CHAPTER

15

The “Invisible Hand” and the First Welfare Theorem

Chapter 14 introduced the *positive* idea of equilibrium in the context of a competitive environment — and Chapter 15 now moves onto the more *normative* assessment of a competitive equilibrium within the context of the first welfare theorem. Put differently, Chapter 14 focuses on *predicting* changes in economic environments in competitive settings while Chapter 15 now focuses on *welfare* as defined by consumer surplus and profit (or producer surplus). In Chapter 14 the consumer side of the market did not play a prominent role — we simply said that the market demand curve arises from the sum of individual demands. This is all we need for prediction. In the Chapter 15, on the other hand, we return to some themes from consumer theory — particularly the insight that welfare is measured on marginal willingness to pay (or compensated demand) curves and that these are the same as regular (or uncompensated) demand curves (that we use for prediction) only in the case of quasilinear tastes.

Chapter Highlights

The main points of the chapter are:

1. It is generally not possible to interpret curves that emerge from aggregating individual consumer demand (or labor supply) curves as if they emerged from an individual's optimization problem. Interpreting aggregate economic relationships that emerge from utility maximization in such a way is possible only if redistributing resources within the aggregated group leads to individually offsetting changes in behavior — i.e. **offsetting income effects**.

2. **It is possible to treat aggregate (or market) demand curves as if they emerged from an individual optimization problem** if there are no income effects — i.e. **if the good of interest is quasilinear**. In that special case, (uncompensated) demand curves are also equal to marginal willingness to pay (or com-

pensated demand) curves, enabling us to **measure consumer surplus on the market demand curve**.

3. Since economic relationships emerging from profit maximization by firms do not involve income effects, there are **no analogous issues with interpreting aggregate or market supply curves** (or labor demand curves) as if they emerged from a single optimization problem. As a result, we can measure **producer surplus (or profit) on the market supply curve** without making any particular assumptions.

4. Under a certain set of conditions, market equilibrium leads to output levels that mirror what would be chosen by omniscient social planners that aim to maximize overall social surplus. This is known as the **first welfare theorem** of economics which **specifies the conditions under which markets allocate resources efficiently**.

5. The advantage of market allocations of resources is that they rely on the **self-interested behavior by individuals who know only their own circumstances and observe the market price** signal that coordinates actions of producers and consumers. The disadvantages of market allocation of resources arise first in real world **violations of the assumptions underlying the first welfare theorem** that lead to violations of efficiency and second on normative judgments about **equity versus efficiency** that may lead us to conclude that some market outcomes, while being efficient, are in some sense “unfair” or “unjust”.

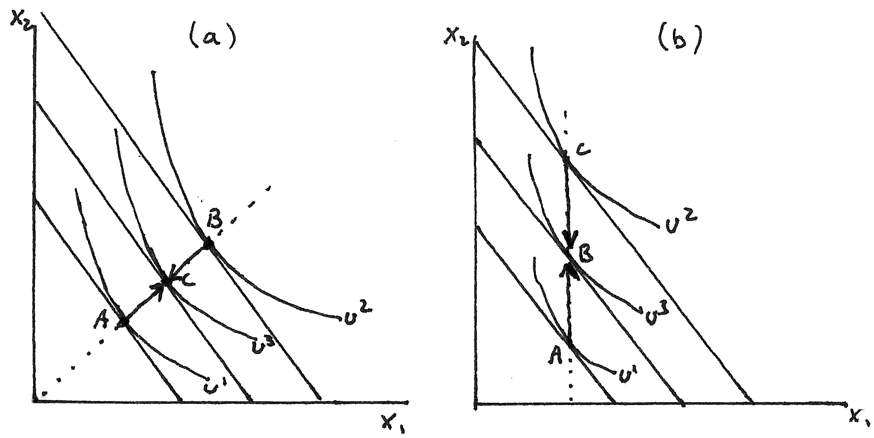
15A Solutions to Within-Chapter-Exercises for Part A

Exercise 15A.1

Suppose that my tastes and my wife’s tastes are exactly identical. If our tastes are also homothetic, does our household behave like a single representative agent? What if our tastes are quasilinear and neither individual is at a corner solution?

Answer: The answer is that, in both cases, our household will behave like an individual agent. This is illustrated in Exercise Graph 15A.1. In panel (a), we assume that our tastes are homothetic and identical. This implies that both my wife and I will optimize along a ray from the origin, with the precise ray depending on the output prices (and thus the slope of the budget constraints). Suppose that I initially have the lowest of these budget constraints and my wife initially has the highest. Then I will optimize at A and she will optimize at B. If you then redistribute income so we both face the same budget constraint, we will both face the middle one — and we will both optimize at C. Thus, my wife’s optimal bundle will move inward along the ray and mine will move outward along the ray — exactly offsetting each other.

Our overall bundle will thus remain the same as you redistribute income. The same is true in panel (b) where our tastes are quasilinear and neither of us is at a corner solution.



Exercise Graph 15A.1 : Individual Agents when Tastes are Identical

Exercise 15A.2

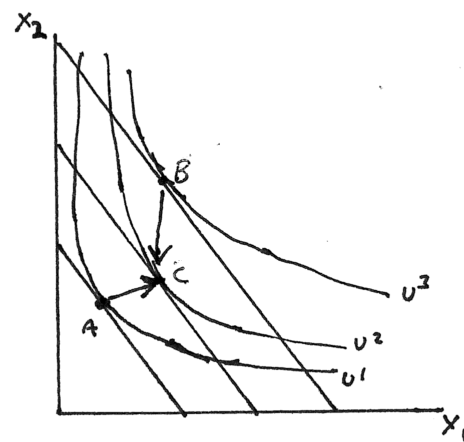
Can you illustrate a case where our tastes are identical but we do not behave as a representative agent?

Answer: One such case is illustrated in Exercise Graph 15A.2. Let's assume that the three indifference curves are drawn from the same map of indifference curves — i.e. the same tastes. Initially I have the low income and my wife has the high income — which means I choose A and she chooses B. Then you redistribute income so we both face the middle income — and we both choose C. The change in my wife's consumption bundle is then clearly not offset by the change in mine — because the arrows are not parallel to one another.

Exercise 15A.3

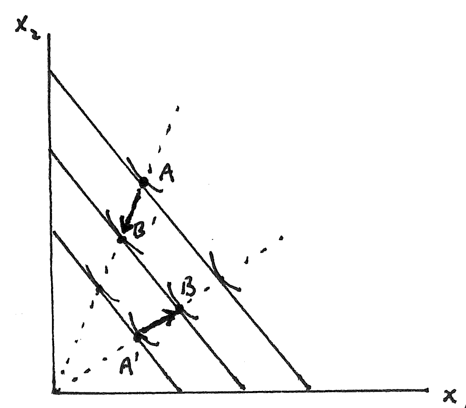
Suppose both my wife and I have homothetic tastes but they are not identical. Does this still imply that we behave like a single representative agent?

Answer: No, we will not behave as a representative agent (unless the tastes are over perfect substitutes). This is illustrated in Exercise Graph 15A.3. Suppose my wife has tastes that cause her to optimize on the steeper ray from the origin and I have tastes that cause me to optimize on the shallower one. If initially I have the low income and she has the high income, she will choose A and I will choose A'. After you redistribute income and we both face the middle budget constraint, she will



Exercise Graph 15A.2 : Individual Agents when Tastes are Identical: Part 2

choose B' and I will choose B. Because we are moving along rays that have different slopes, the changes in our consumption bundles will not offset one another.



Exercise Graph 15A.3 : Homothetic Tastes

Exercise 15A.4

True or False: As long as everyone has quasilinear tastes, the group will behave like a representative agent even if all the individuals do not share the same tastes (assuming no one is at a corner solution). The same is also true if everyone has homothetic tastes.

