CHAPTER

13

Production Decisions in the Short and Long Run

We already saw in Chapter 12 that the single-input model in Chapter 11 was just a slice of the 2-input model with capital held fixed. We now build on this insight — illustrating how short run constraints inhibit immediate adjustments to long run production plans as underlying conditions change. We also find ways of connecting short and long run cost curves — and illustrating the difference between short run and long run profit. Throughout, we try to be very consistent in the following sense: We only call something a *cost* if it is really an economic cost — and we reserve the terms *expenditure* or *expense* for outlays that include sunk costs. This is a departure from the typical way in which textbooks treat costs — but I think it actually makes a lot that follows easier and less confusing while focusing us on what we really mean by economic costs (and economic profit).

Chapter Highlights

The main points of the chapter are:

1. Not every financial outlay by a firm is an **economic cost** for the firm. Economic costs include only opportunity costs — i.e. only those outlays that actually affect economic behavior. **If we include in "costs" only real economic costs**, then it will *always* be the case that the **supply curve is the part of the** MC **curve that lies above** AC.

2. The financial outlays on capital are *fixed* in the short run and are therefore *not* a short run cost — because they have to be paid regardless of what the firm does. These outlays become a *variable* cost in the long run because capital can be varied in the long run. There are really no such things as **fixed costs** in the short run, but there may be such costs (like recurring license fees) in the long run.

3. Firms will produce (on their short run supply curves) and *not* shut down so long as **short run profit** is not negative, and firms will produce (on their long run supply curves) and *not* exit the industry so long as **long run profit** is not negative. Both short and long run profit subtract economic costs from revenue, but what counts as an economic cost differs between the short and long run (because some financial outlays that are costs in the long run are sunk in the short run). As a result, the (short run) **shut down price is lower than the** (long run) **exit price**.

4. Output **supply responses to a change in output price are larger in the long run than in the short run** — implying that long run supply curves are shallower than short run supply curves. Similarly, **input demand curves are shallower in the long run than in the short run**.

5. **Output supply curves shift as input prices change** — to the left as they increase and to the right as they decrease. Similarly, **input demand curves shift as output prices change** — to the left as output price falls and to the right as output price increases.

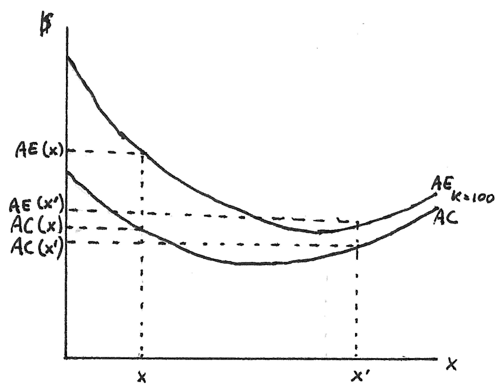
6. **Some long run economic relationships depend on the substitutability of capital and labor in production**. An increase in w may cause a long run increase or decrease in the amount of capital employed — and the long run response of output may be higher or lower than the short run response. Similarly, an increase in r may cause a long run increase or decrease in the amount of labor as well as a larger or smaller long run output response (relative to the short run response) — all depending on the relative substitutability of capital and labor.

13A Solutions to Within-Chapter-Exercises for Part A

Exercise 13A.1

Can you find similar rectangular areas that are equal to $FE_{k=100}$ for other output levels? Given that these rectangular areas have to be equal to one another, can you see why the AC and AE curves must be getting closer and closer as output rises?

Answer: The text graph illustrates the rectangular area for output level x^A, but we could similarly pick any arbitrary output level and illustrate a similar rectangular area that is equal to the fixed expenditure. For instance, in Exercise Graph 13A.1, we pick the output quantities x and x'. At x, the average expenditure is $AE(x)$ and the average cost is $AC(x)$. This implies that the fixed expenditure is the rectangle given by the distance $AE(x) - AC(x)$ times x. At x', the average expenditure is $AE(x')$ and the average cost is $AC(x')$. This implies that the fixed expenditure is the rectangle given by the distance $AE(x') - AC(x')$ times x'. As x' increases, the rectangle representing $FE_{k=100}$ gets longer — which implies that the only way for it to represent the same area as the similar rectangle for smaller output (like x) is for the rectangle to have less height. And this in turn implies that, as a simple matter of geometry, the $AE_{k=100}$ curve has to be getting closer to the $AC_{k=100}$ curve as output increases — i.e. the two curves have to converge. Of course this is simply another way of saying that the fixed expenditure is being spread across more output units on average as output increases, which means that the average fixed expenditure (per unit of output) falls with output.



Exercise Graph 13A.1 : Different Ways of Illustrating Fixed Expenditures

Exercise 13A.2

Can you explain why the *MC* curve intersects both the *AC* and the *AE* curves at their lowest points?

<u>Answer</u>: The *MC* curve has to intersect both the average curves at their lowest point for exactly the same reason — the only way for an average to rise is for the marginal contribution to the average to lie above the average. This is true for both curves because the only cost or expense that is added as output increases is what's included in the marginal cost curve. Once again, the analogy to grade averages used in previous chapters applies — the only way for your average course grade to rise is for your next exam grade to lie above the average.

Exercise 13A.3

Where in the graph would you locate the "marginal expenditure" curve (derived from the total expenditure curve)?

<u>Answer</u>: The marginal expenditure curve includes all those expenditures that increase with output. But the only expenditures that increase with output are those

that are already included in the marginal cost curve. Thus, the marginal expenditure curve is no different than the marginal cost curve — and in fact would be mislabeled as an expenditure curve because it includes only true short run economic costs. You can also see this by noting that the slope of the total expenditure curve is exactly the same everywhere as the slope of the (total) short run cost curve $C_{k=100}$.

Exercise 13A.4

Can you tell from the shape of the long run (total) cost curve whether the production process is increasing, decreasing or constant returns to scale?

Answer: Since the slope is constant, the production process has constant returns to scale.

Exercise 13A.5

Verify the derivation of cost curves in panels (e) and (f) in Graph 13.2. In what sense is the relationship between short run expenditure and long run cost curves similar in this case to the case we derived in the top panels of the graph for constant returns to scale production processes?

Answer: We can use the cost minimizing input bundles A, B, and C to verify the total and average cost numbers in panels (e) and (f). With a wage rate of $20 and a rental rate of $10, A costs 50(20)+100(10)=$2,000 — exactly as indicated in panel (e) in point A'. When 200 units of output cost $2,000 to produce, then the average cost is 2000/200=$10 — exactly as indicated in panel (f) in point A''. The same reasoning can be used to derive the points B' and B'' from B and C' and C'' from C. The result is similar to the constant returns to scale case in that every short run AE curve shares one point in common with the long run AC curve.

Exercise 13A.6

Where would you find the long run marginal cost curve in panel (f) of the Graph?

Answer: The long run marginal cost curve would have the same intercept as the long run AC curve and would intersect the AC curve at its lowest point from below.

Exercise 13A.7

A textbook author (not me!) once told his publisher to produce a graph such as panel (f) of Graph 13.2 and explained that he wanted the short run average expenditure curves corresponding to different levels of fixed capital to *each be tangent at their lowest point* to the U-shaped long run average cost curve. The graphics artist (who knew nothing about economics) came back to the author and sheepishly explained that such a graph cannot logically be drawn. What was wrong in the author's instructions?

Answer: The mistake made by the author was that he insisted the tangencies needed to occur at the lowest point of the average expenditure curves. The economics of the problem only implies that the average expenditure curves be tangent to the long run average cost curve — not that they be tangent at the lowest point of the AE curves. If the long run AC curve is U-shaped, only the average expenditure curve that is tangent at the lowest point of the AC curve has its tangency at its own lowest point. The average expenditure curves with tangencies to the left of the lowest point of AC are tangent to the left of their own lowest points, and the average expenditure curves with tangencies to the right of the lowest point of AC are tangent to the right of their own lowest point.

Exercise 13A.8

Demonstrate that the firm's (long run) profit is zero when $p = 10$.

Answer: When $p = 10$ and the firm produces where $p = MC$, it will produce precisely the quantity at the lowest point of the long run AC curve (where the MC curve crosses) — i.e. it produces 200 output units. Its total revenues are therefore \$2,000. We also know from the long run AC curve that, when the firm produces 200 units, it incurs an average cost of \$10 per unit — or a total cost of \$2,000. Thus, revenues are equal to long run costs — which implies long run profit is zero.

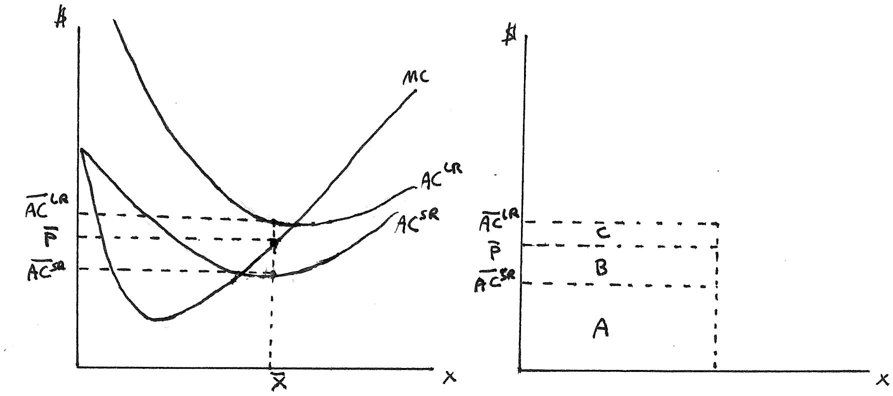
Exercise 13A.9

Can you illustrate that short run economic profits will be positive when price falls between the lowest points of the $AC_{k=100}$ and the AC^{LR} curves even though total expenditures exceed total revenues? What will long run economic profits be in that price range?

Answer: Suppose price is $\overline{p}$ as illustrated in panel (a) of Exercise Graph 13A.9 — i.e. price falls between the lowest point of the short and long run AC curves. If the firm produces, it will then produce where $\overline{p}$ equals MC — i.e. it will produce $\overline{x}$. At that level of production, it incurs a short run average cost of $\overline{AC}^{SR}$ and a long run average cost of $\overline{AC}^{LR}$ as indicated on the vertical axis. This implies a total short run cost of $\overline{AC}^{SR} * \overline{x}$ and a total long run cost of $\overline{AC}^{LR} * \overline{x}$. In panel (b) of the graph, this is illustrated as a total short run cost equal to area A and a total long run cost equal to area $A+B+C$. Total revenue, on the other hand, is $\overline{p} * \overline{x}$ — which is equal to area $A+B$. This implies that short run profit — i.e. revenue minus short run cost — is equal to $A+B-A=B$ while long run profit — i.e. revenue minus long run cost — is equal to $A+B-(A+B+C)=-C$. Thus, short run profit is positive when $\overline{x}$ is produced while long run profit is negative.

Exercise 13A.10

In many beach resorts on the east coast of the U.S., business is brisk in the summers but slow in the winters. In summers, resort rentals are sold out at high



Exercise Graph 13A.9 : Positive SR and Negative LR Profit

weekly rates, but in winters they are only partially rented at much lower rates. If you were to calculate expenses and revenues on a monthly basis, you would almost certainly find these resorts with revenues greater than expenses in the summers and expenses greater than revenues in the winters. How come these resorts don't just shut down in winters?

Answer: Monthly expenses include fixed expenses that are not economic costs in the short run but are fixed costs in the long run. It makes sense for resorts to stay open so long as they can cover at least their short run costs. If they can also recover some of the fixed expenses on top of that, all the better. By staying open in the winter, they may not recover all their fixed expenses — but if they closed, they would lose all of the fixed expenses (which they have to pay regardless of whether they are open or not).

Exercise 13A.11

Compare Graphs 13.4 and 13.5. Why is the supply curve beginning at the higher average curve in 13.5 and on the lower one in 13.4?

Answer: In Graph 13.4, the supply curve is the short run supply curve — while in Graph 13.5 it is the long run supply curve. It therefore begins at the lowest point of the short run average cost curve in the first graph while it begins at the lowest point of the long run average cost curve in the second graph.

Exercise 13A.12

Can we say for sure that the lowest point of the long run *AC* curve will shift to the right when the license fee increases?

Answer: Yes, we can be sure about that. This is because we know that the long run MC curve is unaffected by a change in the cost of the license — and that MC curves always have to cross AC curves at their lowest point.

Exercise 13A.13

True or False: $p'MP_\ell^B = w^A$.

Answer: This is true. In the short run, we can vary labor fully — which means that we can fully reach the short run profit maximizing condition that the marginal revenue product of labor has to be equal to the wage rate. (This is what we found has to be true in the short run production cases we analyzed in Chapter 11.) At the new price p', we will therefore hire labor until the marginal product of labor times p' equals the unchanged wage w^A. If B is truly the profit maximizing production plan in the short run when price increases to p', it therefore has to be the case that we have adjusted labor input to the point where $p'MP_\ell^B = w^A$.

Exercise 13A.14

If the marginal product of labor increases as additional capital is hired in the long run, can you tell whether the producer will hire additional labor (beyond ℓ^B) in the long run? Can you then identify the minimum distance above A on the ray through A the long run optimal isoquant in Graph 13.7a will lie?

Answer: If the marginal product of labor increases as the firm hires more capital in the long run, then this implies that the firm will also want to hire additional labor in the long run. As a result, its long run labor input will exceed the short run adjustment to ℓ^B — placing us onto the steeper ray and to the right of ℓ^B.

Exercise 13A.15

Can you see in panel (a) of Graph 13.8 the cost of not substituting from C to C'? Can you verify that the numbers in panel (b) are correct?

Answer: If the firm were to not substitute labor for capital, it would continue to use the input bundle C to produce 100 units of output. It would then require a "budget" equal to the dashed magenta line which has the slope indicated by the ratio of the new input prices but goes through C. Since this lies above the green "budget" that is cost-minimizing, the difference between the dashed magenta and the green "budgets" is the cost of not substituting. In panel (b), we then have three points to verify: all lie at output level of 100, but one lies on the blue, one on the (dashed) magenta, and one on the green cost curve. On the blue curve, C is derived from the original cost minimizing input bundle C in panel (a) when wage was 20 and the rental rate was 10. Since that input bundle contains 15 units of capital and 7.5 units of labor, the total cost (of producing 100 output units) is then $20 * 7.5 + 10 * 15 = \$300$ — exactly as shown in panel (b). On the (dashed) magenta curve in panel (b), we are assuming the firm did not respond to the change in the wage rate and still used input bundle C from panel (a). The cost, at the lower wage of 10, is then $10 * 7.5 + 10 * 15 = \$225$ — again exactly as shown in panel (b). Finally, the green point C' assumes that the firm has substituted labor for capital and is now using input bundle C' from panel (a). Thus, the cost of producing 100 output units is now $10 * 10.6 + 10 * 10.6 = \$212$ — as again shown in panel (b). The cost of not substituting in response to the wage change when producing 100 output units is therefore equal to $13.

Exercise 13A.16

Are these long-run or short-run cost curves?

Answer: Since the firm is substituting labor and capital, they are long run cost curves (which allow capital to vary.)

Exercise 13A.17

Can you verify that the numbers in panel (c) are correct?

Answer: We have three points in panel (c) to verify — one on the blue, one on the green and one on the (dashed) magenta curve. On the blue curve, $r = 10$. Panel (a) shows that at that rental, the cost minimizing production plan for 100 units of output involves 15 units of capital and 7.5 units of labor. At $w = 20$, this implies a total (long run) cost of $20 * 7.5 + 10 * 15 = \$300$ — exactly as shown in panel (c). On the (dashed) magenta curve, the firm is assumed to not change its input bundle in response to r increasing from 10 to 20. Thus, we still use the input bundle from panel (a) that contains 15 units of capital and 7.5 units of labor — with a total cost of $20 * 7.5 + 20 * 15 = \$450$ — again exactly as shown in panel (c). Finally, on the green curve we assume that the firm has optimally substituted labor for the more expensive capital — switching to the input bundle that contains 10.6 units of labor and 10.6 units of capital in panel (a). This results in a total cost of $20 * 10.6 + 20 * 10.6 = \$424$.

