CHAPTER

16

General Equilibrium

Chapter 14 introduced the *positive* idea of equilibrium in the context of a competitive environment — and Chapter 15 moved onto the more *normative* assessment of a competitive equilibrium within the context of the first welfare theorem. We now extend the basic insights from these chapters to economies that are more "general" in the sense that we don't look just at one isolated market but rather treat the economy as an interconnected web of markets. It admittedly may at times not seem like we are extending the analysis to something more "general" — because the economies we discuss in this chapter seem very "simple" in that they only involve a few individuals and often abstract away from issues like production. But these "simple" economies can be extended to much more complex models with many goods, many individuals, many production processes, etc. And the basic insights that emerge from the "simple" models continue to hold in these more complicated models that indeed are more "general" than the partial equilibrium models of Chapters 14 and 15.

Chapter Highlights

The main points of the chapter are:

1. An **exchange economy** is an economy in which individuals trade what they own but no production takes place. Typically, individuals can improve their welfare in such economies by engaging in mutually beneficial trades. The set of efficient allocations of goods in such an economy is known as the **contract curve**.

2. A **competitive equilibrium** in an exchange economy consists of a set of *prices* and an *allocation* of the goods in the economy such that all individuals would agree to trade to that allocation at these prices. (The equilibrium is "competitive" because all individuals are assumed to be price takers.)

3. The *equilibrium allocation* of goods always lies on the contract curve (and is thus efficient). This is our generalization of the **first welfare theorem**. It

furthermore lies in the **core** of the economy — where the core is defined as the set of allocations under which no coalition of individuals could leave the economy with their endowment and do better on their own. (In the 2-person exchange economy, the core allocations are equal to the contract curve allocations that lie in the region of mutually beneficial trades.)

4. If governments can use *lump sum transfers*, then any allocation on the contract curve can become a competitive equilibrium allocation after appropriate lump sum transfers have been made. This is what is known as the **second welfare theorem**. If, however, the government can only use distortionary taxes, then a tradeoff emerges between notions of "equity" and "efficiency" because it is no longer possible for the government to redistribute and expect an efficient outcome.

5. Production can be introduced into general equilibrium economies, and the same basic welfare results hold. The simplest example of such an economy is the **Robinson Crusoe economy** where a single individual acts as producer, worker and consumer.

16A Solutions to Within-Chapter-Exercises for Part A

Exercise 16A.1

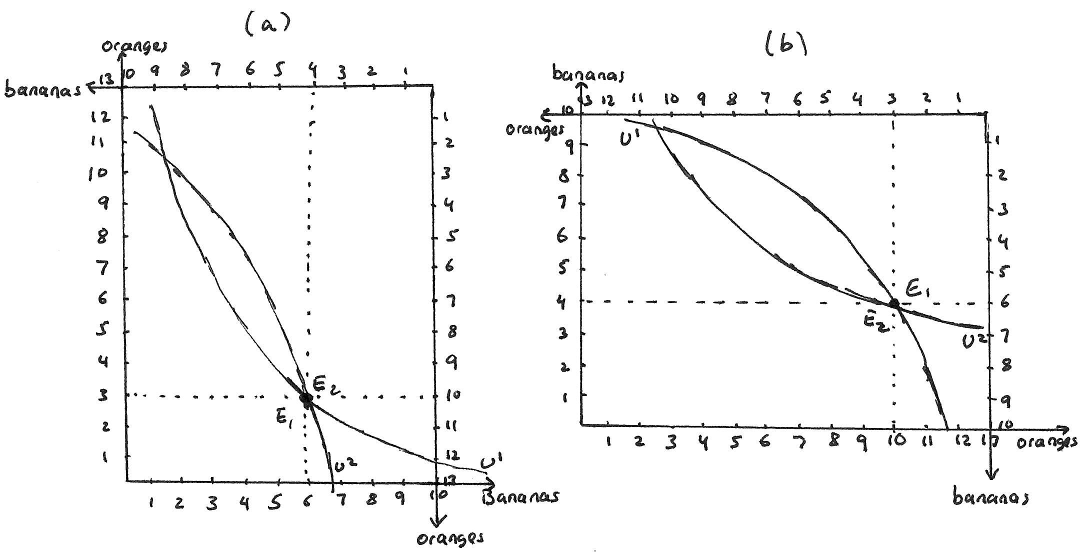
What would the Edgeworth Box for this example look like if oranges appeared on the vertical and bananas on the horizontal axis?

Answer: The Edgeworth Box would then be 10 units (bananas) long and 13 units (oranges) high, with the endowment point as pictured in panel (a) of Exercise Graph 16A.1.

Exercise 16A.2

What would the Edgeworth Box for this example look like if my wife's axes had the origin in the lower left corner and my axes had the origin in the upper right corner?

Answer: The box would have the same dimensions, but the endowment point would be located as pictured in panel (b) of Exercise Graph 16A.1 in the previous within-chapter exercise solution.



Exercise Graph 16A.1 : Edgeworth Boxes

Exercise 16A.3

True or False: Starting at point A, any mutually beneficial trade will involve me trading bananas for oranges, and any trade of bananas for oranges will be mutually beneficial. (*Hint*: Part of the statement is true and part is false.)