Answer: The first part of the statement is true but the second part is false. For quasilinear tastes, the graph in the text in fact has me and my wife having *different* tastes that are quasilinear. So we have demonstrated in the text that the first part is true — as long as tastes are quasilinear, the group will act as an individual agent. In exercise 15A.3, however, we have already demonstrated that the group will not act as an individual if tastes are homothetic but different.

Exercise 15A.5

Suppose that my wife and I share identical homothetic tastes (that are not over perfect substitutes). Will our household demand curve be identical to our marginal willingness to pay curve?

Answer: No. When tastes are homothetic, they give rise to income effects — which implies that individual demand curves and marginal willingness to pay curves will differ, and this difference continues to hold when we consider the aggregate demand curve. Thus, even though we behave as a representative agent, our demand and marginal willingness to pay curves will not be the same.

Exercise 15A.6

Does this measure of long run profit apply also when the firm encounters long run fixed costs?

Answer: Yes. This is because the fixed cost is included in the long run *AC* and is therefore counted, and the presence of fixed costs does not change the *additional* cost incurred by producing more than the quantity at the lowest point of the *AC* curve.

Exercise 15A.7

How would the picture be different if we were depicting an industry in long run equilibrium with all firms facing the same costs? What would long run producer surplus be in that case?

Answer: The long run supply curve would then be flat — which would eliminate the producer surplus area entirely from the graph. This should make sense: In a competitive industry where all firms face the same costs, entry and exit drive long run profit to zero. Thus, while each firm will earn short run profits (because certain long run costs are not costs in the short run), the industry will earn zero profit in the long run. The entire surplus in the market would then be earned by consumers.

Exercise 15A.8

Suppose we were not concerned about identifying producer and worker surplus but instead wanted to only predict the equilibrium wage and the number of workers employed. Would we then also have to assume that leisure is quasilinear for workers?

Answer: No — in order to predict the market equilibrium, we simply need to know the aggregate demand and supply curves in the market. We can aggregate these even if consumers (or workers) do not behave as one single representative agent. Put differently, we need the regular consumer demand or worker supply curves to predict the equilibrium, not the compensated consumer demand and worker supply curves.

Exercise 15A.9

Imagine that you are Barney and that you would like consumers to get a bigger share of the total “pie” than they would get in a decentralized market. How might you accomplish this? (*Hint*: Given your omnipotence, you are not restricted to charging the same price to everyone.)

Answer: All you would have to do is charge a lower price to some of the consumers. You could still give enough to producers so that their surplus is positive — but you could then redistribute some of the surplus from producers to consumers. In the extreme, you would simply cover the costs of producers and hand all the goods to the consumers who value them most, charging them only a price sufficient to raise enough money for you to pay off the producers.

Exercise 15A.10

Suppose the social marginal cost curve is perfectly flat — as it would be in the case of identical producers in the long run. Would you, as Barney, be able to give producers a share of the surplus?

Answer: Sure. All you would have to do is charge the consumers who really value the goods a lot more than the long run equilibrium price that would emerge in the market. That long run price is sufficient to cover all the long run costs for producers — but lots of consumers are willing to pay more. Thus, if you raise this additional revenue from consumers, you can redistribute some of the consumer surplus to producers who would, in the competitive long run market, make zero surplus.

Exercise 15A.11

How would Graph 15.8 look if good x were an inferior good for all consumers?

Answer: In this case the aggregate $MWTP$ curve would be shallower than the market demand curve, causing the actual consumer surplus to be smaller than what we would infer from just looking at the market demand curve.

Exercise 15A.12

True or False: If goods are normal, we will underestimate the consumer surplus if we measure it along the market demand curve, and if goods are inferior we will overestimate it.

Answer: This is true. You can see it for normal goods in the graph in the text — where the $MWTP$ curves are steeper than demand curves. Similarly, $MWTP$ curves are shallower than demand curves in the case of inferior goods — which implies the actual consumer surplus is smaller than what we would measure along the market demand curve.

15B Solutions to Within-Chapter-Exercises for Part B

Exercise 15B.1

Demonstrate that the conditions in equation (15.1) are satisfied for the demand functions in (15.2).

Answer: Taking the first derivatives with respect to I^m, we get

$$\frac{\partial x_i^m}{\partial I^m} = b_i(p_1, p_2) = \frac{\partial x_i^n}{\partial I^n}. \tag{15B.1.i}$$

Then, taking second derivatives, we get

$$\frac{\partial^2 x_i^m}{\partial (I^m)^2} = 0 = \frac{\partial^2 x_i^n}{\partial (I^n)^2}. \tag{15B.1.ii}$$

Exercise 15B.2

Can you see why equation (15.2) represents the most general way of writing demands that satisfy the conditions in equation (15.1)?

Answer: First, the only way the second derivatives can be zero is if income enters linearly and thus drops out when the first derivative is taken. Thus, we know that income can only enter as I multiplied by something that is not also a function of income — i.e. if income enters in the form $Ib(p_1, p_2)$ where the function b is at most a function of the prices (and not income). Second, the only way the first derivatives with respect to income can be the same across individuals is if the term following income is the same for both individuals — because that is the term that remains when we take the first derivative. Thus, the function b cannot be a individual specific — i.e. it cannot have an n or m superscript, but it can vary for goods — i.e. it can have an i subscript. Finally, other terms can enter the demand equations so long as they are not dependent on income — and thus do not affect the first derivative. Thus, we can have an a function that is not dependent on income but depends on prices — and that can vary across goods and individuals (since it drops out when we take the derivative with respect to income).

Exercise 15B.3

What are my household demand functions (for x_1 and x_2) if my wife's and my individual demands are those in equation (15.3)? Do the household demand functions also satisfy the Gorman Form?

Answer: Our household demand functions would simply be the sum of our individual demand functions. Since neither of the individual demand functions for x_1 is a function of income, our household demand function for x_1 will not be a function of income. Thus, our household demands arise from household preferences that are quasilinear in x_1, with household demand functions satisfying the Gorman form.

Exercise 15B.4

Given that the firms encounter a recurring fixed cost of $1,280, which of the above functions should actually be qualified to take account of this fixed cost?

Answer: The short run functions are not impacted, but the long run functions are. For instance, if $w = 20$ and $r = 10$, the lowest point of the AC function gives us a long run exit price of $p = 5$ — a price below which long run production falls to zero.

Exercise 15B.5

Draw the production possibility frontier described above. How would it look differently if the long run market supply curve slopes up? (*Hint*: With an upward-sloping supply curve, society is facing an increasing cost of producing x, implying that the trade-off in the society-wide production possibility frontier must reflect that increasing cost.)

Answer: In panel (a) of Exercise Graph 15B.5, the production possibility frontier given by $I = 5x + y$ is given — with the frontier having slope −5 throughout because the (social) opportunity cost of increasing x by one unit is always that 5 units of y must be sacrificed. When the cost of producing x increases with the level of x (as it does when the supply curve is increasing), then we would get a production possibility frontier with the shape illustrated in panel (b) — where the slope starts shallow (indicating a low opportunity cost for producing x) but increases (in absolute value) as x increases (indicating the increasing opportunity cost.)

Exercise 15B.6

Verify that this is indeed the case.