Exercise 13A.18

Assuming the original cost minimizing input bundle remains C, which of the three curves graphed in Graph 13.8c would be different (and how would it be different) if the inputs in panel (a) of the graph were more substitutable? How would the graph change if the two inputs were perfect complements in productions?

Answer: The green curve would be different. If the inputs were more substitutable, the benefit from substituting labor for capital would be greater — implying that the green curve would lie closer to the blue and farther from the (dashed) magenta. If the inputs were perfect complements, there would be no benefit from substituting labor for capital — which implies the green curve would lie exactly on top of the (dashed) magenta curve.

Exercise 13A.19

In Graph 13.8d, we already derived conditional labor demand curves along which capital is allowed to adjust. Explain why these are *not* long run labor demand curves.

Answer: In Graph 13.8a, we showed how a decrease in the wage rate will cause producers to substitute labor for capital as they consider the economically most efficient way of producing any *fixed quantity of output* (and an increase in the wage rate will do the reverse). As a result, conditional input demand curves slope down (Graph 13.8d). This by itself does not, however, tell us how much labor demand will adjust beyond its short run adjustment when w rises or falls because *conditional* labor demand curves simply tell us how much labor a producer hires *conditional* on wanting to produce a particular level of output. As we see from Graph 13.9, for instance, an increase in w causes the new profit maximizing input bundle to fall below the original isoquant — thus causing not only a decrease in the quantity of labor demanded but also a decrease in the quantity of output supplied. The optimal output level thus changes when the wage rate changes. What we then really want to know is not how labor demand responds *conditional* on the output level remaining the same as it was before but rather how *actual* labor demand responds to changes in input prices given that the profit-maximizing output quantity changes simultaneously.

Exercise 13A.20

Can you tell from just seeing the tangency at (ℓ^A, k^A) of the isocost with the isoquant whether the production plan $A = (\ell^A, k^A, x^A)$ is profit maximizing at prices (w^A, r^A, p^A)?

Answer: No, you cannot — because p^A does not appear anywhere in the graph. We can tell that A involves the cost minimizing input bundle for producing output level x^A at input prices (w^A, r^A), but we cannot tell whether the output quantity x^A is in fact the profit maximizing output quantity. If it is, then A is in fact the profit maximizing production plan; if it is not, then (ℓ^A, k^A) is just the cost minimizing input bundle for producing output level x^A.

Exercise 13A.21

Do you see from Graph 13.9 that long run demand curves for labor (with respect to wage) must slope down, as must long run demand curves for capital (with respect to the rental rate)?

Answer: As wage increases, the graph shows us moving to the left — i.e. less labor; and as w decreases, the graph shows the reverse — i.e. more labor. Thus, the long run quantity of labor demanded increases with a drop in wage and decreases with an increase in wage — which means the long run labor demand curve must slope down. Similarly, as the rental rate increases, the graph shows us moving *down*, and as it decreases it shows us moving *up*. Since capital is measured on the vertical axis. This implies a decrease in r causes an increased use of capital while an increase in r causes less use of capital — which is the same as saying that the long run demand for capital slopes down.

Exercise 13A.22

Does Graph 13.9 tell us anything about whether the cross-price demand curve for labor (with the rental rate on the vertical axis) slopes up or down in the long run?

Answer: No, it does not. As r increases, the region of input bundles where the new profit maximizing production plan might lie contains bundles with more and less labor than ℓ^A. The same is true as r falls. Thus, based on this graph, it would seem that cross-price input demand curves could slope up or down. This is clarified in later graphs in the chapter.

Exercise 13A.23

Where in Graph 13.9 will our new production plan fall after we have made our short run labor adjustment?

Answer: It would fall somewhere along the horizontal line connecting A to k^A.

Exercise 13A.24

We know that we will decrease output in the short run as w increases because we hire fewer workers. In the case of robots and workers, do you think that we will increase or decrease output once we can hire more robots in the long run?

Answer: This will become clearer in the next section. However, note that we increase the number of robots in the long run because the MP_k has gone up — and then we increase it more in order to replace some more of our workers. This suggests that output will be higher in the long run than in the short run.

Exercise 13A.25

Suppose labor and capital were perfect complements in production. What would the analogous graph for an increase in w look like?

Answer: No matter what the ratio of input prices, production would always take place on the 45 degree line that would contain both A and C. But the perfect complementarity between capital and labor implies that the firm would do nothing to change production in the short run when w increases modestly — because capital is fixed in the short run. Thus $B = A$ — which implies the short run labor demand curve would be vertical while the long run labor demand curve would still be downward sloping (and thus again more responsive to changes in w.) The relationship between w and k would be similar to that in panel (i) of the text graph. This logic holds for modest changes in w. If w increases by a lot, then it may be the case that the firm will hire less labor in the short run.

Exercise 13A.26

Demonstrate that $MP_k^B < MP_k^A$ in panel (c) of Graph 13.10.

Answer: In panel (c), the new optimal input bundle C contains *less* capital input than the original bundle A — which implies that the producer cannot immediately switch to the long run optimum when capital is fixed in the short run. Rather, in the short run the producer switches to input bundle B which has the characteristic that the isocost containing B cuts the isoquant containing B from below — i.e. $TRS^B < -w'/r^A$ or equivalently

$$\frac{MP_\ell^B}{MP_k^B} > \frac{w'}{r^A} \text{ which implies } \frac{p^A MP_\ell^B}{p^A MP_k^B} > \frac{w'}{r^A}. \qquad (13A.26)$$

In the short run, we know the firm will adjust labor until $p^A MP_\ell^B = w'$. The above equation then implies that $p^A MP_k^B < r^A$ and thus (since $p^A MP_k^A = r^A$) that $MP_k^B < MP_k^A$.

Exercise 13A.27

How is the long run response in output related to the short run response in output as w increases? What does your answer depend on? (*Hint*: You should be able to see the answer in Graph 13.10.)

Answer: The more substitutable the inputs are in production, the more likely it is that the drop in output will be less in the long run than in the short run (as in panel (a)); and the more complementary the inputs are in production, the more likely it is that the drop in output is greater in the long run than in the short run (as in panel (c).) There is also the in-between case (as in panel (b)) where the long run and short run response in output is identical.

Exercise 13A.28

Can you arrive at these conclusions intuitively using again the examples of robots and computers?

Answer: First, consider the case of robots which are relatively substitutable with workers. If the cost of robots increases, then we will substitute to workers since they are substitutable — causing a relatively small decrease in output. This is because workers and robots do similar things in production. Second, consider the case of computers and graphic artists. These are relatively complementary — each needs the other in order to increase output. Thus, if the cost of computers increases, we would not be able to substitute them easily for graphic artists — and thus, as we reduce our rentals of computers, we would need to let workers go. This causes a decrease in the number of workers (graphic artists) — and a relatively larger decrease in output.

Exercise 13A.29

In panel (a) of Graph 13.7, we determined that the firm will once again end up on the steeper ray once it can adjust capital. Call the new (long-run) input bundle at the higher output price C. Can you now tell what will determine whether C lies to the right or left of B?

Answer: From the extremes illustrated in Graph 13.12, we can tell the following: The more complementary capital and labor are in production, the more likely it will be that C lies to the right of B. It is in those cases that the marginal product of labor increases as capital increases. The more substitutable capital and labor are in production, however, the more likely it will be that C lies to the left of B. In such cases, an increase in capital decreases the marginal product of labor — which causes the firm to shift away from labor as it transitions to the long run when it can use more capital.

13B Solutions to Within-Chapter-Exercises for Part B

Exercise 13B.1

Suppose the long run production function were a function of 3 inputs — labor, capital and land, and suppose that both labor and capital were variable in the short run but land is only variable in the long run. How would we now calculate the short run cost minimizing labor and capital input levels conditional on some (short run) fixed level of land?

Answer: You would have to solve a cost minimization problem with capital and labor as choice variables. The expense associated with land would not factor into the problem. (If you did include the cost of land in the cost minimization problem, it would simply drop out as you solve for the first order conditions.)

Exercise 13B.2

Can you use the expressions above to justify the difference in the (total) cost and total expenditure curves in panel (a) of Graph 13.1 as well as the difference between AC and AE in panel (b) of that graph?

Answer: The difference between $E_{k^A}(x, w^A, r^A)$ and $C_{k^A}(x, w^A)$ is $r^A k^A$ which does not depend on x. Thus, the difference between the short run expense and cost curves is a constant amount — implying that the expenditure curve simply lies above the cost curve by that constant amount. The averages of the two expressions are

$$AE_{k^A}(x, w^A, r^A) = \frac{w^A \ell_{k^A}(x) + r^A k^A}{x} \quad \text{and} \quad AC_{k^A}(x, w^A) = \frac{w^A \ell_{k^A}(x)}{x} \tag{13B.2.i}$$

which implies that the difference between the average expenditure and cost curves is

$$AE_{k^A}(x, w^A, r^A) - AC_{k^A}(x, w^A) = \frac{r^A k^A}{x}. \tag{13B.2.ii}$$

Since $r^A k^A$ is a constant, this difference becomes smaller as x increases — leading to the fact that the short run AE curve converges to the short run AC curve.

Exercise 13B.3

Can you derive from this the relationship between long run average cost and short run average expenses as illustrated graphically in Graph 13.2?

Answer: The relationship implies that

$$\frac{E_{k^A}(x, w^A, r^A)}{x} \geq \frac{C(x, w^A, r^A)}{x} \tag{13B.3}$$

which is the same as writing $AE_{k^A}(x, w^A, r^A) \geq AC(x, w^A, r^A)$, with the expression holding with equality when $x = x^A$. This is exactly what we derived graphically: the short run average expenditure curve lies above the long run average cost curve everywhere except for the output level at which the short run fixed capital is equal to what the cost-minimizing firm would choose in the long run for that output level.

Exercise 13B.4

In the case of U-shaped average cost curves, how can you use the mathematical expressions above to argue that the short run "shut down" price is lower than the long run "exit" price?

Answer: Suppose the lowest point of the long run AC curve occurs at output level $\overline{x}$. At that point, the short run $AE_{\overline{k}}$ curve (where $\overline{k}$ is the long run optimal level of capital for producing $\overline{x}$) is tangent to the long run AC; i.e. $AC(\overline{x}) = AE_{\overline{k}}(\overline{x})$. From the fact that the difference between short run expenditures and short run costs is equal to the fixed expense on capital (in the short run), we also know that $AE_{\overline{k}}(\overline{x}) > AC_{\overline{k}}(\overline{x})$. Together, these results imply

$$AC(\overline{x}) > AC_{\overline{k}}(\overline{x}); \quad (13B.4)$$

i.e. the long run average cost curve lies above the short run AC curve at $\overline{x}$ where the long run AC attains its minimum. The exit price lies at the lowest point of AC while the shutdown price lies at the lowest point of $AC_{\overline{k}}$. We have just shown that the latter lies below the former.

Exercise 13B.5

Verify that these numbers are correct.

Answer: Substituting $(w, r, x) = (20, 10, 1280)$ into the conditional input demands, we get

$$\ell(20, 10, 1280) = \left(\frac{10}{20}\right)^{1/2} \left(\frac{1280}{20}\right)^{5/4} = \left(\frac{1}{4}\right)(64)^{5/4} = 128 \quad (13B.5.i)$$

and

$$k(20, 10, 1280) = \left(\frac{20}{10}\right)^{1/2} \left(\frac{1280}{20}\right)^{5/4} = (2)^{1/2}(64)^{5/4} = 256. \quad (13B.5.ii)$$

Multiplying these by their respective input prices and adding (or, alternatively, evaluating $C(w, r, x)$ at (20,10,1280)), we get that the cost is 20(128)+10(256)=$5,120.

Exercise 13B.6

What is the short run cost (as opposed to expenditure) function?

Answer: The short run cost function is the same as the short run expenditure function except that it does not include the fixed expense on capital. Thus,

$$C_{k^A=256}(x,20) = 20\ell_{k^A=256}(x) = \frac{x^{5/2}}{20^{3/2}256}. \qquad (13B.6)$$

Exercise 13B.7

Verify that, when $x = 1280$, the short run expense is equal to the long run cost.
Answer: The short run expense is

$$E_{k^A=256}(1280,20,10) = \frac{1280^{5/2}}{20^{3/2}256} + 2560 = 2560 + 2560 = \$5,120. \qquad (13B.7.i)$$

The long run cost is

$$C(1280,20,10) = 0.66874(1280)^{5/4} = \$5,120. \qquad (13B.7.ii)$$

Exercise 13B.8

Does the inclusion of a fixed cost cause any change in conditional input demands? What about unconditional input demands?

Answer: In the cost minimization problem (from which conditional input demands are derived), we would now minimize $C = w\ell + rk + FC$ — but since FC enters as a constant, it would drop out as we take the derivatives to get the first order conditions that we solve for the conditional demands. Thus, the inclusion of FC has no effect on conditional input demand functions. (This should make intuitive sense: The least cost way of producing any output level still involves the same input bundle regardless of how high a license fee we have to pay to start producing). In the profit maximization problem (from which we derive the unconditional input demands), we maximize revenues minus costs — which include such fixed costs in the long run. However, when maximizing $\pi = px - w\ell - rk - FC$, we take first order conditions that are derivatives of π — and constants like FC again drop out in the process. Thus, unconditional input demands are unaffected — except to the extent to which the FC affects the exit price and thus the point at which no labor is demanded by the firm. In other words, unconditional input demand functions will remain the same as without the FC except for the fact that they are "shorter".

Exercise 13B.9

Does the inclusion of a fixed cost change either the (short-run) "shut down" price or the (long-run) "exit" price?

Answer: It cannot affect the short run shut-down price since it is not a cost in the short run. It does, however, affect the long run exit price — which is the lowest point of the long run AC curve that includes the FC.

Exercise 13B.10

Would including the fixed expense rk^A in the short run profit maximization problem (so that the objective function becomes $px - w\ell - rk^A$) make any difference as the problem is solved?

Answer: No, it would make no difference because it would drop out as we take first derivatives to get the first order conditions.

Exercise 13B.11

Equation (13.30) can also be read as "the slope of the long run output supply function is larger than the slope of the short run output supply function (with respect to price)." But the long run supply curve in Graph 13.7 appears to have a shallower (and thus smaller) slope than that of the short run supply curve. How can you reconcile what the math and the graphs seem to be telling us?

Answer: The apparent discrepancy arises from the fact that the supply curves we graph are *inverse* slices of the supply function (holding input prices fixed). Thus, when we invert the supply curve picture from part A of the chapter, steeper curves become shallower and shallower curves become steeper. The inverse supply *curves* are therefore such that the long run supply will look steeper than the short run supply — which is consistent with the partial derivatives we derived.

Exercise 13B.12

Verify that these are truly the short run output supply and input demand functions by checking to see if the short run functions give the same answers as the long run functions when (p, w, r)=(5,20,10).

Answer: Plugging in the output price of \$5 and the wage of \$20, we get

$$x_{k=256}(5,20) = 3225\left(\frac{5}{20}\right)^{2/3} \approx 1280 \;\text{ and }\; \ell_{k=256}(5,20) = 1290\left(\frac{5}{20}\right)^{5/3} \approx 128 \qquad (13B.12)$$

when capital is fixed at $k = 256$.

Exercise 13B.13

Panels (a) and (b) of Graph 13.16 are analogous to panels (b) and (e) of Graph 13.8. Now calculate the relevant curves and graph them for the case that is analogous to panels (c) and (f) of Graph 13.8 where, instead of wage falling from \$20 to \$10, the rental rate of capital rises from \$10 to \$20.