Answer: The first part of the statement is true — any mutually beneficial trade from A will involve me giving up bananas and getting oranges. This is because all the allocations of bananas and oranges that lie within the lens shape (and thus above both of our indifference curves through A) involve more oranges and fewer bananas for me. But it is not true that *any* trade of bananas for oranges will be mutually beneficial — only those lying above both our indifference curves. Put differently, the allocations that involve mutually beneficial trades from A do lie to the southeast of A (with fewer bananas and more oranges for me), but there are allocations that lie to the southeast of A that do not lie within the lens that represents allocations which are better for both me and my wife.

Exercise 16A.4

In Chapter 6, we argued that consumers leave Wal-Mart with the same tastes "at the margin" — i.e. with the same marginal rates of substitution between goods that they have purchased, and that this fact implies that all gains from trade have been exhausted. How is this similar to the condition for an efficient distribution of an economy's endowment in the exchange economy?

Answer: Once gains from trade have been exhausted in the Edgeworth Box, it is similarly true that the marginal rates of substitution of the two individuals will be equal to one another — just as when they come out of Wal-Mart after maximizing subject to facing the same prices. Thus, in both cases, the tastes are the same at the margin once all gains from trade have been exhausted. (This presumes that both individuals will optimize at an interior solution — but the efficiency logic extends to the case where individuals end up at corner solutions).

Exercise 16A.5

Starting at the point where my wife gets the entire endowment of the economy, are there points in the Edgeworth Box that make my wife worse off without making me better off (assuming that bananas and oranges are both essential goods for me)?

Answer: If both goods are essential, this means that the only way I can become better off than I am at the bundle (0,0) is for me to get at least some of each of the two goods. Thus, I do not become better off if you just take bananas away from my wife and give them to me, nor am I better off if you take oranges away from her and give them to me. Thus, any movement from (0,0) along either of the axes that refer to me would make my wife worse off without making me better off.

Exercise 16A.6

Is the point on the upper right hand corner of the Edgeworth Box Pareto efficient?

Answer: Yes — it is the point at which I get all bananas and oranges and my wife gets none. If that is the allocation, then any move away from this point within the box will make me worse off — and there is therefore no way to make someone better off without making anyone else worse off.

Exercise 16A.7

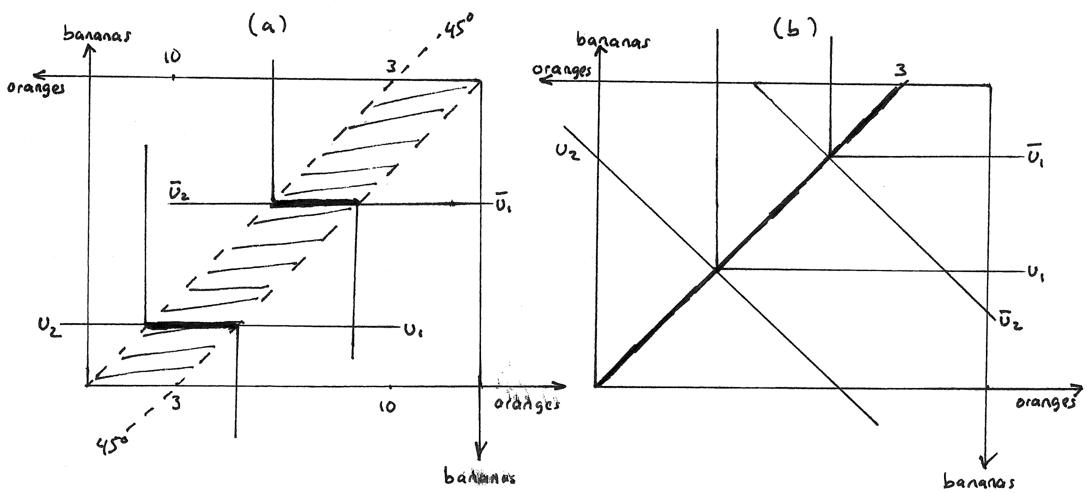
If bananas and oranges are essential goods for both me and my wife, can any points on the axes (other than those at the upper right and lower left corners of the Edgeworth Box) be Pareto efficient?

Answer: No. Consider a bundle that lies on one of the axes. If both goods are essential, it means that one of us is no better off than he/she would be if he/she had nothing. That means that we could simply give everything to the other person — make him/her better off without making the other worse off.

Exercise 16A.8

What would the contract curve look like if bananas and oranges were perfect complements for both me and my wife? (*Hint*: It is an area rather than a "curve".) What if they were perfect complements for me and perfect substitutes for my wife?

Answer: The case of perfect complements for both of us is depicted in panel (a) of Exercise Graph 16A.8 where u_1 and $\overline{u}_1$ are two indifference curves for me and u_2 and $\overline{u}_2$ are two indifference curves for my wife. These are "tangent" along the darkened parts which lie in the shaded region. And the shaded region is the region in which all such "tangencies" between two of our indifference curves lie — i.e. it is the contract curve. (Note that the region is bounded by the two 45 degree lines that emanate from our two origins of the two sets of axes — because it is along these 45 degree lines that the corners of our indifference curves lie.) Panel (b) shows the case where bananas and oranges are perfect complements for me and perfect substitutes for my wife. Again, u_1 and $\overline{u}_1$ are two indifference curves for me, and u_2 and $\overline{u}_2$ are two indifference curves for my wife. These are now "tangent" just at the corner of my indifference curves — making the 45 degree line emanating from *my* origin the contract curve.



