asked to model the savings decisions of a household that has an income of $100,000 this year but expects to have no income a period into the future.*

A: *Suppose the interest rate is 10% over this period and we consider the tradeoff between consuming now and consuming one period from now.*

(a) *On a graph with "Consumption now" on the horizontal and "Future Consumption" on the vertical axis, illustrate how an increase in the interest rate to 20% over the relevant period would change the household's choice set.*

Answer: This is illustrated in panel (a) of the graph where $100,000 in current consumption (with no future consumption) is the "endowment bundle" — the bundle that is always available regardless of what the interest rate is.



Graph 8.10: Savings and Subsidies

(b) *Suppose that you know that the household's tastes can accurately be modeled as perfect complements over consumption now and consumption in the future period. Can you tell whether the household will save more or less as a result of the increase in the interest rate?*

Answer: If consumption now and consumption in the future are perfect complements, then the consumer always optimizes along the 45 degree line regardless of what the interest rate is (because the consumer always seeks to consume a dollar in the future for every dollar she consumes now). Thus, the consumer would move from an original optimal bundle A to a new optimal bundle C in panel (a) of the graph — which implies more consumption now. More consumption now in turn implies less money put into savings — so the household will save less.

(c) *You are asked to advise Congress on a proposed policy of subsidizing savings in order to increase the amount of money people save. Specifically, Congress proposes to provide 5% in interest payments in addition to the interest households earn in the market. You are asked to evaluate the following statement: "Assuming that consumption is always a normal good, small substitution effects make it likely that savings will actually decline as a result of this policy, but large substitution effects make it likely that savings will increase."*

Answer: The policy proposal is one of increasing the interest rate consumers can get on their savings accounts — it thus induces the same kind of change in the budget set as was graphed in panel (a). In panels (b) and (c), two possibilities are illustrated. In (b), the initial indifference curve through the original optimum A has relatively little substitutability built into it — i.e. it looks more like perfect complements than like perfect substitutes. This results in a relatively small substitution effect to B — an effect that can easily be overcome by a wealth effect in the opposite direction to arrive at C that lies to the right of A. Thus, while the substitution effect says consume less now (and thus save more), the wealth effect says consume more now — and the latter outweighs the former for an overall increase in current consumption. In panel (c), the initial indifference curve that contains A is "flatter" around A — i.e. it has more substitutability built into it. As a result, the substitution effect to B is larger — and not as easy to overcome by a wealth effect in the opposite direction. As a result, we get C to the left of A — implying less consumption now and thus more savings. Thus, the statement you are asked to comment on is true — small substitution effects (as in panel (b) or in panel (a) where there is no substitution effect) will cause overall savings to decline as the interest rate increases, while large substitution effects (as in panel (c)) can cause savings to increase as the interest rate rises.

(d) True or False*: If the purpose of the policy described in the previous part of the problem is to increase the amount of consumption households have in the future, then the policy will succeed so long as consumption is always a normal good.*

Answer: This is true. The substitution effect unambiguously points to greater consumption in the future, and the wealth effect points in the same direction (on the vertical axis) so long as future consumption is a normal good. Thus, an increase in the interest rate will definitely increase future consumption even though it will not definitely increase the amount that is saved today.

B: *Now suppose that tastes over consumption now, c_1, and consumption in the future, c_2, can be represented by the Constant Elasticity of Substitution utility function $u(c_1,c_2) = (c_1^{-\rho} + c_2^{-\rho})^{-1/\rho}$.*

(a) *Write down the constrained optimization problem assuming that the real interest rate is r and no government programs dealing with savings are in effect.*

Answer: The problem is

$$\max_{c_1,c_2} \left(c_1^{-\rho} + c_2^{-\rho}\right)^{-1/\rho} \text{ subject to } c_2 = (100000 - c_1)(1+r). \tag{8.53}$$

(b) *Solve for the optimal level of c_1 as a function of ρ and r. For what value of ρ is the household's savings decision unaffected by the real interest rate?*

Answer: Setting up the Lagrange function for problem (8.53) and solving the first two first order conditions, we get $c_2 = (1+r)^{1/(\rho+1)}c_1$. Plugging this into the budget constraint $c_2 = (100000 - c_1)(1+r)$ and solving for c_1, we get

$$c_1 = \frac{100,000}{(1+r)^{-\rho/(\rho+1)} + 1}. \tag{8.54}$$

The savings decision by the household is unaffected by r if c_1 — current consumption — is unaffected. The only way r disappears from the equation for c_1 we derived in (8.54) is for $\rho = 0$. In particular, when $\rho = 0$, $c_1 = 100,000/2 = 50,000$.

(c) *Knowing the relationship betwen ρ and the elasticity of substitution, can you make the statement quoted in (c) of part A more precise?*

Answer: We know that the elasticity of substitution for CES utility functions is $\sigma = 1/(1+\rho)$. Thus, for $\rho = 0$, $\sigma = 1$. As ρ approaches -1, σ approaches ∞, and as ρ approaches ∞, σ approaches 0.

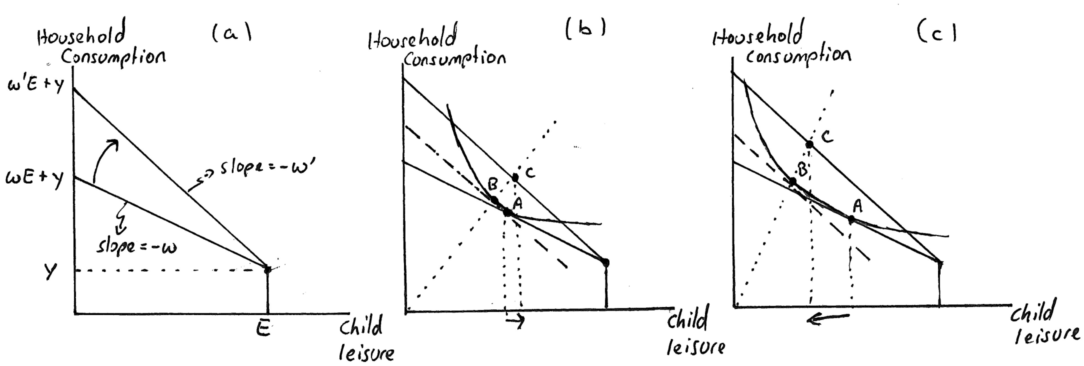
We know that an increase in the interest rate causes a substitution effect that increases savings and a wealth effect that decreases savings. We have shown in the previous part that these exactly offset each other when $\rho = 0$ (i.e. when $\sigma = 1$). As the elasticity of substitution gets larger than 1, the substitution effect increases and will result in an increase in savings — thus, when ρ falls between -1 and 0, savings will increase. Conversely, when ρ falls between 0 and ∞, the elasticity of substitution falls below 1 and the substitution effect is therefore outweighed by the wealth effect — resulting in less savings as the interest rate increases.