Answer: The cost function is

$$C(w, r, x) = 2(wr)^{1/2}\left(\frac{x}{20}\right)^{5/4}. \qquad (13B.13.i)$$

When $w = 20$ and $r = 10$, the cost function becomes $C(20,10,x) = 0.66874x^{5/4}$,

and when $w = 20$ and $r = 20$, it becomes $C(20,20,x) = 0.94574x^{5/4}$. Thus, the cost function shifts *up* by $0.277x^{5/4}$. Taking partial derivatives of these two "slices" of the cost function with respect to x, we get the marginal cost function shifting from $MC(20,10,x) = 0.83593x^{1/4}$ to $MC(20,20,x) = 1.18218x^{1/4}$ — i.e. it shifts up by $0.34625x^{1/4}$. These shifts include both the *direct* and the *indirect* effects from substitutions of labor for capital as r increases. To isolate the *direct* effect, we would have to keep the input demands equal to what they were originally when $r = 10$. We can calculate these from the conditional input demand functions

$$\ell(w,r,x) = \left(\frac{r}{w}\right)^{1/2}\left(\frac{x}{20}\right)^{5/4} \text{ and } k(w,r,x) = \left(\frac{w}{r}\right)^{1/2}\left(\frac{x}{20}\right)^{5/4}. \quad (13B.13.ii)$$

At $w = 20$ and $r = 10$, these are $\ell(20,10,x) = 0.01672x^{5/4}$ and $k(20,10,x) = 0.03344x^{5/4}$. Isolating the direct effect assumes these remain unchanged when r increases to 20 — which would imply a "cost" function of

$$\begin{aligned}\overline{C}(20,20,x) &= 20\ell(20,10,x) + 20k(20,10,x) \\ &= 20\left(0.01672x^{5/4}\right) + 20\left(0.03344x^{5/4}\right) = 1.00311x^{5/4}\end{aligned} \quad (13B.13.iii)$$

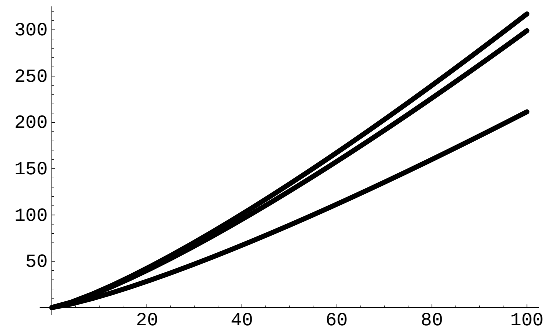
with corresponding marginal cost of $\overline{MC}(20,20,x) = 1.25389x^{1/4}$. Without taking account of the fact that firms will substitute away from capital when r increases, both the cost and marginal cost curves would therefore be higher than they actually are once substitution effects have been taken into account (just as we illustrated in Graph 13.8 of the text in panels (c) and (f)). The three cost functions we derived are then graphed in Exercise Graph 13B.13(1) with the lowest corresponding to $C(20,10,x) = 0.66874x^{5/4}$, the middle corresponding to $C(20,20,x) = 0.94574x^{5/4}$ and the highest corresponding to $\overline{C}(20,20,x)$.

Similarly, the three marginal cost curves we derived are graphed in Exercise Graph 13B.13(2), with the lowest corresponding to $MC(20,10,x) = 0.83593x^{1/4}$, the middle corresponding to $MC(20,20,x) = 1.18218x^{1/4}$ and the highest corresponding to $\overline{MC}(20,20,x) = 1.25389x^{1/4}$.

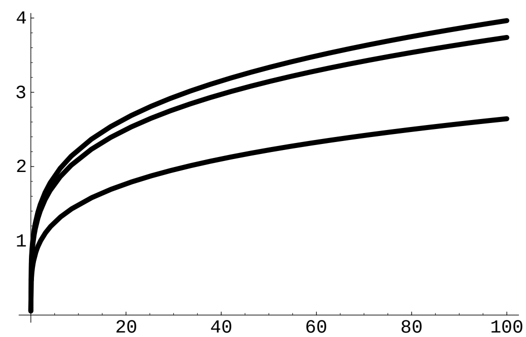
Exercise 13B.14

If the generalized CES function was used as a utility function instead of the version where A and β are set to 1, would the underlying tastes represented by that function be changed?

Answer: No, regardless of what values A and β took (so long as they are positive), the shapes of the indifference curves would be unaffected. Only their labeling would change, but the ordering would be preserved.



Exercise Graph 13B.13(1) : Costs and Substitution Effects



Exercise Graph 13B.13(2) : Marginal Costs and Substitution Effects

Exercise 13B.15

Explain why the direct effect in the table does not depend on the degree of substitutability between capital and labor in production.

Answer: The direct effect of a drop in w is the change in costs associated with just the drop in the wage and not due to any substituting behavior by the firm. Thus, the direct affect assumes the firm will continue to use the same input bundle as before to produce 5,000 units of output — in effect cutting the cost of using the same amount of labor by half when the wage drops by half. Thus, since the direct effect explicitly excludes the substitutions of capital and labor, its size does not depend on the substitutability of capital and labor.

Exercise 13B.16

Show that the short and long run input demand curves calculated for the production function $f(\ell,k) = 20\ell^{2/5}k^{2/5}$ in equation (13.33) and (13.31) are downward sloping.

Answer: We simply have to show that the derivatives with respect to input prices are less than zero. For the short run labor demand curve, this derivative is

$$\frac{\partial \ell_{k=256}(p,w)}{\partial w} = -\left(\frac{2}{3}\right)\frac{2150}{p^{1/3}w^{5/3}} = -\frac{4300}{3p^{1/3}w^{5/3}} < 0. \tag{13B.16.i}$$

For the long run, the derivatives of the input demands with respect to their prices are

$$\frac{\partial \ell(p,w,r)}{\partial w} = -98304\frac{p^5}{r^2w^4} < 0 \text{ and } \frac{\partial k(p,w,r)}{\partial r} = -98304\frac{p^5}{w^2r^4} < 0. \tag{13B.16.ii}$$

Exercise 13B.17

Can you make sense of the fact that the demand for labor falls less (both in the short and long run) the more complementary labor and capital are in production?

Answer: In the short run, the firm has a fixed amount of capital that is already paid for. If capital and labor are relatively complementary in production, then using the capital implies the firm has to hold onto much of its labor in the short run. In the long run, the firm has the opportunity to substitute away from labor and into more capital — but again, if the two inputs are relatively complementary, the firm cannot employ much of such substituting behavior. If, on the other hand, capital and labor are very substitutable, then it is easier for firms to adjust labor in the short run (because it is not that needed to keep the fixed capital productively employed) as well as the long run (because it is now easier to substitute away from the more expensive labor and toward capital when the latter can be adjusted.)

Exercise 13B.18

What value of ρ — and what implied elasticity of substitution between capital and labor — corresponds to the "in between case"?

Answer: The "in between case" happens when the firm does not adjust its capital in the long run (following a change in the wage) — which occurs in the table when $\rho = -0.5$. In this case, the firm reduces its labor in the short run but makes no further adjustments (to either capital or labor) in the long run because it happens to be the case that the marginal revenue product of capital is exactly equal to the rental rate after the short run labor adjustment has been made. We learned in Chapter 5 that the elasticity of substitution for a CES function is $1/(1+\rho)$ — thus, when $\rho = -0.5$, the elasticity of substitution is 2.

Exercise 13B.19

Can you identify in Table 13.4 the relationship of the substitutability of capital and labor to the degree of short versus long run response in labor demand from an increase in output price? Is this consistent with what emerges in Graph 13.12?

Answer: As ρ approaches -1, labor and capital approach perfect substitutes in production; whereas as ρ approaches ∞, labor and capital approach perfect complements. The table shows that, the more substitutable are labor and capital, the more labor demand ($\ell_k(25,10,10)$) responds in the short run (compared to $\ell(20,10,10)$.) In the next column ($\ell(25,10,10)$), the table shows that labor falls from the short to the long run when capital and labor are relatively substitutable whereas it rises from the short to the long run when capital and labor are relatively complementary. Since the long run adjustment of capital is the same regardless of the substitutability of capital and labor (because an increase in price does not change the slopes of isocosts and thus keeps the firm maximizing along the same ray from the origin in the isocost map), this implies that the long run response of increasing capital causes the firm to reduce its labor from the short to the long run when the inputs are relatively substitutable but increase it when they are relatively complementary. This should make intuitive sense: When the inputs are relatively substitutable, the (long run) increase in capital makes labor less productive on the margin — causing the firm to let go of some of its labor. When inputs are relatively complementary, on the other hand, the (long run) increase in capital makes labor more productive on the margin — causing firms to want to hire more. In the first case, the firm does not need additional workers to work the additional machines that come on line in the long run (and can in fact replace some of them with machines) — while in the latter case the firm needs additional workers to work the new machines that come on line.

13C Solutions to Odd Numbered End-of-Chapter Exercises

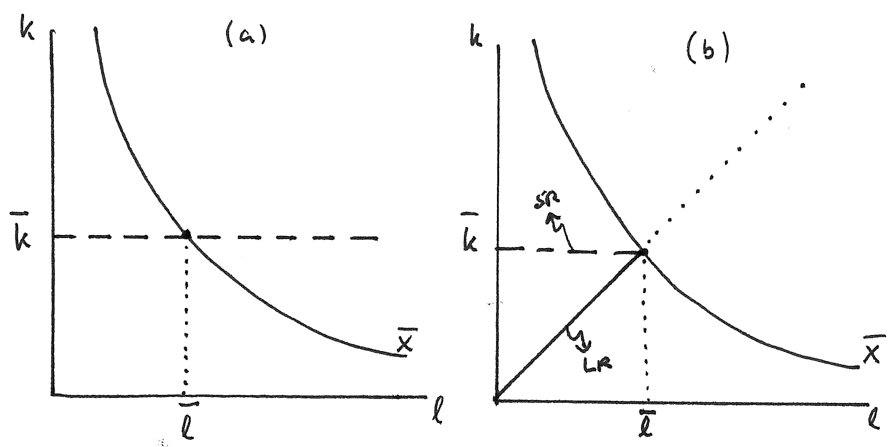
Exercise 13.1

The following problem explores the relationship between maximizing profit in the short and long run when capital is fixed in the short run.

A: *Suppose you have a homothetic production technology and you face output price p and input prices (w, r).*

(a) *On a graph with labor ℓ on the horizontal and capital k on the vertical axis, draw an isoquant and label a point on that isoquant as $(\bar{\ell}, \bar{k})$.*

Answer: This is done in both panels of Exercise Graph 13.1.



Exercise Graph 13.1 : Short Run and Long Run Profit Maximization

(b) *Suppose that the point in your graph represents a profit maximizing production plan. What has to be true at this point?*

Answer: It must be true that $pMP_k = r$ and $pMP_\ell = w$ — i.e. the marginal revenue products of the inputs must be equal to their input prices.

(c) *In your graph, illustrate the slice along which the firm must operate in the short run.*

Answer: This slice is indicated by the horizontal (dashed) line in panel (a) of Exercise Graph 13.1. Capital is fixed at $\bar{k}$ along this line.

(d) *Suppose that the production technology has decreasing returns to scale throughout. If p falls, can you illustrate all the possible points in your graph where the new profit maximizing production plan will lie in the long run? What about the short run?*

Answer: This is illustrated in panel (b) of Exercise Graph 13.1. The horizontal dashed line is a portion of the short run production slice — that portion that yields less output than the output along the isoquant $\overline{x}$. Since we know from Chapter 11 that short run production falls as price falls, any point along this dashed line can result from a decrease in p. The solid portion of the ray emanating from the origin represents the production plans that could result in the long run when firms can adjust capital. Again, we know that output will fall with a decrease in price, but in the long run capital will adjust so that the ratio of capital to labor remains constant. This is because input prices have not changed. Thus the ratio of input prices remains unchanged, which means all cost minimizing input bundles lie on this ray from the origin through the original profit maximizing bundle $(\overline{\ell}, \overline{k})$. (And, of course, it must be the case that any new long run profit maximizing production plan — while involving less output — still produces output in the least costly way.)

(e) *What condition that is satisfied in the long run will typically not be satisfied in the short run?*

Answer: Since the firm can only adjust labor in the short run, the firm will let go of labor until $pMP_\ell = w$ — i.e. until the marginal revenue product of labor is equal to the wage. However, the firm cannot adjust capital in the short run — which implies that the marginal revenue product condition will not hold for capital in the short run. In fact, since the firm will release capital in the long run, we know that it must be the case that $pMP_k < r$ at the short run profit maximum.

(f) *What qualification would you have to make to your answer in (d) if the production process had initially increasing but eventually decreasing returns to scale?*

Answer: In that case, there is a portion of the solid part of the ray (in panel (b) of the Graph) along which the firm will not profit maximize no matter how much price falls — because the firm would make a negative profit along that portion. Without knowing more, we cannot tell exactly where the solid part of the ray in the graph should end in this case — but we do know it should not extend all the way down to the origin in the graph.

B: *Consider the Cobb-Douglas production function $x = f(\ell, k) = A\ell^\alpha k^\beta$.*

(a) *For input prices (w, r) and output price p, calculate the long run input demand and output supply functions assuming $0 < \alpha, \beta \leq 1$ and $\alpha + \beta < 1$.*

Answer: Solving the usual profit maximization problem

$$\max_{\ell,k} \; pA\ell^\alpha k^\beta - w\ell - rk, \tag{13.1.i}$$

we get the input demand functions

$$\ell(w,r,p) = \left(\frac{pA\alpha^{(1-\beta)}\beta^\beta}{w^{(1-\beta)}r^\beta}\right)^{1/(1-\alpha-\beta)} \text{ and } k(w,r,p) = \left(\frac{pA\alpha^\alpha\beta^{(1-\alpha)}}{w^\alpha r^{(1-\alpha)}}\right)^{1/(1-\alpha-\beta)}. \tag{13.1.ii}$$

Plugging these into the production function and simplifying, we then get the output supply function

$$x(w,r,p) = \left(\frac{Ap^{(\alpha+\beta)}\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)^{1/(1-\alpha-\beta)}. \tag{13.1.iii}$$

(b) *How would your answer change if $\alpha+\beta \geq 1$?*

Answer: If $\alpha+\beta=1$, then the production function has constant returns to scale — which implies that either all output levels are profit maximizing or only one of the corners (i.e. zero or infinity) is optimal. If $\alpha+\beta>1$, the production function has increasing returns to scale — which implies that the firm should produce an infinite amount of output. (Of course this does not make sense in light of the fact that we are assuming price-taking behavior — i.e. we are assuming firms small enough relative to the market such that they cannot influence price.)

(c) *Suppose that capital is fixed at $\overline{k}$ in the short run. Calculate the short run input demand and output supply functions.*

Answer: We need to use the short run slice of the production function that holds $\overline{k}$ fixed — i.e. $x = \left[A\overline{k}^{\beta}\right]\ell^{\alpha}$. The short run profit maximization problem is then

$$\max_{\ell} \; p\left[A\overline{k}^{\beta}\right]\ell^{\alpha} - w\ell, \tag{13.1.iv}$$

where we do not take into account the *expense* of capital that is fixed and we treat the bracketed term as a constant. (Even if we did include it, it would drop out since only ℓ is a choice variable and the capital expense term would simply drop out as we take first order conditions). Solving this in the usual way, we get the short run labor demand function

$$\ell_{\overline{k}}(p,w) = \left(\frac{\alpha pA\overline{k}^{\beta}}{w}\right)^{1/(1-\alpha)}. \tag{13.1.v}$$

Substituting this back into the production function and simplifying, we get the short run output supply function

$$x_{\overline{k}}(p,w) = \left(A\overline{k}^{\beta}\right)^{1/(1-\alpha)}\left(\frac{\alpha p}{w}\right)^{\alpha/(1-\alpha)}. \tag{13.1.vi}$$

(d) *What has to be true about α and β for these short run functions to be correct?*

Answer: It has to be the case that the production process has decreasing returns to scale — i.e. $\alpha+\beta<1$. Otherwise, the true solution is a corner solution that will not be picked up by our usual optimization method.