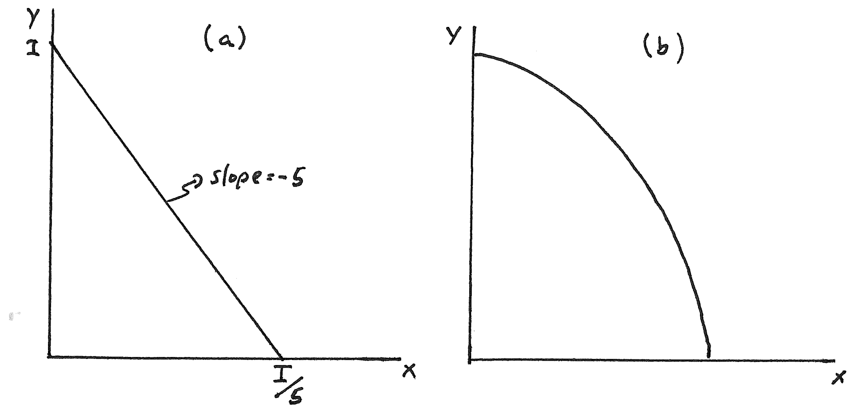
Answer: The Lagrange function is

$$\mathscr{L} = 12,649.11x^{1/2} + y + \lambda(I - 5x - y) \quad (15B.6.i)$$

which gives rise to the first order conditions

$$\frac{12,649.11}{2x^{1/2}} - 5\lambda = 0 \text{ and } 1 - \lambda = 0. \quad (15B.6.ii)$$

Plugging the latter into the former and solving for x, we get $x = 1,600,000$.



Exercise Graph 15B.5 : Production Possibility Frontiers

Exercise 15B.7

One way to verify that the representative consumer's utility function is truly "representative" is to calculate the implied demand curve and see whether it is equal to the aggregate demand curve $D^M(p) = 40,000,000/p^2$ that we are trying to represent. Illustrate that this is the case for the utility function $U(x,y) = 12,649.11x^{1/2} + y$.

Answer: To derive the implied demand curve for the representative consumer, we solve the problem

$$\max_{x,y} 12,649.11x^{1/2} + y \text{ subject to } I = px + y. \quad (15B.7.i)$$

Setting up the lagrange function

$$\mathcal{L} = 12,649.11x^{1/2} + y + \lambda(I - px - y), \quad (15B.7.ii)$$

we can derive the first order conditions

$$\frac{12,649.11}{2x^{1/2}} - \lambda p = 0 \text{ and } 1 - \lambda = 0. \quad (15B.7.iii)$$

Substituting the latter into the former and solving for x, we get

$$x(p) = \frac{40,000,000}{p^2}, \quad (15B.7.iv)$$

precisely the aggregate demand function we are trying to represent with the representative consumer.

15C Solutions to Odd Numbered End-of-Chapter Exercises

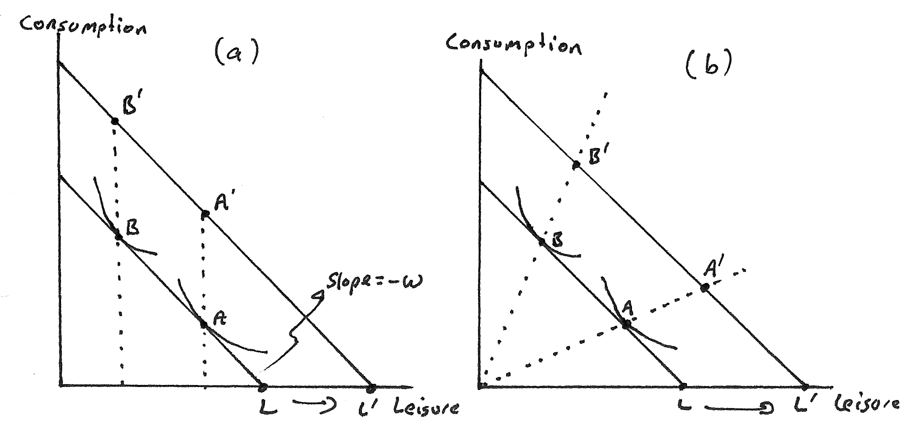
Exercise 15.1

Everyday Application: *Labor Saving Technologies*: *Consider inventions such as washing machines or self-propelled vacuum cleaners. Such inventions reduce the amount of time individuals have to spend on basic household chores — and thus in essence increase their leisure endowments.*

A: *Suppose that we wanted to determine the aggregate impact such labor saving technologies will have on a particular labor market in which the wage is w.*

(a) *Draw a graph with leisure on the horizontal axis and consumption on the vertical and assume an initially low level of leisure endowment for worker A. For the prevailing wage w, indicate this worker's budget constraint and his optimal choice.*

Answer: This is illustrated in panel (a) of Exercise Graph 15.1 as the lower of the two parallel budgets where worker A optimizes at bundle A.



Exercise Graph 15.1 : Labor Saving Household Technologies

(b) *On the same graph, illustrate the optimal choice for a second worker B who has the same leisure endowment and the same wage w but chooses to work more.*

Answer: This is also illustrated in panel (a) where worker B optimizes at bundle B — consuming less leisure and thus working more.

(c) *Now suppose that a household-labor saving technology (such as an automatic vacuum cleaner) is invented and both workers experience the same increase in their leisure endowment. If leisure is quasilinear for both workers, will there be any impact on the labor market?*

Answer: The increase in leisure endowment is indicated as an increase from L to L' in panel (a) of the graph. Since wage remains the same, this results in a parallel shift out of the budget constraints. If tastes are quasi-linear in leisure, then worker A will optimize at A' and worker B will optimize at B'. Since their leisure consumption remains unchanged, this implies that workers will increase their labor supply by exactly the increase in leisure $(L' - L)$.

(d) *Suppose instead that tastes for both workers are homothetic. Can you tell whether one of the workers will increase his labor supply by more than the other?*

Answer: This is illustrated in panel (b) of Exercise Graph 15.1 where worker A will choose bundle A' and worker B will choose bundle B'. Worker A will therefore increase his leisure consumption by more than worker B — with neither worker committing the entire increase in leisure $(L' - L)$ to increased work hours. However, because worker B increases his leisure consumption by less than worker A, we know that worker B will increase his labor supply by more than worker A.

(e) *How does your answer suggest that workers in an economy cannot generally be modeled as a single "representative worker" even if they all face the same wage?*

Answer: In order for us to be able to use a "representative worker", it would have to be the case that, when leisure endowments are redistributed between workers, the overall amount of labor supplied remains unchanged. We can see in panel (a) of Exercise Graph 15.1 that, when leisure is quasi-linear, leisure demand remains unchanged as leisure endowments are changed. Thus, were we to redistribute leisure endowments between individuals, the one who gets more leisure endowment would supply all of it as labor while the one who loses it would reduce his labor hours by the same amount. Thus, the actions of the two workers would exactly offset each other. The same is not, however, true in panel (b) where tastes are homothetic. Thus, a redistribution of leisure among workers would cause an increase in labor hours for the worker who receives more leisure endowment and reduce the labor hours of the worker who receives less — but the two would not offset each other unless the tastes were also identical.

B: *Consider the problem of aggregating agents in an economy where we assume individuals have an exogenous income.*

(a) *In a footnote in this chapter, we stated that, when the indirect utility for individual m can be written as* $V^m(p_1, p_2, I^m) = \alpha^m(p_1, p_2) + \beta(p_1, p_2)I^m$, *then demands can be written as in equation (15.2). Can you demonstrate that this is correct by using Roy's Identity?*

Answer: Applying Roy's identity, we get

$$x_i^m(p_1,p_2,I) = -\frac{\partial V/\partial p_i}{\partial V/\partial I} = -\frac{\left(\partial \alpha^m(p_1,p_2)/\partial p_i\right) + I^m\left(\partial \beta(p_1,p_2)/\partial p_i\right)}{\beta(p_1,p_2)}. \tag{15.1.i}$$

If we now define

$$a_i^m(p_1,p_2) = -\frac{\partial \alpha^m(p_1,p_1)/\partial p_i}{\beta(p_1,p_2)} \text{ and } b_i(p_1,p_2) = -\frac{\partial \beta(p_1,p_2)/\partial p_i}{\beta(p_1,p_2)}, \tag{15.1.ii}$$

we can write the demand function for good i by consumer m as

$$x_i^m(p_1,p_2,I) = a_i^m(p_1,p_2) + I^m b_i(p_1,p_2). \tag{15.1.iii}$$

Note that we can do this because the first term on the right hand side of equation (15.1.i) contains both an m superscript and an i subscript — thus causing the a function to contain both. But the second term contains (aside from I^m) only an i subscript (and no m superscript) — thus allowing us to write the b function without the m superscript.