8.9 Policy Application: *International Trade and Child Labor: The economist Jagdish Bhagwati explained in one of his public lectures that international trade causes the wage for child labor to increase in developing countries. He then discussed informally that this might lead to more child labor if parents are "bad" and less child labor if parents are "good".*

A: *Suppose that households in developing countries value two goods: "Leisure time for Children in the Household" and "Household Consumption." Assume that the adults in a household are earning \$y in weekly income regardless of how many hours their children work. Assume that child wages are w per hour and that the maximum leisure time for children in a household is E hours per week.*

(a) *On a graph with "weekly leisure time for children in the household" on the horizontal axis and "weekly household consumption" on the vertical, illustrate the budget constraint for a household and label the slopes and intercepts.*

Answer: This initial budget is illustrated in panel (a) of Graph 8.11 where the bundle (E, y) is effectively the "endowment" bundle for the household — i.e. the bundle that does not depend on child wages.



Graph 8.11: Child Labor and International Trade

(b) *Now suppose that international trade expands and, as a result, child wages increase to w'. Illustrate how this will change the household budget.*

Answer: This is also illustrated in panel (a) of the graph — the budget rotates outward around the "endowment" bundle (E, y).

(c) *Suppose that household tastes are homothetic and that households require their children to work during some but not all the time they have available. Can you tell whether children will be asked to work more or less as a result of the expansion of international trade?*

Answer: You cannot tell — it depends on the size of the substitution effect and thus on the degree of substitutability between child leisure and household consumption. We know that tastes can be homothetic with little or no substitutability between goods (as in perfect complements), and tastes can be homothetic with perfect substitutability. Of course there are lots of in between cases. In panel (b), we illustrate the case of relatively little substitutability where the substitution effect from A to B is small and outweighed by the wealth effect from B to C to result in an overall increase in leisure for children. In panel (c), on the other hand, we illustrate the case where the substitution effect outweighs the wealth effect — resulting in a decrease in leisure for children.

The substitution effect here simply says that, as child wages increase, the opportunity cost of giving leisure to children increases and households will therefore give less leisure. The wealth effect, on the other hand, says that increasing child wages make the household richer — and richer households will consume more of all normal goods, including child leisure.

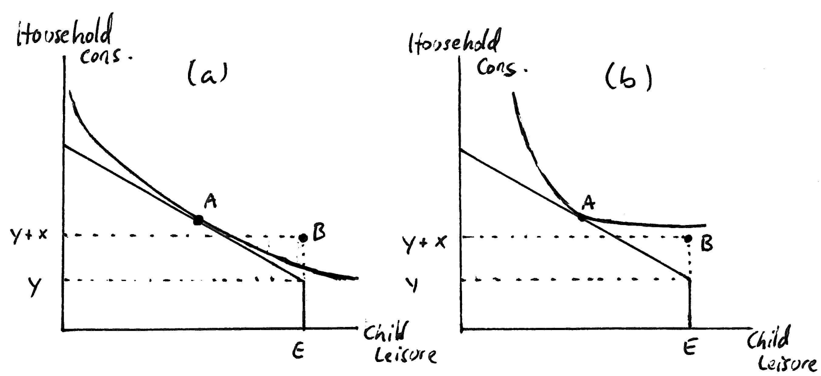
(d) *In the context of the model with homothetic tastes, what distinguishes "good" parents from "bad" parents?*

Answer: Good parents are those whose tastes look more like those in panel (b) while bad parents are those whose tastes look more like panel (c). Put differently, parents become "better" in this model the more they view child leisure and household consumption as complements. This has a certain amount of intuitive appeal: Good parents are those that essentially say that they can only become better off when household consumption goes up if child welfare (i.e. child leisure) also goes up — they are complements and have to go together. Bad parents are those that view household consumption as a substitute for child welfare.

(e) *When international trade increases the wages of children, it is likely that it also increases the wages of other members of the household. Thus, in the context of our model, y — the amount brought to the household by others — would also be expected to go up. If this is so, will we observe more or less behavior that is consistent with what we have defined as "good" parent behavior?*

Answer: This would cause a parallel shift in the budget beyond the initial rotation that results from the increase child wages. Such a parallel shift gives rise to a pure wealth effect. So long as child leisure is a normal good, increases in y would therefore cause increases in consumption of all goods — including child leisure. This would strengthen the wealth effect from the increase in w and thus cause more parents to reduce the amount of work their children have to undertake. Put differently, the more y is also increased by international trade, the more substitutable child leisure and household consumption can be and still cause parents to be "good".

(f) *In some developing countries with high child labor rates, governments have instituted the following policy: If the parents agree to send a child to school instead of work, the government pays the family an amount x. (Assume the government can verify that the child is in fact sent to school and does in fact not work, and assume that the household views time at school as leisure time for the child.) How does that alter the choice set for parents? Is the policy more or less likely to succeed the more substitutable the household tastes treat child "leisure" and household consumption?*



Graph 8.12: Child Labor and International Trade: Part II

Answer: Under this policy, the government in essence makes one additional bundle available to the household — a bundle in which the child's "leisure" or "non-work" hours are E and the household's consumption is y plus the payment x the government is providing in order for the child to attend school. This new bundle is depicted as bundle B in both panels of Graph 8.12. In each panel, A is the original optimal bundle before this policy was introduced. But in panel (a), the original optimal indifference curve is relatively flat and therefore passes below B while in panel (b) it is closer to the shape of perfect complements which makes it pass above B. Thus, conditional on A being the original optimum, the policy is more likely to induce the household to choose B (and thus send their child to school) the more substitutable are household consumption and child leisure.