(e) *Suppose $\overline{k} = k(w,r,p)$ (where $k(w,r,p)$ is the long run capital demand function you calculated in part (a).) What is your optimal short run labor demand and output supply in that case?*

Answer: If we plug our long run capital demand function in for $\overline{k}$ in the short run labor demand function, we get, after simplifying the expression,

$$\ell_{k(w,r,p)}(p,w) = \left(\frac{pA\alpha^{(1-\beta)}\beta^{\beta}}{w^{(1-\beta)}r^{\beta}}\right)^{1/(1-\alpha-\beta)}. \quad (13.1.vii)$$

Similarly, if we plug the long run capital demand function in for $\overline{k}$ in short run supply function, we get

$$x_{k(w,r,p)}(p,w) = \left(\frac{Ap^{(\alpha+\beta)}\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}.\right)^{1/(1-\alpha-\beta)}. \quad (13.1.viii)$$

(f) *How do your answers compare to the long run labor demand function $\ell(w,r,p)$ and the long run supply function $x(w,r,p)$ you calculated in part (a)? Can you make intuitive sense of this?*

Answer: The short run labor demand and output supply functions we calculated are exactly equal to the long run labor demand and output supply functions calculated earlier; i.e.

$$\ell_{k(w,r,p)}(p,w) = \ell(w,r,p) \text{ and } x_{k(w,r,p)}(p,w) = x(w,r,p). \quad (13.1.ix)$$

This should make intuitive sense: If I provide you in the short run with the optimal level of capital for you to reach the long run profit maximum at current input and output prices, then you'll choose your labor input exactly as you would in the long run. Put differently, we are "fixing" capital at exactly the long run quantity — which means there is nothing to keep you from implementing the long run profit maximizing production plan immediately.

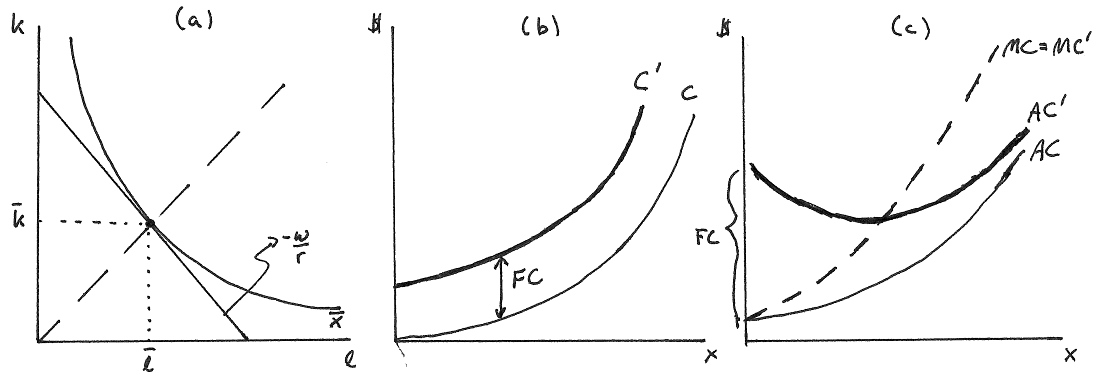
Exercise 13.3

In this exercise we add a (long run) fixed cost to the analysis.

A: *Suppose the production process for a firm is homothetic and has decreasing returns to scale.*

(a) *On a graph with labor ℓ on the horizontal and capital k on the vertical axis, draw an isoquant corresponding to output level $\overline{x}$. For some wage rate w and rental rate r, indicate the cost minimizing input bundle for producing $\overline{x}$.*

Answer: This is illustrated in panel (a) of Exercise Graph 13.3 where the input bundle $(\overline{\ell},\overline{k})$ is the cost minimizing input bundle to produce $\overline{x}$.



Exercise Graph 13.3 : Fixed Costs

(b) *Indicate in your graph the slice of the production frontier along which all cost minimizing input bundles lie for this wage and rental rate.*

Answer: Since the production process is homothetic, all tangencies of isocosts with slope $-w/r$ with isoquants for different output levels will lie on the ray emanating from the origin and passing through $(\bar{\ell}, \bar{k})$. This is also illustrated in panel (a) of Exercise Graph 13.3.

(c) *In two separate graphs, draw the (total) cost curve and the average cost curve with the marginal cost curve.*

Answer: This is illustrated in panels (b) and (c) of Exercise Graph 13.3 as C, AC and MC. Since the slice of the production frontier indicated by the dashed ray in panel (a) has decreasing returns, the shape of the cost function must be such that cost increases at an increasing rate as x goes up. The same then holds for AC, with the MC beginning at the same point as AC but lying above AC throughout.

(d) *Suppose that, in addition to paying for labor and capital, the firm has to pay a recurring fixed cost (such as a license fee). What changes in your graphs?*

Answer: Nothing changes in panel (a) — because the fact that the firm has to pay some cost to begin producing does not change how much labor and capital will be needed to reach different isoquants. The cost curve, however, shifts up to C', with C and C' parallel to each other and the difference being the FC. The marginal cost curve, however, remains unchanged since fixed costs do not enter the *additional* cost of producing output. Finally, the average cost curve moves up but, unlike the cost curve, not in a parallel fashion. It increases by the FC when $x = 1$ (because the average fixed cost is FC/x). As x increases, however, FC/x falls — which causes the new average cost curve AC' to converge to the original AC as x gets large.

(e) *What is the firm's exit price in the absence of fixed costs? What happens to that exit price when a fixed cost is added?*

Answer: In the absence of fixed costs, the firm's exit price is equal to the marginal cost of producing the first unit of output — because that is where the marginal cost curve crosses the AC curve. When fixed costs are introduced, however, the exit price rises to the lowest point of the new U-shaped average cost curve AC' where the unchanged MC curve crosses it. Thus, the exit price increases.

(f) *Does the firm's supply curve shift as we add a fixed cost?*

Answer: No, the supply curve does not shift, but it does become "shorter". It does not shift because the MC curve does not shift. It becomes "shorter" because the exit price increases. Thus, the supply curve before the introduction of the fixed cost is the entire MC curve in panel (c) of Exercise Graph 13.3, but after the FC is introduced, it shrinks to only the portion of the MC curve that lies above AC'.

(g) *Suppose that the cost minimizing input bundle for producing $\overline{x}$ that you graphed in part (a) is also the profit maximizing production plan before a fixed cost is considered. Will it still be the profit maximizing production plan after we include the fixed cost in our analysis?*

Answer: This will still be the profit maximizing production plan *if* it is optimal for the firm not to exit. In that case, price is sufficiently high relative to w and r such that it crosses MC above AC' in panel (c) of Exercise Graph 13.3. However, it may be the case that the introduction of FC implies that it is no longer profit maximizing to produce — and that a corner solution of producing nothing is optimal. This occurs if the price falls below AC'.

B: *As in exercises 13.1 and 13.2, suppose the production process is again characterized by the production function $x = f(\ell, k) = A\ell^{\alpha}k^{\beta}$ with $0 < \alpha, \beta \leq 1$ and $\alpha + \beta < 1$.*

(a) *If you have not already done so in a previous exercise, derive the (long run) cost function for this firm.*

Answer: The long run cost function (solved from the cost minimization problem) is

$$C(w,r,x) = w\ell(w,r,x) + r(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)}. \quad (13.3.i)$$

(b) *Now suppose that, in addition to the cost associated with inputs, the firm has to pay a recurring fixed cost of FC. Write down the cost minimization problem that includes this FC. Will the conditional input demand functions change as a result of the FC being included?*

Answer: The cost minimization problem would now be

$$\min_{\ell,k} \; w\ell + rk + FC \text{ subject to } x = A\ell^{\alpha}k^{\beta}. \quad (13.3.ii)$$

When we now write down the Lagrange function and take derivatives to get the first order conditions from which we derive the conditional input demand functions, the FC term will disappear since it enters the objective function as a constant. Thus, the conditional input demand functions will be unchanged by the addition of a FC term. This should make intuitive sense: Just because the firm has to pay something like a license fee to start producing does not mean it doesn't need exactly as much capital and labor to produce any given level of output as it did before.

(c) *Write down the new cost function and derive the marginal and average cost functions from it.*

Answer: Since the conditional input demands are no different than they were before the FC, the new cost function is the same as the one derived in (a) except that we also have to include the fixed cost; i.e. it now becomes

$$C(w,r,x) = w\ell(w,r,x) + r(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + FC. \tag{13.3.iii}$$

From this, we can derive the marginal cost function

$$MC(w,r,x) = \frac{\partial C(w,r,x)}{\partial x} = \left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} \tag{13.3.iv}$$

and the average cost function

$$AC(w,r,x) = \frac{C(w,r,x)}{x} = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{FC}{x}. \tag{13.3.v}$$

Note that the marginal cost function is the same as it would be without the FC but the AC function is not.

(d) *What is the shape of the average cost curve? How does its lowest point change with changes in the FC?*

Answer: We can infer the shape of the AC curve by checking whether its derivative with respect to x is positive or negative. The derivative is

$$\frac{\partial AC(w,r,x)}{\partial x} = \left[(1-\alpha-\beta)\left(\frac{w^{\alpha}r^{\beta}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} x^{(1-2(\alpha+\beta))/(\alpha+\beta)}\right] - \frac{FC}{x^2}. \tag{13.3.vi}$$

The bracketed term is positive (since $\alpha+\beta<1$) but the FC term is negative. Furthermore, as x approaches zero, the first term is smaller than the absolute value of the second term — implying an initially negative sign and thus an initially downward sloping AC curve. As x gets larger, however, the absolute value of the FC term gets smaller, with the positive bracketed term eventually outweighing the negative FC term. Thus, at some point, the slope of the AC curve becomes positive. This implies a U-shape to the AC curve. And, as FC increases, only the second term changes while the bracketed first term remains the same. Thus, the AC curve will have negative slope for a larger range of x — implying that the bottom of the U moves to the right as FC increases. We can of course also infer this from the fact that the upward sloping MC curve is unchanged as FC increases — because the MC curve must cross AC no matter how high FC gets. Thus, as FC pushes up the AC curve, it must be that its lowest point slides up along the unchanged MC curve. And, to calculate the level of output at which the AC reaches its lowest point, we can set equation (13.3.vi) to zero and solve for x to get

$$x = \left(\frac{A\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)\left(\frac{FC}{(1-\alpha-\beta)}\right)^{(\alpha+\beta)} \quad (13.3.\text{vii})$$

and note again that x increases as FC increases.

(e) *Does the addition of a FC term change the (long run) marginal cost curve? Does it change the long run supply curve?*

Answer: No, the FC term does not appear in the MC equation and thus has no impact on the marginal cost curve. This, of course, makes intuitive sense — a fixed cost is paid before production even begins and therefore does not impact the cost of producing additional units of output. Since the long run supply curve is a portion of the long run MC curve, we know that the supply curve is therefore not shifted by changes in the fixed cost. However, since the supply curve is that portion of MC that lies above the AC curve, and since AC shifts up with an increase in FC, the supply curve "shrinks" as FC increases.

(f) *How would you write out the profit maximization problem for this firm including fixed costs? If you were to solve this problem, what role would the FC term play?*

Answer: The profit maximization problem would then be

$$\max_{\ell,k} \; pA\ell^{\alpha}k^{\beta} - w\ell - rk - FC. \quad (13.3.\text{viii})$$

When we take first order conditions, we are taking derivatives of the objective function above — and thus FC disappears from the first order conditions that are used to solve the maximization problem. As a result, none of the input demand or output supply functions will change when FC is included in the profit maximization problem.

(g) *Considering not just the math but also the underlying economics, does the addition of the FC have any implications for the input demand and output supply functions?*

Answer: We see from the math that the functions produced by our optimization problem are unchanged. However, we also know that the math does not identify corner solutions. In the absence of a fixed cost, we do not have to worry about such corner solutions so long as the production function has decreasing returns to scale — but when we add a fixed cost, we know from what we did in the earlier parts of the exercise that the AC curve takes on a U-shape as a result of the addition of the FC. Thus, while the exit price before a FC term is added is zero, it is now at the lowest point of the AC curve. This implies that the functions produced by the math are correct only for sufficiently high prices and/or sufficiently low wages and rental rates. If p gets too high relative to w and r, the firm should exit and produce nothing rather than the positive amount indicated by the math. This is analogous to our conclusion that only a portion of the MC curve will be the supply curve when there is a FC while the entire MC curve is the supply curve (under decreasing returns to scale) when there is no fixed cost.

Exercise 13.5

We will often assume that a firm's long run average cost curve is U-shaped. This shape may arise for two different reasons which we explore in this exercise.

A: *Assume that the production technology uses labor ℓ and capital k as inputs, and assume throughout this problem that the firm is currently long run profit maximizing and employing a production plan that is placing it at the lowest point of its long run AC curve.*

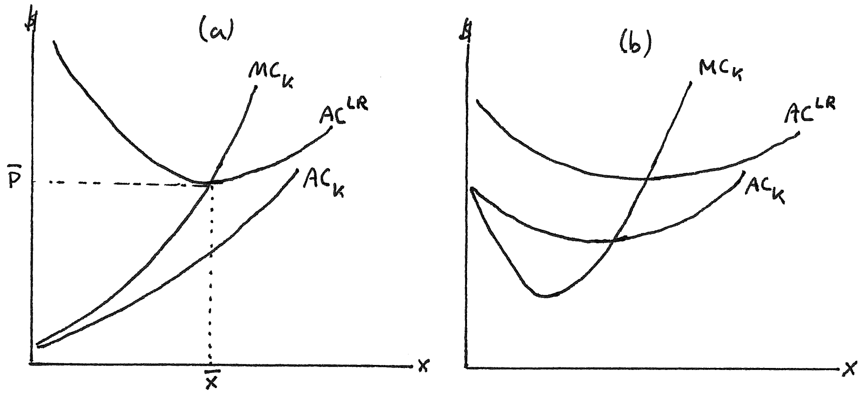
(a) *Suppose first that the technology has decreasing returns to scale but that, in order to begin producing each year, the firm has to pay a fixed license fee F. Explain why this causes the long run AC curve to be U-shaped.*

Answer: The long run AC curve is U-shaped in this case because the fixed cost F, while only an expense in the short run and therefore not included in short run AC curves, it is a real economic cost in the long run. At low levels of output, the average fixed cost F/x is large (because x is small) — causing AC to be high. As output increases, the average fixed cost F/x falls (because x gets large) — and therefore becomes a diminishing factor in the long run AC curve. Instead, the fact that the production process has decreasing returns to scale pushes AC up as x increases.

(b) *Draw a graph with the U-shaped AC curve from the production process described in part (a). Then add to this the short run MC and AC curves. Is the short run AC curve also U-shaped?*

Answer: This is illustrated in panel (a) of Exercise Graph 13.5(1).

The fact that the firm is currently profit maximizing at the lowest point of its long run AC curve implies that output price must be $\overline{p}$. The short run MC_k curve must then go through this lowest point on the AC^{LR} curve. Furthermore, since the production technology has decreasing returns to



Exercise Graph 13.5(1) : U-shaped Average Cost Curves

scale, it must be that any slice that holds capital fixed must also have decreasing marginal product of labor. Thus, the short run production function (that holds capital fixed) has decreasing returns to scale, and there are no fixed costs in the short run, only fixed expenses. This implies that the MC_k curve must be upward sloping, causing the short run AC_k curve to lie below it and be similarly upward sloping.

(c) *Next, suppose that there are no fixed costs in the long run. Instead, the production process is such that the marginal product of each input is initially increasing but eventually decreasing, and the production process as a whole has initially increasing but eventually decreasing returns to scale. (A picture of such a production process was given in Graph 12.16 in the previous chapter.) Explain why the long run AC curve is U-shaped in this case.*

Answer: The fact that the production process has initially increasing but eventually decreasing returns to scale implies that the long run average costs must initially fall but will eventually increase in the decreasing returns to scale portion of the production process. The reasoning is identical to that for the single input case with production frontiers that initially get steeper but eventually get shallower.

(d) *Draw another graph with the U-shaped AC curve. Then add the short run MC and AC curves.*

Answer: This is done in panel (b) of Exercise Graph 13.5(1). The U-shape of the short run MC_k and AC_k curves is due to the fact that, for any fixed level of capital, the short run production function has initially increasing but eventually decreasing returns to scale. That, in turn, arises from the fact that the production process has initially increasing but eventually decreasing marginal product of labor. The short run MC_k curve again intersects the lowest point of the long run AC^{LR} curve because the firm is initially profit maximizing at the lowest point of the long run AC curve.