(b) *Now consider the case of workers who choose between consumption (priced at 1) and leisure. Suppose they face the same wage w but different workers have different leisure endowments. Letting the two workers be superscripted by n and m, can you derive the form that the leisure demand equations $l^m(w,L^m)$ and $l^n(w,L^n)$ would have to take in order for redistributions of leisure endowments to not impact the overall amount of labor supplied by these workers (together) in the labor market?*

Answer: In order for redistributions in leisure endowments to have offsetting effects, it must be the case that the first derivative of $l^m(w,L^m)$ with respect to L^m is equal to the first derivative of $l^n(w,L^n)$ with respect to L^n and that the second derivative of each is zero. (This gives us the parallel linear (and offsetting) changes in consumption bundles as endowments are redistributed.) In order for this to be the case, the functions have to take the form

$$l^m(w,L^m) = a^m(w) + b(w)L^m \text{ and } l^n(w,L^n) = a^n(w) + b(w)L^n. \tag{15.1.iv}$$

The first derivatives with respect to the leisure endowments are then equal to $b(w)$, and the second derivatives are zero. Were the b functions superscripted by m and n, this would not be the case, nor would it be the case if leisure entered the b or a functions directly.

(c) *Can you re-write these in terms of labor supply equations $\ell^m(w,L^m)$ and $\ell^n(w,L^n)$?*

Answer: Since labor supply is just the leisure endowment minus leisure demand, we get

$$\ell^m(w, L^m) = L^m - \left(a^m(w) + b(w)L^m\right) = (1 - b(w))\,L^m - a^m(w) \quad (15.1.v)$$

and

$$\ell^n(w, L^n) = L^n - \left(a^n(w) + b(w)L^n\right) = (1 - b(w))\,L^n - a^n(w). \quad (15.1.vi)$$

(d) *Can you verify that these labor supply equations have the property that redistributions of leisure between the two workers do not affect overall labor supply?*

Answer: The first derivative of the labor supply functions with respect to the leisure endowments are now equal to $(1 - b(w))$ and thus equal to each other — and the second derivatives are zero. Thus, a redistribution of endowments indeed causes an increase in labor supply by the worker who receives more endowment which is exactly offset by the decrease in labor supply by the worker who receives less endowment.

Exercise 15.3

Business and Policy Application: *License Fees and Surplus without Income Effects: In previous chapters, we explored the impact of recurring license fees on an industry's output and price. We now consider their impact on consumer and producer surplus.*

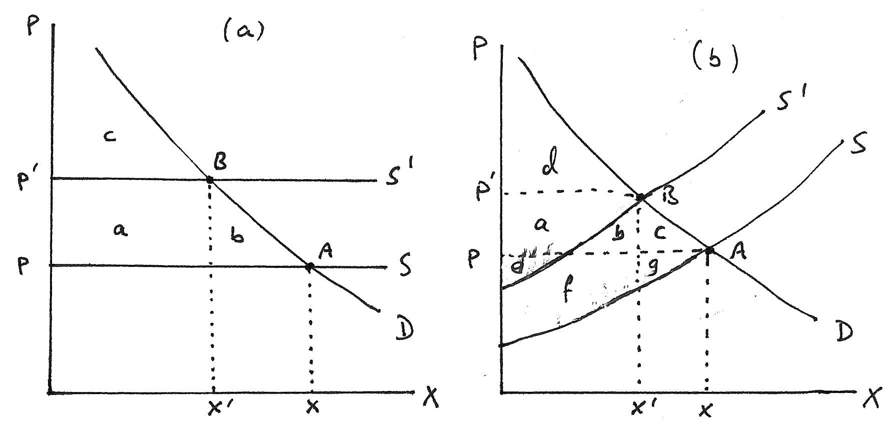
A: *Suppose that all firms in the fast food restaurant business face U-shaped average cost curves prior to the introduction of a recurring license fee. The only output they produce is hamburgers. Suppose throughout that hamburgers are a quasilinear good for all consumers.*

(a) *First, assume that all firms are identical. Illustrate the long run market equilibrium and indicate how large consumer and long run producer surplus (i.e. profit) are in this industry.*

Answer: This is illustrated in panel (a) of Exercise Graph 15.3. The long run supply curve S is flat because of entry and exit decisions by identical firms — leading to equilibrium price p and equilibrium output level x at point A. Producer surplus (or long run profit) is simply zero. Consumer surplus can be measured on the uncompensated demand curve because of the quasilinearity of x (that causes compensated demand curves to lie on top of the uncompensated demand curve). Thus, long run consumer surplus is $(a + b + c)$.

(b) *Illustrate the change in the long run market equilibrium that results from the introduction of a license fee.*

Answer: The long run supply curve shifts up (to S') because the long run average cost curves of each firm shift up. At the new equilibrium, price is p' and output is x' at point B. (Each firm ends up producing more, but the industry produces less as firms exit.)



Exercise Graph 15.3 : License Fees and Quasilinear Tastes

(c) *Suppose that the license fee has not yet been introduced. In considering whether to impose the license fee, the government attempts to ascertain the cost to consumers by asking a consumer advocacy group how much consumers would have to be compensated (in cash) in order to be made no worse off. Illustrate this amount as an area in your graph.*

Answer: Ordinarily, this would be measured along the compensated demand curve that goes through A. Since x is quasilinear, however, the compensated demand curves lie on the uncompensated demand curve that goes through A and B. Thus, the compensation is given by area $(a+b)$.

(d) *Suppose instead that the government asked the consumer group how much consumers would be willing to pay to avoid the license free. Would the answer change?*

Answer: Ordinarily this would be measured on the compensated demand curve that goes through B. However, all compensated demand curves lie on the uncompensated demand curve because of the quasilinearity of x. Thus, the amount consumers would be willing to pay is $(a+b)$.

(e) *Finally, suppose the government simply calculated consumer surplus before and after the license fee is imposed and subtracted the latter from the former. Would the government's conclusion of how much the license fee costs consumers change?*

Answer: No. The consumer surplus at A is $(a+b+c)$ and the consumer surplus at B is c — making the difference area $(a+b)$. (Again, this is only true because consumer surplus can, because of the quasilinearity of x, be measured on the uncompensated demand curve.)

(f) *What in your answers changes if, instead of all firms being identical, some firms had higher costs than others (but all have U-shaped average cost* *curves)?*

Answer: Not very much would be different — except for the fact that producer surplus would now be positive given that firms who are more cost effective can earn positive profit. This is illustrated in panel (b) of Exercise Graph 15.3 where supply shifts from S to S'. The change in consumer surplus is $(a+b+c)$ (which is equivalent to $(a+b)$ in panel (a)) — and the same measure would give the compensation required to consumers or the amount consumers would be willing to pay to prevent the fee from going into effect. Producer surplus, or long run profit, would be $(e+f+g)$ before and $(e+a)$ after.