Exercise Graph 16A.8 : Contract Curves

Exercise 16A.9

What does the contract curve look like if bananas and oranges are perfects substitutes (one for one) for both me and my wife? (*Hint*: You should get a large area within the Edgeworth Box as a result.)

Answer: The entire Edgeworth Box is then the contract curve. Because our indifference curves have the same slopes, there will be — for any arbitrary indifference curve for me — an indifference curve for my wife that lies right on top of mine and is thus "tangent" everywhere. Every allocation in the Edgeworth Box is therefore efficient. This should make intuitive sense: Suppose the two goods are Coke and Pepsi and neither of us can tell Pepsi apart from Coke. In that case, Coke and

Pepsi are perfect substitutes — and the only thing we care about is how much Coke and Pepsi we have *in total*. For any allocation in the Edgeworth Box, we will then not be able to make one of us better off without making the other worse off — because we either keep the overall quantity of soft drinks the same by changing only the mix of Coke and Pepsi in each of our bundles (thus making neither of us better or worse off), or we will increase the total quantity of soft drinks for one of us (thus making the other worse off). There is no way to make one of us better off without making the other worse off.

Exercise 16A.10

What are the intercepts of this budget on my wife's axes for oranges and bananas?

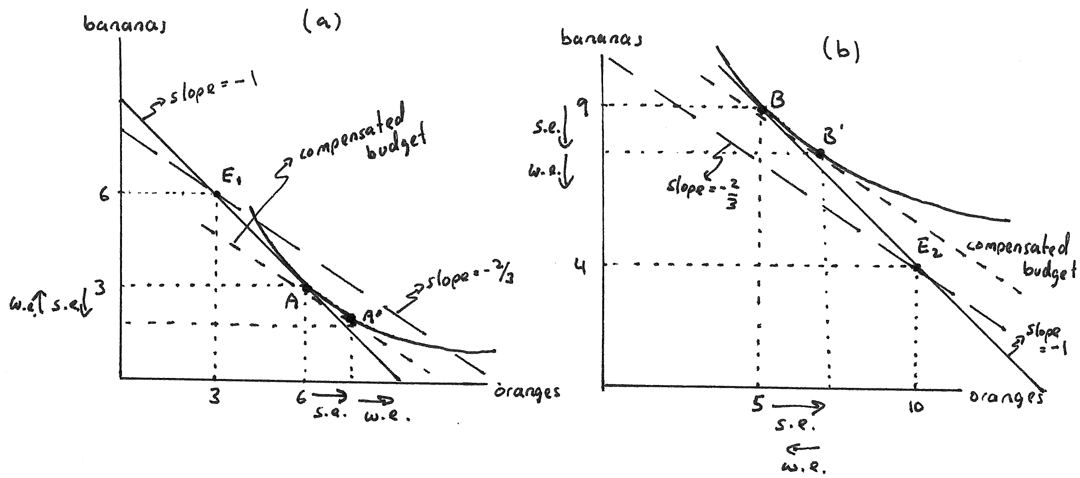
Answer: She begins at her endowment point $E_2 = (10, 4)$, with 10 oranges and 4 bananas. If she sells the 4 bananas at a price of 1.5, she collects \$6 — for which she can buy 6 oranges at a price of 1. Thus, the intercept of her budget on the oranges axis is 16. Alternatively, she could sell her 10 oranges for \$10 at a price of 1, allowing her to buy 6.67 bananas at a price of 1.5. Her bananas intercept is therefore 10.67.

Exercise 16A.11

Suppose both oranges and bananas are normal goods for both me and my wife. Draw separate graphs for me and my wife — with the initial budget constraint when the prices were both equal to 1 and the new budget constraint when the price of bananas is raised to 1.5. Illustrate — using substitution and wealth effects — why my demand for oranges will unambiguously increase and my wife's demand for bananas will unambiguously decrease. Can you say unambiguously what will happen to my demand for bananas and my wife's demand for oranges?

Answer: This is illustrated in Exercise Graph 16A.11.

In panel (a), the graph for "me" is drawn — where the endowment point E_1 is the initial 3 oranges and 6 bananas and the solid original budget has slope -1 and passes through E_1. The new slope when bananas are priced at 1.5 (relative to oranges at 1) becomes $-2/3$ and must still pass through E_1. The bundle A is optimal on the original budget, and the move from A to A' is a substitution effect that implies more consumption of oranges and less of bananas. Since both goods are normal, the movement from the compensated budget to the new budget leads to an increase in consumption of both goods relative to A' — i.e. a wealth effect in the same direction as the substitution effect for oranges but not for bananas. We can therefore unambiguously say that I will consume more oranges when the price of bananas increases, but we cannot say whether I will consume more bananas (since we have offsetting substitution and wealth effects on the bananas axis). In panel (b), the graph for "my wife" is drawn, with the solid budget through E_2 again representing the initial budget with slope -1. The new budget again passes through E_2 but has shallower slope $-2/3$ — giving us a substitution effect that implies more oranges and fewer bananas (just as in panel (a)). For my wife, however, the move-



Exercise Graph 16A.11 : Price Changes in the Edgeworth Box

ment from the compensated budget to the new budget is a decrease in income — so the wealth effect points in the opposite direction (relative to the substitution effect) for oranges and in the same direction for bananas. Thus, for my wife we can unambiguously say that her consumption of bananas will fall as the price of bananas increases, but we cannot be sure whether her consumption of oranges will increase or decrease.