(e) *Is it possible for short run AC curves to* not *be U-shaped if the production process has initially increasing but eventually decreasing returns to scale?*

Answer: Yes, this is possible. In order for the AC^{LR} curve to assume a U-shape (in the absence of fixed costs), the production process must have initially increasing returns to scale (that eventually turn into decreasing returns to scale). In order for the short run AC_k curve to have a U-shape, it must be that the short run production function with fixed capital initially has increasing but eventually decreasing returns to scale. But this will occur only if the marginal product of labor is initially increasing and eventually decreasing. If the marginal product of labor for the fixed capital level k is diminishing throughout, then the short run AC_k curve will be upward sloping throughout because there are no fixed costs in the short run. Since it is possible for the marginal product of each input to be decreasing but still to have increasing returns to scale of the entire production process (that varies both capital and labor), it is possible to have the U-shaped AC curve in the long run but not the short run (in the absence of fixed costs).

B: *Suppose first that the production process is Cobb-Douglas, characterized by the production function $x = f(\ell, k) = A\ell^{\alpha}k^{\beta}$ with $\alpha, \beta > 0$ and $\alpha + \beta < 1$.*

(a) *In the absence of fixed costs, you should have derived in exercise 13.2 that the long run cost function for this technology is given by*

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)}. \tag{13.5}$$

If the firm has long run fixed costs F, what is its long run average cost function? Is the average cost curve U-shaped?

Answer: The long run average cost function is then simply $C(w,r,x)$ divided by x plus FC/x — which gives us

$$AC(w,r,x) = \frac{C(w,r,x)}{x} = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{FC}{x}. \tag{13.5.i}$$

In B(d) of exercise 13.3, we already argued that this must be U-shaped. Its derivative is

$$\frac{\partial AC(w,r,x)}{\partial x} = \left[(1-\alpha-\beta)\left(\frac{w^{\alpha}r^{\beta}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} x^{(1-2(\alpha+\beta))/(\alpha+\beta)}\right] - \frac{FC}{x^2}. \tag{13.5.ii}$$

which, when set to zero, gives us the output level at which the long run AC curve reaches its lowest point:

$$x = \left(\frac{A\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)\left(\frac{FC}{(1-\alpha-\beta)}\right)^{(\alpha+\beta)} \tag{13.5.iii}$$

(b) *What is the short run cost curve for a fixed level of capital $\overline{k}$? Is the short run average cost curve U-shaped?*

Answer: The short run production function with fixed $\overline{k}$ is $x = f(\ell) = \left[A\overline{k}^{\beta}\right]\ell^{\alpha}$ which, when solved for ℓ, gives us the conditional labor demand of

$$\ell_{\overline{k}}(x) = \left(\frac{x}{A\overline{k}^{\beta}}\right)^{1/\alpha} \tag{13.5.iv}$$

Multiplying by wage w, we then get the short run cost function

$$C_{\overline{k}}(x) = w\left(\frac{x}{A\overline{k}^{\beta}}\right)^{1/\alpha} \tag{13.5.v}$$

from which we can derive the short run $MC_{\overline{k}}$ and $AC_{\overline{k}}$ functions

$$MC_{\overline{k}}(x) = \frac{w}{\alpha}\left(\frac{x^{(1-\alpha)}}{A\overline{k}^{\beta}}\right)^{1/\alpha} \quad \text{and} \quad AC_{\overline{k}}(x) = w\left(\frac{x^{(1-\alpha)}}{A\overline{k}^{\beta}}\right)^{1/\alpha}. \tag{13.5.vi}$$

These are both increasing in x — and thus the short run MC and AC curves slope up. They also converge to zero as x goes to zero. Thus, they give rise to a picture such as the on in panel (a) of Exercise Graph 13.5(1).

(c) *Now suppose that the production function is still $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ but now $\alpha+\beta > 1$. Are long run average and marginal cost curves upward or downward sloping? Are short run average cost curves upward or downward sloping? What does your answer depend on?*

Answer: The long run MC curve is downward sloping because of increasing returns to scale. The long run AC curve is similarly downward sloping (and starts above MC because of the long run fixed costs). (You can see this from the equation (13.5.i) where the exponent on x is now negative in the first term — implying that both terms decline in x.) Whether or not the short run MC and AC curves slope down depends on whether α is less than or greater than 1. If it is less than 1, then x enters the short run MC and AC functions in equation (13.5.vi) with positive exponent — implying that these costs increase with x. When $\alpha > 1$, on the other hand, x enters with negative exponent — causing the cost curves to fall with x. Thus, increasing returns to scale is consistent with both upward and downward sloping short run MC and AC curves — the key is whether the marginal product of labor increases or decreases.

(d) *Next, suppose that the production technology were given by the equation*

$$x = f(\ell, k) = \frac{\alpha}{1 + e^{-(\ell-\beta)} + e^{-(k-\gamma)}} \tag{13.5.vii}$$

where e is the base of the natural logarithm. (We first encountered this in exercises 12.5 and 12.6.) If capital is fixed at $\overline{k}$, what is the short run production function and what is the short run cost function?

Answer: The short run production function is

$$x = f_{\overline{k}}(\ell) = \frac{\alpha}{\left[1 + e^{-(\overline{k}-\gamma)}\right] + e^{-(\ell-\beta)}} \tag{13.5.viii}$$

The short run cost function is then simply this production function solved for ℓ. We can multiply both sides by the denominator of the right hand side, divide both sides by x and then subtract the bracketed term from both sides to get

$$e^{-(\ell-\beta)} = \frac{\alpha}{x} - \left[1 + e^{-(\overline{k}-\gamma)}\right] = \frac{\alpha - \left[1 + e^{-(\overline{k}-\gamma)}\right]x}{x}. \tag{13.5.ix}$$

Taking natural logs of both sides, we can then solve for the conditional short run labor demand function

$$\ell_{\overline{k}}(w, x) = \beta - \ln\left(\frac{\alpha - \left[1 + e^{-(\overline{k}-\gamma)}\right]x}{x}\right) \tag{13.5.x}$$

which, when multiplied by w, gives us the short run cost function

$$C_{\overline{k}}(w, x) = w\beta - w\ln\left(\frac{\alpha - \left[1 + e^{-(\overline{k}-\gamma)}\right]x}{x}\right). \tag{13.5.xi}$$

(e) *What is the short run marginal cost function?*

Answer: Taking the derivative of the short run cost function with respect to x, we get

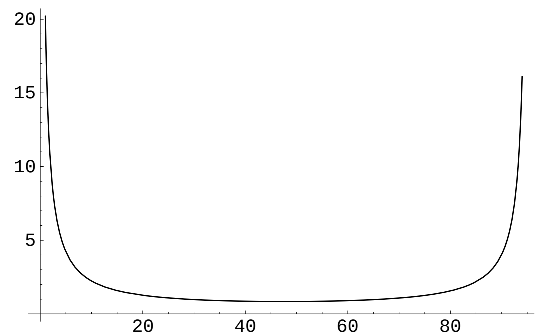
$$MC_{\overline{k}}(w, x) = \frac{w\alpha}{\left(\alpha - \left[1 + e^{-(\overline{k}-\gamma)}\right]x\right)x}. \tag{13.5.xii}$$

(f) *You should have concluded in exercise 12.6 that the long run MC function is $MC(w,r,x) = \alpha(w+r)/(x(\alpha - x))$ and demonstrated that the MC curve (and thus the long run AC curve) is U-shaped for the parameters $\alpha = 100$, $\beta = 5 = \gamma$ when $w = r = 20$. Now suppose capital is fixed at $\overline{k} = 8$. Graph the short run MC curve and use the information to conclude whether the short run AC curve is also U-shaped.*

Answer: The short run MC curve then becomes

$$MC_{\overline{k}=8} \approx \frac{2,000}{(100-1.05x)x} \quad (13.5.xiii)$$

which is plotted in Exercise Graph 13.5(2). This is obviously U-shaped — which causes the short run AC curve to be U-shaped as well.



Exercise Graph 13.5(2) : Short Run MC when $\alpha = 100$, $\beta = \gamma = 5$, $w = r = 20$ and $\overline{k} = 8$

(g) *What characteristic of the this production function is responsible for your answer in part (f)?*

Answer: The characteristic that is causing the U-shaped curves in the short run is the initially increasing marginal product of labor that causes the short run production function to display initially increasing and eventually decreasing returns to scale.

Exercise 13.7

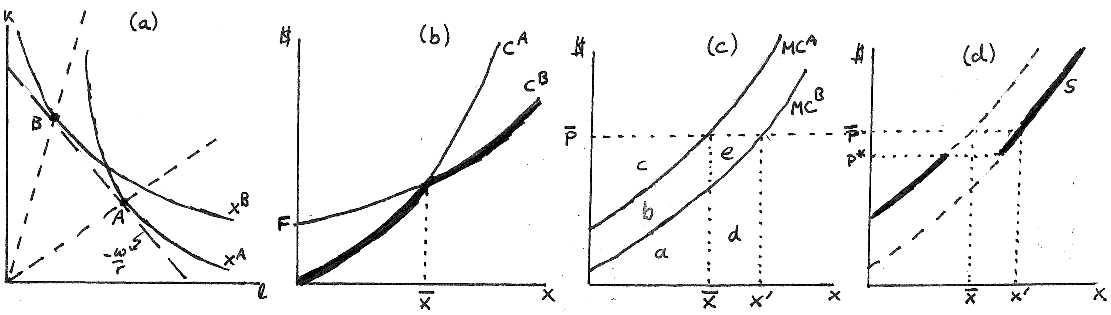
Business Application: *Switching Technologies*: *Suppose that a firm has two different homothetic, decreasing returns to scale technologies it could use, but one of these is patented and requires recurring license payments F to the owner of the patent. In this exercise, assume that all inputs — including the choice of which technology is used — are viewed from a long run perspective.*

A: *Suppose further that both technologies take capital k and labor ℓ as inputs but that the patented technology is more capital intensive.*

(a) *Draw two isoquants, one from the technology representing the less capital intensive and one representing the more capital intensive technology. Then illustrate the slice of each map that a firm will choose to operate on assuming the wage w and rental rate r are the same in each case.*

Answer: This is illustrated in panel (a) of Exercise Graph 13.7 where x^A labels an isoquant from the non-patented (labor intensive) technology and x^B labels an isoquant from the patented (capital intensive) technology. The isocost with slope $-w/r$ is tangent to the labor intensive technology at A and to the capital intensive technology at B. Since the production technologies are homothetic, the ray passing from the origin through A

represents the slice of the labor intensive production technology along which cost minimizing input bundles lie, and the ray passing from the origin through B represents the slice of the capital intensive technology along which cost minimizing bundles lie. Thus, the ratio of capital to labor used in production is greater in the patented (capital intensive) technology.



Exercise Graph 13.7 : Switching Technologies

(b) *Suppose that the patented technology is sufficiently advanced such that, for any set of input prices, there always exists an output level $\overline{x}$ at which it is (long run) cost effective to switch to this technology. On a graph with output x on the horizontal and dollars on the vertical, illustrate two cost curves corresponding to the two technologies and then locate $\overline{x}$. Then illustrate the cost curve that takes into account that a firm will switch to the patented technology at $\overline{x}$.*

Answer: This is illustrated in panel (b) of Exercise Graph 13.7. The non-patented technology gives rise to the cost curve labeled C^A and the patented technology gives rise to the cost curve C^B. Since there is an $\overline{x}$ such that it is cost effective to switch to the patented technology at $\overline{x}$, the two cost curves must intersect. The C^B curve must furthermore have positive intercept of F because use of the patented technology requires a fixed payment of F before production begins. The bold curve that connects C^A below $\overline{x}$ to C^B above $\overline{x}$ then represents the real cost curve for a firm in this industry — because the firm will, for any given output level, use the technology that minimizes costs.

(c) *What happens to $\overline{x}$ if the license cost F for using the patented technology increases? Is it possible to tell what happens if the capital rental rate r increases?*

Answer: If F increases, then C^B shifts up while C^A remains unchanged. Thus, it must be that the two cost curves intersect at a higher level of output — i.e. an increase in F causes an increase in the production level $\overline{x}$ at which a firm would switch from the non-patented to the patented technology. If the capital rental rate r increases, however, we cannot tell what will happen to $\overline{x}$. This is because now both cost curves are affected, with the upward shift in C^A by itself causing $\overline{x}$ to decrease while the upward shift of C^B by itself would cause $\overline{x}$ to increase. Which of these effects dominates when both curves shift will depend on the precise nature of the underlying technology as well as the ratio of w to r.

(d) *At $\overline{x}$, which technology must have a higher marginal cost of production? On a separate graph, illustrate the marginal cost curves for the two technologies.*

Answer: In panel (b) of Exercise Graph 13.7, it is clear that the slope of C^A is steeper at $\overline{x}$ than the slope of C^B at that output level. Thus, the marginal cost of production at $\overline{x}$ is higher under the non-patented technology than under the patented technology. This is likely to be true for all output levels — leading to marginal cost curves such as those depicted in panel (c) of Exercise Graph 13.7 where MC^A indicates the marginal cost curve under the non-patented technology and MC^B indicates the marginal cost curve under the patented technology. (It is in principle possible that these marginal cost curves cross in some places — but that would require unusually shaped cost curves in panel (b) where C^B must have an intercept of F, C^A has no such intercept and the two curves cross at $\overline{x}$.)

(e) *At $\overline{x}$, the firm is cost-indifferent between using the two technologies. Recognizing that the marginal cost curves capture all costs that are not fixed — and that total costs excluding fixed costs can be represented as areas under marginal cost curves, can you identify an area in your graph that represents the recurring fixed license fee F?*

Answer: The total cost of producing $\overline{x}$ under the non-patented technology is simply the area under the MC^A curve in panel (c) of Exercise Graph 13.7 — i.e. area $a+b$. (This is because the marginal cost of each unit of output is the additional cost incurred — and when we sum all these "additional costs" we get the total cost if there is no fixed cost of production). Similarly, the total cost minus the fixed cost F under the patented technology is the area under MC^B — i.e. area a. These areas differ by b — i.e. not counting the fixed cost F under the patented technology, the cost of producing $\overline{x}$ is smaller under the patented technology by area b. At $\overline{x}$, however, the total cost (including fixed costs) is equal for the two technologies (as seen in panel (b) of the graph); i.e. $a+b=a+F$. Thus, $F=b$.

(f) *Suppose output price p is such that it is profit maximizing* under the non-patented technology *to produce $\overline{x}$. Denote this as $\overline{p}$. Can you use marginal cost curves to illustrate whether you would produce more or less if you switched to the patented technology?*

Answer: In order for it to be profit maximizing to produce $\overline{x}$ under the non-patented technology, $\overline{p}$ must fall as depicted in panel (c) of Exercise Graph 13.7 — i.e. $\overline{p}$ must intersect MC^A at $\overline{x}$. But $\overline{p}$ intersects MC^B at x' — which implies that, were the firm to switch to the patented technology, it would produce more.

(g) *Would profit be higher if you used the patented or non-patented technology when output price is $\overline{p}$. (*Hint*: Identify the total revenues if the firm produces at $\overline{p}$ under each of the technologies. Then identify the total cost of using the non-patented technology as an area under the appropriate marginal cost curve and compare it to the total costs of using the patented technology as an area under the other marginal cost curve and add to it the fixed fee F.)*

Answer: When selling $\overline{x}$ at $\overline{p}$, total revenue is equal to $\overline{p}$ times $\overline{x}$ — which is equal to the area $a+b+c$ in panel (c) of Exercise Graph 13.7. If the firm uses the non-patented technology to produce $\overline{x}$, its total costs are equal to the area under MC^A — which is equal to area $a+b$. Thus, the profit for a firm using the non-patented technology is $(a+b+c)-(a+b)=c$. If the firm uses the patented technology at price $\overline{p}$, it produces x' and thus earns revenues of $\overline{p}$ times x' — which is equal to area $a+b+c+d+e$. Its costs (not including the fixed F) are equal to the area under MC^B — which is $a+d$. We also concluded that the fixed F is equal to area b. Thus, total costs (including F) are $a+b+d$. Subtracting this from total revenue, we get profit of $a+b+c+d+e-(a+b+d)=c+e$. When price is $\overline{p}$, the firm would therefore earn profit of c by producing $\overline{x}$ under the non-patented technology and profit $c+e$ producing x' using the patented technology. Profit is therefore higher if the firm uses the patented technology when price is $\overline{p}$.