B: *Suppose that each firm's cost function is given by* $C(w,r,x) = 0.047287w^{0.5}r^{0.5}x^{1.25} + F$ *where* F *is a recurring fixed cost.*[1]

(a) *What is the long run equilibrium price for hamburgers* x *(as a function of* F*) assuming wage* $w = 20$ *and rental rate* $r = 10$*?*

Answer: Each firm's cost function would then be

$$C(x,F) = 0.047287(20)^{0.5}(10)^{0.5}x^{1.25} + F = 0.66873917x^{1.25} + F. \quad (15.3.i)$$

From this, we can derive the long run average cost function as

$$AC(x,F) = 0.66873917x^{0.25} + \frac{F}{x}. \quad (15.3.ii)$$

To find the lowest point of this average cost function, we take the derivative with respect to x, set it to zero and solve for x to get $x = 4.18256389F^{0.8}$. Plugging this back into the average cost function, we get the long run equilibrium price (as a function of F):

$$p(F) = 0.66873917\left(4.18256389F^{0.8}\right)^{0.25} + \frac{F}{4.18256389F^{0.8}} = 1.195439F^{0.2}. \quad (15.3.iii)$$

(b) *Suppose that, prior to the imposition of a license fee, the firm's recurring fixed cost* F *was \$1,280. What is the pre-license fee equilibrium price?*

Answer: Using the equation $p(F)$, we can determine the initial equilibrium price

$$p(1280) = 1.195439\left(1280^{0.2}\right) = 5. \quad (15.3.iv)$$

(c) *What happens to the long run equilibrium price for hamburgers when a \$1,340 recurring license fee is introduced?*

Answer: Again, using the equation $p(F)$ and substituting the new fixed cost $F = 1280 + 1340 = 2620$, we get

$$p(2620) = 1.195439\left(2620^{0.2}\right) = 5.77. \quad (15.3.v)$$

[1] You can check for yourself that this is the cost function that arises from the production function $f(\ell,k) = 20\ell^{0.4}k^{0.4}$.

(d) *Suppose that tastes for hamburgers x and a composite good y can be characterized by the utility function* $u(x,y) = 20x^{0.5} + y$ *for all 100,000 consumers in the market, and assume that all consumers have budgeted \$100 for x and other goods y. How many hamburgers are sold before and after the imposition of the license fee?*

Answer: The demand function derived from this utility function is $x(p) = 100/p^2$. Summing over 100,000 consumers, we get a market demand function of

$$X(p) = \frac{10,000,000}{p^2}. \qquad (15.3.\text{vi})$$

Substituting the before and after prices of \$5 and \$5.77, this implies that 2,000,000 hamburgers were sold before the license fee and about 1,733,100 hamburgers are sold afterwards.

(e) *Derive the expenditure function for a consumer with these tastes.*

Answer: We need to solve the expenditure minimization problem

$$\min_{x,y} \; px + y \; \text{ subject to } \; u = 20x^{0.5} + y. \qquad (15.3.\text{vii})$$

This gives us the compensated demand functions

$$x(p) = \frac{100}{p^2} \;\text{ and }\; y(p,u) = u - \frac{200}{p}. \qquad (15.3.\text{viii})$$

Substituting this into the expenditure equation $px + y$, we get the expenditure function

$$E(p,u) = p\left(\frac{100}{p^2}\right) + u - \frac{200}{p} = u - \frac{100}{p}. \qquad (15.3.\text{ix})$$

(f) *Use this expenditure function to answer the question in A(c).*

Answer: First, we have to figure out how much utility consumers get in the absence of the license fee when $p = 5$. In that case, they consume 4 of x and 80 of y (given that they have budgeted \$100 for both goods) — which gives utility $u = 20(4^{0.5}) + 80 = 120$. In order to reach this utility level at the higher price $p = 5.77$, we have to evaluate the expenditure function $E(p,u)$ at $p = 5.77$ and $u = 120$; i.e.

$$E(5.77, 120) = 120 - \frac{100}{5.77} \approx 102.67. \qquad (15.3.\text{x})$$

Since each consumer has \$100 budgeted to start with, this implies that the government would have to compensate each consumer by \$2.67 — or a total of \$267,000 for the 100,000 consumers.

(g) *Use the expenditure function to answer the question in A(d).*

Answer: If consumers are asked how much they are willing to pay to not have the license fee implemented, they would first need to know how much utility they will get if the license fee in fact does get implemented. At $p = 5.77$, each consumer demands approximately 3 hamburgers (x) — down from 4 — and consumes \$82.67 of other goods (y) — up from 80 before. This implies that each consumer gets utility $u(3, 82.67) = 20(3^{0.5}) + 82.67 = 117.31$ if the license fee is implemented. (Had we not rounded a bit, this would actually be 117.33.) If the fee is not implemented, price falls to $p = 5$ — thus, in order to determine how much of a budget each consumer will need to be as well off without the fee as they are with it, we need to evaluate the expenditure function $E(p, u)$ at $p = 5.77$ and $u = 117.31$. This gives us

$$E(5.77, 117.31) = 117.31 - \frac{100}{5} = 97.31. \qquad (15.3.xi)$$

Thus, a consumer with a current budget of \$100 would be willing to pay \$2.69 — or, had we not rounded the utility figure and used 117.33, we would get that they are willing to pay \$2.67 each. Thus, the answer is the same as what we derived in the previous part — and consumers overall would be willing to pay approximately \$267,000 to avoid the license fee being implemented.

(h) *Take the integral of the demand function that gives you the consumer surplus before the license fee and repeat this to get the integral of the consumer surplus after the license fee is imposed.*

Answer: The consumer surplus before the license fee is

$$\int_5^{\infty} \frac{100}{p^2} dp = -\frac{100}{p}\Big|_5^{\infty} = 0 - \left(-\frac{100}{5}\right) = 20, \qquad (15.3.xii)$$

and the consumer surplus after the license fee is

$$\int_{5.77}^{\infty} \frac{100}{p^2} dp = -\frac{100}{p}\Big|_{5.77}^{\infty} = 0 - \left(-\frac{100}{5.77}\right) = 17.33. \qquad (15.3.xiii)$$

You could of course also have used the aggregate demand curve — and you would then have gotten the same answers (multiplied by 100,000).

(i) *How large is the change in consumer surplus from the price increase? Compare your answer to what you calculated in parts (f) and (g).*

Answer: The change in consumer surplus is therefore $20 - 17.33 = 2.66$ or (up to rounding errors) identical to what we calculated in parts (f) and (g). This is because, under quasilinear tastes, the (uncompensated) demand curve lies on top of the compensated demand curves — and we can thus use the (uncompensated) demand curve to measure changes in consumer surplus. (It furthermore implies that the two measures of changes in consumer surplus derived in (f) and (g) are identical because, even though they are measured on different compensated demand curves, they are identical because the compensated demand curves for different utility levels lie on top of one another.)

Exercise 15.5

Policy Application: *Redistribution of Income without Income Effects*: *Consider the problem a society faces if it wants to both maximize efficiency while also insuring that the overall distribution of "happiness" in the society satisfies some notion of "equity".*

A: *Suppose that everyone in the economy has tastes over x and a composite good y, with all tastes quasilinear in x.*

(a) *Does the market demand curve (for x) in such an economy depend on how income is distributed among individuals (assuming no one ends up at a corner solution)?*

Answer: If everyone's tastes are quasilinear in x, this means that each person's demand for x is independent of income (unless someone is at a corner solution). Thus, the aggregate demand curve in the market for x does not depend on the distribution of income in the population. Since the supply curve also does not depend on the distribution of income, the market equilibrium in the x market is independent of the income distribution.

(b) *Suppose you are asked for advice by a government that has the dual objective of maximizing efficiency as well as insuring some notion of "equity". In particular, the government considers two possible proposals: Under proposal A, the government redistributes income from wealthier individuals to poorer individuals before allowing the market for x to operate. Under proposal B, on the other hand, the government allows the market for x to operate immediately and then redistributes money from wealthy to poorer individuals after equilibrium has been reached in the market. Which would you recommend?*

Answer: Since the market outcome in the x market is independent of the distribution of income, it does not matter whether income is redistributed before or after the market equilibrium has been reached. The end result will be exactly the same. Thus, you should tell the government it does not matter which policy is put in place.