B: *Suppose parental tastes can be captured by the utility function $u(c,\ell) = c^{0.5}\ell^{0.5}$. For simplicity, suppose further that $y = 0$.*

(a) *Specify the parents' constrained optimization problem and set up the appropriate Lagrange function.*

Answer: The problem to be solved is

$$\max_{c,\ell} c^{0.5}\ell^{0.5} \text{ subject to } c = w(E-\ell). \tag{8.55}$$

The Lagrange function is

$$\mathscr{L}(c,\ell,\lambda) = c^{0.5}\ell^{0.5} + \lambda(w(E-\ell)-c). \tag{8.56}$$

(b) *Solve the problem you have set up to determine the level of leisure the parents will choose for their children. Does w have any impact on this decision?*

Solving the first two first order conditions, you get $c = w\ell$. Plugging this into the budget constraint, you can then solve for $\ell = E/2$ and plugging this back into $c = w\ell$, you can get $c = wE/2$. The level of leisure parents choose for their children ($\ell = E/2$) is independent of wage — so w has no impact on their decision in this case.

(c) *Explain intuitively what you have just found. Consider the CES utility function (that has the Cobb-Douglas function you just worked with as a special case). For what ranges of ρ would you expect us to be able to call parents "good" in the way that Bhagwati informally defined the term?*

Answer: For the Cobb-Douglas tastes that are modeled, the substitution effect (that causes parents to reduce their children's leisure when w increases) is exactly offset by the wealth effect (which causes parents to increase their children's leisure as w increases). We know that Cobb-Douglas tastes are CES tastes with $\rho = 0$ and elasticity of substitution of 1. As ρ falls below zero, the goods become more substitutable and as ρ rises above zero they become more complementary. In part A we determined that parents are more likely to be "good" if they view child leisure as relatively complementary to household consumption — thus, for CES utility functions, parents are "bad" if $-1 \leq \rho < 0$ and parents are "good" if $0 < \rho \leq \infty$.

(d) *Can parents for whom household consumption is a quasilinear good ever be "good"?*

Anwer: Yes, if substitution effects are sufficiently small, such parents can be "good". This is because tastes that are quasilinear *in consumption* would only give rise to substitution effects with no wealth effect for household consumption (i.e. *on the vertical axis*). Thus, while the substitution effect points to an increase in household consumption and a decrease in child leisure, the wealth effect points to no further change in household consumption and an increase in child leisure. Put differently, while the quasilinearity of household consumption implies no wealth effect on the vertical axis, it also implies the entire wealth effect happens on the child leisure axis in the direction opposite to the substitution effect.

Be careful in this answer to pay attention to the fact that the question states that household consumption, not child leisure, is the quasilinear good. Had the question asked whether parents can be "good" if child leisure is the quasilinear good, the answer would have been an unambiguous no. This is because we would then only have a substitution effect on the horizontal axis — which implies that child leisure decreases and thus child labor increases with an increase in w.

(e) *Now suppose (with the original Cobb-Douglas tastes) that $y > 0$. If international trade pushes up the earnings of other household members — thus raising y, what happens to child leisure?*

Answer: Solving for the optimal leisure time (in the same way as we did above), we get

$$\ell = \frac{wE+y}{2w}. \tag{8.57}$$

The derivative of this with respect to y is positive — i.e. as y increases, so does the amount of leisure chosen for the child.

(f) *Suppose again that $y=0$ and the government introduces the policy described in part A(f). How large does x have to be in order to cause our household to send their child to school (assuming again that the household views the child's time at school as leisure time for the child)?*

Answer: Without participating in the policy, the household consumes $c = wE/2$ and $\ell = E/2$ — and therefore gets utility $(wE/2)^{0.5}(E/2)^{0.5} = w^{0.5}E/2$. If the household participates in the policy, it's child would get leisure of E and the household consumption would be x. Thus, participating in the policy means utility of $x^{0.5}E^{0.5}$. The household will be indifferent between the two options if the utility of participating and not participating are equal; i.e. if

$$\frac{w^{0.5}E}{2} = x^{0.5}E^{0.5}. \tag{8.58}$$

Solving this for x we get $x = wE/4$. For any x greater than this, the household is therefore better off choosing to send their child to school.

(g) *Using your answer to the previous part, put into words what fraction of the market value of the child's time the government has to provide in x in order for the family to choose schooling over work for their child?*

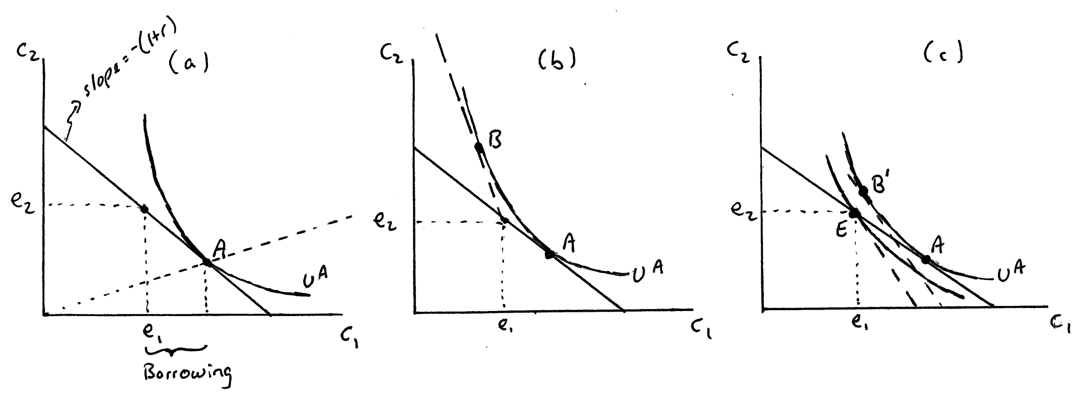
Answer: We concluded above that x has to be at least $wE/4$ in order for the household to be willing to send the child to school. The market value of the child's time endowment is wE. The amount that is required for the child to be sent to school is therefore equivalent to one quarter of the market value of the leisure time of the child.