(h) True or False*: Although the total cost of production is the same under both technologies at output level $\overline{x}$, a profit maximizing firm will choose the patented technology if price is such that $\overline{x}$ is profit maximizing under the non-patented technology.*

Answer: This is true. We had identified $\overline{x}$ as the output level at which the two cost curves cross in panel (b) of Exercise Graph 13.7 — and thus total costs are the same under both technologies when firms produce $\overline{x}$. But we also just concluded that, at price $\overline{p}$ at which it is profit maximizing under the non-patented technology to produce $\overline{x}$, the firm can earn more profit by using the patented technology and producing x' (in panel (c) of the graph.)

(i) *Illustrate the firm's supply curve. (*Hint*: The supply curve is not continuous, and the discontinuity occurs at a price below $\overline{p}$.)*

Answer: This is illustrated in panel (d) of Exercise Graph 13.7. Up to some price level below $\overline{p}$, the profit maximizing firm will choose the non-patented technology. Over that range of prices, MC^A therefore forms the supply curve. But at some price — indicated as p^* in the graph — the firm will earn the same profit under both technologies but will produce more under the patented technology. We know that $p^* < \overline{p}$ because of our conclusion that profit using the patented technology is higher at $\overline{p}$ than profit under the non-patented technology. At prices higher than p^*, the firm will then have switched to the patented technology — causing the supply curve from then on to lie on MC^B.

B: *Suppose that the two technologies available to you can be represented by the production functions $f(\ell,k) = 19.125\ell^{0.4}k^{0.4}$ and $g(\ell,k) = 30\ell^{0.2}k^{0.6}$, but technology g carries with it a recurring fee of F.*

(a) *In exercise 13.2 you derived the general form for the 2-input Cobb-Douglas conditional input demands and cost function. Use this to determine the ratio of capital to labor (as a function of w and r) used under these two technologies. Which technology is more capital intensive?*

Answer: Plugging $\alpha = \beta = 0.4$ and $A = 19.125$ into the previously derived formula for input demands, we get that the f technology gives rise to conditional input demands

$$\ell_f(w,r,x) = 0.025\left(\frac{r}{w}\right)^{1/2} x^{5/4} \text{ and } k_f(w,r,x) = 0.025\left(\frac{w}{r}\right)^{1/2} x^{5/4}, \tag{13.7.i}$$

and plugging in $\alpha = 0.2$, $\beta = 0.6$ and $A = 30$, we get that the g technology gives rise to conditional input demands

$$\ell_g(w,r,x) = 0.00625\left(\frac{r}{w}\right)^{3/4} x^{5/4} \text{ and } k_g(w,r,x) = 0.01875\left(\frac{w}{r}\right)^{1/4} x^{5/4}. \tag{13.7.ii}$$

The ratio of capital to labor under the technologies f an g are then (respectively)

$$\frac{k_f(w,r,x)}{\ell_f(w,r,x)} = \frac{w}{r} \text{ and } \frac{k_g(w,r,x)}{\ell_g(w,r,x)} = 3\frac{w}{r}; \tag{13.7.iii}$$

i.e. the capital to labor ratio under technology g is three times as high as under f. (These ratios correspond to the slopes of the rays in panel (a) of Exercise Graph 13.7.) The g technology is therefore more capital intensive.

(b) *Determine the cost functions for the two technologies (and be sure to include F where appropriate).*

Answer: For the f technology, we plug in $\alpha = \beta = 0.4$ and $A = 19.125$ into the previously derived cost function for Cobb-Douglas production; and for the g technology we plug in $\alpha = 0.2$, $\beta = 0.6$ and $A = 30$ — and then we add the fixed technology fee F which has to be paid if g is used. This gives us

$$C_f(w,r,x) = 0.05w^{1/2}r^{1/2}x^{5/4} \text{ and } C_g(w,r,x) = 0.025w^{1/4}r^{3/4}x^{5/4} + F. \tag{13.7.iv}$$

(c) *Determine the output level $\overline{x}$ (as a function of w, r and F) at which it becomes cost effective to switch from the technology f to the technology g. If F increases, is it possible to tell whether $\overline{x}$ increases or decreases? What if r increases?*

Answer: To calculate $\overline{x}$, we need to find where the cost curves intersect (as in panel (b) of Exercise Graph 13.7). We therefore set the two cost functions in equation (13.7.iv)equal to each other and solve for x to get

$$\overline{x} = \left(\frac{40F}{2w^{1/2}r^{1/2} - w^{1/4}r^{3/4}}\right)^{4/5}. \tag{13.7.v}$$

If F increases, $\overline{x}$ unambiguously increases as well — which makes sense since the fixed cost of using g has increased, it will not be cost effective to switch until a higher level of output. But if r increases, we cannot tell whether $\overline{x}$ will increase or decrease. (We gave some intuition for this in the answer to (c) of part A of this exercise.)

(d) *Suppose $w = 20$ and $r = 10$. Determine the price $\overline{p}$ (as a function of F) at which a firm using technology f would produce $\overline{x}$.*

Answer: From the first cost function in equation (13.7.iv), we can derive the marginal cost under technology f as $MC_f(w,r,x) = 0.0625w^{0.5}r^{0.5}x^{0.25}$. Plugging in $w = 20$ and $r = 10$, we then get $MC_f(20,10,x) = 0.8838835x^{0.25}$. Plugging these same input prices into equation (13.7.v), we get $\overline{x} = 2.0414474F^{0.8}$. Thus, the marginal cost of producing $\overline{x}$ under technology f is

$$MC_f(20,10,\overline{x}) = 0.8838835\left(2.0414474F^{0.8}\right)^{0.25} = 1.0565245F^{1/5} = \overline{p}, \tag{13.7.vi}$$

where the last equality simply emerges from the fact that $p = MC$ for profit maximizing firms.

(e) *How much would the firm produce with technology g if it faces $\overline{p}$? Can you tell whether, regardless of the size of F, this is larger or smaller than $\overline{x}$ (which is the profit maximizing quantity when the firm used technology f and faces $\overline{p}$)?*

Answer: From the second cost function in equation (13.7.iv) we can derive the marginal cost function under technology g as $MC_g(w,r,x) = 0.03125w^{0.25}r^{0.75}x^{0.25}$ which becomes $MC_g(20,10,x) = 0.3716272x^{0.25}$ when $w = 20$ and $r = 10$. Setting this equal to $\overline{p} = 1.0565245F^{1/5}$ from equation (13.7.vi) and solving for x, we get $x \approx 65.33F^{0.8}$. We can then conclude that

$$x \approx 65.33F^{0.8} > 2.0414474F^{0.8} = \overline{x}; \tag{13.7.vii}$$

i.e. regardless of what value F takes (as long as $F > 0$), the profit maximizing production level will be higher using technology g than when using technology f when the price is such that $\overline{x}$ is profit maximizing under technology f. We illustrated this intuitively in panel (c) of Exercise Graph 13.7.

(f) *The (long run) profit function for a Cobb-Douglas production function $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ is*

$$\pi(w,r,p) = (1-\alpha-\beta)\left(\frac{Ap\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)^{1/(1-\alpha-\beta)}. \tag{13.7.viii}$$

Can you use this to determine (as a function of p, w and r) the highest level of F at which a profit maximizing *firm will switch from f to g? Call this $\overline{F}(w,r,p)$.*

Answer: Plugging $\alpha = \beta = 0.4$ and $A = 19.125$ into the profit function, we get the profit function for the technology f; and plugging in $\alpha = 0.2$, $\beta = 0.6$ and $A = 30$, we get the profit function for technology g. This gives us

$$\pi_f(w,r,p) = 13,100\left(\frac{p^5}{w^2r^2}\right) \text{ and } \pi_g(w,r,p) = 209,952\left(\frac{p^5}{wr^3}\right). \quad (13.7.\text{ix})$$

For the production function g, however, we also need to take into account the fixed cost F — thus subtract F from $\pi_g(w,r,p)$. The fixed cost $\overline{F}$ at which the firm will switch to the technology g is the fixed cost at which profit is equal for both technologies. Thus, we need to solve

$$13,100\left(\frac{p^5}{w^2r^2}\right) = 209,952\left(\frac{p^5}{wr^3}\right) - F \quad (13.7.\text{x})$$

for F. This gives us

$$\overline{F} = 209,952\left(\frac{p^5}{wr^3}\right) - 13,100\left(\frac{p^5}{w^2r^2}\right). \quad (13.7.\text{xi})$$

(g) *From your answer to (f), determine (as a function of w, r and F) the price p^* at which a profit maximizing firm will switch from technology f to technology g.*

Answer: We simply need to solve equation (13.7.x) for p which gives us

$$p^* = \left(\frac{Fw^3r^3}{209,952w^2 - 13,100wr}\right)^{1/5}. \quad (13.7.\text{xii})$$

(h) *Suppose again that $w = 20$, $r = 10$. What is p^* (as a function of F)? Compare this to $\overline{p}$ you calculated in part (d) and interpret your answer in light of what you did in A(i).*

Answer: Plugging $w = 20$ and $r = 10$ into equation (13.7.xii), we get $p^* = 0.6288325F^{0.2}$. Comparing this to our answer in part (d), we conclude that

$$p^* = 0.6288325F^{1/5} < 1.0562545F^{1/5} = \overline{p}. \quad (13.7.\text{xiii})$$

Thus, the price p^* at which a profit maximizing firm switches from technology f to technology g lies below the price $\overline{p}$ at which a firm using production technology f would produce $\overline{x}$ at which the cost of production is equal for the two technologies. This implies that the supply curve switches from the MC_f curve to MC_g at p^* and below $\overline{p}$ — which we illustrated intuitively in panel (c) of Exercise Graph 13.7.

(i) *Suppose (in addition to the values for parameters specified so far) that* $F = 1000$. *What is* $\overline{p}$ *and* p^**? At the price at which the profit maximizing firm is indifferent between using technology* f *and technology* g*, how much does it produce when it uses* f *and how much does it produce when it uses* g*?*

Answer: Plugging $F = 1000$ into the equations for $\overline{p}$ and p^*, we get $\overline{p} = 1.0562545(1000)^{1/5} \approx \$4.21$ and $p^* = 0.6288325(1000)^{1/5} = 2.5034 \approx \$2.50$. Plugging $\alpha = \beta = 0.4$, $A = 19.125$, $w = 20$, $r = 10$ and $p = 2.50$ into the supply function $x(w, r, p)$, we get $x_f^* \approx 64.32$; and, plugging $\alpha = 0.2$. $\beta = 0.6$, $A = 19.125$, $w = 20$, $r = 10$ and $p = 2.50$ in the supply function, we get $x_g^* \approx 2{,}062$.

(j) *Continuing with the values we have been using (including* $F = 1000$*), can you use your answer to (a) to determine how much labor and capital the firm hires at* p^* *under the two technologies? How else could you have calculated this?*

Answer: Plugging $r = 10$, $w = 20$ and $x_f^* = 64.32$ into the conditional labor and capital demands from equation (13.7.i), we get $\ell_f^* = 3.22$ and $k_f^* = 6.44$ — the cost minimizing labor and capital inputs if the firm uses the f technology to produce $x_f^* = 64.32$ (which in turn is the profit maximizing output level at $p^* = 2.50$.) Similarly, if we plug $r = 10$, $w = 20$ and $x_g^* = 2{,}062$ into the conditional labor and capital demands from equation (13.7.ii), we get $\ell_g^* = 51.6$ and $k_g^* = 310$ — the cost minimizing labor and capital inputs if the firm uses the g technology to produce $x_g^* = 2{,}062$ (which in turn is the profit maximizing output level at $p^* = 2.50$.) You could also of course have derived the unconditional labor demand and capital demand functions — either by doing the profit maximization problems or using Hotelling's lemma.

(k) *Use what you have calculated in (i) and (j) to verify that profit is indeed the same for a firm whether it uses the* f *or the* g *technology when price is* p^* *(when the rest of the parameters of the problem are as we have specified them in (i) and (j).) (Note: If you rounded some of your previous numbers, you will not get exactly the same profit in both cases — but if the difference is small, it is almost certainly just a rounding error.)*

Answer: Profit is simply revenue minus costs. We can then calculate the profit under each technology as

$$\pi_f = p^* x_f^* - w\ell_f^* - rk_f^* = 2.5034(64.32) - 20(3.22) - 10(6.44) \approx \$32 \tag{13.7.xiv}$$

and

$$\pi_g = p^* x_g^* - w\ell_g^* - rk_g^* = 2.5034(2062) - 20(51.6) - 10(310) - 1000 \approx \$32. \tag{13.7.xv}$$

Exercise 13.9

Business and Policy Application: *Fixed amount of Land for Oil Drilling: Suppose that your oil company is part of a competitive industry and is using three rather than two inputs — labor ℓ, capital k and land L — to produce barrels of crude oil denoted by x. Suppose that the government, due to environmental concerns, has limited the amount of land available for oil drilling — and suppose that it has assigned each oil company $\overline{L}$ acres of such land. Assume throughout that oil sells at a market price p, labor at a market wage of w and capital at a rental rate r — and these prices do not change as government policy changes.*

A: *Assume throughout that the production technology is homothetic and has constant returns to scale.*

(a) *Suppose that, once assigned to an oil company, the company is not required to pay for using the land to drill for oil (but it cannot do anything else with it if it chooses not to drill.) How much land will your oil company use?*

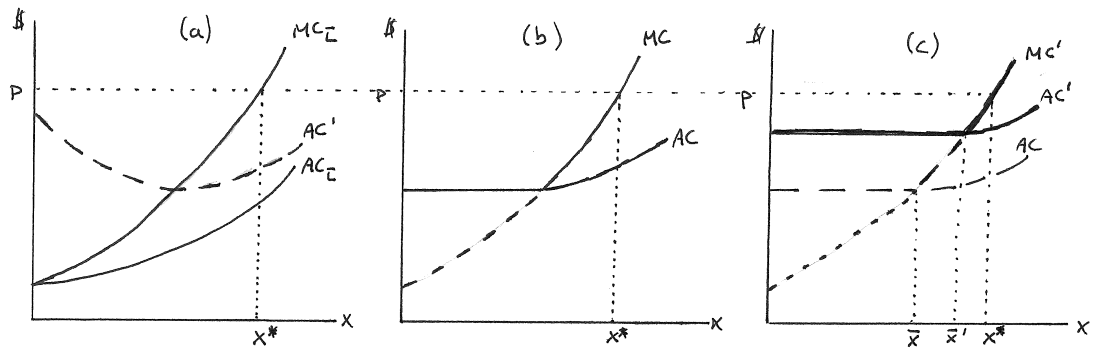
Answer: If land (up to $\overline{L}$ acres) is free to the company, it will use all of it so long as the marginal product of land is positive. This is because profit maximization implies that a firm hires inputs until the marginal revenue product equals the price of the input — and the price of land in this case is zero up to $\overline{L}$ (and effectively infinite thereafter).

(b) *While the 3-input production frontier has constant returns to scale, can you determine the effective returns to scale of production once you take into account that available land is fixed?*

Answer: Land essentially becomes a fixed input in both the short and long run. Constant returns to scale of the 3-input production frontier implies that we can double output by doubling all 3 of our inputs. But if land cannot be increased, doubling labor and capital will be insufficient to double output. The 2-input production frontier that holds land fixed at $\overline{L}$ therefore has decreasing returns to scale. (One way to think of this 2-imput production frontier is as a 3-dimensional "slice" (that holds land fixed at $\overline{L}$) of the original 4-dimensional production frontier that plotted production plans (x, ℓ, k, L)).