(c) *Suppose next that the government has been replaced by an omniscient social planner who does not rely on market processes but who shares the previous government's dual objective. Would this planner choose a different output level for x than is chosen under proposal A or proposal B in part (b)?*

Answer: No, the social planner would do exactly what the government would do under either of the two policies. This is because the social planner is not restricting his ability to achieve different notions of equity by allowing surplus in the x market to be maximized — which happens when the competitive equilibrium quantity of x is produced.

(d) True or False: *As long as money can be easily transferred between individuals, there is no tension in this economy between achieving many different notions of "equity" and achieving efficiency in the market for x.*

Answer: This is true (as already explained in the previous part).

(e) *To add some additional realism to the exercise, suppose that the government has to use distortionary taxes in order to redistribute income between individuals. Is it still the case that there is no tradeoff between efficiency and different notions of equity?*

Answer: In this case, a tradeoff does emerge — because redistribution through distortionary taxes implies the creation of deadweight losses as income is transferred between individuals. Thus, more redistribution implies a loss of social surplus — thus the tension between "equity" and efficiency.

B: *Suppose there are two types of consumers: Consumer type 1 has utility function $u^1(x,y) = 50x^{1/2} + y$, and consumer type 2 has utility function $u^2(x,y) = 10x^{3/4} + y$. Suppose further that consumer type 1 has income of 800 and consumer type 2 has income of 1,200.*

(a) *Calculate the demand functions for x for each consumer type assuming the price of x is p and the price of y is 1.*

Answer: Using the utility function $u(x,y) = Ax^{\alpha} + y$, we can solve for the demand function for x as

$$2x(p) = \left(\frac{\alpha A}{p}\right)^{1/(1-\alpha)}. \tag{15.5.i}$$

Substituting for the terms in the two utility functions for the two types, this implies demand functions

$$x^1(p) = \left(\frac{0.5(50)}{p}\right)^{1/(1-0.5)} = \frac{625}{p^2} \text{ and } x^2(p) = \left(\frac{0.75(10)}{p}\right)^{1/(1-0.75)} = \frac{3{,}164.0625}{p^4} \tag{15.5.ii}$$

for type 1 and 2 respectively.

(b) *Calculate the aggregate demand function when there are 32,000 of each consumer type.*

Answer: Multiplying each demand function by 32,000 and adding, we get

$$X(p) = \frac{32{,}000(625)}{p^2} + \frac{32{,}000(3{,}164.0625)}{p^4} = \frac{20{,}000{,}000p^2 + 101{,}250{,}000}{p^4}. \tag{15.5.iii}$$

(c) *Suppose that the market for x is a perfectly competitive market with identical firms that attain zero long run profit when $p = 2.5$. Determine the long run equilibrium output level in this industry.*

Answer: Substituting $p = 2.5$ into the equation $X(p)$, we get $X(2.5) = 5{,}792{,}000$.

(d) *How much x does each consumer type consume?*

Answer: Type 1 consumers consume $625/(2.5^2) = 100$ units of x and type 2 consumers consume $3,164.0625/(2.5^4) = 81$ units of x.

(e) *Suppose the government decides to redistribute income in such a way that, after the redistribution, all consumers have equal income — i.e. all consumers now have income of 1,000. Will the equilibrium in the x market change? Will the consumption of x by any consumer change?*

Answer: Income does not enter any demand function (because the good x is quasilinear) — which implies that the income distribution does not enter the aggregate demand function $X(p)$. Thus, redistributing income in this way does not change either the equilibrium level of output in the market or the level of x consumption of any individual.

(f) *Suppose instead of a competitive market, a social planner determined how much x and how much y every consumer consumes. Assume that the social planner is concerned about both the absolute welfare of each consumer as well as the distribution of welfare across consumers — with more equal distribution more desirable. Will the planner produce the same amount of x as the competitive market?*

Answer: Yes — social surplus is still maximized at the same output level regardless of how the planner decides to redistribute income (so long as no one ends up at a corner solution). Thus, the planner would want to maximize the surplus in the x market by picking the same output level as the market — and he can then worry about redistributing income to the desired level.

(g) True or False*: The social planner can achieve his desired outcome by allowing a competitive market in x to operate and then simply transferring y across individuals to achieve the desired distribution of happiness in society.*

Answer: This is true. In other words, in an economy where all tastes are quasilinear in x, the planner does not actually have to calculate the optimal quantity of x but can rather allow the market to determine that quantity since it is unaffected by how income is distributed. By shifting y from some people to others, the planner can then achieve whatever desired level of "equity" he desires.

(h) *Would anything in your analysis change if the market supply function were upward sloping?*

Answer: Since the market demand curve is unaffected by redistribution of income, the market demand would continue to intersect market supply at the same point regardless of whether or not the supply curve slopes up. Thus, nothing changes fundamentally in the problem if we assume an upward sloping supply curve.

(i) *Economists sometimes refer to economies in which all individuals have quasilinear tastes as "transferable utility economies" — which means that in economies like this, the government can transfer happiness from one* *person to another. Can you see why this is the case if we were using the utility functions as accurate measurements of happiness?*

Answer: If we use the two utility functions in this problem as accurate measurements of happiness, then the planner will increase utility by 1 unit for a person of type 1 and lower it by 1 unit for a person of type 2 if he transfers one unit of y from person 1 to person 2. Thus, he is in essence able to transfer utility between individuals.

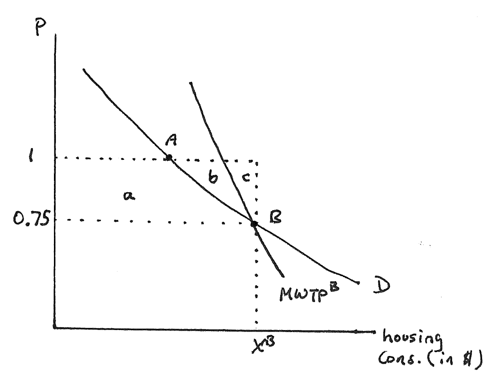
Exercise 15.7

Policy Application: *Dead Weight Loss from Subsidy of Mortgage Interest: The U.S. tax code subsidizes housing through a deduction of mortgage interest. For new homeowners, mortgage interest makes up the bulk of their housing payments which tend to make up about 25% of a household's income. Assume throughout that housing is a normal good.*

A: *For purposes of this problem, we will assume that all housing payments made by a household represent mortgage interest payments. If a household is in a 25% tax bracket, allowing the household to deduct mortgage interest on their taxes then is equivalent to reducing the price of $1 worth of housing consumption to $0.75.*

(a) *Illustrate a demand curve for a consumer, indicating both the with- and without-deductibility housing price.*

Answer: This is illustrated in Exercise Graph 15.7 where the demand curve is given by D. Without tax deductibility of housing costs, the consumer would locate at A where the price of a dollar of housing consumption is $1. Under deductibility, however, the consumer faces a price of $0.75 for every dollar in housing consumption — which implies she will locate at B.



Exercise Graph 15.7 : Tax Deductibility of Housing Costs

(b) *On the same graph, illustrate the compensated (or $MWTP$) curve for this consumer assuming that housing costs are deductible.*

Answer: If housing costs are deductible, the consumer locates at B. Thus, we would need to draw the $MWTP$ curve that runs through B — indicated as $MWTP^B$ in the graph. This is steeper than the uncompensated demand curve because housing is assumed to be a normal good.

(c) *On your graph, indicate where you would locate the amount that a consumer would be willing to accept in cash instead of having the subsidy of housing through the tax code.*

Answer: The consumer is equally happy all along $MWTP^B$ — which implies that we would need to give her the area $(a+b)$ in cash in order for her to be indifferent between the cash and the price subsidy.