8.10 Policy Application: *Subsidizing Savings versus Taxing Borrowing: In end-of-chapter exercise 6.10 we analyzed cases where the interest rates for borrowing and saving are different. Part of the reason they might be different is because of government policy.*

A: *Suppose banks are currently willing to lend and borrow at the same interest rate. Consider an individual who has income e_1 now and e_2 in a future period, with the interest rate over that period equal to r. After considering the tradeoffs, the individual chooses to borrow on his future income rather than save. Suppose in this exercise that the individual's tastes are homothetic.*

(a) *Illustrate the budget constraint for this individual — and indicate his optimal choice.*

Answer: This is illustrated in panel (a) of Graph 8.13.



Graph 8.13: Saving and Borrowing under Different Policies

(b) *Now suppose the government would like to encourage this individual to save for the future. One proposal might be to subsidize savings (through something like a 401K plan) — i.e. a policy that increases the interest rate for saving without changing the interest rate for borrowing. Illustrate how this changes the budget constraint. Will this policy work to accomplish the government's goal?*

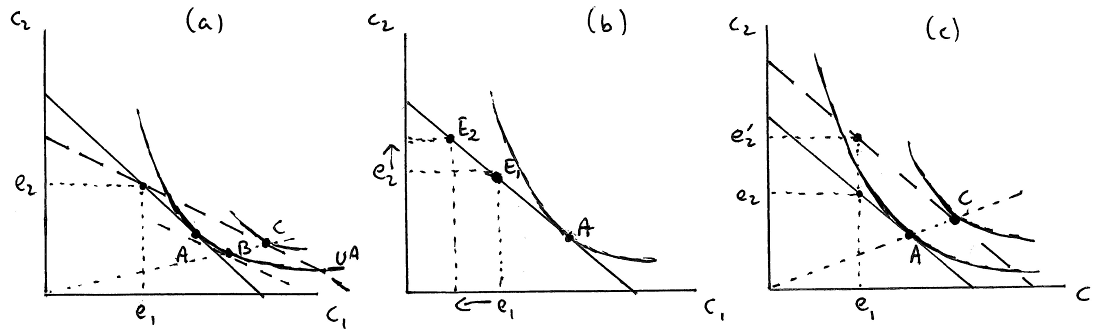
Answer: This policy will work if the increase in the interest rate for savings is sufficiently high. In panel (b) of Graph 8.13, we illustrate the smallest possible increase in the interest rate for savings that is necessary to induce a change in the individual's behavior. The increase in the interest rate for savings causes the slope of the budget to become steeper to the left of the bundle (e_1, e_2) where the individual saves rather than borrows. If it only becomes a little steeper, A is still optimal and no change in behavior takes place. But if it becomes sufficiently steeper (as graphed), B becomes optimal — which implies the individual switches from borrowing to saving.

(c) *Another alternative would be to penalize borrowing by taxing the interest the banks collect from loans — thus raising the effective interest rate for borrowing. Illustrate how this changes the budget. Will this policy cause the individual to borrow less? Can it cause him to start saving?*

Answer: An increase in the interest rate for borrowing makes the slope of the budget past (e_1, e_2) steeper. This causes a substitution effect in the direction of less borrowing (from A to B' in panel (c) of Graph 8.13). But with homothetic tastes, we can be sure that a tangency will never happen on the solid portion of the budget to the left of (e_1, e_2) — because initially (in panel (a)), when the interest rate was r for both borrowing and saving, the individual choose A. When tastes are homothetic, this means that the MRS is equal to this slope along the ray from the origin — which never intersects the budget to the left of (e_1, e_2). Thus, the most this policy might do is to cause the individual to optimize at the kink point of the new budget — point E in panel (c) of the graph. Thus, if the interest rate for borrowing increases enough, the individual might choose to stop borrowing — but he will not start saving.

(d) *In reality, the government often does the opposite of these two policies: Savings (outside qualified retirement plans) are taxed while some forms of borrowing (in particular borrowing to buy a home) are subsidized. Suppose again that initially the interest rate for borrowing and saving is the same — and then suppose that the combination of taxes on savings (which lowers the effective interest rate on savings) and subsidies for borrowing (which lowers the effective interest rate for borrowing) reduce the interest rate to $r' < r$ equally for both saving and borrowing. How will this individual respond to this combination of policies?*

Answer: This combination of policies causes the budget to rotate counter-clock wise through the bundle (e_1, e_2) — with a new slope that is shallower. This is illustrated in panel (a) of Graph 8.14. Such a change in the budget gives rise to a substitution effect from A to B and a wealth effect from B to C — both pointing toward greater consumption now and thus an increase in borrowing.



Graph 8.14: Saving and Borrowing under Different Policies: Part II

(e) *Suppose that, instead of taxing or subsidizing interest rates, the government simply "saves for" the individual by taking some of the individual's current income e_1 and putting it into the bank to collect interest for the future period. How will this change the individual's behavior?*

Answer: This is illustrated in panel (b) of Graph 8.14. All that the policy does is to transfer some of e_1 to e_2 — moving us from E_1 to E_2 in the graph. Since the interest rate is unchanged, the budget remains exactly as it was before. Thus, if A was optimal before, A is still optimal for the consumer — it just now means that consumer has to increase his borrowing by the amount of saving the government did for him. In other words, the consumer will undo what the government is doing on his behalf.

(f) *Now suppose that, instead of taking some of the person's current income and saving it for him, the government simply raises the social security benefits (in the future period) without taking anything away from the person now. What will the individual do?*

Answer: This policy essentially shifts e_2 up — but it does not change the interest rate. Thus, it causes a parallel shift in the budget — giving rise to a pure wealth effect that is illustrated in panel (c) of Graph 8.14. As a result, the individual will increase the amount he borrows.

