CHAPTER

14

Competitive Market Equilibrium

Until now, we have only proceeded through the first two of three steps of the "economic way of thinking" — the crafting of a model and the process of optimizing within that model. We now proceed to the final step: to illustrate how an equilibrium emerges from the optimizing behavior of individuals — how the "economic environment" that individuals in a competitive setting take as given emerges from their actions. In the process, we begin to get a sense of how order can emerge "spontaneously" — an idea introduced briefly in the introduction as one of the big ideas that we should not loose as we dive into technical details of economic models.

Chapter Highlights

The main points of the chapter are:

1. An **equilibrium** arises in an economic model when no individual has an incentive to change behavior given what everyone else is doing. In a competitive model, it means that no individual has an incentive to change behavior given the economic environment that has emerged "spontaneously".

2. The **short run for an industry** is the time over which the number of firms in the industry is fixed because firms have not had an opportunity to enter or exit the industry. The **short run industry (or market) supply curve** is therefore the sum of the firm supply curves for the (short run) fixed number of firms in the industry, and the **short run equilibrium** is driven by the price at which the short run industry supply curve intersect the market demand curve (which is simply the sum of all individual demand curves).

3. The **long run for an industry** is the time it takes for sufficient numbers of firms to enter or exit the industry as conditions change. The **long run industry (or market) supply curve** therefore arises from the condition that the *marginal* firm in the industry must make zero profits so that no firm in the industry has an incentive to exit and no firm outside the industry has an incentive to enter. When all firms face identical costs, this implies a horizontal

long run industry supply curve at the price which falls at the lowest point of each firm's long run AC curve. The **long run equilibrium** then emerges at the intersection of market demand and long run industry supply.

4. To analyze what happens as conditions change in a competitive market, the most important curves to keep track of on the firm side are the (1) **long run** AC **curve** and (2) the **short run firm supply curve that crosses the long run** AC **curve at its lowest point** (but extends below it because shut down prices are lower than exit prices.) Any change that impacts short run firm supply curves will impact the short run industry supply curve, and any change that impacts the long run AC curve will impact the long run industry supply curve.

5. Changes that affect only a single firm in an industry do not affect the market equilibrium in either the short or the long run.

14A Solutions to Within-Chapter-Exercises for Part A

Exercise 14A.1

Can you explain why there is always a natural tendency for wage to move toward the equilibrium wage if all individuals try to do the best they can?

Answer: If wage were to drift below the equilibrium, firms would not be able to fill all their job vacancies because not enough workers are willing to work at a below-market wage. Thus, it is in each firm's interest to offer a slightly higher wage in order to fill its positions — and this continues to be true until all positions are filled at the equilibrium wage. If, on the other hand, the wage were to drift above the equilibrium, some workers who want to work will be unable to find a position. It would be in their interest to offer to work for slightly less so that they can get employed when there are fewer jobs than workers wishing to take them — and this continues until the wage falls at the equilibrium where the number of jobs is exactly equal to the number of workers willing to take them.

Exercise 14A.2

Suppose your firm only used labor inputs (and not capital) and that labor is always a variable input. If your firm had to renew an annual license fee, would the AC^{SR} and the long-run AC curves ever cross in this case?

Answer: No, in this case the only difference between AC and AC^{SR} is the cost of the license fee — which does not vary with output. Thus, AC lies above AC^{SR} but converges to it as x increases.

Exercise 14A.3

Why might the AC^{SR} and the long-run AC curves cross when the difference emerges because of an input (like capital) that is fixed in the short run? (*Hint*: Review Graphs 13.2 and 13.3.)

Answer: This is because the fixed *expense* associate with the input that is fixed in the short run becomes a *variable* cost in the long run. It is therefore different than a license fee that does not change with the level of output — the cost associated with the input increases as output increases. Suppose, for instance, that capital is the fixed input in the short run and therefore is not part of short run average cost. For high levels of output, the fixed capital level may be sufficiently low such that very high levels of labor are necessary to make up for it. This may cause the short run labor costs to exceed the long run costs of both labor and capital if the firm in the long run can substitute a lot of the labor for relatively little capital.

Exercise 14A.4

Explain why the MC curve in Graph 14.4 would be the same in the long and short run in the scenario of exercise 14A.2 but not in the scenario of exercise 14A.3.

Answer: This is because in the case of a fixed cost (like a license fee), the cost does not change with output — which implies the MC curve does not change even if the AC curve shifts up. But if the fixed expense in the short run becomes a variable cost in the long run (as happens with a fixed input that becomes variable), then the MC curve changes because the cost associated with the input changes with output as the input becomes variable in the long run.

Exercise 14A.5

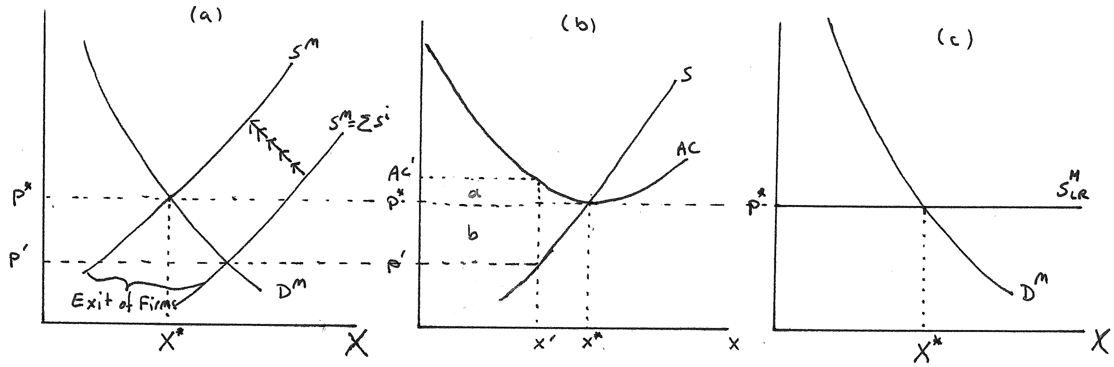
Can you draw the analogous sequence of graphs for the case when the short run equilibrium price falls below p^*?

Answer: This is illustrated in Exercise Graph 14A.5. The long run equilibrium price p^* falls at the lowest point of (long run) AC for the firms (where profit is zero). If the short run equilibrium price p' falls below p^* as drawn in panel (a), each firm produces x' along its short run supply curve as illustrated in panel (b). This implies that the average cost AC' is higher than p', leading to long run profit that is negative and equal to $(-a-b)$ in the graph. As a result, firms will exit — shifting the short run market supply curve in panel (a) to the left until price reaches p^*.

Exercise 14A.6

How does the full picture of equilibrium in Graph 14.2 look different in the long run?

Answer: The long run price would settle at the lowest point of the (long run) AC curve of firms. Thus, panel (f) would only need to show the long run AC curve, and the intersection of D^M and S^M in panel (e) would occur at the price equal to the lowest level of the AC curves of firms.



Exercise Graph 14A.5 : Movement to Long Run Equilibrium when price is below AC

Exercise 14A.7

How would you think the time-lag between short and long run changes in labor markets is related to the "barriers to entry" that workers face, where the barrier to entry into the PhD economist market, for instance, lies in the cost of obtaining a PhD.

Answer: The greater the barriers to entry, the longer it will take for the labor market to reach the long run equilibrium.

Exercise 14A.8

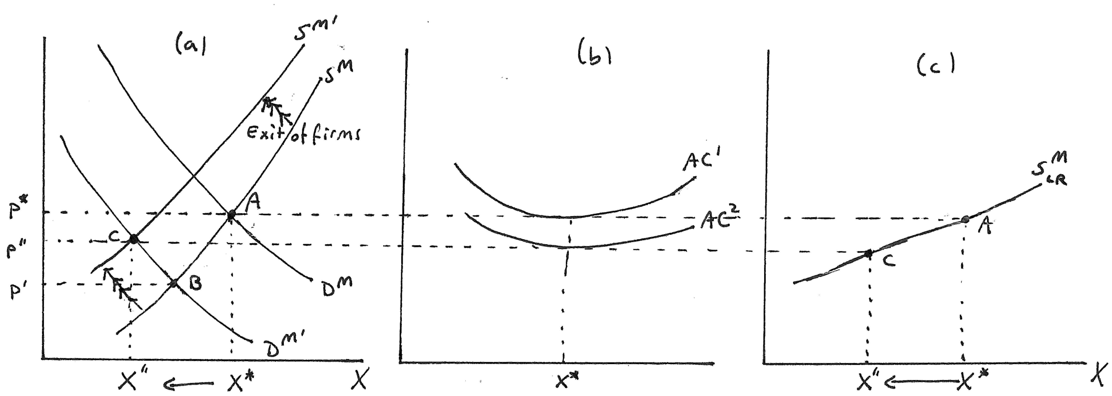
Can you explain why the previous sentence is true?

Answer: As long as the lowest points of the AC curves of firms are all at the same vertical height in the graph, all firms have the same exit (or entry) price — and thus exit and entry decisions by firms will drive long run price to that level. It does not matter whether these lowest points of AC curves occur at different output levels for different firms — i.e. whether these lowest points are horizontally different. If they are, it simply implies that different firms will produce different levels of output in long run equilibrium, but their exit/entry prices are all the same (so long as the AC curves do not differ vertically.)

Exercise 14A.9

Suppose market demand shifts inward instead of outward. Can you illustrate what would happen in graphs similar to those of Graph14.6?

Answer: Exercise Graph 14A.9 illustrates this. In panel (a), D^M and S^M intersect at the original equilibrium A — with price at p^* and industry output at X^*. At that price, any firm with AC at or below p^* is producing x^* — as, for instance, both the firms in panel (b). (This is assuming the lowest point of all AC curves occurs at the same output quantity). When demand shifts to $D^{M'}$ as illustrated in panel (a), the initial short run equilibrium shifts to B. Since the marginal firms were making zero profit before, they are now making negative long run profit — implying that they will begin to exit, which in turn causes the short run market supply curve in panel (a) to shift to the left. This continues to happen until the marginal firm left in the industry makes zero profit. This is illustrated as firm 2 with average cost curve AC^2 in panel (b). Since the highest cost firms exit, the new equilibrium price p'' will fall below p^* — leading to the upward sloping long run market supply curve in panel (c).



Exercise Graph 14A.9 : Inward shift in D^M

Exercise 14A.10

True or False: The entry and exit of firms in the long run insures that the long run market supply curve is always shallower than the short run market supply curve.

Answer: This is true. One way to see this is to think about changes in demand for an industry that is initially in long run equilibrium (before the change in demand). Suppose demand increases. This implies that industry output will rise as each firm produces more at the higher price (that results from the new intersection of (short run) market supply and demand. But, since we started in long run equilibrium, all firms that were initially making zero profit must now be making higher profit — which gives an incentive to firms outside the industry to *enter*. This will shift the short run market supply curves, driving down the price and increasing industry output until the marginal firm makes zero profit again. Thus, the entry of new firms causes the long run output increase to be larger than the short run increase. The reverse happens when demand falls. In that case, firms will produce less as price falls to the new intersection of demand and short run market supply. But since they were initially making zero profit, this implies they will not make negative (long run) profit — which implies some of the firms will exit the industry. This will shift the market supply curves inward, raising price back to the lowest point of the firms' (long run) AC curves. That shift then causes a further decrease in industry output. Thus, whether demand increases or decreases, the long run response is larger than the short run response — meaning that long run industry supply curves are shallower than short run industry supply curves because of entry and exit of firms in the long run.

Exercise 14A.11

True or False: While long run industry supply curves slope up (in increasing cost industries) because firms have different cost curves, long run industry supply curves in decreasing cost industries slope down even if firms have identical cost curves.

Answer: This is true. The text demonstrates how upward sloping long run industry supply curves arise from firms having different cost curves — with higher cost firms entering industries as industries expand — and thus price increasing to insure zero profit for marginal firms. In decreasing cost industries, however, the downward slope of industry supply curves is not due to different costs for firms — and in fact, if firms did have different cost curves, it would be much more likely that an industry could ever have a downward sloping long run supply curve. Rather, the downward sloping industry supply curve arises because *all* firms experience lower costs as the industry expands — i.e. firm costs are changing as the industry expands and input prices fall.