Exercise 16A.12

Suppose you had decided to leave the price of bananas at 1 and to rather change the price of oranges. What price (for oranges) would you have to set in order to achieve the same equilibrium outcome?

Answer: The equilibrium outcome will be achieved for *any* set of prices that result in a slope of the budget equal to $-2/3$. With oranges denoting good 1 and bananas denoting good 2, the slope of the budget constraint is $-p_1/p_2$. If we keep the price of bananas at 1, we therefore have to set the price of oranges (p_1) to 2/3 in order to get the right slope for the budget constraint.

Exercise 16A.13

Suppose you set the price of oranges equal to 2 instead of 1. What price for bananas will result in the same equilibrium outcome?

Answer: The equilibrium outcome will be achieved for *any* set of prices that result in a slope of the budget equal to $-2/3$. With oranges denoting good 1 and bananas denoting good 2, the slope of the budget constraint is $-p_1/p_2$. If we keep the price of oranges at $p_1 = 2$, we therefore have to set the price of bananas (p_2) to 3 in order to get the right slope for the budget constraint.

Exercise 16A.14

True or False: When the First Welfare Theorem holds, competitive equilibria in an exchange economy result in allocations that lie on the contract curve but not necessarily in the core.

Answer: This is false — the competitive equilibria in an exchange economy result in allocations that lie in the core but not necessarily on the contract curve. This is because no one would agree to trade at equilibrium prices if she became worse off — and lots of allocations on the contract curve have one person worse off than she is at the endowment point. Competitive equilibrium allocations *are*, however, efficient in addition to making no one worse of than she is at the endowment point — which implies that such allocations are on the portion of the contract curve that lies in the lens between the indifference curves that pass through the endowment point — i.e. in the core.

Exercise 16A.15

Can you think of other redistributions of oranges and bananas that would be "appropriate" for insuring that D is the competitive equilibrium outcome?

Answer: Any redistribution that results in a new endowment point that lies on the green budget (that is tangent to both indifference curves at D) would work. For instance, transferring 4 bananas and 3 oranges to me would leave my wife with 7 oranges and no bananas — which is the intercept of the green budget with her horizontal axis (oranges). Alternatively, you could give me 10 oranges and take 3 bananas away — leaving my wife with 7 bananas and no oranges. This would correspond to the allocation on the bananas axis. Of course there are many other possibilities in between these that would also work.

Exercise 16A.16

Does this production frontier exhibit increasing, decreasing or constant returns to scale? Is the marginal product of labor increasing, constant or decreasing?

Answer: This production frontier has decreasing returns to scale and diminishing marginal product of labor throughout.

Exercise 16A.17

As drawn, which of our usual assumptions about tastes — rationality, convexity, monotonicity, continuity — are violated?

Answer: The monotonicity assumption is violated — more is not better since I prefer less labor over more, all else being equal.

Exercise 16A.18

In votes on referenda on school vouchers, researchers have found that renters vote differently than homeowners. Consider a renter and homeowner in a bad public school district. Who do you think will be more likely to favor the introduction of school vouchers and who do you think will be more likely to be opposed?

Answer: The homeowner should be more likely to favor vouchers as demand for housing in bad public school districts will increase and thus drive up housing prices. Put differently, the homeowner in the bad public school district may favor private school vouchers because he expects it to increase the value of his home. The renter, on the other hand, would experience higher rents without the capital gain that comes from homeownership. Thus, the renter in the bad public school district is less likely to favor private school vouchers.

Exercise 16A.19

How do you think the elderly (who do not have children in school but who typically do own a home) will vote differently on school vouchers depending on whether they currently live in a good or bad public school district?

Answer: Demand for housing in bad public school districts will increase while demand for housing in good school districts will decrease — implying that housing prices will rise in bad school districts and fall in good school districts. The elderly would therefore be more likely to favor vouchers if they own a house in bad public school districts than if they owned a house in a good public school district.

Exercise 16A.20

If you were considering opening up a private school following the introduction of private school vouchers, would you be more likely to open your school in poor or in rich districts?

Answer: You would be more likely to open it in a poor district where public schools are likely to be relatively poor. This is because you will be able to attract no only parents who currently send their children to the local public school, but you will also be able to attract parents who currently live in better school districts but are willing to move to get better housing deals if they can send their children to private schools. (Some research suggests that the latter demand for private schools may be twice as large as the former.)

Exercise 16A.21

Suppose two different voucher proposals were on the table: The first proposal limits eligibility for vouchers only to families below the poverty line, while the second limits eligibility to those families who live in bad public school districts. Which policy is more likely to lead to general equilibrium effects in housing markets?

Answer: The second policy that targets geographically to bad (typically poor) public school districts will cause general equilibrium effects in housing markets because families — regardless of income — can qualify for the vouchers by moving to the targeted areas. The first policy targets primarily people who already tend to live in bad public school districts — and is therefore less likely to induce families to move (since middle income families could not qualify for the vouchers even if they moved.)

16B Solutions to Within-Chapter-Exercises for Part B

Exercise 16B.1

Can you see how the Edgeworth Box we drew in Section A contains all the allocations in this set?