B: *Suppose your tastes can be captured by the utility function $u(c_1, c_2) = c_1^{\alpha} c_2^{(1-\alpha)}$.*

(a) *Assuming you face a constant interest rate r for borrowing and saving, how much will you consume now and in the future (as a function of e_1, e_2 and r.)*

Answer: You would solve the problem

$$\max_{c_1, c_2} c_1^{\alpha} c_2^{(1-\alpha)} \text{ subject to } (1+r)e_1 + e_2 = (1+r)c_1 + c_2. \tag{8.59}$$

Solving the first two first order conditions, you get $c_2 = (1+r)(1-\alpha)c_1/\alpha$, and plugging this into the budget constraint, you can solve for c_1. Plugging the solution back into $c_2 = (1+r)(1-\alpha)c_1/\alpha$ you can then solve for c_2. You should get

$$c_1 = \frac{\alpha[(1+r)e_1 + e_2]}{(1+r)} \text{ and } c_2 = (1-\alpha)[(1+r)e_1 + e_2]. \tag{8.60}$$

(b) *For what values of α will you choose to borrow rather than save?*

Answer: In order for you to borrow, it must be that your optimal c_1 is greater than your current income e_1 — i.e. $c_1 > e_1$. Using the optimal c_1 just derived above, this implies

$$\frac{\alpha[(1+r)e_1 + e_2]}{(1+r)} > e_1. \tag{8.61}$$

Solving for α, we get that

$$\alpha > \frac{(1+r)e_1}{e_1(1+r) + e_2}. \tag{8.62}$$

(c) *Suppose that $\alpha = 0.5$, $e_1 = 100,000$, $e_2 = 125,000$ and $r = 0.10$. How much do you save or borrow?*

Answer: Using our solutions in equation (8.60), we get $c_1 = \$106,818.18$ and $c_2 = \$117,500$. Thus, you would borrow \$6,818.

(d) *If the government could come up with a "financial literacy" course that changes how you view the tradeoff between now and the future by impacting α, how much would this program have to change your α in order to get you to stop borrowing?*

Answer: Using equation (8.62), $\alpha = 0.4681$ would be necessary in order for you to neither borrow nor save. For α greater than that, you would continue to borrow.

(e) *Suppose the "financial literacy" program had no impact on α. How much would the government have to raise the interest rate for saving (as described in A(b)) in order for you to become a saver? (*Hint*: You need to first determine c_1 and c_2 as a function of just r. You can then determine the utility you receive as a function of just r — and you will not switch to saving until r is sufficiently high to give you the same utility you get by borrowing.)*

Answer: Using our answers from above, the utility you get from borrowing is

$$u(106818.18, 117500) = 106818.18^{0.5}\,117500^{0.5} \approx 112,031.85. \tag{8.63}$$

Put differently, the label on the indifference curve that you end up at if you borrow is 112,031.85. This indifference curve, and the optimal bundle when you borrow, is illustrated as point A in Graph 8.15.

We know that, as the interest rate for savings goes up, the budget to the left of $(e_1, e_2) = (100000, 125000)$ increases. To find how much the government must subsidize the interest rate in order to cause you to switch from borrowing to saving, we have to determine the interest rate that makes this steeper budget tangent to the same indifference curve that contains A — i.e. the indifference curve with utility 112,031.85.

With $e_1 = 100,000$, $e_2 = 125,000$ and $\alpha = 1/2$, equation (8.60) tells us you will consume

$$c_1 = \frac{100000(1+r) + 125000}{2(1+r)} \text{ and } c_2 = \frac{[100000(1+r) + 125000]}{2} \tag{8.64}$$

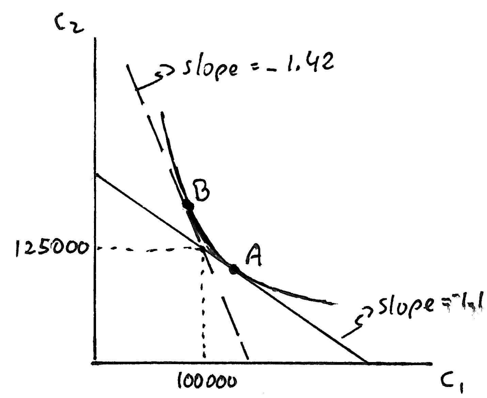
for any given interest rate r. This can also be written

$$c_1 = \frac{50,000(1+r) + 62,500}{(1+r)} \text{ and } c_2 = 50,000(1+r) + 62,500. \tag{8.65}$$

Using the results from equation (8.65), the utility you get at interest rate r is then

$$u(c_1, c_2) = \left(\frac{50,000(1+r) + 62,500}{(1+r)}\right)^{0.5} (50,000(1+r) + 62,500)^{0.5}. \tag{8.66}$$

Thus, to solve for the interest rate r that gets the same utility as the utility you currently get by borrowing, we have to solve



Graph 8.15: Subsidizing Saving

$$\left(\frac{50,000(1+r)+62,500}{(1+r)}\right)^{0.5}(50,000(1+r)+62,500)^{0.5}=112,031.85. \quad (8.67)$$

After some algebra and after applying the quadratic formula, this gives

$$r=0.1 \text{ and } r\approx 0.42. \quad (8.68)$$

The first of these results simply confirms that you reach the indifference curve labeled $u = 112,031.85$ when the interest rate is 0.1 — when we know you borrow at bundle A in the graph. The second interest rate is what the rate would have to rise to in order for you to get to the same indifference curve by saving — i.e. the interest rate that generates B in the graph as an optimum. For this particular individual, the government must increase the interest rate from saving to 42% in order to get the individual to save rather than borrow.

(f) *Verify your conclusion about the impact of the policy proposal outlined in A(c).*

Answer: In equation(8.65), we concluded that consumption now is

$$c_1=\frac{50,000(1+r)+62,500}{(1+r)}=50,000+\frac{62,500}{(1+r)}. \quad (8.69)$$

This expression decreases with an increase in r — which implies consumption now will fall as the interest rate for borrowing increases. Put differently, an increase in the interest rate for borrowing causes you to borrow less. In fact, when r reaches 0.25, c_1 becomes $100,000 — the amount of this period's income e_1. When r increases above 0.25, you would begin saving if r was both the interest rate for saving and borrowing. However, in this problem we only consider a policy in which the government raises the interest rate for borrowing and leaves the interest rate for saving unchanged. Thus, as r rises above 0.25, you simply engage in neither borrowing nor saving — optimizing instead at the kink point in your budget. (You can also show that at this point the MRS is larger in absolute value than the slope of the savings portion of the budget — which is another way of showing that the kink point is the optimal point.)