Exercise 14A.12

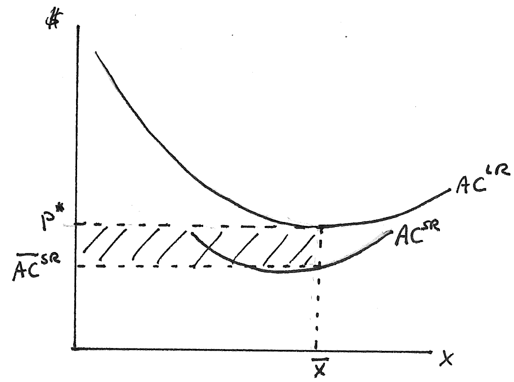
True or False: In the presence of fixed costs (or fixed expenditures), short-run profit is always greater than zero in long run equilibrium.

Answer: This is true. This is because short run profits do not include these fixed (long-run) costs while long run profits do.

Exercise 14A.13

Can you illustrate graphically the short and long run profits of the marginal firm in long run equilibrium? (*Hint*: You can do this by inserting into the graph the AC^{SR} curve as previously pictured in Graph 14.4.)

Answer: This is illustrated in Exercise Graph 14A.13 where the equilibrium price p^* causes the marginal firm to produce $\overline{x}$ at the lowest point of its long run average cost curve AC^{LR}. This implies short run average costs of AC^{SR}. From the short run perspective, the firm therefore incurs costs of $\overline{AC}^{SR} * \overline{x}$ but earns revenues of $p^* * \overline{x}$. The difference between these — indicated by the shaded area in the graph — is the short run profit earned by the marginal firm in long run equilibrium.



Exercise Graph 14A.13 : Short Run Profit in Long Run Equilibrium

Exercise 14A.14

Why does the increase in the fee result in a new (green) AC' curve that converges to the original (blue) AC curve?

Answer: This is because the increase in the fixed fee does not depend on the level of output — so, on average, the additional fee becomes less as output increases. Put differently, if the increased fixed fee is F, the average increased fixed fee is F/x — which is F when $x = 1$ but converges to zero as x gets large.

Exercise 14A.15

If you add the firm's long-run supply curve into panel (b) of the graph, where would it intersect the two average cost curves? Would the same be true for the firm's initial short-run supply curve? (*Hint*: For the second question, keep in mind that the firm will change its level of capital as its output increases.)

Answer: The firm's long run supply curve would intersect the two long run AC curves at their lowest points (because the long run supply curve for a firm is the portion of its long run MC curve that lies above its long run AC curve.) In fact, it would initially begin at the lowest point of the blue AC curve — and would get shorter as a result of the increase in the recurring fixed cost, starting at the lowest point of the green AC' curve after the increase in the recurring fixed cost.

The firm's initial short run supply curve would also cross the lowest point of the initial blue long run AC curve (since the industry is initially in long run equilibrium). But it would (almost certainly) not cross the lowest point of the green AC' curve. This is because the level of capital that the firm has in the initial long run equilibrium is unlikely to be the level of capital it will end up with in the new long run equilibrium — and its short run supply curve therefore takes a (long-run) sup-optimal level of capital as fixed.

Exercise 14A.16

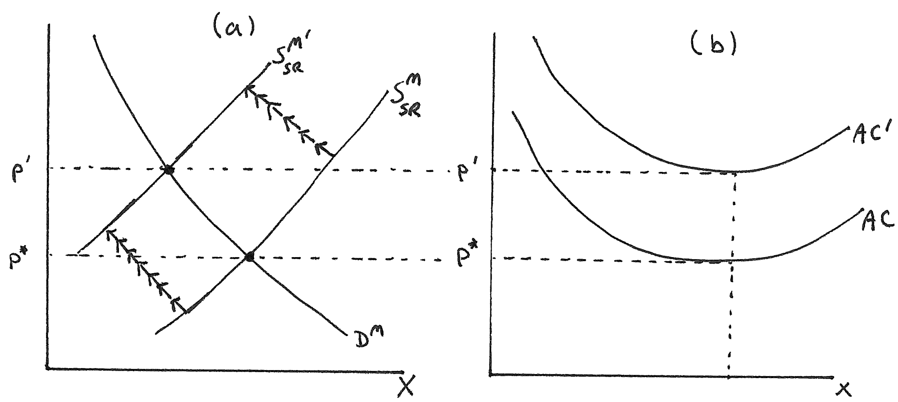
Could the *AC* curve shift similarly in the case where the increase in cost was that of a long run fixed cost?

Answer: No, it could not. In the case of a long run fixed cost, the (long run) *MC* curve remains unchanged because the fixed cost does not change the *additional* cost of producing any of the units of output. The *MC* curve also has to cross the *AC* curve both before and after the increase in the fixed cost — which means that the lowest point of the *AC* curve must shift to the right when the fixed cost increases.

Exercise 14A.17

How would you illustrate the transition from the short run to the long run using graphs similar to those in panels (a) and (b) in Graph 14.8?

Answer: When long run costs increase, firms exit the industry. This shifts the short run supply curve to the left — driving price up until it reaches the lowest point of the firms' *AC* curves. This is illustrated in Exercise Graph 14A.17.



Exercise Graph 14A.17 : Transitioning to Long Run Equilibrium

Exercise 14A.18

Consider two scenarios: In both scenarios, the cost of capital increases, causing the long run *AC* curve to shift up, with the lowest point of the *AC* curve shifting up by the same amount in each scenario. But in Scenario 1, the lowest point of the *AC* curve shifts to the right while in Scenario 2 it shifts to the left. Will the long run equilibrium price be different in the two scenarios? What about the long run equilibrium number of firms in the industry?

Answer: The only thing that matters for where the long run equilibrium price will settle is the vertical height of the lowest point of the long run *AC* curves of firms. Since this is the same in both scenarios, the long run equilibrium price will be the same for both cases. Thus, the long run market supply curve will intersect the market demand curve at the same point — which implies industry output will also be the same in both scenarios. But each firm will *increase* its production in the new equilibrium in Scenario 1 while each firm will *decrease* its production in the new equilibrium in Scenario 2. Thus, in order for the industry to produce the same in both scenarios, it must be the case that the equilibrium number of firms will be larger in Scenario 2 than in Scenario 1.

Exercise 14A.19

The previous side-quote makes the statement that a per-unit tax on output will be fully passed on to consumers in the long run. Can you explain what that means in the context of Graph 14.10? Does it also hold in the short run?

Answer: A per-unit tax will shift the long run *AC* curve up exactly as shown in panel (d) of Graph 14.10 – with the lowest point remaining at the same output level x^*. This is because a per-unit tax raises the marginal cost of each output unit by exactly the same amount – and thus causes every point on the average cost curve to shift up by that amount. Since the new long run zero-profit price therefore jumps from p^* to p'' – a difference exactly equal to the per unit tax, the tax is fully incorporated into the price that consumers pay in the long run. In the short run, however, the price rises by less – only to p' in panel (c) of Graph 14.10. Thus, the tax is not fully passed on to consumers in the short run, only in the long run as the exit of firms drives price higher.

Exercise 14A.20

If an increase in the wage causes an increase in the number of firms in an industry, does this give you enough information to know whether the lowest point of the long-run *AC* for firms shifted to the left or right? What if instead an increase in the wage causes a decrease in the number of firms in the industry?

Answer: An increase in the wage causes in increase in long run *AC* and thus an increase in the long run equilibrium price. As price increases, consumers will demand less – so total output in the industry falls. If the number of firms goes up, that implies more firms will be producing a smaller overall industry output – implying each firm is producing less than a firm produced before the wage increase. That tells us that the lowest point of long run *AC* must have shifted to the left.

If, on the other hand, the increase in the wage causes a decline in the number of firms, then a smaller number of firms is producing a smaller overall level of industry output. From this we cannot tell whether each individual firm is producing more or less than before the wage increase. It could be that industry output fell very little (due to a steep market demand curve) and that, since there are fewer firms, each firm actually produces more. Or it could be that industry output fell a lot (due to a shallow market demand curve), with each firm producing less than originally despite there being fewer firms. In the first case the lowest point of long run *AC* shifted to the right while in the second case it shifted to the left.

Exercise 14A.21

True or False: Regardless of what cost it is, if it increases for only one firm in a competitive industry, that firm will exit in the long run but it might not shut down in the short run.

Answer: This is true. Before the increase in costs, this firm was making zero long run profit. The increase in its costs does not change the equilibrium price since the firm is “small” — and thus long run profit is negative after the increase in the cost. This implies the firm will exit in the long run. In the short run, the firm continues to produce if the cost that increases is one associated with a short-run fixed input or a long run fixed cost — because these increases do not affect short run economic costs. If the increase in costs is an increase in a true short run cost (such as the cost of labor), then the firm's short run *MC* curve shifts to the left, causing it to either produce less (if short run profit does not become negative) or to shut down (if short run profit has become negative.)

Exercise 14A.22

What would a fifth row for an increase in per-unit taxes on output look like? Can you then also replicate Table 14.1 for the cases where the demand and the various cost examples decrease rather than increase.

Answer: An increase in per unit taxes would affect *AC* and *MC* in both the short and long run. Market price would rise in both the short and long run, but more in the long run than in the short run. Industry output would fall in both the short and long run, but more in the long run than in the short run. Firm output would fall in the short run but then return (for those firms that don't exit) to the original output level in the long run. The number of firms would decline.

The replication of the table follows.

Example	Affected Costs SR	Affected Costs LR	Market Price	Industry Output	Firm Output	LR # of Firms
↓ License Fee	None	*AC*	$-^{SR} \downarrow^{LR}$	$-^{SR} \uparrow^{LR}$	$-^{SR} \downarrow^{LR}$	↑
↓ r	None	*AC*, *MC*	$-^{SR} \downarrow^{LR}$	$-^{SR} \uparrow^{LR}$	$-^{SR} ?^{LR}$	?
↓ w	*AC*, *MC*	*AC*, *MC*	$\downarrow^{SR} \downarrow^{LR}$	$\uparrow^{SR} \Uparrow^{LR}$	$\uparrow^{SR} ?^{LR}$	?
↓ Demand	None	None	$\downarrow^{SR} -^{LR}$	$\downarrow^{SR} \Downarrow^{LR}$	$\downarrow^{SR} -^{LR}$	↓

14B Solutions to Within-Chapter-Exercises for Part B

Exercise 14B.1

Why is the demand function not a function of income?

Answer: This is because the utility function from which it was derived is quasi-linear — which removes income effects from demand.

Exercise 14B.2

Demonstrate that the average cost of production is U-shaped and reaches its lowest point at x =1280 where AC=5. (*Hint*: You can illustrate the U-shape by showing that the derivative of AC is zero at 1280, negative for output less than 1280 and positive for output greater than 1280.)

Answer: Taking the derivative of $AC(x)$, we get

$$\frac{\partial AC(x)}{\partial x} = \frac{0.167185}{x^{3/4}} - \frac{1280}{x^2}. \qquad (14B.2.i)$$

Substituting $x = 1280$, we get $AC(1280) = 0$, and, for $x < 1280$, the function is indeed negative while for $x > 1280$ it is positive. We therefore have a U-shaped AC curve that reaches its lowest point at $x = 1280$. At that output level, the average cost is

$$AC(1280) = 0.66874(1280)^{1/4} + \frac{1280}{1280} = 5. \qquad (14B.2.ii)$$

Exercise 14B.3

Verify these individual production and consumption quantities.

Answer: Substituting $p = 5$ into the consumer demand equation, we get

$$x^d(5) = \frac{625}{5^2} = 25, \qquad (14B.3.i)$$

and substituting $p = 5$ into the firm supply equation $x^s(p)$, we get

$$x^s(5) = 437.754(5)^{2/3} = 1280. \qquad (14B.3.ii)$$

Exercise 14B.4

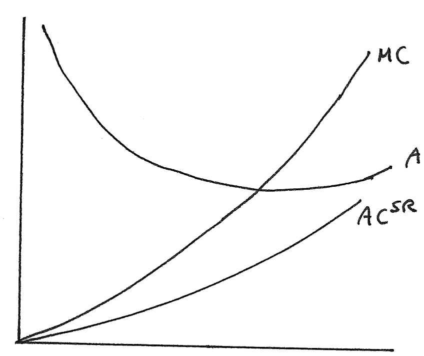
We have already indicated that $k = 256$ is in fact the optimal long run quantity of capital when (p, w, r)=(5,20,10). Can you then conclude that the industry is in long run equilibrium from the information in the previous paragraph? (*Hint*: This can only be true if no firm has an incentive to enter or exit the industry.)