Answer: The Edgeworth Box has length equal to the sum of the endowments of good 1 and height equal to the sum of the endowments of good 2 — i.e. length equal to $x_1^1 + x_1^2 = 13$ and height equal to $x_2^1 + x_2^2 = 10$ — just as is stated in the definition for FA. The set FA is then simply the set of all the possible ways of dividing 13 units of x_1 and 10 units of x_2 between two people — which is exactly what the Edgeworth Box is.

Exercise 16B.2

True or False: The Edgeworth Box represents a graphical technique that allows us to graph in a 2-dimensional picture points that lie in four dimensions.

Answer: This is true as is easily seen in the definition of the set FA which is in fact the definition of the Edgeworth Box. This definition has points that have four components — i.e. that lie in four dimensions. The clever trick used in graphing the Edgeworth Box is that we use two different two-dimensional axes to assign four values to each point — thus graphing a four dimensional space in 2 dimensions.

Exercise 16B.3

What are the reservation utilities for me and my wife in our example (given the utility functions specified above)?

Answer: The utility functions are $u^1(x_1, x_2) = x_1^{3/4} x_2^{1/4}$ and $u^2(x_1, x_2) = x_1^{1/4} x_2^{3/4}$, and our endowments are $(e_1^1, e_2^1) = (3, 6)$ and $(e_1^2, e_2^2) = (10, 4)$. Thus,

$$U^1 = 3^{3/4} 6^{1/4} \approx 3.5676 \text{ and } U^2 = 10^{1/4} 4^{3/4} = 5.0297. \tag{16B.3}$$

Exercise 16B.4

For the example of me and my wife, write the set of mutually beneficial allocations in the form of equation (16.6). Can you see that the lens-shaped area identified in the Edgeworth Box in Graph 16.2 is equivalent to this set?

Answer: The set would be

$$MB = \{(x_1^1, x_2^1, x_1^2, x_2^2) \in FA \mid \left(x_1^1\right)^{3/4} \left(x_2^1\right)^{1/4} \geq 3.5676 \text{ and } \left(x_1^2\right)^{1/4} \left(x_2^2\right)^{3/4} \geq 5.0297\}, \tag{16B.4.i}$$

where

$$FA = \left\{(x_1^1, x_2^1, x_1^2, x_2^2) \in \mathbb{R}_+^4 \mid x_1^1 + x_1^2 = 13 \text{ and } x_2^1 + x_2^2 = 10\right\}. \tag{16B.4.ii}$$

Exercise 16B.5

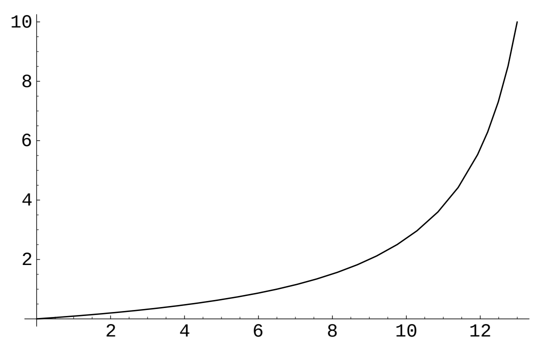
Can you see that no allocation in the set PE of an Edgeworth Box could have indifference curves pass through it in a way that creates a lens-shaped area between them?

Answer: Such a lens shape between indifference curves would imply a region that is feasible and contains bundles that lie above both indifference curve — i.e. there exist ways of making both people better off. If an allocation is such that we can make both people better off, it cannot lie in the PE set.

Exercise 16B.6

Verify that the contract curve we derived goes from one corner of the Edgeworth Box to the other.

Answer: Substituting $x_1^1 = 0$ into the equation for the contract curve, we get $x_2^1 = 0$ — i.e. the contract curve passes through the lower left hand corner of the Edgeworth Box at (0,0). Substituting $x_1^1 = 13$ into the equation for the contract curve, we get $x_2^1 = 10$ — i.e. the contract curve passes through (13,10), the upper left corner of the Edgeworth Box. The rest of the function is pictured in Exercise Graph 16B.6.



Exercise Graph 16B.6 : Contract Curve

Exercise 16B.7

A different way to find the contract curve would be to maximize my utility subject to the constraint that my wife's utility is held constant at utility level u^* and that her consumption bundle is whatever is left over after I have been given my consumption bundle. Put mathematically, this problem can be written as $\max_{x_1,x_2} x_1^{3/4}x_2^{1/4}$

s.t. $u^* = (13 - x_1)^{1/4}(10 - x_2)^{3/4}$ (where we drop the superscripts given that all variables refer to my consumption.) Demonstrate that this leads to the same solution as that derived in equation (16.9).

Answer: Setting up the Lagrange function for this problem, we get

$$\mathcal{L} = x_1^{3/4}x_2^{1/4} + \lambda\left(u^* - (13 - x_1)^{1/4}(10 - x_2)^{3/4}\right). \qquad (16B.7.i)$$

Taking first order conditions, we get

$$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{3x_2^{1/4}}{4x_1^{1/4}} + \lambda\frac{(10 - x_2)^{3/4}}{4(13 - x_1)^{3/4}} = 0 \text{ and } \frac{\partial \mathcal{L}}{\partial x_1} = \frac{x_1^{3/4}}{4x_2^{3/4}} + \lambda\frac{3(13 - x_1)^{1/4}}{4(10 - x_2)^{1/4}} = 0. \qquad (16B.7.ii)$$

Collecting the λ terms on one side and dividing the two first order conditions by one another, we then get

$$\frac{3x_2}{x_1} = \frac{(10 - x_2)}{3(13 - x_1)} \qquad (16B.7.iii)$$

which is identical to the condition in the text from which we derived the contract curve

$$x_2 = \frac{10x_1}{(117 - 8x_1)}. \qquad (16B.7.iv)$$

Exercise 16B.8

Verify that these are the correct demands for this problem.