(g) *Verify you conclusion to A(d).*

Answer: Again, we already derived the consumption this period as a function of the interest rate as

$$c_1=50,000+\frac{62,500}{(1+r)}. \quad (8.70)$$

A decrease in the general interest rate for both saving and borrowing will thus increase c_1. Thus, you will increase your borrowing under these policies.

(h) *Verify your conclusion to A(e); i.e. suppose the government takes x of your current income e_1 and saves it — thus increasing e_2 by $x(1+r)$.*

Answer: The original budget constraint $(1+r)e_1 + e_2 = (1+r)c_1 + c_2$ is thus changed to $(1+r)(e_1 - x) + (e_2 + (1+r)x) = (1+r)c_1 + c_2$. The left hand side of the latter reduces to the left hand side of the former — thus the budget constraint remains the same. With the same budget constraint, the optimal choice remains unchanged.

(i) *Finally, suppose the increase in social security benefits outlined in A(f) is implemented; i.e. suppose e_2 is increased by x. How and by how much does your borrowing change?*

Answer: In equation (8.60), we determined

$$c_1 = \frac{\alpha[(1+r)e_1 + e_2]}{(1+r)}. \tag{8.71}$$

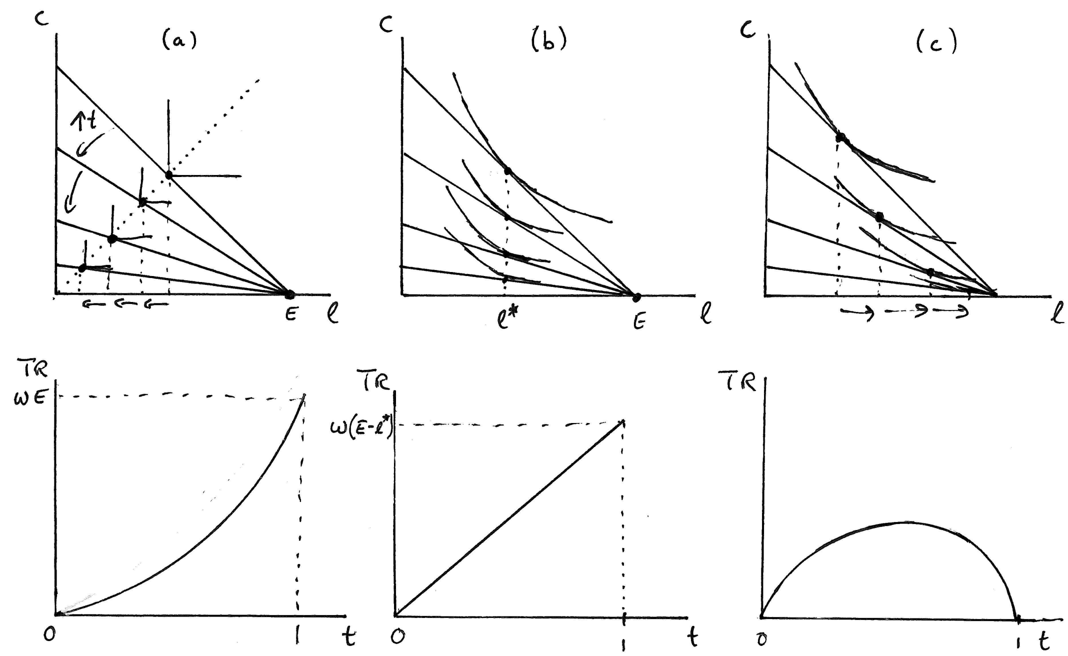
An increase in e_2 by x causes an increase in consumption c_1 by $\alpha x/(1+r)$ — which implies an increase in borrowing by that amount.

8.11 Policy Application: *Tax Revenues and the Laffer Curve: In this exercise, we will consider how the tax rate on wages relates to the amount of tax revenue collected.*

A: *As introduced in Section B, the* Laffer Curve *depicts the relationship between the tax rate on the horizontal axis and tax revenues on the vertical. (See the footnote in Section 8B.2.2 for background on the origins of the name of this curve.) Because people's decision on how much to work may be affected by the tax rate, deriving this relationship is not as straightforward as many think.*

(a) *Consider first the extreme case in which leisure and consumption are perfect complements. On a graph with leisure hours on the horizontal and consumption dollars on the vertical, illustrate how increases in the tax on wages affect the consumer's optimal choice of leisure (and thus labor).*

Answer: The top portion of panel (a) of Graph 8.16 illustrates multiple budget lines with diminishing slopes as the tax rate on wages increases (thus causing the slope of the budget line, $-(1-t)w$, to become smaller in absolute value). When leisure and consumption are perfect complements, all optimal bundles will always occur on a single ray from the origin. As the tax rate increases, optimal leisure consumption decreases — thus causing labor supply to increase. At the tax rate goes to 1, the entire leisure endowment E is spent on work.



Graph 8.16: Substitution Effects and the Laffer Curve

(b) *Next, consider the less extreme case where a change in after-tax wages gives rise to substitution and wealth effects that exactly offset one another on the leisure axis. In which of these cases does tax revenue rise faster as the tax rate increases?*

Answer: The top portion of panel (b) of the graph illustrates this case — as the tax rate increases (and the budget becomes shallower), the optimal leisure consumption remains unchanged at ℓ^*. When wealth and substitution effects cancel each other out, as in panel (b), tax revenues increase as tax rates increase only because more money is collected for each hour the worker works. When consumption and leisure are perfectly complementary (as in panel (a)), tax revenues rise as tax rates increase not only because more is collected for each hour that is worked but also because more hours are worked. Thus, one would expect tax revenues to rise faster under the tastes in (a) than in (b).