(d) *On your graph, indicate the area of the deadweight loss.*

Answer: Under the subsidy implicit in the tax deductibility provision of the tax code, the government in essence pays $0.25 for every $1 in housing the consumer chooses. Under the subsidy the consumer chooses x^B — which implies that the total cost of the subsidy to the government is $0.25x^B$ — which is equal to the area $(a+b+c)$. Thus, the tax deductibility costs the government c more than the cash subsidy that would make the consumer just as well off — which implies c is the deadweight loss.

(e) *If you used the regular demand curve to estimate the deadweight loss, by how much would you over- or under-estimate it?*

Answer: If we used the regular demand curve to estimate the cash amount necessary to make the consumer just as happy, we would implicitly assume that housing is quasilinear (which it is not). As a result, we would conclude that the area a is how much cash the consumer would accept instead of tax deductibility of housing — which would lead us to conclude that the deadweight loss from tax deductibility is $(b+c)$ when it is actually just c. Thus, we would over-estimate the deadweight loss by area b.

B: *Suppose that a household earning $60,000 (after taxes) has utility function* $u(x,y) = x^{0.25}y^{0.75}$, *where x represents dollars worth of housing and y represents dollars worth of other consumption. (Thus, we are implicitly setting the price of x and y to $1.)*

(a) *How much housing does the household consume in the absence of tax deductibility?*

Answer: Letting p equal the price of housing, the demand function for this consumer is $x(p) = 0.25(60,000)/p$. When $p = 1$, this implies that $x = 15,000$.

(b) *If the household's marginal tax rate is 25% (and if all housing payments are deductible), how much housing will the household consume?*

Answer: The housing price for this household now falls to $0.75 — which implies the household will choose $x = 0.25(60,000)/0.75 = 20,000$ in housing.

(c) *How much does the implicit housing subsidy cost the government for this consumer?*

Answer: Since the government effectively pays a quarter of the housing bill, it costs the government \$5,000.

(d) *Derive the expenditure function for this household (holding the price of other consumption at \$1 but representing the price of housing as p.)*

Answer: For a Cobb-Douglas utility function of the form $u(x_1, x_2) = x_1^{\alpha} x_2^{(1-\alpha)}$, we showed in Chapter 10 that the expenditure function takes the form

$$E(p_1, p_2, u) = \frac{u p_1^{\alpha} p_2^{(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}. \quad (15.7.i)$$

Setting $p_1 = p$, $p_2 = 1$ and $\alpha = 0.25$, this gives us

$$E(p, u) = 1.75476535 p^{0.25} u. \quad (15.7.ii)$$

(e) *Suppose the government contemplates eliminating the tax deductibility of housing expenditures. How much would it have to compensate this household for the household to agree to this?*

Answer: At the subsidized housing price, the household consumes \$20,000 in housing and \$45,000 in other goods. This gives utility of $u(20000, 45000) \approx 36,742$. To reach this level of utility at a non-subsidized price, the household's budget would have to be

$$E(1, 36742) = 1.75476535(36742) \approx 64,474. \quad (15.7.iii)$$

Since the consumer begins with \$60,000, this means the government would have to pay the household \$4,474.

(f) *Can you derive the same amount as an integral on a compensated demand function?*

Answer: For a Cobb-Douglas utility function of the form $u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$, we calculated in Chapter 10 that the compensated demand functions for x_1 is

$$h_1(p_1, p_2, u) = \left(\frac{\alpha p_2}{(1-\alpha)p_1}\right)^{(1-\alpha)} u. \quad (15.7.iv)$$

Plugging in $\alpha = 0.25$, $p_2 = 1$ and $p_1 = p$, we get the compensated demand function

$$h_x(p, u) = \left(\frac{1}{3p}\right)^{0.75} u. \quad (15.7.v)$$

Evaluating the integral of this between the prices 0.75 and 1 when utility is 36,742, we get

$$\begin{aligned}\int_{0.75}^{1} \left(\frac{1}{3p}\right)^{0.75} (36742)\,dp &= 4(36742)\left(\frac{1}{3}\right)^{0.75} p^{0.25}\Big|_{0.75}^{1} \\ &= 64474\left(1 - 0.75^{0.25}\right) \approx 4,474.\end{aligned} \quad (15.7.vi)$$

(g) *Suppose you only knew this household's (uncompensated) demand curve and used it to estimate the change in consumer surplus from eliminating the tax deductibility of housing expenditures. How much would you estimate this to be?*

<u>Answer</u>: You would take the integral of the uncompensated demand curve between prices 0.75 and 1 to get

$$\int_{0.75}^{1} \frac{15000}{p} dp = 15000 \ln(p)|_{0.75}^{1} = 15000(\ln(1) - \ln(0.75)) \approx 4,315. \tag{15.7.vii}$$

(h) *Are you over- or under-estimating the deadweight loss from the subsidy if you use the (uncompensated) demand curve?*

<u>Answer</u>: If we use the uncompensated demand curve, we estimate the dead-weight loss from the subsidy as $5,000 - 4,315 = \$685$. If we use the compensated demand curve, we get $5,000 - 4,474 = \$526$. Thus, we are over-estimating the deadweight loss if we use the uncompensated demand function.

(i) *Suppose that all 50,000,000 home-owners in the U.S. are identical to the one you have just analyzed. What is the annual deadweight loss from the deductibility of housing expenses? By how much would you over- or under-estimate this amount if you used the aggregate demand curve for housing in this case?*

<u>Answer</u>: The annual deadweight loss is $526(50,000,000) = \$26,300,000,000$ or $26.3 billion dollars. If we used the uncompensated demand curve to estimate the deadweight loss, we would get $34.25 billion instead. Thus, by using the wrong demand curve, we would overestimate the deadweight loss by $7.95 billion.

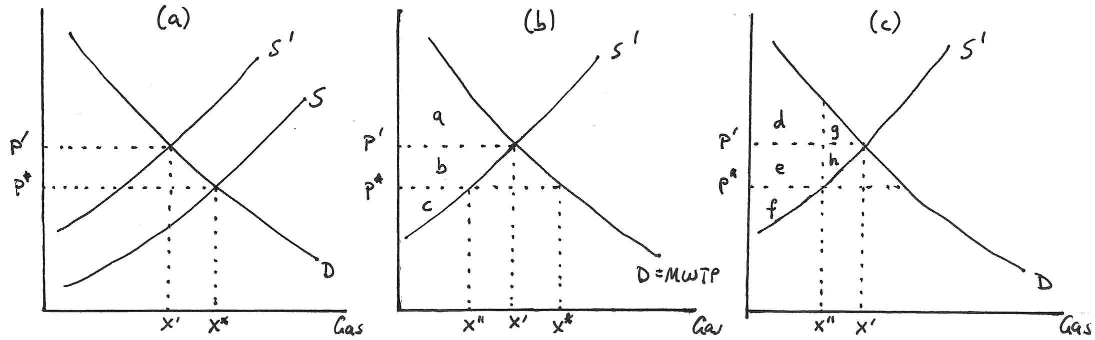
Exercise 15.9

<u>Policy Application</u>: *Anti-Price Gauging Laws*: *As we will discuss in more detail in Chapter 18, governments often interfere in markets by placing restrictions on the price that firms can charge. One common example of this is so-called "anti-price gauging laws" that restrict profits for firms when sudden supply shocks hit particular markets.*

A: *A recent hurricane disrupted the supply of gasoline to gas stations on the East Coast of the U.S. Some states in this region enforce laws that prosecute gasoline stations for raising prices as a result of natural disaster-induced drops in the supply of gasoline.*

(a) *On a graph with weekly gallons of gasoline on the horizontal and price per gallon on the vertical, illustrate the result of a sudden leftward shift in the supply curve (in the absence of any laws governing prices.)*

<u>Answer</u>: This is illustrated in panel (a) of Exercise Graph 15.9 where S is the original supply curve and S' is the new supply curve. The equilibrium



Exercise Graph 15.9 : Anti-Price Gauging Laws

shifts from one where price was p^* and gasoline consumption x^* to one where the price is p' and gasoline consumption is x'.