Answer: Consider the utility function $u(x_1, x_2) = x_1^{\alpha}x_2^{(1-\alpha)}$ for an individual with endowments e_1 and e_2. At prices p_1 and p_2, the value of the endowment for this individual is $p_1e_1 + p_2e_2$, which allows us to write the utility maximization problem as

$$\max_{x_1,x_2} x_1^{\alpha}x_2^{(1-\alpha)} \text{ subject to } p_1x_1 + p_2x_2 = p_1e_1 + p_2e_2. \qquad (16B.8.i)$$

We can then write the Lagrange function as

$$\mathcal{L} = x_1^{\alpha}x_2^{(1-\alpha)} + \lambda\left(p_1e_1 + p_2e_2 - p_1x_1 - p_2x_2\right). \qquad (16B.8.ii)$$

The first two first order conditions are then

$$\frac{\partial \mathcal{L}}{\partial x_1} = \alpha x_1^{(\alpha-1)}x_2^{(1-\alpha)} - \lambda p_1 = 0 \text{ and } \frac{\partial \mathcal{L}}{\partial x_2} = (1-\alpha)x_1^{\alpha}x_2^{-\alpha} - \lambda p_2 = 0, \qquad (16B.8.iii)$$

which solve to give us

$$x_2 = \frac{(1-\alpha)p_1x_1}{\alpha p_2}. \qquad (16B.8.iv)$$

Substituting this into the budget constraint and solving for x_1, we then get

$$x_1 = \frac{\alpha(p_1e_1 + p_2e_2)}{p_1}, \quad (16B.8.v)$$

and substituting this back into (16B.8.iv), we get

$$x_2 = \frac{(1-\alpha)(p_1e_1 + p_2e_2)}{p_2}. \quad (16B.8.vi)$$

For me, $\alpha = 3/4$ and $(e_1, e_2) = (3,6)$ whereas for my wife $\alpha = 1/4$ and $(e_1, e_2) = (10,4)$. Substituting these into equations (16B.8.v) and (16B.8.vi), we can verify that these become equivalent to the equations in the text.

Exercise 16B.9

Write down the equilibrium condition in the x_2 market (from the second equation in expression (16.14)) using the appropriate expressions from (16.15) and solve for the equilibrium price ratio. You should get the same answer.

Answer: Equilibrium in the x_2 market is reached when

$$\frac{(3p_1 + 6p_2)}{4p_2} + \frac{3(10p_1 + 4p_2)}{4p_2} = 6 + 4. \quad (16B.9)$$

Solving this for p_2, we again get $p_2 = 3p_1/2$.

Exercise 16B.10

Demonstrate that the same equilibrium allocation of goods will arise if $p_1 = 2$ and p_2 is one and a half times p_1 — i.e. $p_2 = 3$.

Answer: Plugging the prices into the demand equations for the two individuals, we get

$$x_1^1(2,3)) = \frac{3(3(2) + 6(3))}{4(2)} = 9 \text{ and } x_2^1(2,3) = \frac{(3(2) + 6(3))}{4(3)} = 2$$
$$x_1^2(2,3) = \frac{(10(2) + 4(3))}{4(2)} = 4 \text{ and } x_2^2(2,3) = \frac{3(10(2) + 4(3))}{4(3)} = 8. \quad (16B.10)$$

Exercise 16B.11

Can you demonstrate that the equilibrium allocation we derived for me and my wife lies in the core that we defined in equation (16.12)?

Answer: To do this, we have to demonstrate that the allocation $(x_1^1, x_2^1, x_1^2, x_2^2) = (9,2,4,8)$ lies in the PE set as well as the MB set. To demonstrate that it lies in the PE set, we have to show that

$$x_2^1 = \frac{10x_1^1}{(117 - 8x_1^1)}, x_1^2 = 13 - x_1^1 \text{ and } x_2^2 = 10 - x_2^1. \quad (16B.11.i)$$

Plugging $x_1^1 = 9$ into the first two of these equations, we get

$$x_2^1 = \frac{10(9)}{(117-8(9))} = 2 \text{ and } x_1^2 = 13-9 = 4, \quad (16B.11.ii)$$

and plugging $x_2^1 = 2$ into the last of the three equations, we get $x_2^2 = 10-2 = 8$. Thus, the allocation $(x_1^1, x_2^1, x_1^2, x_2^2) = (9,2,4,8)$ indeed lies in the PE set. To verify that it also lies in the MB set, we just need to verify that the utility for each person lies above their reservation utilities of $U^1 = 3.57$ and $U^2 = 5.03$. Evaluating utility for each of the individuals at the allocation $(x_1^1, x_2^1, x_1^2, x_2^2) = (9,2,4,8)$, we get

$$u^1(9,2) = 9^{3/4}2^{1/4} \approx 6.18 \text{ and } u^2(4,8) = 4^{1/4}8^{3/4} \approx 6.73. \quad (16B.11.iii)$$

Exercise 16B.12

In our proof, we began by assuming that there exists a $(y_1^1, y_2^1, y_1^2, y_2^2)$ that is strictly preferred by everyone to $(x_1^1, x_2^1, x_1^2, x_2^2)$ and showed that there cannot be such an allocation within this economy. Can you see how the same logic also goes through if we assume that there exists an allocation $(z_1^1, z_2^1, z_1^2, z_2^2)$ that is strictly preferred by one of the individuals while leaving the other indifferent to $(x_1^1, x_2^1, x_1^2, x_2^2)$?