Answer: Since we know $k = 256$ is the long run optimal quantity of capital, the short run expenditures of \$3,840 become economic costs in the long run. (If we did not know $k = 256$ was long run optimal, we would not be able to conclude this since the firm would further adjust capital and therefore the long run cost of capital would differ from the short run expense on capital.) Thus, from a long run perspective, the firm has \$3,840 more in costs than it has in the short run. Since we concluded that short run profit is \$3,840, this implies long run profit is \$0. The industry is therefore in long run equilibrium — with no firm in the industry wanting to exit and no firm outside the industry wanting to enter.

Exercise 14B.5

Can you graph the AC^{SR} into panel (c) of Graph 14.12?

Answer: This is illustrated in Exercise Graph 14B.5 where the short run average cost curve must lie below the short run MC curve.



Exercise Graph 14B.5 : AC^{SR} with Decreasing Returns to Scale Production

Exercise 14B.6

Why does the long-run profit become negative \$960 if nothing changes?

Answer: Since we started in long run equilibrium, we know that initially the firms were making zero long run profit. When the license fees are increased by \$960, this then implies that long run profit must fall to minus \$960 if nothing changes.

Exercise 14B.7

Verify these calculations.

Answer: The lowest point of the AC' curve occurs where its derivative is zero — i.e. where

$$\frac{\partial AC'(x)}{\partial x} = \frac{0.167185}{x^{3/4}} - \frac{2240}{x^2} = 0 \tag{14B.7.i}$$

which solves to $x = 2002.8$ or approximately $x = 2000$. At $x = 2000$, the average cost is

$$AC'(2000) = 0.66874(2000)^{1/4} + \frac{2240}{2000} \approx 5.59. \tag{14B.7.ii}$$

At that price, the market demand curve tells us that the consumers' demand is $D^M(5.59) = 40,000,000/(5.59^2) \approx 1,280,000$, with each individual consumer demanding $x^d(5.59) = 625/(5.59^2) \approx 20$. Each firm will supply about 2000 units of output — and with a total of about 1,280,000 produced by the industry, this implies that the new number of firms in the industry will be 1,280,000/2000≈ 640.

Exercise 14B.8

Compare the changes set off by an increase in the license fee to those predicted in Graph 14.8.

Answer: The graph suggests that we will see an increase in the quantity supplied by each firm with a decrease in quantity supplied by the industry at a higher price. Here we have calculated price increasing from \$5 to \$5.59, the industry quantity falling from 1,600,000 to 1,280,000 and and the amount produced by each firm *increasing* from 1,280 to 2,000. This is consistent with the directions of changes identified in the graph.

Exercise 14B.9

Verify these calculations.

Answer: The lowest point of the AC' curve occurs where its derivative is zero — i.e. where

$$\frac{\partial AC'(x)}{\partial x} = \frac{0.204759}{x^{3/4}} - \frac{1280}{x^2} = 0 \tag{14B.9.i}$$

which solves to $x = 1088.36$ or approximately $x = 1088$. At $x = 1088$, the average cost is

$$AC'(1088) = 0.819036(1088)^{1/4} + \frac{1280}{1088} \approx 5.88. \tag{14B.9.ii}$$

At that price, the market demand curve tells us that the consumers' demand is $D^M(5.88) = 40,000,000/(5.88^2) \approx 1,156,925$, with each individual consumer demanding $x^d(5.88) = 625/(5.88^2) \approx 18$. Each firm will supply 1088 units of output, which implies that the total number of firms will be 1,156,925/1088≈ 1063.

Exercise 14B.10

Are these results consistent with Graph 14.9?

Answer: The graph suggests that industry output will fall while firm output remains unchanged and price increases. We calculated that industry output falls from 1,600,000 to 1,156,925 and price rises from $5 to $5.88. These results are consistent with the graph. We also calculated that each firm's output will fall from 1,250 to 1,088 which is different from what is shown in the graph. However, in developing the graph, we noted that the lowest point of the long run *AC* curve might shift to the left or right as the rental rate of capital increases — and we just happened to draw it as shifting in neither direction. So, while the mathematical results in this example are not consistent with how we drew the graph, they are consistent with our discussion in part A of the chapter.

Exercise 14B.11

How much capital and labor are hired in the industry before and after the increase in r?

Answer: In Chapters 12 and 13, we calculated the input demand functions for this technolgoy to be

$$\ell(p,w,r) = 32768\frac{p^5}{r^2 w^3} \quad \text{and} \quad k(p,w,r) = 32768\frac{p^5}{w^2 r^3}. \tag{14B.11.i}$$

When $p = 5$ and $w = 20$, these become

$$\ell(r) = \frac{12800}{r^2} \quad \text{and} \quad k(r) = \frac{256000}{r^3}. \tag{14B.11.ii}$$

Evaluate at $r = 10$ and $r = 15$, this gives us $\ell = 128$ and $k = 256$ when $r = 10$ going to $\ell = 58.89$ and $k = 75.85$ when r increases to 15.

Exercise 14B.12

Verify these calculations.

Answer: Setting short run market supply equal to market demand implies

$$417,586p^{2/3} = \frac{40,000,000}{p^2}, \tag{14B.12}$$

which implies $p^{8/3} = 95.789$ or $p = 5.533409 \approx \$5.53$. Substituting this into the firm's new short run supply function $x^{s'}(p) = 334.069p^{2/3}$, we get that each firm produces $xs'(5.53) \approx 1,045$. The industry output can be calculated by substituting the new price into either the market demand or supply curves — both of which tell us that overall industry output will rise to 1,306,395. (Because of rounding error, you will actually get slightly different answers depending on whether you plug the new price into the market demand or short run market supply functions — the output level 1,306,395 arises from using the price $p = 5.533409$ we calculated before rounding. Up to a rounding error, this is also the same as what we get if we multiply each firm's output of 1,045 by the total number of firms in the short run equilibrium (1,250).)Total revenue for each firm will then simply be the price times the output level 1,045 — or approximately \$5,782 if we use the un-rounded price or \$5,779 if we use $p = 5.53$ which is slightly rounded down.

Exercise 14B.13

How much does the industry production change in the short run?

Answer: Industry production falls from the original 1,600,000 calculated earlier to the 1,306,395 we calculated in the previous exercise — a short run drop of a little less than 300,000 output units.

Exercise 14B.14

Verify these calculations and compare the results to our graphical analysis of an increase in the wage rate in Graph 14.10.

Answer: First, to calculate the long run equilibrium price, we need to determine the lowest point of the long run average cost curve. The cost curve $C(w,r,x) = 2(wr)^{1/2}(x/20)^{5/4} + 1280$ (given at the beginning of part B of the text) becomes $C(30,10,x) = 0.819036x^{5/4} + 1280$ when the new wage (and original rental rate) are substituted for w and r. Thus, the average (long run) cost curve after the wage increase is

$$AC(x) = 0.819036x^{1/4} + \frac{1280}{x}. \tag{14B.14.i}$$

This reaches its lowest point when

$$\frac{\partial AC(x)}{\partial x} = \frac{0.204759}{x^{3/4}} - \frac{1280}{x^2} = 0. \tag{14B.14.ii}$$

Solving for x, we get that $x = 1088.36 \approx 1088$ — and the lowest average cost level reached at that output level is $AC(1088.36) \approx \$5.88$. Thus, the price rises from the original \$5.00 to \$5.53 in the short run to \$5.88 in the long run. Each firm, which originally produced 1280 units, reduces its output to 1045 in the short run, but that output level increases to 1088 in the long run for those firms that remain in the industry. At the new long run equilibrium price, the market demands $D^M = 40,000,000/(5.88^2) \approx 1,156,925$ — down from the original 1,600,000 and from the short run equilibrium industry output of 1,306,395. This implies that the number of firms that remain in the industry falls from the original 1,250 to 1,156,925/1088$\approx$ 1,063 firms.

The graph in part A predicted that the industry would produce less in the short run and even less in the long run, and that the price will rise in the short run and even more in the long run. Both these predictions hold up in this example. The graph furthermore predicted that output by each firm will initially fall in the short run but will rise back to its original quantity in the long run for firms that stay in the industry. The latter does not hold in this example, but we had pointed out in part A that the long run output could in principle go up or down — and we simply graphed it as unchanged from the original quantity solely for convenience. Thus, the prediction in this example that firm output for those that remain in the industry will initially go down and then recover somewhat but not fully is not inconsistent with the discussion surrounding the graph in part A.

Exercise 14B.15

How much does individual consumption by consumers who were originally in the market change in the short run?

Answer: Individual demand was derived at the beginning of the chapter to be $x^d(p) = 625/p^2$. Substituting in $p = 5.91$, we then get that $x^d(5.91) = 625/(5.91^2) = 17.894 \approx 18$.

Exercise 14B.16

Verify these calculations and compare the results with Graph 4.11 where we graphically illustrated the impact of an increase in market demand.

Answer: Since the average cost curves for firms have not changed, the long run price falls to the previously calculated lowest point of the AC curve — which is \$5. Thus, firms will enter until price falls from the short run equilibrium of \$5.91 to the long run equilibrium of \$5. To meet market demand of 2,500,000 at that price, the new number of firms in the industry must be $2,500,000/1280 \approx 1,953$, up from the initial 1,250, with each firm producing at its original equilibrium quantity of 1,280 units of output. Thus, initially industry quantity rises in the short run because each firm produces more at a higher price, but in the long run each firm returns to its original quantity, more firms enter and price falls to its original level, with the industry increasing production beyond the short run increase. This is exactly what is demonstrated in Graph 14.11 of the text.

14C Solutions to Odd Numbered End-of-Chapter Exercises

Exercise 14.1

In Table 14.1, the last column indicates the predicted change in the number of firms within an industry when economic conditions change.

A: *In two cases, the table makes a definitive prediction, whereas in two other cases it does not.*

(a) *Explain first why we can say definitively that the number of firms falls as a recurring fixed cost (i.e. license fee) increases? Relate your answer to what we know about firm output and price in the long run.*

Answer: When a fixed cost increases, the long run MC curve does not change but the long run AC curve shifts up. Since the MC curve always crosses the lowest point of the AC curve, we know that this implies that the lowest point of the long run AC curve shifts to the right — i.e. to a higher level of output. This implies that the output level of firms that remain in the industry will increase in the new equilibrium — as will the price (since the AC curve has shifted up). But an increase in price means that, for any downward sloping demand curve, consumers will demand less of the good. The industry therefore produces less at a higher price — with each firm in the industry producing more. The only way this is possible is if some firms have exited — i.e. the number of firms in the industry has decreased.

(b) *Repeat (a) for the case of an increase in demand.*

Answer: When demand increases, none of the cost curves for firms change — with the long run AC curve in particular remaining unchanged. Thus, each firm in the new equilibrium will be producing at the same lowest point of its AC curve — and at the same price. The only thing that has changed is that demand has shifted — which implies that, at the same price, more output will be produced in the industry. With each firm in the industry producing the same output quantity, the only way more can be produced in the industry is for more firms to have entered — i.e. the number of firms in the industry has increased.

(c) *Now consider an increase in the wage rate and suppose first that this causes the long run AC curve to shift up without changing the output level at which the curve reaches its lowest point. In this case, can you predict whether the number of firms increases or decreases?*

Answer: In this case, the output level produced by each firm in the industry will remain the same but will be sold at a higher price. When price increases, however, less will be demanded (assuming a downward sloping market demand curve) — which implies the industry produces less. With each firm in the industry producing the same as before, the only way for the industry to produce less is for some firms to have exited. Thus, the number of firms in the industry falls in this case.