(c) *On a graph with the tax rate (ranging from 0 to 1) on the horizontal and tax revenues on the vertical, how does this relationship differ for tastes in (a) and (b)?*

Answer: This is depicted in the lower graphs. The graph below panel (a) illustrates that tax revenues increase at a faster rate as the tax rate increases when leisure and consumption are perfect complements. Tax revenues reach the highest point when the tax rate approaches 1 — as the worker approaches working all the time and paying all his salary in taxes.[4] The graph below panel (b) illustrates the relationship between tax rates and tax revenues when wealth and substitution effects offset each other. Tax revenues increase at the same rate (since work hours remain unaffected) — with tax revenue approaching $w(E-\ell^*)$ as the tax rate approaches 1.

(d) *Now suppose that the substitution effect outweighs the wealth effect on the leisure axis as after-tax wages change. Illustrate this and determine how it changes the relationship between tax rates and tax revenue.*

Answer: This is illustrated in panel (c) of Graph 8.16. Since the substitution effect (which says to consume more leisure as the after-tax wage falls) outweighs the wealth effect (that says to consume less leisure as the after-tax wage falls), increasing tax rates result in increasing leisure consumption. As tax rates increase, we therefore have two competing effects on tax revenues: On the one hand, more is collected for every hour worked. On the other hand, however, fewer hours are worked. Thus, it is quite plausible for tax rates to reach a point where additional increases in rates imply decreases in tax revenues. This relationship — which is the one that has come to be associated with the term "Laffer curve", is illustrated below panel (c).

(e) *Laffer suggested (and most economists agree) that the curve relating tax revenue (on the vertical axis) to tax rates (on the horizontal) is initially upward sloping but eventually slopes down — reaching the horizontal axis by the time the tax rate goes to 1. Which of the preferences we described in this problem can give rise to this shape?*

Answer: Only the preferences in panel (c) — those where substitution effects outweigh wealth effects on the leisure axis — can result in such a shape. Most economists agree that eventually — as tax rates approach 100 percent, tax revenue falls to zero. The only disagreement is at what point the downward sloping part of the curve begins. Part of the reason Laffer became known for this curve is that he popularized the notion that the peak of the Laffer curve may, in some instances, occur at rates considerably lower than 100 percent. To the extent that this is true, it is possible to cut tax rates and increase tax revenues. Most economists believe that, at least in the U.S., federal tax rates are now to the left of the peak on the Laffer curve — implying that tax revenues cannot be increased through tax cuts. At the same time, the top marginal tax rates were once 90% in the U.S. (in the 1960's) and 70% in 1980. It is when rates are that high (as opposed to top rates around 40% as is the case in the U.S. today) that it is considerably more likely that we are on the "wrong side of the Laffer curve."

(f) *True or False: If leisure is a normal good, the Laffer Curve can have an inverted U-shape only if leisure and consumption are (at least at some point) sufficiently substitutable such that the substitution effect (on leisure) outweighs the wealth effect (on leisure).*

Answer: This is true — see the answers above. (It is plausible, though, that the effect is more like that in panel (a) for low tax rates and then increasingly becomes like panel (c) for higher and higher rates.)

B: *In Section 8B.2.2, we derived a Laffer Curve for the case where tastes were quasilinear in leisure. Now consider the case where tastes are Cobb-Douglas — taking the form* $u(c,\ell)=c^{\alpha}\ell^{(1-\alpha)}$. *Assume*

[4] The problem is well defined as the tax rate increases all the way up to but not including 1. When the tax rate hits 1, it makes no difference to the worker whether he works or does not — because both leisure and consumption are "essential" goods.

that a worker has 60 hours of weekly leisure endowment that he can sell in the labor market for wage w.

(a) *Suppose the worker's wages are taxed at a rate t. Derive his optimal leisure choice.*

Answer: We need to solve

$$\max_{c,\ell} c^{\alpha}\ell^{(1-\alpha)} \text{ subject to } c=(1-t)w(60-\ell). \tag{8.72}$$

Solving this in the usual way, we get $\ell = 60(1-\alpha)$.

(b) *For someone with these tastes, does the Laffer Curve take the inverted U-shape described in Section 8B.2.2. Why or why not? Which of the cases described in A does this represent?*

Answer: We just derived that $\ell = 60(1-\alpha)$ — which means that the number of weekly hours worked is equal to 60α. Thus, work hours are not impacted by the tax rate t — which means substitution and wealth effects exactly offset each other as in the case described in A(b). The relationship between tax rates t and tax revenues TR is then quite straightforward:

$$TR = w(60\alpha)t, \tag{8.73}$$

which has the shape depicted in the lower graph of panel (b) in Graph 8.16, a straight line with intercept of zero and slope $w(60\alpha)$.

(c) *Now consider the more general CES function $(\alpha c^{-\rho}+(1-\alpha)\ell^{-\rho})^{-1/\rho}$. Again derive the optimal leisure consumption.*

Answer: We now need to solve the problem

$$\max_{c,\ell} \left(\alpha c^{-\rho}+(1-\alpha)\ell^{-\rho}\right)^{-1/\rho} \text{ subject to } c=(1-t)w(60-\ell). \tag{8.74}$$

The usual first two first order conditions simplify to

$$c=\left(\frac{\alpha(1-t)w}{(1-\alpha)}\right)^{1/(\rho+1)}\ell. \tag{8.75}$$

Substituting this into the budget constraint $c=(1-t)w(60-\ell)$, we get

$$\ell=\frac{60(1-t)w}{\left(\frac{\alpha(1-t)w}{(1-\alpha)}\right)^{1/(\rho+1)}+(1-t)w} = 60\left[\left(\frac{\alpha}{(1-\alpha)}\right)^{1/(\rho+1)}((1-t)w)^{-\rho/(\rho+1)}+1\right]^{-1}. \tag{8.76}$$

(d) *Does your answer simplify to what you would expect when $\rho=0$?*

Answer When $\rho=0$, equation (8.76) reduces to

$$\ell=60\left[\left(\frac{\alpha}{(1-\alpha)}\right)^{1}((1-t)w)^{0}+1\right]^{-1}=60\left[\left(\frac{\alpha}{(1-\alpha)}\right)+1\right]^{-1}=60(1-\alpha). \tag{8.77}$$

This is exactly what we derived for the Cobb-Douglas tastes in (a) — which makes sense since CES utility functions become Cobb-Douglas when ρ approaches 0.