Answer: Suppose the allocation $(z_1^1, z_2^1, z_1^2, z_2^2)$ is strictly preferred by individual 1 to the allocation $(x_1^1, x_2^1, x_1^2, x_2^2)$ and that individual 2 is indifferent between the two allocations. Then (z_1^1, z_2^1) could not have been affordable for individual 1 under the equilibrium prices (p_1, p_2) (or else he would have chosen it) — i.e.

$$p_1 z_1^1 + p_2 z_2^1 > p_1 x_1^1 + p_2 x_2^1. \quad (16B.12.i)$$

If individual 2 is indifferent between $(z_1^1, z_2^1, z_1^2, z_2^2)$ and $(x_1^1, x_2^1, x_1^2, x_2^2)$, then we can conclude that the bundle (z_1^2, z_2^2) could not have been cheaper at the equilibrium prices than (x_1^2, x_2^2). This is because if (z_1^2, z_2^2) actually were to lie strictly inside the budget set (and not on the budget line), the indifference curve that contains (z_1^2, z_2^2) would cut the budget line — leaving some bundles on higher indifference curves that still lie on the budget constraint — which in turn implies that the optimal choice (x_1^2, x_2^2) could not have been one that yields the same utility as (z_1^2, z_2^2). Concluding that (z_1^2, z_2^2) could not have been cheaper at the equilibrium prices than (x_1^2, x_2^2) is then the same as concluding that

$$p_1 z_1^2 + p_2 z_2^2 \geq p_1 x_1^2 + p_2 x_2^2 \quad (16B.12.ii)$$

Adding equations (16B.12.i) and (16B.12.ii), we get

$$p_1(z_1^1 + z_1^2) + p_2(z_2^1 + z_2^2) > p_1(x_1^1 + x_1^2) + p_2(x_2^1 + x_2^2) \quad (16B.12.iii)$$

just as in the proof in the text (where we assumed that there exists a bundle which is strictly preferred by *both* individuals). The rest of the proof is then identical to the one in the text — with the ultimate contradiction that the allocation $(z_1^1, z_2^1, z_1^2, z_2^2)$ is not feasible.

Exercise 16B.13

Can you demonstrate that this is in fact the case for an N-person, M-good economy?

Answer: The proof is virtually identical to what we have done for the 2-person case. We begin with the equilibrium defined by prices $(p_1, p_2, ..., p_M)$ and the allocation

$$(x_1^1, x_2^1, ..., x_M^1, x_1^2, x_2^2, ..., x_M^2, ..., x_1^N, x_2^N, ...x_M^N) \in \mathbb{R}_+^{NM}. \quad (16B.13.i)$$

To say that this allocation is an equilibrium allocation under prices $(p_1, p_2, ..., p_M)$ is the same as saying that each individual n, taken his/her endowment and prices as given, in fact maximizes utility by choosing his/her portion of the allocation — i.e. $(x_1^n, x_2^n, ..., x_M^n)$. We are trying to prove that this equilibrium allocation must be efficient — i.e. there does not exist an alternative allocation

$$(y_1^1, y_2^1, ..., y_M^1, y_1^2, y_2^2, ..., y_M^2, ..., y_1^N, y_2^N, ...y_M^N) \in \mathbb{R}_+^{NM} \quad (16B.13.ii)$$

that is strictly preferred by at least one person and viewed as at least as good by all others. So suppose that such an allocation *did* exist, and suppose we identify the person for whom this allocation is strictly better as person 1. We then know that $(y_1^1, y_2^1, ..., y_M^1)$ could not have been affordable for person 1 at prices (p_1, p_2) (or else it would have been chosen) — i.e. we know

$$p_1 y_1^1 + p_2 y_2^1 + ... + p_M y_M^1 > p_1 x_1^1 + p_2 x_2^1 + ... + p_M x_M^1. \quad (16B.13.iii)$$

Similarly — and for reasons analogous to those explained in the previous exercise — it must be the case that

$$p_1 y_1^n + p_2 y_2^n + ... + p_M y_M^n \geq p_1 x_1^n + p_2 x_2^n + ... + p_M x_M^n \text{ for all } n = 2, 3, ..., N. \quad (16B.13.iv)$$

Adding equation (16B.13.iii) to the $(N-1)$ equations in expression (16B.13.iv), we get

$$p_1 \sum_{n=1}^N y_1^n + p_2 \sum_{n=1}^N y_2^n + ... + p_M \sum_{n=1}^N y_M^n > p_1 \sum_{n=1}^N x_1^n + p_2 \sum_{n=1}^N x_2^n + ... + p_M \sum_{n=1}^N x_M^n. \quad (16B.13.v)$$