(d) *Repeat (c) but assume that the lowest point of the AC curve shifts up and to the right.*

Answer: If the lowest point of the *AC* curve shifts up and to the right, it means that firms that remain in the industry will produce *more* at a higher price — but the higher price implies that less will be demanded and thus the industry produces *less*. The only way each firm can produce more while the industry produces less is if some firms exited — and the number of firms in the industry therefore declines.

(e) *Repeat (c) again but this time assume that the lowest point of the AC curve shifts up and to the left.*

Answer: In this case, the lowest point of the *AC* curve occurs at a lower level of output and higher price — which means that firms in the industry will produce less and sell at that higher price. A higher price in turn means that consumers will demand less. Thus, each firm produces *less* as does the industry. Whether this implies more or fewer firms now depends on how much less each firm produces relative to how much the quantity demanded falls with the increase in price. Suppose each firm produces x% less and the industry as a whole produces y% less. Then if $x = y$, the number of firms stays exactly the same; if $x < y$, the number of firms falls and if $x > y$, the number of firms in the industry has to increase.

(f) *Is the analysis regarding the new equilibrium number of firms any different for a change in r?*

Answer: No, the analysis is no different for a change in r. Even if capital is fixed in the short run, it is variable in the long run — and treated just like the input labor.

(g) *Which way would the lowest point of the AC curve have to shift in order for us not to be sure whether the number of firms increases or decreases when w* falls*?*

Answer: When w falls, we know the long run *AC* curve will shift down — which implies that the long run equilibrium price will fall. At a lower price, the quantity demanded will increase — which implies that industry output will *increase*. Were each firm to continue to produce the same amount as before — or were each firm to produce less, then the only way for the industry to produce more would be for the number of firms to increase. Thus, in order for us not to be sure of whether the number of firms increases, it would have to be that each firm produces more (just as the industry produces more) — and this only happens if the lowest point of the *AC* curve shifts to the right as w falls.

B: *Consider the case of a firm that operates with a Cobb-Douglas production function $f(\ell, k) = A\ell^{\alpha}k^{\beta}$ where $\alpha, \beta > 0$ and $\alpha + \beta < 1$.*

(a) *The cost function for such a production process — assuming no fixed costs — was given in equation (13.45) of exercise 13.5. Assuming an additional recurring fixed cost F, what is the average cost function for this firm?*

Answer: Including the fixed cost F, the total cost function becomes

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^\alpha r^\beta x}{A\alpha^\alpha \beta^\beta}\right)^{1/(\alpha+\beta)} + F \qquad (14.1.i)$$

which gives us an AC function

$$AC(w,r,x) = \frac{C(w,r,x)}{x} = (\alpha+\beta)\left(\frac{w^\alpha r^\beta x^{(1-\alpha-\beta)}}{A\alpha^\alpha \beta^\beta}\right)^{1/(\alpha+\beta)} + \frac{F}{x}. \qquad (14.1.ii)$$

(b) *Derive the equation for the output level x^* at which the long run AC curve reaches its lowest point.*

Answer: The AC curve reaches its lowest point where its derivative with respect to x is zero — i.e. where

$$\frac{\partial AC(w,r,x)}{\partial x} = \left[(1-\alpha-\beta)\left(\frac{w^\alpha r^\beta}{A\alpha^\alpha \beta^\beta}\right)^{1/(\alpha+\beta)} x^{(1-2(\alpha+\beta))/(\alpha+\beta)}\right] - \frac{F}{x^2} = 0. \qquad (14.1.iii)$$

Solving this for x, we then get the output level at the lowest point of the long run AC curve:

$$x^* = \left(\frac{A\alpha^\alpha \beta^\beta}{w^\alpha r^\beta}\right)\left(\frac{F}{1-\alpha-\beta}\right)^{(\alpha+\beta)}. \qquad (14.1.iv)$$

(c) *How does x^* change with F, w and r?*

Answer: Given the expression for x^*, it is easy to see that x^* increases with F and decreases with w and r. Put differently, the lowest point of the AC curve occurs at higher output levels as the fixed cost increases and at lower output levels when input prices increase.

(d) True or False*: For industries in which firms face Cobb-Douglas production processes with recurring fixed costs, we can predict that the number of firms in the industry increases with F but we cannot predict whether the number of firms will increase or decrease with w or r.*

Answer: This is true. As F increases, output price rises as does output by each firm. The higher output price, however, means that the quantity demanded — and thus the quantity supplied by the industry — decreases. The only way the industry output can decrease when firm output increases is if some firms have left the industry. When input prices increase, the equilibrium output price similarly rises (as the AC shifts up) — causing the industry to produce less. But, in the case analyzed here, each firm also produces less — so we cannot immediately tell whether the number of firms will increase or decrease.

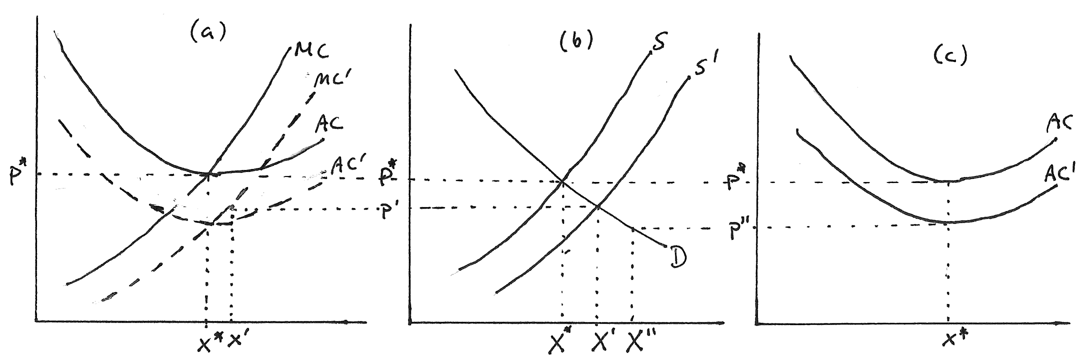
Exercise 14.3

Everyday and Business Application: *Fast Food Restaurants and Grease (cont'd): In exercise 12.8, you investigated the impact of hybrid vehicles that can run partially on grease from hamburger production on the number of hamburgers produced by a fast food restaurant. You did so, however, in the absence of considering the equilibrium impact on prices — assuming instead that prices for hamburgers are unaffected by the change in demand for grease.*

A: *Suppose again that you use a decreasing returns to scale production process for producing hamburgers using only labor and that you produce 1 ounce of grease for every hamburger. In addition, suppose that you are part of a competitive industry and that each firm also incurs a recurring fixed cost F every week.*

(a) *Suppose that the cost of hauling away grease is $q > 0$ per ounce. Illustrate the shape of your marginal and average cost curve (given that you also face a recurring fixed cost.)*

Answer: These are illustrated in panel (a) of Exercise Graph 14.3 as the solid curves labeled *MC* and *AC*. The marginal curve is upward sloping because it is unaffected by the recurring fixed cost. The average cost curve, however, is U-shaped as a result of *F*.



Exercise Graph 14.3 : Hamburgers and Hybrid Vehicles

(b) *Assuming all restaurants are identical, illustrate the number of hamburgers you produce in long run equilibrium.*

Answer: In long run equilibrium, you will be making zero profit — which means the long run price is p^* as illustrated in panel (a) of Exercise Graph 14.3. As a result, you produce x^*.

(c) *Now suppose that, as a result of the increased use of hybrid vehicles, the company you previously hired to haul away your grease is now willing to pay for the grease it hauls away. How do your cost curves change?*

Answer: The marginal and average cost curves will now shift down by the change in q. This is illustrated in panel (a) of Exercise Graph 14.3 with the dashed MC and AC curves.

(d) *Describe the impact this will have on the equilibrium price of hamburgers and the number of hamburgers you produce in the short run.*

Answer: In panel (b) of Exercise Graph 14.3, we illustrate the initial short run market supply curve S that intersects the demand curve D at the original price p^*. As a result of each firm's marginal cost curve shifting down, the short run market supply curve shifts down to S' — resulting in a decrease in the price to p'. The industry ends up producing more (X' rather than X^* in panel (b)) — which means each restaurant is producing more in the short run when the number of restaurants is fixed. This is illustrated as x' in panel (a).

(e) *How does your answer change in the long run?*

Answer: In the long run, the equilibrium price must settle at a point where all restaurants again make zero profit — i.e. to the lowest point of the new AC' curve. This is illustrated in panel (c) of Exercise Graph 14.3. Because the cost of each hamburger produced decreases by the same amount, the average cost curve shifts down in a parallel way — leaving the lowest point at that same output quantity x^* as before. Thus, in the long run, each restaurant will produce the same number of hamburgers as originally (x^*) and will sell them at price p''. The price has in essence fallen by the full decrease in the marginal cost. In the new equilibrium, there will be more restaurants than before.

(f) *Would your answers change if you instead assumed that restaurants used both labor and capital in the production of hamburgers?*

Answer: No — the cost curves would shift in exactly the same way.

(g) *In exercise 12.8, you concluded that the cholesterol level in hamburgers will increase as a result of these hybrid vehicles if restaurants can choose more or less fatty meat. Does your conclusion still hold?*

Answer: Yes, this conclusion still holds — if firms can profit from using fattier beef, they will all do so in equilibrium. If a firm did not do so, it would make negative profit.

B: *Suppose, as in exercise 12.8, that your production function is given by $f(\ell) = A\ell^\alpha$ (with $0 < \alpha < 1$) and that the cost of hauling away grease is q. In addition, suppose now that each restaurant incurs a recurring fixed cost of F.*

(a) *Derive the cost function for your restaurant.*

Answer: Solving the production function $x = A\ell^\alpha$ for ℓ, we get the conditional labor demand function

$$\ell(w, x) = \left(\frac{x}{A}\right)^{1/\alpha}. \tag{14.3.i}$$

Multiplying this by w and adding the cost of hauling grease as well as the fixed cost, we get

$$C(w,x,q) = w\left(\frac{x}{A}\right)^{1/\alpha} + qx + F. \tag{14.3.ii}$$

(b) *Derive the marginal and average cost functions.*

Answer: Taking the derivative of the cost function with respect to x, we get

$$MC(w,x,q) = \left(\frac{w}{\alpha A^{1/\alpha}}\right) x^{(1-\alpha)/\alpha} + q. \tag{14.3.iii}$$

Dividing the cost function by x, we get the average cost function

$$AC(w,x,q) = \left(\frac{w}{A^{1/\alpha}}\right) x^{(1-\alpha)/\alpha} + q + \frac{F}{x}. \tag{14.3.iv}$$

(c) *How many hamburgers will you produce in the long run?*

Answer: In the long run, you produce at the lowest point of your average cost curve. To determine the output quantity at which this occurs, we take the first derivative of our AC function, set it to zero and solve for x. This gives us

$$x^* = A\left(\frac{\alpha F}{(1-\alpha)w}\right)^{\alpha}. \tag{14.3.v}$$

(d) *What is the long run equilibrium price of hamburgers?*

Answer: To determine the long run equilibrium price, we plug x^* back into the average cost function and solve it; i.e.

$$p^* = AC(w,x^*,q) = \left(\frac{w}{A^{1/\alpha}}\right)\left[A\left(\frac{\alpha F}{(1-\alpha)w}\right)^{\alpha}\right]^{(1-\alpha)/\alpha} + q + \frac{F}{\left[A\left(\frac{\alpha F}{(1-\alpha)w}\right)^{\alpha}\right]} \tag{14.3.vi}$$

which, after some algebra, reduces to

$$p^* = \frac{w^{\alpha}F^{(1-\alpha)}}{A(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}} + q. \tag{14.3.vii}$$

(e) *From your results, determine how the long run equilibrium price and output level of each restaurant changes as q changes.*

Answer: The term q does not appear in our equation for x^* — which implies that it has no impact on the number of hamburgers produced by each restaurant in the long run. This is consistent with what we determined graphically. Our equation for p^*, on the other hand, has q simply entering as an additive term. Thus, in long run equilibrium, q is simply passed onto the consumer — as it falls (and even becomes negative), consumers therefore get the entire benefit.