(e) *Determine the range of values of ρ such that leisure consumption increases with t.*

Answer: We are interested in the change in ℓ with a change in t — i.e. we are interested in the derivative of ℓ in equation (8.76) with respect to t. This derivative is

$$\frac{\partial\ell}{\partial t}=-60\left[\left(\frac{\alpha}{(1-\alpha)}\right)^{\frac{1}{\rho+1}}((1-t)w)^{-\frac{\rho}{\rho+1}}+1\right]^{-2}\left(\frac{-\rho}{(\rho+1)}\right)\left(\frac{\alpha}{(1-\alpha)}\right)^{\frac{1}{\rho+1}}((1-t)w)^{-\frac{2\rho+1}{\rho+1}}(-w). \tag{8.78}$$

The equation is a mess — but determining whether it is positive or negative is not too difficult. First, we can cancel the negative sign at the end with the negative sign in the middle of the expression. This gives us

$$\frac{\partial \ell}{\partial t} = -60\left[\left(\frac{\alpha}{(1-\alpha)}\right)^{\frac{1}{\rho+1}}((1-t)w)^{-\frac{\rho}{\rho+1}}+1\right]^{-2}\left(\frac{\rho}{(\rho+1)}\right)\left(\frac{\alpha}{(1-\alpha)}\right)^{\frac{1}{\rho+1}}((1-t)w)^{-\frac{2\rho+1}{\rho+1}}(w). \tag{8.79}$$

None of the terms in this expression can be negative — with the exception of the middle term $(\rho/(\rho+1))$ that is negative if $-1<\rho<0$ and positive if $\rho>0$. Given the negative sign that remains at the front of the expression, we can then conclude that

$$\frac{\partial \ell}{\partial t} > 0 \text{ if and only if } -1<\rho<0, \tag{8.80}$$

$$\frac{\partial \ell}{\partial t} = 0 \text{ if and only if } \rho=0, \text{ and} \tag{8.81}$$

$$\frac{\partial \ell}{\partial t} < 0 \text{ if and only if } \rho>0. \tag{8.82}$$

In other words, when $-1\leq\rho<0$, leisure consumption goes up as the tax rate increases, and when $\rho>0$ the reverse is true. This exactly mirrors the graphs in Graph 8.16 — with $\rho=0$ representing the middle panel (b), $\rho>0$ representing the case where consumption and leisure are relatively complementary (as in panel (a)), and $-1<\rho<0$ representing the case where consumption and leisure is relatively substitutable (as in panel (c)).

(f) *When ρ falls in the range you have just derived, what happens to leisure consumption as t approaches 1? What does this imply for the shape of the Laffer Curve?*

Answer: We just derived that leisure consumption increases with t if and only if $-1<\rho<0$. Leisure consumption is given in the expression (8.76) which is

$$\ell == 60\left[\left(\frac{\alpha}{(1-\alpha)}\right)^{1/(\rho+1)}((1-t)w)^{-\rho/(\rho+1)}+1\right]^{-1}. \tag{8.83}$$

Note that as t approaches 1, $(1-t)w$ approaches zero. The exponent on the term $((1-t)w)$ is $-\rho/(\rho+1)$ — which is positive when $-1<\rho<0$.[5] Thus, the term

$$\left(\frac{\alpha}{(1-\alpha)}\right)^{1/(\rho+1)}((1-t)w)^{-\rho/(\rho+1)} \tag{8.84}$$

goes to zero as t approaches 1. This leaves us, as t goes to 1, with

$$\ell = 60[0+1]^{-1} = 60. \tag{8.85}$$

Thus, for any $-1<\rho<0$, leisure consumption goes to the entire leisure endowment of 60 as t approaches 1 — which means that labor supply goes to zero as t approaches 1. This further implies that tax revenues will fall to zero as the tax rate approaches 1 — as depicted in the lower portion of panel (c) in Graph 8.16. The Laffer curve therefore takes the inverted U-shape that we typically expect.

(g) *Suppose $\alpha=0.25$, $w=20$ and $\rho=-0.5$. Calculate the amount of leisure a worker would choose as a function of t. Then derive an expression for this worker's Laffer Curve and graph it.*

Answer: Plugging these values into expression (8.76), we get

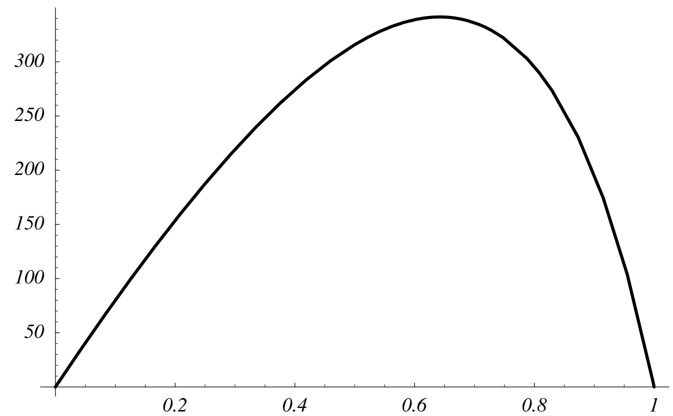
$$\ell = 60\left[\left(\frac{0.25}{0.75}\right)^{\frac{1}{0.5}}((1-t)20)^{\frac{-(-0.5)}{0.5}}+1\right]^{-1} = 60\left[\left(\frac{1}{3}\right)^{2}(1-t)20+1\right]^{-1} = \frac{540}{20(1-t)+9}. \tag{8.86}$$

[5] This is because the denominator $(\rho+1)$ is positive, and the numerator is positive since it is a negative number multiplied by a negative sign.

The government collects $TR = tw(60-\ell) = 20t(60-\ell)$ in revenue — which gives us the Laffer curve

$$TR = 20t\left(60 - \frac{540}{20(1-t)+9}\right) = \frac{24000t(1-t)}{20(1-t)+9}. \tag{8.87}$$

This is plotted in Graph 8.17 (with t on the horizontal and TR on the vertical).



Graph 8.17: The Laffer Curve when $\rho = -0.5$, $\alpha = 0.25$ and $w = 20$