(c) *What do average and marginal cost curves look like for your company over the time frame when both labor and capital can be varied?*

Answer: Since the effective 2-input production frontier (that holds $\overline{L}$ fixed) has decreasing returns to scale, the average and marginal cost curves are upward sloping with marginal cost above average cost throughout. These curves are pictured as the solid curves $AC_{\overline{L}}$ and $MC_{\overline{L}}$ in panel (a) of Exercise Graph 13.9.



Exercise Graph 13.9 : Land Rented for Oil Drilling

(d) *Now suppose that the government begins to charge a per-acre rental price q for use of land that is assigned to your company, but an oil company that is assigned $\overline{L}$ acres of land only has the option of renting all $\overline{L}$ acres or none at all. Given that it takes time to relocate oil drilling equipment, you cannot adjust to this change in the short run. Will you change how much oil you produce?*

Answer: No. In the short run, the charge for renting land is simply an expense until the company can actually vacate the land.

(e) *In the long run (when you can move equipment off land), what happens to average and marginal costs for you company? Will you change your output level?*

Answer: If your company does not exit the industry, it has to pay $q\overline{L}$ to rent all the land assigned to it — which makes it exactly like a fixed recurring license fee. Once paid, the marginal cost of drilling for oil is no different than it was when the land was free. Thus, the $MC_{\overline{L}}$ curve in panel (a) of Exercise Graph 13.9 does not change and continues to be the marginal cost curve for this company. The long run average cost curve, however, incorporates the fixed payment for the land — and thus takes on the U-shape depicted by the dashed AC' curve in the graph. The difference between $AC_{\overline{L}}$ and AC' in panel (a) is the average fixed cost of the land — which is high when oil production is low but, as an average, falls as oil production increases. Thus, $AC_{\overline{L}}$ and AC' converge as x gets large. If you continue to produce in the long run, you will produce where the price of oil p intersects the marginal cost curve that has not changed as a result of the government's policy of charging for use of the land. Thus, *if* you continue to stay in the oil business, you will not change your output level in the long run. This is depicted for the price p in panel (a) of the graph — with production x^* unchanged when the government charges a land rental fee sufficiently low to keep p above AC'. However, if p lies below the dashed AC' curve, you will exit the industry in the long run (once you can vacate the land).

(f) *Suppose the government had employed a different policy that charges a per-acre rent of q but allowed companies to rent any number of acres between 0 and $\overline{L}$. What do long run average and marginal cost curves look like in that case? Would it ever be the case that a firm will rent fewer than $\overline{L}$ acres?(*Hint*: These curves should have a flat as well as an upward sloping portion.)*

Answer: In this case, your company would initially not be limited in how much land it can use for oil drilling — not until it drills enough to run into the $\overline{L}$ constraint imposed by the government. Thus, for low levels of x, your company is operating with full (long run) discretion in terms of how much land, labor and capital to use — and thus faces a constant returns to scale production process. This implies that, for low levels of x, the average cost function takes the form it does for constant returns to scale production functions — i.e. it is constant. However, once the output level reaches the point where your company would ordinarily want to rent more than $\overline{L}$ acres of land in order to produce output level x at minimum cost, the rent on $\overline{L}$ becomes a fixed cost, land becomes a fixed input and production from here on out has decreasing returns to scale. This implies that, at some output level, the constant AC curve begins to slope up. This is pictured in panel (b) of Exercise Graph 13.9 as the initially flat and eventually upward sloping AC curve. Since constant returns to scale implies a constant marginal cost and decreasing returns to scale implies an increasing marginal cost, the MC curve in the graph is similarly flat initially but upward sloping after some output level. It therefore overlaps with AC until the land constraint binds — at which point it slopes up and lies above AC causing AC to slope up as well.

(g) *How much will you produce now compared to the case analyzed in (d)?*

Answer: You will produce exactly the same as in part (d) — because the portion of the upward sloping (solid) portion of the MC curve in panel (b) is exactly the same as the $MC_{\overline{L}}$ curve in panel (a). (In both cases, this portion of the marginal cost curve is derived from the 2-input production frontier that holds land fixed at $\overline{L}$.) Thus, as shown in Exercise Graph 13.9, price will again intersect marginal cost at x^*.

(h) *Suppose that under this alternative policy the government raises the rental price to q'. Will your company change its output level in the short run?*

Answer: No — in the short run, your company is unable to move its oil drilling equipment — and thus forced to rent the land. Since there is nothing you can do about it — i.e. you cannot affect the expense by anything that you do — it is not an economic cost and therefore affects nothing.

(i) *How do long run average and marginal cost curves change? If you continue to produce oil under the higher land rental price, will you increase or decrease your output level, or will you leave it unchanged*

Answer: Panel (c) of Exercise Graph 13.9 illustrates the new marginal and average cost curves as the bold curves labeled MC' and AC' (with the two curves overlapping along the flat hold portion). Underneath these bold curves, the AC and MC curves from panel (b) are replicated for comparison. Under the higher land rental price q', the constant returns to scale portion extends to higher level of output — because when the government charges more for land, firms will substitute away from land and toward more capital and/or labor. (For instance, you might think of a firm that produces relatively little choosing to drill horizontally from one spot rather than drilling vertically from two spots that require more land.) As a result, the land constraint $\overline{L}$ does not bind until a higher output level is reached — $\overline{x}'$ as opposed to $\overline{x}$ when the land rental rate was lower. But, although the firm conserves on land when q increases, the average cost per barrel of oil still increases — which is why the flat portion of the AC' curve lies above AC. *Once the constraint of $\overline{L}$ is reached*, however, the firm operates on the 2-dimensional production frontier that holds land fixed and thus experiences decreasing returns to scale. This implies the upward sloping MC' curve — which lies on the previously derived $MC_{\overline{L}}$ curve (in panel (a)) and the MC curve in panel (b). You will continue to produce only if p lies above the lowest point of AC' as drawn in panel (c) of the graph — but this means you will produce at the intersection of price and the same marginal cost as in the previous panels. Thus, *if you continue to produce*, you will not change your output level as a result of the increase in the rental fee of land. The only possible change in your production plan would arise if p fell between the intercepts of the dashed AC and the bold AC' curves — in which case you would have produced before but would exit after the increase in q.

(j) True or False*: The land rental rate q set by the government has no impact on oil production levels so long as oil companies do not exit the industry. Explain. (*Hint*: This is true.)*

Answer: This is true as already explained. In fact, none of the policy changes we investigated has an impact on oil production unless it causes a firm to exit the industry. You can see this in Exercise Graph 13.9 by simply observing that p always intersects the same marginal cost curve to give us output level x^*.

B: *Suppose that your production technology for oil drilling is characterized by the production function $x = f(\ell, k, L) = A\ell^{\alpha} k^{\beta} L^{\gamma}$ where $\alpha + \beta + \gamma = 1$ (and all exponents are positive).*

(a) *Demonstrate that this production function has constant returns to scale.*

Answer: To demonstrate this, we simply need to show that multiplying all inputs by t results in a t-fold increase in output; i.e.

$$f(t\ell, tk, tL) = A(t\ell)^{\alpha}(tk)^{\beta}(tL)^{\gamma} = t^{(\alpha+\beta+\gamma)}A\ell^{\alpha}k^{\beta}L^{\gamma} = tA\ell^{\alpha}k^{\beta}L^{\gamma} = tf(\ell, k, L). \tag{13.9.i}$$

(b) *Suppose again that the government assigns $\overline{L}$ acres of land to your company for oil drilling, that there is no rental fee for the land but you cannot use the land for any other purpose. Given the fixed level of land available, what is your production function now? Demonstrate that it has decreasing returns to scale.*

Answer: The production function now is

$$x = f_{\overline{L}}(\ell, k) = \left[A\overline{L}^{\gamma}\right]\ell^{\alpha}k^{\beta}, \tag{13.9.ii}$$

where the term in brackets simply enters as a constant. This is simply a 2-input Cobb-Douglas production function with $\alpha + \beta < 1$ — which makes it a decreasing returns to scale production function. We can demonstrate this simply by showing

$$f_{\overline{L}}(t\ell, tk) = \left[A\overline{L}^{\gamma}\right](t\ell)^{\alpha}(tk)^{\beta} = t^{(\alpha+\beta)}\left[A\overline{L}^{\gamma}\right]\ell^{\alpha}k^{\beta} < t\left[A\overline{L}^{\gamma}\right]\ell^{\alpha}k^{\beta} = tf_{\overline{L}}(\ell, k). \tag{13.9.iii}$$

(c) *In exercise 13.2, you were asked to derive the (long run) cost function for a 2-input Cobb-Douglas production function. Can you use your result — which is also given in equation (13.5) of exercise 13.5 — to derive the cost function for your oil company? What is the marginal cost function associated with this?*

Answer: Since the production function $f_{\overline{L}}(\ell, k)$ is simply a 2-input Cobb-Douglas function with constant $\left[A\overline{L}^{\gamma}\right]$ (rather than just A) in the front, we can simply use the cost function previously derived and replace A with $\left[A\overline{L}^{\gamma}\right]$ to get

$$C_{\overline{L}}(w, r, x) = (\alpha + \beta)\left(\frac{w^{\alpha}r^{\beta}x}{\left[A\overline{L}^{\gamma}\right]\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)}. \tag{13.9.iv}$$

The marginal cost function is then

$$MC_{\overline{L}}(w, r, x) = \frac{\partial C_{\overline{L}}(w, r, x)}{\partial x} = \left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{\left[A\overline{L}^{\gamma}\right]\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} = \left(\frac{w^{\alpha}r^{\beta}x^{\gamma}}{\left[A\overline{L}^{\gamma}\right]\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)}. \tag{13.9.v}$$

(d) *Next, consider the scenario under which the government charges a per-acre rental fee of q but only gives you the option of renting all $\overline{L}$ acres or none at* *all. Write down your new (long run) cost function and derive the marginal and average cost function. Can you infer the shape of the marginal and average cost curves?*

Answer: In order to drill for oil, you now need to pay a fixed cost of $q\overline{L}$ to rent the land. Thus, the cost function now is

$$C_{\overline{L}}(w,r,q,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{\left[A\overline{L}^{\gamma}\right]\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + q\overline{L}. \tag{13.9.vi}$$

This gives us a marginal cost curve

$$MC_{\overline{L}}(w,r,x) = \frac{\partial C_{\overline{L}}(w,r,q,x)}{\partial x} = \left(\frac{w^{\alpha}r^{\beta}x^{\gamma}}{\left[A\overline{L}^{\gamma}\right]\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} \tag{13.9.vii}$$

where the $q\overline{L}$ term drops out and the MC therefore is not a function of q. Finally, we get the average cost function

$$AC_{\overline{L}}(w,r,q,x) = \frac{C_{\overline{L}}(w,r,q,x)}{x} = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{\gamma}}{\left[A\overline{L}^{\gamma}\right]\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{q\overline{L}}{x}. \tag{13.9.viii}$$

Since the marginal cost function is unchanged, the marginal cost curve with $q>0$ is the same upward sloping marginal cost curve as that with $q=0$; i.e. the land rental fee has no impact on marginal cost. (You can verify that the marginal cost curve is upward sloping by simply checking that the derivative of the marginal cost function with respect to x is positive.) The average cost function, however, does change with q — but only because of the last term $q\overline{L}/x$. For small x, this term is large — but as x increases, it converges to zero. This creates the U-shape of the AC' curve in panel (a) of Exercise Graph 13.9.

(e) *Does the (long run) marginal cost function change when the government begins to charge for use of the land in this way?*

Answer: As already demonstrated, it does not.

(f) *Now suppose that the government no longer requires your company to rent all $\overline{L}$ acres but instead agrees to rent you up to $\overline{L}$ acres at the land rental rate q. What would your conditional input demands and your (total) cost function be in the absence of the cap on how much land you can rent?*

Answer: To get the conditional input demands without a cap on how much land you can rent, you simply solve the problem

$$\min_{\ell,k,L} \; w\ell + rk + qL \text{ subject to } x = A\ell^{\alpha}k^{\beta}L^{\gamma}. \tag{13.9.ix}$$

Solving first order conditions in the usual way, we get

$$\ell(w,r,q,x) = \left(\frac{\alpha}{w}\right)^{(1-\alpha)}\left(\frac{r}{\beta}\right)^{\beta}\left(\frac{q}{\gamma}\right)^{\gamma}\frac{x}{A}\ ;\ k(w,r,q,x) = \left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{\beta}{r}\right)^{(1-\beta)}\left(\frac{q}{\gamma}\right)^{\gamma}\frac{x}{A} \tag{13.9.x}$$

and

$$L(w,r,q,x) = \left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{r}{\beta}\right)^{\beta}\left(\frac{\gamma}{q}\right)^{(1-\gamma)}\frac{x}{A}. \tag{13.9.xi}$$

Multiplying these by their input prices, adding and simplifying, we then get the cost function

$$C(w,r,q,x) = w\ell(w,r,q,x) + rk(w,r,q,x) + qL(w,r,q,x) = \frac{w^{\alpha}r^{\beta}q^{\gamma}x}{A\alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}}. \tag{13.9.xii}$$

Note that this is a constant returns to scale cost function that has the property that

$$MC = \frac{w^{\alpha}r^{\beta}q^{\gamma}}{A\alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}} = AC, \tag{13.9.xiii}$$

where neither MC nor AC is dependent on x. In other words, marginal and average cost curves are overlapping and flat.

(g) *From now on, suppose that $A = 100$, $\alpha = \beta = 0.25$, $\gamma = 0.5$, $\overline{L} = 10{,}000$. Suppose further that the weekly wage rate is $w = 1000$, the weekly capital rental rate is $r = 1000$ and the weekly land rent rate is $q = 1000$. At what level of output $\overline{x}$ will your production process no longer exhibit constant returns to scale (given the land limit of $\overline{L}$)? What is the marginal and average cost of oil drilling prior to reaching $\overline{x}$ (as a function of x)?*

<u>Answer</u>: To determine at what level of output we reach the $\overline{L}$ constraint, we simply have to set equation (13.9.xi) to $\overline{L}$ and solve for x. This gives us

$$\overline{x} = A\overline{L}\left(\frac{\alpha}{w}\right)^{\alpha}\left(\frac{\beta}{r}\right)^{\beta}\left(\frac{q}{\gamma}\right)^{(1-\gamma)}. \tag{13.9.xiv}$$

Plugging in the values $A = 100$, $\alpha = \beta = 0.25$, $\gamma = 0.5$, $\overline{L} = 10{,}000$ and $w = r = q = 1000$, this gives us $\overline{x} \approx 707{,}107$. Plugging the same values into equation (13.9.xiii), we get that prior to reaching $\overline{x}$, $MC = AC \approx 28.28$.

(h) *After reaching this $\overline{x}$, what is the marginal and average long run cost of oil drilling (as a function of x)? Compare the marginal cost at $\overline{x}$ to your marginal cost answer in (g) and explain how this translates into a graph of the marginal cost curve for the firm in this scenario.*

<u>Answer</u>: After $\overline{x}$, we are employing the decreasing returns to scale production function $f_{\overline{L}=10000}$ for which we calculated marginal and average costs

in equations (13.9.vii) and (13.9.viii). Substituting the various values into these equations, we get

$$MC_{\overline{L}=10000}(x) = 0.00004x \quad \text{and} \quad AC_{\overline{L}=10000}(x) = 0.00002x + \frac{10,000,000}{x}. \tag{13.9.xv}$$

Evaluating these at $\overline{x} = 707,107$, we get $AC = MC \approx 28.28$ — which is exactly what we arrived at in the previous part. This justifies the picture in panel (b) of Exercise Graph 13.9 where the constant returns to scale portion meets the upward sloping portion of the MC curve at $\overline{x}$.

(i) *What happens to $\overline{x}$ as q increases? How does that change the graph of marginal and average cost curves?*

Answer: From equation (13.9.xiv), we can easily see that $\overline{x}$ increases as q increases. This changes the graph for marginal and average cost curves as illustrated in panel (c) of Exercise Graph 13.9.

(j) *If the price per barrel of oil is $p = 100$, what is your profit maximizing oil production level?*

Answer: Setting p of 100 equal to $MC_{\overline{L}} = 0.00004x$ and solving for x, we get $x = 2,500,000$. You will therefore produce 2,500,000 barrels of oil per week.