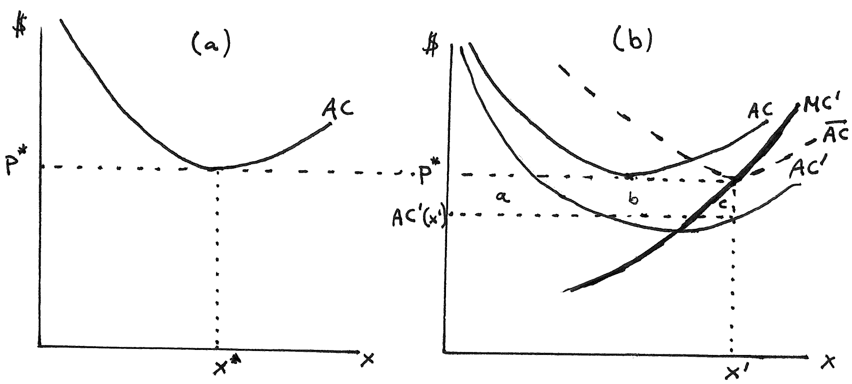
Exercise 14.5

Business Application: *"Economic Rent" and Profiting from Entrepreneural Skill: Suppose, as in exercise 14.4, that you are operating a hamburger restaurant that is part of a competitive industry. Now you are also the owner, and suppose throughout that the owner of a restaurant is also one of the workers in the restaurant and collects the same wage as other workers for the time he/she puts into the business each week. (In addition, of course, the owner keeps any weekly profits.)*

A: *Again, assume that all the restaurants are using the same homothetic decreasing returns to scale technology, but now the inputs include the level of entrepreneural capital c in addition to weekly labor ℓ and capital k. As in exercise 14.4, assume also that all restaurants are required to pay a recurring weekly fixed cost F.*

(a) *First assume that all restaurant owners possess the same level of entrepreneural skill c. Draw the long run AC curve (for weekly hamburger production) for a restaurant and indicate how many weekly hamburgers the restaurant will sell and at what price assuming that the industry is in long run equilibrium.*

Answer: This is illustrated in panel (a) of Exercise Graph 14.5 where the long run average cost curve of each firm is U-shaped because of the recurring fixed cost. Each restaurant will produce x^* and sell it at p^*.



Exercise Graph 14.5 : Rent on Entrepreneural Skill

(b) *Suppose next that you are special and possess more entrepreneural and management skill than all those other restaurant owners. As a result of your higher level of c, the marginal product of labor and capital is 20% greater for any bundle of ℓ and k than it is for any of your competitors. Will the long run equilibrium price be any different as a result?*

Answer: No, a single firm in a competitive industry is not large enough to affect market prices.

(c) *If your entrepreneural skill causes the marginal product of capital and labor to be 20% greater for any combination of ℓ and k than for your competitors, how does your isoquant map differ from theirs? For a given wage and rental rate, will you employ the same labor to capital ratio as your competitors?*

Answer: We know that the slopes of isoquants are $TRS = -MP\ell/MP_k$. If both marginal products increase by the same percentage, then the ratio is unchanged — which implies that your isoquant map looks exactly the same as your competitors' except that it is differently labeled because you can produce more with less capital and labor. Since the shapes of the isoquants are the same as those for your competitors', isocosts will be tangent along the same ray from the origin — which implies that you will employ the same labor to capital ratio as you minimize your costs. You will simply require less capital and labor for any given level of output.

(d) *Will you produce more or less than your competitors? Illustrate this on your graph by determining where the long run MC and AC curves for your restaurant will lie relative to the AC curve of your competitors.*

Answer: Since you need less labor and capital for any given level of output, your average costs are lower. Similarly, the lowest point of your AC curve will occur at a higher level of output than for your competitors. This is illustrated in panel (b) of Exercise Graph 14.5 where AC is your competitors' average cost curve and AC' is yours. Finally, we can put your long run MC' curve into the graph (making sure it crosses your average cost curve AC' at its lowest point. Since the market price is unchanged at p^*, we know you will profit maximize where p^* intersects MC' — at output level x'. You will therefore produce more than your competitors.

(e) *Illustrate in your graph how much weekly profit you will earn from your unusually high entrepreneural skill.*

Answer: In panel (b) of Exercise Graph 14.5, two rectangular boxes emerge from the dotted lines combined with the axes. The larger of these is equal to total revenues for your firm (price times output); the smaller one is your total cost (average cost times output); and the difference — area $a+b+c$ — is the difference between the two. Since those without your entrepreneural skill make zero profit, the profit you derive from your skill is therefore $a+b+c$.

(f) *Suppose the owner of MacroSoft, a new computer firm, is interested in hiring you as the manager of one of its branches. How high a weekly salary would it have to offer you in order for you to quit the restaurant business assuming you would work for 36 hours per week in either case and assuming the wage rate in the restaurant business is $15 per hour.*

Answer: It would have to offer you a salary equal to the level of compensation you currently get for spending your time in the restaurant business. Since you are one of the workers drawing an hourly wage $w = 15$, you are making $540 per week as one of the workers in your restaurant *plus* you earn a profit of $a+b+c$ as indicated in Exercise Graph 14.5.

MacroSoft would therefore have to offer you a minimum weekly salary of $a+b+c+540$.

(g) *The benefit that an entrepreneur receives from his skill is sometimes referred to as the* economic rent *of that skill — because the entrepreneur could be renting his skill out (to someone like MacroSoft) instead of using it in his own business. Suppose MacroSoft is willing to hire you at the rate you determined in part (f). If the economic rent of entrepreneural skill is included as a cost to the restaurant business you run, how much profit are you making in the restaurant business?*

Answer: You would then be making zero profit because $a+b+c$ in Exercise Graph 14.5 would become an additional periodic fixed cost in your restaurant business.

(h) *Would counting this economic rent on your skill as a cost in the restaurant business affect how many hamburgers you produce? How would it change the AC curve in your graph?*

Answer: Since the economic rent is a recurring long run fixed cost, it does not impact the long run MC curve in panel (b) of Exercise Graph 14.5. Thus, price p^* continues to intersect MC' at x' — implying you will produce exactly the same amount as if we did not count economic rent on your skill as a cost. The only thing that would change in panel (b) of the graph is that the average cost curve would be higher because $a+b+c$ is now included as a recurring average cost — and this average cost curve (denoted $\overline{AC}$ in the graph) — would reach its lowest point as it intersects the unchanged MC'. Thus, price equals MC' at the lowest point of $\overline{AC}$ — giving us zero profit for the firm if economic rents on entrepreneural skill are counted as a cost for the firm (and a payment to the owner).

B: *Suppose that all restaurants are employing the production function* $f(\ell,k,c) = 30\ell^{0.4}k^{0.4}c$ *where* ℓ *stands for weekly labor hours, k stands for weekly hours of rented capital and c stands for the entrepreneural skill of the owner. Note that, with the exception of the c term, this is the same production technology used in exercise 14.4. The weekly demand for hamburgers in your city is, again as in exercise 14.4,* $x(p) = 100{,}040 - 1{,}000p$.

(a) *First, suppose that* $c = 1$ *for all restaurant owners, that* $w = 15$ *and* $r = 20$*, that there is a fixed weekly cost $4,320 of operating a restaurant, and the industry is in long run equilibrium. Determine the weekly number of hamburgers sold in each restaurant, the price at which hamburgers sell, and the number of restaurants that are operating. (If you have done exercise 14.4, you should be able to use your results from there.)*

Answer: Since $c = 1$ for all restaurants, the production function becomes identical to that used in exercise 14.4. In parts (a) through (c) of exercise 14.4, you calculated that each restaurant will produce 4,320 hamburgers per week, that the long run equilibrium price will be $5 per hamburger and that there will be 22 restaurants in your city.

(b) *Next, suppose that you are the only restaurant owner that is different from all the others in that you are a better manager and entrepreneur and that this is reflected in* $c = 1.24573$ *for you. Determine your long run AC and MC functions. (Be careful not to use the cost function given in exercise 14.4 since c is no longer equal to 1. You can instead rely on the cost function derived for Cobb-Douglas technologies given in equation (13.45) in exercise 13.5 (and remember to add the fixed cost).)*

Answer: Your production function can then be written as

$$f(\ell,k) = [30(1.24573)]\,\ell^{0.4}k^{0.4} = 37.3719\ell^{0.4}k^{0.4}. \tag{14.5.i}$$

The cost function for a Cobb-Douglas production process $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ is

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)}. \tag{14.5.ii}$$

Substituting in $\alpha = \beta = 0.4$, $A = 37.3719$, $w = 15$ and $r = 20$, and adding the fixed cost of \$4,320, this gives us the cost function

$$C(x) = 0.374895x^{1.25} + 4320. \tag{14.5.iii}$$

The marginal and average cost functions are then

$$MC(x) = \frac{dC(x)}{dx} = 0.468619x^{0.25} \text{ and } AC(x) = \frac{C(x)}{x} = 0.374895x^{0.25} + \frac{4320}{x}. \tag{14.5.iv}$$

(c) *How many hamburgers will you produce in long run equilibrium?*

Answer: The long run equilibrium price must still be equal to \$5 per hamburger because the rest of the firms must be operating at zero long run profit. Your restaurant will, however, produce where price equals your MC curve which is different from those of the other firms. Thus, we set price equal to MC — i.e. $5 = 0.4686x^{0.25}$ and solve for x to get $x \approx 12,960$.

(d) *How many restaurants will there be in long run equilibrium given your higher level of c?*

Answer: Your production of 12,960 hamburgers per week is 3 times the production of the 4,320 hamburgers per week in the other restaurants. Before, there were 22 restaurants — which implies that now there will only be 20 including your restaurant. Thus, your entry into the restaurant market drives two of the other restaurants out of business.

(e) *How many workers (including yourself) and units of capital are you hiring in your business compared to those hired by your competitors? (Recall that the average worker is assumed to work 36 hours per week.)*

Answer: You can either solve the profit maximization problem to derive the labor and capital demand curves and use these to determine how many hours of labor and capital will be used. Alternatively, we could differentiate the cost function with respect to the input prices to get the conditional labor and capital demand functions — then plug in the input prices and output levels to get to the answer. Either way, we get that the other firms are hiring 576 labor hours and 432 capital hours per week, and your firm is hiring 1,728 labor hours and 1,296 hours of capital per week. At a work week of 36 hours, this implies that other firms hire 16 workers and your firm hires 36 workers.

(f) *How does your restaurant's weekly long run profit differ from that of the other restaurants?*

Answer: Other restaurants are selling 4,320 hamburgers at a price of \$5 to make total weekly revenues of \$21,600; and they pay a weekly fixed cost of \$4,320 and hire 576 worker hours at wage \$15 and 432 capital hours at rental rate \$20 for total cost of $4320 + 576(15) + 432(20) = \$21,600$. Thus, profits of other restaurants are zero (as we know has to be in long run equilibrium). Your firm, on the other hand, is selling 12,960 hamburgers at a price of \$5 for a total revenue of \$64,800. Your costs include the \$4,320 weekly fixed cost plus the cost of 1,728 hours of labor hired at a wage of \$15 and 1,296 hours of capital hired at a rental rate of \$20 for a total cost of $4320 + 1728(15) + 1296(20) = \$56,160$. This implies a profit for you of $64,800 - 56,160 = \$8,640$ per week.

(g) *Suppose Macrosoft is interested in hiring you as described in part A(f). How high a weekly salary would MacroSoft have to offer you in order for you to quit the restaurant business and accept the MacroSoft offer?*

Answer: Since you are also one of the workers who works 36 hours per week in your restaurant, your overall compensation is your labor income of $36(15) = \$540$ plus the profit of \$8,640 per week — for a total of \$9,180 per week. This is the least that MacroSoft would have to offer you in weekly compensation in order to attract you away from your restaurant business.

(h) *If you decide to accept the MacroSoft offer and you exit the restaurant business, will total employment in the restaurant business go up or down?*

Answer: With you in the restaurant business, we have 36 workers working in your business and 16 in each of the 19 others — for a total of 340 restaurant workers. With you out of the business, there are 22 restaurants employing 16 workers each — for a total of 352 workers. Thus, if you accept the MacroSoft offer, the number of workers (including owners) in restaurants increases by 12.