(b) *Suppose that gasoline is a quasilinear good for consumers. Draw a graph similar to the one in part (a) but include only the post-hurricane supply curve (as well as the unchanged demand curve). Illustrate consumer surplus and producer profit if price is allowed to settle to its equilibrium level.*

Answer: This is illustrated in panel (b) of Exercise Graph 15.9. Consumer surplus would be equal to area a and producer profit would be equal to area $(b+c)$.

(c) *Now consider a state that prohibits price adjustments as a result of natural disaster-induced supply shocks. How much gasoline will be supplied in this state? How much will be demanded?*

Answer: This is also illustrated in panel (b). At the pre-crisis price of p^*, firms would supply x'' — but consumers would want to buy x^*.

(d) *Suppose that the limited amount of gasoline is allocated at the pre-crisis price to those who are willing to pay the most for it. Illustrate the consumer surplus and producer profit.*

Answer: This is illustrated in panel (c) of Exercise Graph 15.9. If the limited amount of gasoline x'' is bought at p^* by those who value it the most, then consumer surplus is $(d+e)$. Producer profit is area f.

(e) *On a separate graph, illustrate the total surplus achieved by a social planner who insures that gasoline is given to those who value it the most and sets the quantity of gasoline at the same level as that traded in part (c). Is the social surplus different than what arises under the scenario in (d)?*

Answer: The social surplus would then be the same as in part (d) — equal to area $(d+e+f)$.

(f) *Suppose that instead the social planner allocates the socially optimal amount of gasoline. How much greater is social surplus?*

Answer: The socially optimal quantity is x'. If that much is produced, the total surplus is $(d+e+f+g+h)$ — which is greater than the surplus under the restricted quantity x'' by area $(g+h)$.

(g) *How does the total social surplus in (f) compare to what you concluded in (b) that the market would attain in the absence of anti-price gauging laws?*

Answer: It is identical.

(h) True or False*: By interfering with the price signal that communicates information about where gasoline is most needed, anti-price gauging laws have the effect of restricting the inflow of gasoline to areas that most need gasoline during times of supply disruptions.*

Answer: This is true, as demonstrated in the problem. The areas where gasoline would be most needed are those where the price would rise most in the absence of anti-price gauging laws. Thus, it is in these areas that the greatest shortages would emerge.

B: *Suppose again that the aggregate demand function $X^D(p) = 250{,}000/p^2$ arises from 10,000 local consumers of gasoline with quasilinear tastes (as in exercise 15.8).*

(a) *Suppose that the industry is in long run equilibrium — and that the short run industry supply function in this long run equilibrium is $X^S(p) = 3{,}906.25p$. Calculate the equilibrium level of (weekly) local gasoline consumption and the price per dollar.*

Answer: Setting $X^D(p) = X^S(p)$, we get $p = 4$. Substituting this back into either the demand or supply equation, we get $x = 15{,}625$.

(b) *What is the size of the consumer surplus and (short run) profit?*

Answer: The consumer surplus is

$$\int_4^\infty \frac{250{,}000}{p^2}dp = -\frac{250{,}000}{p}\Big|_4^\infty = 0 - (-62{,}500) = \$62{,}500. \qquad (15.9.i)$$

The firm (short run) profits are

$$\int_0^4 3{,}906.25p\,dp = 1{,}953.125p^2\Big|_0^4 = 31{,}250 - 0 = \$31{,}250. \qquad (15.9.ii)$$

(c) *Next suppose that the hurricane-induced shift in supply moves the short run supply function to $\overline{X}^S = 2{,}000p$. Calculate the new (short run) equilibrium price and output level.*

Answer: We solve for the new equilibrium price by setting $X^D(p) = \overline{X}^S(p)$ and solving for $p = 5$. Plugging this back into either the demand or supply functions, we get $x = 10{,}000$.

(d) *What is the sum of consumer surplus and (short run) profit if the market is allowed to adjust to the new short run equilibrium?*

Answer: Consumer surplus is now

$$\int_5^{\infty} \frac{250,000}{p^2} dp = -\frac{250,000}{p}|_5^{\infty} = 0 - (-50,000) = \$50,000. \quad (15.9.iii)$$

Profits for firms are

$$\int_0^5 2,000 p dp = 1,000 p^2 |_0^5 = 25,000 - 0 = \$25,000. \quad (15.9.iv)$$

Thus, the sum of consumer surplus and (short run) firm profits is $75,000.

(e) *Now suppose the state government does not permit the price of gasoline to rise above what you calculated in part (a). How much gasoline will be supplied?*

Answer: At a price of $p = 4$, the gallons of gasoline supplied will be

$$\overline{X}^S(4) = 2,000(4) = 8,000. \quad (15.9.v)$$

(f) *Assuming that the limited supply of gasoline is bought by those who value it the most, calculate overall surplus (i.e. consumer surplus and (short run) profit) under this policy.*

Answer: The easiest way to calculate this is to find the area under the demand *curve* that lies above the supply curve up to $x = 8,000$. The area under the demand curve is

$$\int_0^{8000} \frac{500}{x^{0.5}} dx = 1,000 x^{0.5} |_0^{8000} \approx \$89,442.72. \quad (15.9.vi)$$

The supply *curve* is the supply function solved for p — i.e. $p = 0.0005x$. The area under the supply curve up to $x = 8000$ is

$$\int_0^{8000} 0.0005 x dx = 0.00025 x^2 |_0^{8000} = \$16,000. \quad (15.9.vii)$$

Thus, the overall surplus is $89442.72 - 16000 = \$73,442.72$.

(g) *How much surplus is lost as a result of the government policy to not permit price increases in times of disaster-induced supply shocks?*

Answer: In the absence of the policy, total surplus was $75,000 — which is $1,557.28 greater than the total surplus under the policy.

Conclusion: Potentially Helpful Reminders

1. The idea of representing different sides of the market as if they emerged from the behavior of a "representative agent" is a powerful one because it allows us to treat certain market curves using our insights from the development of consumer and producer theory.

2. It is only when a market relationship emerges from consumer theory — as it does in the case of consumer demand and labor supply — that we have to be careful as we are tempted to think about these market relationships as if they emerged from a single optimization problem. This is because of the presence of income effects that, when assumed away, remove the difficulty. It is for this reason that the analysis of welfare becomes significantly more straightforward when we assume quasilinear tastes.

3. Remember that you can always use market relationships to predict market outcomes — regardless of whether the condition of quasilinearity holds. It is only when we then try to determine exact welfare measures that we have to be careful if quasilinearity does not hold — which implies that it is only when quasilinearity does not hold that we have to worry about separately thinking about compensated rather than uncompensated relationships.

4. The complications of introducing income effects are explored in a particularly revealing way in end-of-chapter exercises 15.5 and 15.6 where we show how redistribution of income does not alter the equilibrium in a market under some conditions (quasilinearity) but does do so under other conditions (i.e. when there are income effects). Nevertheless, we show that the market equilibrium will retain its efficiency property under the assumptions of the first welfare theorem even in the presence of income effects — even though the nature of the equilibrium will depend on the initial distribution of income in that case.

5. It is important from the outset to be aware of the limitations of the first welfare theorem — limitations that arise from the underlying assumptions listed in the chapter and developed throughout the remainder of the text. End-of-chapter exercises 15.2, 15.7, 15.8 and 15.9 begin to explore these.