(k) *Suppose the government now raises q from 1,000 to 10,000. What happens to your production of oil? What if the government raises q to 15,000?*

Answer: If you produce, you will still produce where p of 100 equals $MC_{\overline{L}} = 0.00004x$ — which implies you will still produce 2,500,000 barrels of oil per week. The question is whether the government has raised the land rental rate so high that it is more advantageous to exit and produce nothing. To determine at what q this happens, we have to determine at what q the lowest point of the average cost curve is equal to $p = 100$. The lowest point of average cost occurs along the flat, constant returns to scale portion where average cost is given by equation (13.9.xiii). Substituting the various values into that equation, we get $AC \approx 0.894427q^{0.5}$. Setting this equal to 100 and solving for q, we get $q = 12,500$. Thus, for any $q \leq 12,500$, output will remain unchanged at 2,500,000 barrels of oil per week, but for $q > 12,500$, output falls to zero as the firm exits the industry.

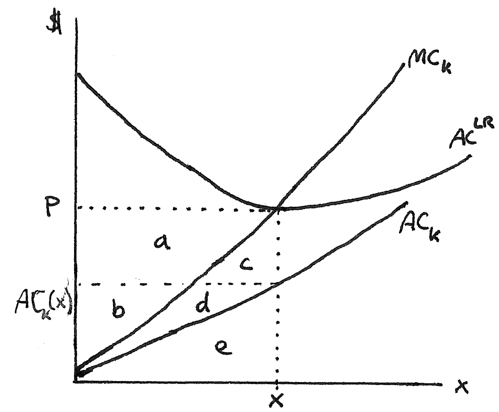
Exercise 13.11

Policy and Business Application: *Business Taxes: In this exercise, suppose that your hamburger business "McWendy's" has a homothetic decreasing returns to scale production function that uses labor ℓ and capital k to produce hamburgers x. You can hire labor at wage w and capital at rental rate r but also have to pay a fixed annual franchise fee F to the McWendy parent company in order to operate as a McWendy's restaurant. You can sell your McWendy's hamburgers at price p.*

A: *Suppose that your restaurant, by operating at its long run profit maximizing production plan (ℓ^*, k^*, x^*), is currently making zero long run profit. In each* *of the policy proposals in parts (b) through (h), suppose that prices w, r and p remain unchanged. In each part, beginning with (b), indicate what happens to your optimal production plan in the short and long run.*

(a) *Illustrate the short run AC and MC curves as well as the long run AC curve. Where in your graph can you locate your short run profit — and what is it composed of?*

Answer: These curves are illustrated in panel (a) of Exercise Graph 13.11 where the U-shape of the long run average cost curve derives from the fixed franchise fee.



Exercise Graph 13.11 : Short Run Profit

Since there are no fixed costs — only fixed expenses — in the short run, and since the decreasing returns to scale of the long run production process also implies decreasing returns to scale for the short run production process, the short run *MC* and *AC* curves have to be upward sloping (with the former above the latter). The short run profit is then composed of the expense on capital rk and the fixed franchise fee F — i.e. short run profit is $rk + F$ which disappears in the long run as these turn into costs. In the graph, there are two ways of seeing this short run profit. In both cases, we begin by identifying total revenue as the area $a + b + c + d + e$ — i.e. the price p times output x. (We know price has to be at the lowest point of the long run *AC* curve since we know the firm is making zero long run profit.) The short run costs can then be identified as the average (short run) cost of producing x — which is $AC_k(x)$ — times the output level x, or just area $b + d + e$. Alternatively, short run costs can also be seen as the area under the short run marginal cost curve — area $c + d + e$. Subtracting short run costs from revenues, we then get that short run profit is $a + c$ or, equivalently, $a + b$. (Logically this of course implies that $b = c$.)

(b) *Suppose the government determined that profits in your industry were unusually high last year — and imposes a one-time "windfall profits tax" of*

50% on your business's profits from last year.

Answer: There is nothing you can do in your business to avoid paying this windfall profits tax — it is a sunk "cost", an expense in the short run and irrelevant in the long run. Therefore you will not change your production plan.

(c) *The government imposes a 50% tax on short run profits from now on.*

Answer: If you are currently maximizing short run profits (which you are), you will not change anything in the short run if the government takes half of your short run profit. Half of the most you can make is still more than half of less than the most you can make in your business. In the long run, however, your profit will now no longer be positive — which means you will exit in the long run and stop producing (absent any changes in prices in the industry).

(d) *The government instead imposes a 50% tax on long run profits from now on.*

Answer: Your long run profit is zero — so the government will not collect any taxes from you. If what you were doing before was profit maximizing, you are still profit maximizing by doing exactly the same as you were doing before. This would be true even if your long run profits were positive. If you now only make half as much long run profit, you are still making a positive profit — which means you are still making more in this business than you could anywhere else.

(e) *The government instead taxes franchise fees causing the blood sucking McWendy's parent company to raise its fee to $G > F$.*

Answer: This will increase your long run costs — which means that, since you were making zero long run profit before, you will now be making negative long run profit. In the absence of any other changes (such as a change in price), you will therefore exit and stop producing hamburgers.

(f) *The government instead imposes a tax t on capital (which is fixed in the short run) used by your restaurant — causing you to have to pay not only r but also tr to use one unit of capital.*

Answer: Since capital is fixed in the short run, nothing changes for you in the short run (assuming you still are committed to the capital you have for now). Put differently, the tax payment tr is a short run expense, not a cost. In the long run, however, your average and marginal cost curves increase. If you were to continue to produce, this implies you will produce less (as p intersects MC at a lower quantity) — but you will in fact exit in the long run because your long run profit — which was zero before the increase in costs — must now be negative.

(g) *Instead of taxing capital, the government taxes labor in the same way as it taxed capital in part (f).*

Answer: Since labor is variable in the short run, this tax is an immediate cost since you affect you overall tax payment by changing how many workers you hire. Thus, the MC_k shifts up. If you continue to produce in the short run, you will then produce less because the new MC_k intersects price at a lower quantity. It is not clear, however, whether you will not fully shut down even in the short run. The crucial question is whether the tax on labor is sufficiently high for short run profit (which was positive at the outset) to fall below zero. If so, you will shut down. Depending on how large the tax rate t, you will therefore either produce less or not at all in the short run. In the long run, however, you will exit (unless something else changes) — because your previously zero long run profit is now negative.

(h) *Finally, instead of any of the above, the government imposes a "health tax" t on hamburgers — charging you \$$t$ for every hamburger you sell.*

Answer: The answer is similar to that given to part (g) — in the short run, you may stay open and produce fewer hamburgers or you may shut down depending on whether short run profits under the lower production level remain positive. In the long run, however, you will exit (unless something else changes).

B: *In previous exercises, we gave the input demand functions for a a firm facing prices (w,r,p) and technology $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ (with $\alpha,\beta > 0$ and $\alpha+\beta < 1$) in equation (13.50) and the long run output supply function in equation (13.49) — both given in footnotes to earlier end-of-chapter exercises in this chapter.*

(a) *When you add a recurring fixed cost F, how are these functions affected? (*Hint*: You will have to restrict the set of prices for which the functions are valid — and you can use the profit function given in exercise 13.7) to do this strictly in terms of A, α, β and the prices (w,r,p).) What are the short run labor demand and output supply functions for a given $\bar{k}$?*

Answer: We now have to include the role of the recurring fixed cost F in the long run input demand and output supply functions. But we know from our graphical work that this simply causes these functions to become "shorter" — i.e. these functions remain valid but only for the set of input and output prices at which long run profit (which incorporates F) is not negative. In the absence of F, the profit function for Cobb-Douglas production functions was given in exercise 13.7 as

$$\pi(w,r,p) = (1-\alpha-\beta)\left(\frac{Ap\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)^{1/(1-\alpha-\beta)}. \qquad (13.11.i)$$

To say that long run profit is positive at a given set of prices (p,w,r) is therefore to say that

$$(1-\alpha-\beta)\left(\frac{Ap\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)^{1/(1-\alpha-\beta)} \geq F \qquad (13.11.ii)$$

or, rearranging terms, that

$$p \geq \left(\frac{w^\alpha r^\beta}{A\alpha^\alpha \beta^\beta}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(1-\alpha-\beta)}. \tag{13.11.iii}$$

We can thus write the long run labor demand, capital demand and output supply functions as

$$\ell(w,r,p) = \left(\frac{pA\alpha^{(1-\beta)}\beta^\beta}{w^{(1-\beta)}r^\beta}\right)^{1/(1-\alpha-\beta)} \text{ if } p \geq \left(\frac{w^\alpha r^\beta}{A\alpha^\alpha \beta^\beta}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(1-\alpha-\beta)} \tag{13.11.iv}$$

$$k(w,r,p) = \left(\frac{pA\alpha^\alpha\beta^{(1-\alpha)}}{w^\alpha r^{(1-\alpha)}}\right)^{1/(1-\alpha-\beta)} \text{ if } p \geq \left(\frac{w^\alpha r^\beta}{A\alpha^\alpha \beta^\beta}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(1-\alpha-\beta)} \tag{13.11.v}$$

$$x(w,r,p) = \left(\frac{Ap^{(\alpha+\beta)}\alpha^\alpha\beta^\beta}{w^\alpha r^\beta}\right)^{1/(1-\alpha-\beta)} \text{ if } p \geq \left(\frac{w^\alpha r^\beta}{A\alpha^\alpha \beta^\beta}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(1-\alpha-\beta)}, \tag{13.11.vi}$$

with all three equations equaling zero if the condition does not hold.

In the short run, however, F plays no role since it is not an economic cost. The short run production function given fixed $\overline{k}$ is $f_{\overline{k}}(\ell) = \left[A\overline{k}^\beta\right]\ell^\alpha$. Solving the short run profit maximization problem

$$\max_{\ell}\ p\left[A\overline{k}^\beta\right]\ell^\alpha - w\ell, \tag{13.11.vii}$$

we get the short run labor demand function

$$\ell_{\overline{k}}(w,p) = \left(\frac{\alpha p\left[A\overline{k}^\beta\right]}{w}\right)^{1/(1-\alpha)}. \tag{13.11.viii}$$

Substituting this back into the short run production function, we get the short run supply function

$$x_{\overline{k}}(w,p) = \left[A\overline{k}^\beta\right]^{1/(1-\alpha)}\left(\frac{\alpha p}{w}\right)^{\alpha/(1-\alpha)}. \tag{13.11.ix}$$

Each of the following correspond to the respective parts (b) through (h) in part A of the question:

(b) *For each of (b) through (h) in part A of the exercise, indicate whether (and how) the functions you derived in part (a) are affected.*

Answer: In (b), none of the functions are affected since last year's profits do not enter any of them.

In (c), the short run functions are not affected when a 50% tax on short run profits is imposed. To see more clearly why, you can write the short run profit maximization problem to include the 50% short run profits tax — and you would get

$$\max_{\ell} \ 0.5\left(pA\overline{k}^{\beta}\ell^{\alpha} - w\ell\right), \tag{13.11.x}$$

which has first order conditions identical to those in the original problem in equation (13.11.vii).

In the long run, however, the 50% short run profit tax becomes a recurring fixed cost of doing business and would thus increase the F term in equations (13.11.iv), (13.11.v) and (13.11.vi). While this does not affect the functions themselves directly, it affects the range of prices under which the functions are not simply equal to zero (because the firm exits). If long run profit is originally zero, for instance, the 50% short run profit tax would then cause the input demand and output supply functions to go to zero because the inequality in each expression no longer holds.

In (d), none of the functions are affected. You can again see that the long run functions are unaffected by realizing that a tax on long run profits drops out as we solve the profit maximization problem

$$\max_{\ell,k} (1-t)\left(pf(\ell,k) - w\ell - rk\right) \tag{13.11.xi}$$

where t stands for the tax rate applied to long run profit. Put differently, since the government taking *a fraction* of long run profit does not cause long run profit to become negative, this tax will never cause the inequality in equations (13.11.iv), (13.11.v) and (13.11.vi) to not hold.

In (e), since F does not appear in the short run equations, the new fixed cost G will also not appear — leaving the short run curves unaffected. F does, however, appear in equations (13.11.iv), (13.11.v) and (13.11.vi) — or, to be more precise, it appears in the inequality that restricts the prices for which the functions are applicable. As F increases, the inequality will no longer hold for some range of prices at the lower end — thus raising the price at which the firm exits. If, for instance, the firm was initially making zero long run profit, it would exit with an increase in F to G because the inequality in (13.11.iv), (13.11.v) and (13.11.vi) no longer holds.

In (f), since r appears in equations (13.11.iv), (13.11.v) and (13.11.vi), we know that the long run functions are affected. They are affected in two ways: First, the equations themselves are affected, with an increase in r causing a decrease in ℓ, k, and x; and second, the inequality is affected in the sense that the inequality now no longer holds for some range of prices at the lower end. This implies that, in the long run, the firm will reduce its output and its demand for labor and capital — and it will reduce these to zero if the inequality no longer holds. For instance, if long run profit is initially zero, the firm will exit (unless something else changes). In the short run, however, r does not appear in either the labor demand or output supply equations — and thus nothing changes in the short run.

In (g), the impact on the long run will mirror what we just described for a capital tax. In the short run, however, there was no impact of the tax on capital because r did not enter the short run labor demand or output supply functions — but w does appear in these, which implies that the labor tax has an immediate short run impact. In particular, an increase in w causes an immediate decrease in both labor demand and output supply.

In (h), the tax on hamburgers will also have short and long run impacts on our derived functions by changing the output price from p to $(p-t)$. This lower output price will shift short run supply and short run labor demand in the respective short run functions, reducing the quantity in each. In the long run, p appears in both the initial equation as well as the inequality of expressions (13.11.iv), (13.11.v) and (13.11.vi). In the equations to the left of each expression, p changes to $(p-t)$ — causing a drop in each. In the inequalities on the right, p changes to $(p-t)$ on the left-hand side of the inequality, or — alternatively, we can rewrite the inequality as

$$p \geq \left(\frac{w^{\alpha} r^{\beta}}{A\alpha^{\alpha}\beta^{\beta}}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(1-\alpha-\beta)} + t. \qquad (13.11.xii)$$

This implies that the price for which the input demand and output supply functions are valid increases — again "shortening" the input demand and output demand curves. If, for instance, the firm was making zero profit before the implementation of the tax, it will exit after the implementation (unless something else changes).

Conclusion: Potentially Helpful Reminders

1. Always be careful to call something a "cost" only if it is a real opportunity cost of doing business in the environment you are analyzing. Not everything that involves writing a check is a cost of doing business.
2. This is in fact what underlies the conclusion that the shut down price in the short run is (almost) always lower (and never higher) than the long run exit price. You don't have to cover the long run costs that are short run expenditures in order to justify staying open in the short run — but you do have to

cover all your long run costs in order to justify remaining in business in the long run.

3. When all is said and done, there are really only three types of costs we are analyzing: (1) Variable costs associated with inputs that can be changed in both the short and long run — and therefore enter MC and AC in both the short and long run; (2) Costs associated with inputs that are variable only in the long run and thus enter MC and AC only in the long run (while being "sunk" expenditures in the short run); and (3) long run recurring fixed costs that are not associated with inputs. (Later on in the text, we will also introduce one-time fixed entry costs associated with entering a market — but for now we assume that entry and exit are costless.)

4. Output supply curves are always more responsive to output price in the long run (than in the short run), and own-price input demand curves are similarly always more responsive to (own) input prices in the long run (than in the short run).

5. But cross-price input demand curves that map the relationship between an input and *a different input's* price can be more or less responsive in the long run (than in the short run) depending on the substitutability of inputs in production. Similarly, cross-input-price output supply curves — i.e. the response of output to input price changes — can be more or less responsive in the long run depending on the substitutability of inputs in production.

6. The most important part of this chapter in terms of building a foundation for future chapters is the first section. It is possible to make it through the remainder of the text quite easily without fully understanding the nuances of the rest of the chapter. If you do want to tackle the material in the latter sections, understanding Graph 13.9 is crucial.