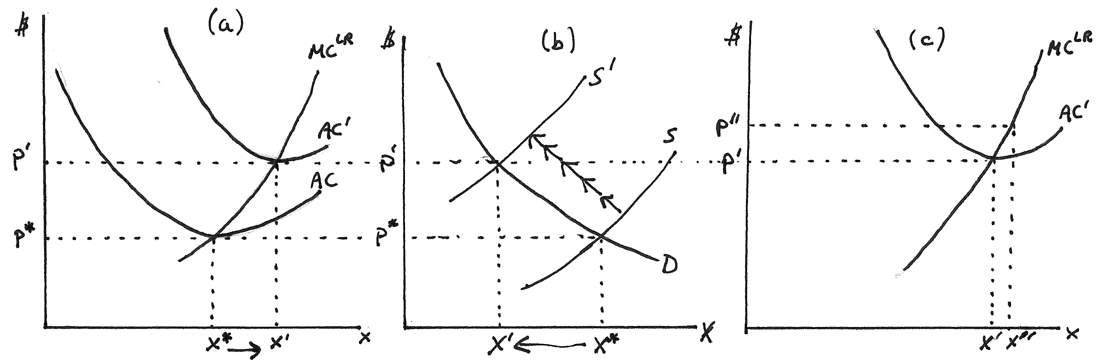
Exercise 14.7

Business and Policy Application: *Using License Fees to Make Positive Profit: Suppose you own one of many identical pharmaceudical companies producing a particular drug x.*

A: *Your production process has decreasing returns to scale but you incur an annually recurring fixed cost F for operating your business.*

(a) *Begin by illustrating your firm's average long run cost curve and identify your output level assuming that the output price is such that you make zero long run profit.*

Answer: This is illustrated in panel (a) of Exercise Graph 14.7 where the initial long run average cost curve — AC — is U-shaped because of the fixed cost F. Zero long run profit implies you are producing at the lowest point of that AC curve — quantity x^* sold at price p^*.



Exercise Graph 14.7 : A Large License Fee

(b) *Next to your graph, illustrate the market demand and short run market supply curves that justify the zero-profit price as an equilibrium price.*

Answer: This is done in panel (b) of the graph where the initial supply S intersects demand D at price p^*.

(c) *Next, suppose that the government introduces an annually recurring license fee G for any firm that produces this drug. Assume that your firm remains in the industry. What changes in your firm and in the market in both the short and long run as a result of the introduction of G and assuming that long run profits will again be zero in the new long run equilibrium?*

Answer: Since this is a fixed cost, it has no short run impact. In the long run, it does not shift the long run MC curve in panel (a) of Exercise Graph 14.7, only the long run AC curve which is illustrated as AC'. This implies that the lowest point of AC' lies to the right of the lowest point of the initial AC. Once the industry settles into a new long run equilibrium where all firms make zero profit, it must then be that the new equilibrium price p' falls at the lowest point of AC' causing each firm to produce x'. The license fee G therefore increases the output level in each firm that remains in the industry. However, overall production in the industry falls (in panel (b)) from X^* to X' as consumers demand less at the higher price p'. The fact that each firm is producing more but the industry is producing less implies that a number of firms must have exited on the way to the new long run equilibrium.

(d) *Now suppose that G is such that the number of firms required to sustain the zero-profit price in the new long run equilibrium is not an integer. In particular, suppose that we would require 6.5 firms to sustain this price as an equilibrium in the market. Given that fractions of firms cannot exist, how many firms will actually exist in the long run?*

Answer: If 7 firms produced, the price would be driven below p' and all firms would make negative long run profit. Thus, it must be that one more firm exits — and only 6 remain in the industry.

(e) *How does this affect the long run equilibrium price, the long run production level in your firm (assuming yours is one of the firms that remains in the market), and the long run profits for your firm?*

Answer: This is illustrated in panel (c) of Exercise Graph 14.7. We begin by replicating the new AC' as well as the (unchanged) long run MC curves from panel (a) — identifying again the zero profit price p' after the new fee G has been introduced. But if only 6 firms exist and it would have taken 6.5 to produce X' (in panel (b)) when each firm produces x', supply shifts further to the left as the 7th firm exits, driving price somewhat above p'. This reduces the quantity demanded and increases the quantity supplied by the remaining firms. In panel (c), the price p'' therefore lies above the zero profit price p' — with each of the remaining firms now producing $x'' > x'$. And, since price is now above the lowest point of the long run AC curve, each firm will make some positive economic profit.

(f) *True or False: Sufficiently large fixed costs may in fact allow identical firms in a competitive industry to make positive long run profits.*

Answer: This is, as we have just demonstrated, true.

(g) True or False*: Sufficiently large license fees can cause a competitive industry to become more* concentrated *— where by "concentrated" we mean fewer firms competing for each customer.*

Answer: This is true. As we have shown, an increase in fixed costs such as license fees will cause an upward and rightward shift of the long run average cost curves of firms — causing each firm to produce a larger quantity at a higher price in the new long run equilibrium. Because of the higher price, the industry produces less. Thus, we'll have fewer firms, with each firm attracting *more customers* — i.e. fewer firms compete for each given customer.

B: *Suppose that each firm in the industry uses the production function* $f(\ell,k) = 10\ell^{0.4}k^{0.4}$ *and each incurs a recurring annual fixed cost of $175,646.*

(a) *Determine how much each firm produces in the long run equilibrium if* $w = r = 20$. *(You can use the cost function derived for Cobb-Douglas technologies given in equation (13.45) in exercise 13.5 (and remember to add the fixed cost).)*

Answer: For a Cobb-Douglas function of $f(\ell,k) = A\ell^{\alpha}k^{\beta}$, the cost function (in the absence of fixed costs) is

$$C(w,r,x) = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)}. \tag{14.7.i}$$

Dividing this by x and adding the average fixed cost FC/x, we then get the long run average cost function

$$AC(w,r,x) = \frac{C(w,r,x)}{x} + \frac{FC}{x} = (\alpha+\beta)\left(\frac{w^{\alpha}r^{\beta}x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{FC}{x}. \tag{14.7.ii}$$

Evaluating this at $\alpha = \beta = 0.4$, $A = 10$, $w = r = 20$ and $FC = 175,646$, this reduces to

$$AC(x) \approx 2.249x^{1/4} + \frac{175,646}{x}. \tag{14.7.iii}$$

The lowest point of this AC curve occurs where its derivative is equal to zero; i.e. where

$$\frac{dAC(x)}{dx} = \frac{0.5623}{x^{3/4}} - \frac{175646}{x^2} = 0. \tag{14.7.iv}$$

Solving this for x, we get $x \approx 24,873$. This is how much each firm is producing in long run equilibrium.

(b) *What price are consumers paying for the drugs produced in this industry?*

Answer: Substituting $x = 24,873$ back into the average cost function $AC(x)$ from equation (14.7.iii), we get that the long run equilibrium price must be approximately \$35.31.

(c) *Suppose consumer demand is given by the aggregate demand function $x(p) = 1,000,000 - 10,000p$. How many firms are in this industry?*

Answer: Plugging in the long run equilibrium price $p = 35.31$, we get consumer demand of $x = 1,000,000 - 10,000(35.31) = 646,900$. With each firm producing 24,873 units, this implies that there are $646,900/24873 = 26$ firms in the industry.

(d) *Suppose the government introduces a requirement that each company has to purchase an annual operating license costing \$824,354. How do your answers to (a), (b) and (c) change in the short and long run?*

Answer: Since this is an added fixed cost that is only a fixed expense in the short run, it does not affect anything in the short run. When added to the fixed cost of \$175,646, this annual fee increases the total fixed costs to \$1,000,000. This changes the long run AC curve to

$$AC(x) = 2.249x^{1/4} + \frac{1,000,000}{x} \tag{14.7.v}$$

and its derivative to

$$\frac{dAC(x)}{dx} = \frac{0.5623}{x^{3/4}} - \frac{1,000,000}{x^2}. \tag{14.7.vi}$$

Setting the derivative to 0 and solving for x, we now get $x = 100,000$, up from the previous 24,873. Plugging this back into $AC(x)$, we get a long run equilibrium price of $p = 50$, up from the previous 35.31. At this price, consumers demand $x = 1,000,000 - 10,000(50) = 500,000$ units. With each firm producing 100,000 units, this leaves room for only 5 firms, down from the previous 26.

(e) *Are any of the firms that remain active in the industry better or worse off in the new long run equilibrium?*

Answer: No — they make zero profit before and again after the change to the new long run equilibrium.

(f) *Suppose instead that the government's annual fee were set at $558,258. Calculate the price at which long run profits are equal to zero.*

Answer: When added to the fixed cost of $175,646, this annual fee increases the total fixed costs to $773,904. This changes the long run AC curve to

$$AC(x) = 2.249x^{1/4} + \frac{773,904}{x} \tag{14.7.vii}$$

and its derivative to

$$\frac{dAC(x)}{dx} = \frac{0.5623}{x^{3/4}} - \frac{773,904}{x^2}. \tag{14.7.viii}$$

Setting the derivative to 0 and solving for x, we now get approximately $x = 78,075$, up from the initial 24,873 but down from the 100,000 under the higher license fee. Plugging this back into $AC(x)$, we get a zero long run profit price of $p = 47$, up from the previous 35.31 but below the 50 under the higher license fee.

(g) *How many firms would this imply will survive in the long run assuming fractions of firms can operate?*

Answer: At a price of $p = 47$, consumers demand $x = 1,000,000 - 10,000(47) = 530,000$ units. With each firm producing 78,075 units, this would imply approximately $530,000/78,075 = 6.79$ firms in the market.

(h) *Since fractions of firms cannot operate, how many firms will actually exist in the long run? Verify that this should imply an equilibrium price of approximately $48.2. (*Hint*: Use the supply function given for a Cobb-Douglas production process in equation (13.49) found in the footnote to exercise 13.7.)*

Answer: Since 7 firms cannot exist in the market, only 6 can survive. But if 6 firm produced 78,075 units at the zero long run profit price $p = 47$, only

468,450 units would be produced — which is 61,550 units less than the 530,000 units demanded by consumers at that price. Thus, in order for the market to clear in the long run, price has to increase. In order to determine by how much, we have to first derive the long run supply curve for each firm. The supply function for a production process $f(\ell,k) = A\ell^{\alpha}k^{\beta}$ is

$$x(w,r,p) = \left(\frac{Ap^{(\alpha+\beta)}\alpha^{\alpha}\beta^{\beta}}{w^{\alpha}r^{\beta}}\right)^{1/(1-\alpha-\beta)} \tag{14.7.ix}$$

which, when evaluated at $\alpha = \beta = 0.4$, $A = 10$, and $w = r = 20$, becomes

$$x(p) = 0.016p^4. \tag{14.7.x}$$

Multiplying this by 6 — which is the number of firms remaining in the industry, we get

$$X^{LR}(p) = 0.096p^4. \tag{14.7.xi}$$

In an equilibrium with 6 firms, the equilibrium price then occurs where this supply function equals the demand function $x(p) = 1,000,000-10,000p$. The equation $0.096p^4 = 1,000,000-10,000p$ holds at $p = 48.197082195$ or approximately $p = 48.2$.

(i) *What does this imply for how much profit each of the remaining firms can actually make?*

Answer: At a price of \$48.2, equation (14.7.x) implies that the firm will produce output of approximately $x = 86,338$ which implies total revenues of $48.2(86,338) \approx 4,161,492$. Using equation (14.7.vii), we can determine its long run average cost at output 86,338 to be

$$AC = 2.249(86,338^{1/4}) + \frac{773,904}{86,338} \approx 47.058. \tag{14.7.xii}$$

This implies total costs of $47.058(86,338) \approx 4,062,894$. Subtracting this from total revenues of 4,161,492, we get long run profit of approximately \$98,598.

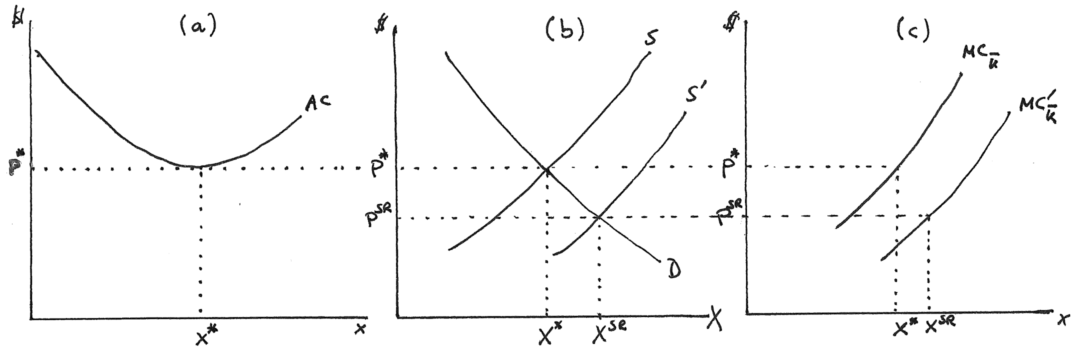
Exercise 14.9

Policy and Business Application: *Minimum Wage Labor Subsidy (cont'd)*: *In exercise 13.10, we investigated the firm's decisions in the presence of a government subsidy for hiring minimum wage workers. Implicitly, we assumed that the policy has no impact on the prices faced by the firm in question.*

A: *Suppose again that you operate a business that uses minimum wage workers ℓ and capital k. The minimum wage is w, the rental rate for capital is r and you are one of many identical businesses in the industry, each using a homothetic, decreasing returns to scale production process and each facing a recurring fixed cost F.*

(a) *Begin by drawing the average cost curve of one firm and relating it to the (short run) supply and demand in the industry assuming we are in long run equilibrium.*

Answer: This is done in panels (a) and (b) of Exercise Graph 14.9 where the long run equilibrium price p^* occurs at the intersection of the original market demand D and supply S curves (in panel (b)) and falls at the lowest point of each firm's average cost curve (in panel (a)).



Exercise Graph 14.9 : Minimum Wage Subsidy

(b) *Now the government introduces a wage subsidy s that lowers the effective cost of hiring minimum wage workers from w to $(1-s)w$. What happens in the firm and in the industry in the short run?*

Answer: This is illustrated in panels (b) and (c) of Exercise Graph 14.9. Each individual firm's short run marginal cost curve (given the fixed level of capital $\bar{k}$) shifts down — from $MC_{\bar{k}}$ to $MC'_{\bar{k}}$ in panel (c). Since this is happening to all firms, this implies that the short run market supply curve (in panel (b)) shifts down — from S to S'. As a result, the new short run equilibrium price falls to p^{SR} (in panel (b)) — and each firm produces more (in panel (c)) at that price. (In principle it is possible to draw these graphs such that the price falls and each firm produces less. However, that is a logical impossibility — because the number of firms is fixed in the short run. If the market overall produces more, it must be that each firm produces more in the short run).

(c) *What happens to price and output (in the firm and the market) in the long run compared to the original quantities?*

Answer: In the long run, the AC curve in panel (a) of Exercise Graph 14.9 shifts down — but we cannot be sure whether the lowest point shifts right or left. (The more likely case is that it shifts to the right). Thus, in the new long run equilibrium, we know the price has to settle below p^* — with each firm producing more or less than originally depending on whether the lowest point of the AC curve falls to the right or left of x^*. Since price is below p^*, we can also be sure that the overall quantity produced in the market will increase.

(d) *Is it possible to tell whether there will be more or fewer firms in the new long run equilibrium?*

Answer: No, it is not. It could be that each firm produces sufficiently more in the new long run equilibrium and the overall quantity demanded increases relatively less such that fewer firms can be sustained in the new equilibrium. But the reverse is also possible.

(e) *Is it possible to tell whether the long run price will be higher or lower than the short run price? How does this relate to your answer to part (d)?*

Answer: No, it is not possible to tell for sure. This relates to (d) in the sense that it relates to whether additional firms will enter or existing firms will exit in the transition from the short run to the long run equilibrium. If conditions are such that the number of firms falls, this implies that the exit of firms from the industry will put upward pressure on price relative to its short run value. If, on the other hand, conditions are such that the number of firms increases, then this implies that the entry of new firms puts additional downward pressure on price — causing the long run price to fall below the short run price.

B: *Suppose that the firms in the industry use the production technology* $x = f(\ell, k) = 100\ell^{0.25}k^{0.25}$ *and pay a recurring fixed cost of* $F = 2,210$. *Suppose further that the minimum wage is \$10 and the rental rate of capital is* $r = 20$.

(a) *What is the initial long run equilibrium price and firm output level?*

Answer: Plugging the production function values and input prices into the cost function for Cobb-Douglas production, we get a cost function of $C(x) = 0.00282843x^2$, and adding the fixed cost F, we get $C(x) = 0.00282843x^2 + 2210$. This gives us the average cost function $AC(x) = 0.00282843x + 2210/x$. Taking the first derivative, setting it to zero and solving for x then gives us the output level at the lowest point of the AC curve — which is $x \approx 884$. Plugging this back into the AC function, we then get that the long run equilibrium price is \$5. Each firm therefore sells 884 output units at a price of \$5 per unit.

(b) *Suppose that* $s = 0.5$ *— implying that the cost of hiring minimum wage labor falls to \$5. How does your answer to (a) change?*

Answer: The new long run equilibrium is then derived exactly as it was in (a) except that $w = 5$ is substituted in the first step when we derive the cost function from the general Cobb-Douglas form of the cost function. This gives us $C(x) = 0.002x^2$ and, once we go through the remaining steps, $x \approx 1,051$ as the output quantity at the lowest point of the AC curve. The long run (zero profit) price is approximately \$4.20.

(c) *How much more or less of each input does the firm buy in the new long run equilibrium compared to the original one? (The input demand functions for a Cobb-Douglas production process were previously derived and given in equation (13.50) of exercise 13.8.)*

Answer: Substituting $A = 100$, $\alpha = 0.25 = \beta$ and $r = 20$ into the labor and capital demand equations, we get

$$\ell(w,p) \approx 139.75\left(\frac{p^2}{w^{3/2}}\right) \text{ and } k(w,p) \approx 6.9877\left(\frac{p^2}{w^{1/2}}\right). \tag{14.9.i}$$

In the initial equilibrium, $(w,p) = (10,5)$ while in the new equilibrium, $(w,p) = (5,4.2)$. Substituting these into the equations, we then get that an initial input bundle $(\ell,k) = (110.5,55.25)$ and a new input bundle $(\ell,k) = (221,55.25)$. (Answers may differ slightly due to rounding errors.) Labor input therefore doubled but capital input remained unchanged.

(d) *If price does not affect the quantity of x demanded very much, will the number of firms increase or decrease in the long run?*

Answer: If the quantity demanded (and thus the quantity produced by the industry) remains roughly the same, the number of firms in the industry must decline since each firm is now producing 1,051 units of output rather than the initial 884.

(e) *Suppose that demand is given by $x(d) = 200,048 - 2,000p$. How many firms are there in the initial long run equilibrium?*

Answer: In the initial long run equilibrium, $p = 5$. This implies that the total quantity demanded is $200,048 - 2,000(5) = 190,048$. Each firm produces 884 units initially, which implies that we have 215 firms operating. (Your answer may be slightly below 215 because of rounding error when we use 884 units per firm rather than 883.84 which is the more exact number returned by the math.)

(f) *Derive the short run market supply function and illustrate that it results in the initial long run equilibrium price.*

Answer: To derive the short run market supply function, we need to first determine the short run supply function of each of the 215 firms in the initial equilibrium. We concluded in (c) that each firm is using 55.25 units of capital in that initial equilibrium. In the short run, when capital is fixed, each firm is therefore operating on the slice

$$f_{k=55.25}(\ell) = \left[100(55.25)^{0.25}\right]\ell^{0.25} = 272.636\ell^{0.25}. \tag{14.9.ii}$$

Solving the short run profit maximization problem

$$\max_{\ell}\ p\left(272.636\ell^{0.25}\right) - w\ell, \tag{14.9.iii}$$

we get

$$\ell_{k=55.25}(w,p) = 278.42\left(\frac{p}{w}\right)^{3/4} \text{ and } x_{k=55.25}(w,p) = 1113.67\left(\frac{p}{w}\right)^{1/3}, \tag{14.9.iv}$$

the latter of which is the firm's short run supply function. Multiplying this by the number of firms in the industry (which we derived as 215), we get a short run market supply function

$$X^{SR}(w,p) = 239,440\left(\frac{p}{w}\right)^{1/3} \qquad (14.9.v)$$

which becomes

$$X^{SR}(p) = 111,138p^{1/3} \text{ when } w = 10. \qquad (14.9.vi)$$

At the original equilibrium, it must be that demand is equal to this short run market supply — i.e.

$$200,048 - 2,000p = 111,138p^{1/3}, \qquad (14.9.vii)$$

which holds (approximately, due to some rounding) for our initial long run equilibrium price $p = 5$.

(g) *Verify that the short run equilibrium price falls to approximately \$2.69 when the wage is subsidized.*

Answer: When wage falls to $w = 5$, the short run market supply curve in equation (14.9.v) becomes

$$X^{SR}(p) = 140,025p^{1/3}. \qquad (14.9.viii)$$

The short run equilibrium then occurs where supply equals demand; i.e. where

$$140,025p^{1/3} = 200,048 - 2,000p \qquad (14.9.ix)$$

which (approximately) holds when $p = 2.69$.

(h) *How much does each firm's output change in the short run?*

Answer: Plugging the price $p = 2.69$ and subsidized wage $w = 5$ into the short run supply function for each firm, we get $x_{k=55.25}(5,2.69) \approx 906$. Note that this sums approximately to the overall quantity transacted in the market when there are (the initial) 215 firms in the market.

(i) *Determine the change in the long run equilibrium number of firms when the wage is subsidized and make sense of this in light of the short run equilibrium results.*

Answer: We previously determined that the long run equilibrium price will be roughly \$4.20 — which is lower than the initial price of \$5 but higher than the short run price of \$2.69. At \$4.20, we can determine the total level of output in the industry by substituting this price into the demand function to get $x = 191,648$ — and with each firm producing 1,051 in the new long run equilibrium, this implies approximately 182 firms — down from the initial 215. We can see the dynamics of what makes firms choose to exit by observing that the short run equilibrium price of $2.69 lies below the long run zero-profit price of $4.20 — thus firms are making negative long run profits in the short run equilibrium (while still making positive short run profits since the expense on capital and the fixed costs are not real costs in the short run).

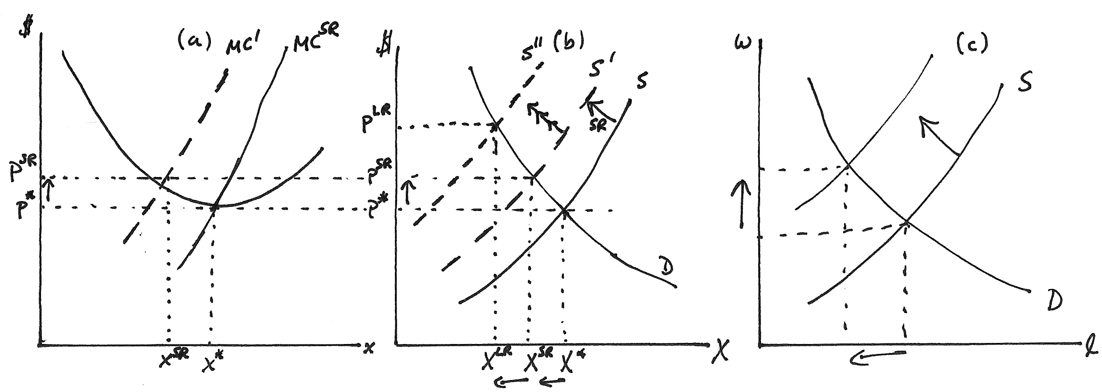
Exercise 14.11

Policy Application: *Public School Teacher Salaries, Class Size and Private School Markets: In exercise 14.10, we noted that private schools that charge tuition operate alongside public schools in U.S. cities. There is much discussion in policy circles regarding the appropriate level of public school teacher salaries (which are set by the local or state government) as well as the appropriate number of public school teachers (that determines class size in public schools).*

A: *Suppose again that private schools face U-shaped long-run AC curves for providing seats to children and that the private school market is currently in long run equilibrium.*

(a) *Begin by drawing two graphs — one with the long run AC curve for a representative private school and a second with the demand and (short run) aggregate supply curves (for private school seats) that are consistent with the private school market being in long run equilibrium (with private school tuition p on the vertical axis).*

Answer: This is done in panels (a) and (b) of Exercise Graph 14.11. In order for the private school market to be in long run equilibrium, each school makes zero profit and thus operates on the lowest point of its long run *AC* curve. Thus, the equilibrium price is p^* as reflected by the intersection of demand *D* and supply *S* in panel (b).



Exercise Graph 14.11 : Public School Teacher Salaries and Private School Markets

(b) *Now suppose the government initiates a major investment in public education by raising public school teacher salaries. In the market for private* *school teachers (with private school teacher salaries on the vertical and private school teachers on the horizontal), what would you expect to happen as a result of this public school investment?*

Answer: This is illustrated in panel (c) of Exercise Graph 14.11. I would expect the supply of private school teachers (of a given quality) to decrease as they are more attracted to the public schools. Thus, the equilibrium teacher wage in the private school market should increase.

(c) *How will this impact private school tuition levels, the number of seats in private schools and the overall number of children attending private schools in the short run?*

Answer: This increase in teacher salaries causes the short run MC curve of each private school to immediately shift to the left — indicated by MC' in panel (a) of Exercise Graph 14.11. Since this happens for all existing private schools, the market supply curve in panel (b) shifts immediately from S to S'. As a result, tuition price increases to p^{SR}, with the number of children attending private schools decreasing from the initial X^* to X^{SR} in panel (b). Each individual school will also admit fewer children (as shown in pane (a)). We know this because the same number of schools is admitting fewer students — and all schools are assumed to be roughly identical.

(d) *How does your answer change in the long run as private schools can enter and exit the industry?*

Answer: In principle, it could be that private schools either enter or exit as a result of these changes. Exercise Graph 14.11 illustrates the (more likely) case of schools exiting. If some private schools exit the market (because the short run increase in price is insufficient to cover long run costs), the supply curve in panel (b) of Exercise Graph 14.11 will shift further to S'', causing tuition price to increase further to p^{LR}. Thus, the market will serve fewer children in the long run — down from the initial drop to X^{SR} to X^{LR} in panel (b) of the graph. How many students are admitted by individual schools relative to the short run change is ambiguous — it depends on where the lowest point of the new long run AC falls, but it would typically be at a size smaller than the initial x^* (though it is in principle possible for it to be larger). If the lowest point of long run AC shifts sufficiently far to the left, private schools become very small — and if the demand curve is not too flat, this implies that the number of schools would actually *increase* in the long run. (This seems less likely and is not pictured in the graph.)

(e) *Suppose that instead of this teacher salary initiative, the city government decides to channel its public school investment initiative into hiring more public school teachers (as the city government is simply recruiting additional teachers from other states) and thus reducing class size. Assuming that this has no impact on the equilibrium salaries for teachers but does cause parents to feel more positively about public schools, how will the private school market be impacted in the short and long run?*

Answer: This would cause a decrease in demand for private schools — i.e. a leftward shift. The result would be the mirror image of what we concluded for vouchers in exercise 14.10: The lower demand would cause an initial drop in tuition levels in the short run, with each private school serving fewer children. In the long run, some private schools would exit, shifting the market supply curve to the left and raising tuition prices back to their original zero-profit level. In the long run, each remaining private school would therefore admit as many children as it did initially and would charge the same tuition it initially charged, but the private school market as a whole would serve fewer children.

(f) *How will your long run answer to (e) be affected if the government push for more public school teachers also causes equilibrium teacher salaries to increase?*

Answer: The shift in demand we just analyzed would shrink the private school sector but not change what the remaining private schools do (in terms of how many children they serve and what tuition they charge). If, however, there is an additional upward pressure on private school teacher salaries, then the smaller private school sector would change along the same lines as illustrated in Exercise Graph 14.11 — it would, in the long run, experience a further decrease in size, tuition levels would increase and existing private schools might be somewhat smaller or larger depending on how the increase in teacher salaries affects the lowest point of the *AC* curve.

B: *As in exercise 14.10, assume a total city-wide demand function* $x(p) = 24,710 - 2500p$ *for private school seats and let each private school's average long run cost function be given by* $AC(x) = 0.655x^{1/3} + (900/x)$. *Again, interpret all dollar values in thousands of dollars.*

(a) *If you have not already done so, calculate the initial long run equilibrium size of each school, what tuition price they charge and how many private schools there are in the market.*

Answer: As demonstrated in exercise 14.10 (b) and (d), each school has 515 students and charges $7,000 in tuition. There are 14 private schools in the city.

(b) *If you did B(a) in exercise 14.10 you have already shown that this* $AC(x)$ *curve arises from the Cobb-Douglas production function* $x = f(\ell, k) = 35\ell^{0.5}k^{0.25}$ *when* $w = 50$ *and* $r = 25$ *and when private schools face a fixed cost of 900. If you have not already done so, use this information to determine how many teachers and how much capital each school hires.*

Answer: With the labor and capital demand functions from (c) and (f) in exercise 14.10, we calculated that the initial long run profit maximizing production plan includes approximately 36 teachers per school as well as 36 units of capital per school.

(c) *Suppose that the increased pay for public school teachers drives up the equilibrium wage for private school teachers from 50 to 60 (i.e. from $50,000* *to \$60,000 per year). What happens to the equilibrium tuition price in the short run?*

Answer: The short run production function for fixed capital $\overline{k} = 36$ is

$$x = f_{\overline{k}}(\ell) = \left[35\left(36^{0.25}\right)\right]\ell^{0.5} \approx 85.73\ell^{0.5}. \qquad (14.11.i)$$

The short run profit maximization problem is then

$$\max_{\ell}\ p\left(85.73\ell^{0.5}\right) - w\ell. \qquad (14.11.ii)$$

Solving this, we get the short run labor demand function, and substituting it back into equation (14.11.i), we get the short run supply function:

$$\ell_{\overline{k}}(w,p) = 1{,}837.4\left(\frac{p}{w}\right)^2 \text{ and } x_{\overline{k}}(w,p) = 3674.8\left(\frac{p}{w}\right). \qquad (14.11.iii)$$

Setting w equal to the new level of 60 in the supply function $x_{\overline{k}}(w,p)$, we get each school's short run supply curve $x_{\overline{k}}(p) \approx 61.25p$, and multiplying it by 14 (i.e. the number of schools in the market), we get the market short run supply curve

$$X^{SR}(p) = 857.5p. \qquad (14.11.iv)$$

Setting this equal to the market demand curve $x(p) = 24710 - 2500p$ and solving for p, we get $p = 7.36$ for a tuition level of \$7,360, up from the initial \$7,000.

(d) *What happens to school size and class size?*

Answer: Plugging $w = 60$ and $p = 7.36$ into the expressions in equation (14.11.iii), we get

$$\ell_{\overline{k}} = 1{,}837.4\left(\frac{7.36}{60}\right)^2 \approx 27.65 \text{ and } x_{\overline{k}} = 3674.8\left(\frac{7.36}{60}\right) \approx 450.8. \qquad (14.11.v)$$

School size (i.e. the number of children in each school) therefore shrinks from the initial 515 to about 451, and the number of teachers per school shrinks from the initial 36 to around 27.65. This implies that class size increases from 14.3 to 16.3.

(e) *How will your answers on school size, tuition level and class size change in the long run? (*Hint*: You can use the cost function given in equation (13.45) of exercise 13.5 to derive the AC function — just make sure you keep track of the fixed cost of 900!)*

Answer: In the long run, schools again have to end up on the lowest point of their average cost curves. However, since w has increased, their average cost curves have shifted up. Dividing the cost function from exercise 13.5 by x and adding the average fixed cost $900/x$, we get

$$AC(w,r,x) = \frac{C(w,r,x)}{x} + \frac{FC}{x} = (\alpha+\beta)\left(\frac{w^{\alpha} r^{\beta} x^{(1-\alpha-\beta)}}{A\alpha^{\alpha}\beta^{\beta}}\right)^{1/(\alpha+\beta)} + \frac{FC}{x}. \tag{14.11.vi}$$

Substituting $\alpha = 0.5$, $\beta = 0.25$, $A = 35$, $w = 60$, $r = 25$ and $FC = 900$, we get

$$AC(x) = 0.74x^{1/3} + \frac{900}{x}. \tag{14.11.vii}$$

The lowest point on this U-shaped average cost curve arises when the derivative of AC is zero; i.e. when

$$\frac{dAC(x)}{dx} = \frac{0.247}{x^{2/3}} - \frac{900}{x^2} = 0. \tag{14.11.viii}$$

Solving this for x, we get $x \approx 469$. Thus, the school size, which started at 515 students and fell to 451 students in the short run, goes to 469 students in the long run. Plugging 469 back into the $AC(x)$ function in equation (14.11.vii), we can then determine that the bottom of the U-shaped AC curve occurs at an average cost of approximately 7.668 — which has to be the tuition price in the new long run equilibrium. Thus, tuition, which started at \$7,000 per student and went to \$7,360 in the short run, rises to to \$7,668 in the long run. Evaluating the long run labor demand equation at $p = 7.668$ and $w = 60$, we can determine that the number of teachers — which began at 36 and fell to 27.65 per school in the short run — goes to approximately 30 teachers per school. This causes class size — which began at 14.3 and rose to 16.3 in the short run — to go to 15.65.

(f) *How many private schools will remain in the market in the long run?*

Answer: We first have to determine the total quantity of school seats demanded — which we can do by evaluating the demand function at the new long run equilibrium price $p = 7.668$. This gives us

$$x(7.668) = 24710 - 2500(7.668) = 5,540. \tag{14.11.ix}$$

With each school serving about 469 children, this implies 11.8 or approximately 12 schools will remain. (Actually, given that 11.8 falls between 11 and 12, 11 schools would remain, with each producing at a slightly higher tuition price serving somewhat more students and making a small profit.)

Conclusion: Potentially Helpful Reminders

1. Students often find it irritating that we have defined the “short run” differently for firms than for industries — with firms operating in the “short run” as long as their capital level is fixed, and industries operating in the “short

run" so long as entry and exit of firms is not possible. You might be less irritated if you recognize that the two definitions actually boil down to the same definition: A firm needs to be able to invest in capital in order to enter (or to get rid of its capital in order to exit) the industry. Thus, a firm can enter or exit only if it can vary capital — which links our two ways of thinking about the "short run".

2. Of all the firm cost and expenditure curves we derived in the previous three chapters, only two are crucial in this chapter: (1) The long run *AC* curve (of the marginal firm) — or, to be even more specific, the lowest point of that long run *AC* curve. This determines long run price. (2) The short run *MC* curve for the fixed level of capital each firm has — which gives rise to the short run supply curve.

3. We often start out our analysis by assuming that an industry is initially in long run equilibrium. That implies that we start with a picture in which the marginal firm's short run supply curve crosses its long run *AC* curve at the lowest point of the *AC* curve. It extends below the *AC* curve because the short run shut-down price lies below the long run exit price. *Avoid drawing unnecessary curves that don't matter for your analysis.*

4. When you analyze a change that occurs within an industry that is initially in long run equilibrium, you therefore start with the initial long run picture of the marginal firm (with its long run *AC* curve and the short run supply curve) — and then ask whether any of the changes altered either of these curves. Then you can trace out what happens in the short run and the long run.

5. You can conclude what happens to the overall number of firms in the industry if you know what happens to each firm's output when we go from the initial to the new long run equilibrium and you know whether the overall quantity demanded at the new long run equilibrium price is higher or lower than it was originally. (The answer to the question of whether the number of firms increases or decreased will often be ambiguous if the change affecting the industry is a change in input prices). In the short run, the number of firms is fixed.