Walras' Law tells us that the right hand side is equal to the value of the endowments — which implies we can re-write this as

$$p_1 \sum_{n=1}^N y_1^n + p_2 \sum_{n=1}^N y_2^n + ... + p_M \sum_{n=1}^N y_M^n > p_1 \sum_{n=1}^N e_1^n + p_2 \sum_{n=1}^N e_2^n + ... + p_M \sum_{n=1}^N e_M^n \quad (16B.13.vi)$$

or as

$$p_1\left(\sum_{n=1}^{N} y_1^n - \sum_{n=1}^{N} e_1^n\right) + p_2\left(\sum_{n=1}^{N} y_2^n - \sum_{n=1}^{N} e_2^n\right) + ... + p_M\left(\sum_{n=1}^{N} y_M^n - \sum_{n=1}^{N} e_M^n\right) > 0. \quad (16B.13.vii)$$

But, since all prices are positive, this implies that at least one of the bracketed terms in equation (16B.13.vii) must be greater than zero — i.e.

$$\sum_{n=1}^{N} y_m^n - \sum_{n=1}^{N} e_m^n > 0 \text{ for some good } m. \quad (16B.13.viii)$$

But that implies that the y allocation is not feasible — i.e. the assumption of inefficiency of the equilibrium allocation has led to a contradiction. Thus, the equilibrium allocation is efficient.

Exercise 16B.14

Verify the results in equation (16.27).

Answer: We can solve this either by setting up the Lagrange function or simply by substituting the constraint into the objective. Doing the former, we get the Lagrange function

$$\mathscr{L} = x^{\alpha}(L-\ell)^{(1-\alpha)} + \lambda\left(x - A\ell^{\beta}\right). \quad (16B.14.i)$$

The first two first order conditions are then

$$\frac{\partial \mathscr{L}}{\partial x} = \alpha x^{(\alpha-1)}(L-\ell)^{(1-\alpha)} + \lambda = 0 \text{ and}$$
$$\frac{\partial \mathscr{L}}{\partial \ell} = -(1-\alpha)x^{\alpha}(L-\ell)^{-\alpha} - \lambda\beta A\ell^{(\beta-1)} = 0. \quad (16B.14.ii)$$

which can also be written as

$$\alpha x^{(\alpha-1)}(L-\ell)^{(1-\alpha)} = -\lambda \text{ and } (1-\alpha)x^{\alpha}(L-\ell)^{-\alpha} = -\lambda\beta A\ell^{(\beta-1)}. \quad (16B.14.iii)$$

Dividing the two equations in expression (16B.14.iii) by one another and solving for x, we get

$$x = \frac{\alpha\beta A(L-\ell)\ell^{(\beta-1)}}{(1-\alpha)}, \quad (16B.14.iv)$$

and substituting this into the constraint from the optimization problem, we get

$$A\ell^{\beta} = \frac{\alpha\beta A(L-\ell)\ell^{(\beta-1)}}{(1-\alpha)} \quad (16B.14.v)$$

which we can in turn solve for ℓ to verify

$$\ell = \frac{\alpha\beta L}{1-\alpha(1-\beta)}. \quad (16B.14.vi)$$

Plugging this into (16B.14.iv) and solving for x, we furthermore verify the text's expression for x. This involves a bit of algebra — the quickest way to get there is as follows: Begin by substituting (16B.14.vi) into $(L-\ell)$ to get

$$(L-\ell) = L - \frac{\alpha\beta L}{1-\alpha(1-\beta)} = \frac{[1-\alpha(1-\beta)]L - \alpha\beta L}{1-\alpha(1-\beta)} = \frac{(1-\alpha)L}{1-\alpha(1-\beta)}. \quad (16B.14.vii)$$

Substituting this for $(L-\ell)$ into equation (16B.14.iv), we get

$$x = \frac{\alpha\beta A\ell^{(\beta-1)}}{(1-\alpha)}\left(\frac{(1-\alpha)L}{1-\alpha(1-\beta)}\right) = \frac{\alpha\beta AL\ell^{(\beta-1)}}{1-\alpha(1-\beta)}. \quad (16B.14.viii)$$

Substituting (16B.14.vi) into this for ℓ, we then get

$$x = \frac{\alpha\beta AL}{1-\alpha(1-\beta)}\left(\frac{\alpha\beta L}{1-\alpha(1-\beta)}\right)^{(\beta-1)} = A\left(\frac{\alpha\beta L}{1-\alpha(1-\beta)}\right)^{\beta}. \quad (16B.14.ix)$$

Exercise 16B.15

Verify that this is the correct solution.

Answer: Taking the first derivative of $(pA\ell^{\beta} - w\ell)$ (with respect to ℓ) and setting it to zero, we get

$$\beta pA\ell^{(\beta-1)} - w = 0, \quad (16B.15.i)$$

and solving for ℓ, we get

$$\ell = \left(\frac{w}{\beta pA}\right)^{1/(\beta-1)} = \left(\frac{\beta pA}{w}\right)^{1/(1-\beta)}. \quad (16B.15.ii)$$

Substituting this back into the production function, we get

$$x = A\left(\left(\frac{\beta pA}{w}\right)^{1/(1-\beta)}\right)^{\beta} = A\left(\frac{\beta pA}{w}\right)^{\beta/(1-\beta)} \quad (16B.15.iii)$$

Exercise 16B.16

Verify that these solutions for labor supply and banana demand are correct.

Answer: The Lagrange function for the maximization problem is

$$\mathscr{L} = x^{\alpha}(L-\ell)^{(1-\alpha)} + \lambda(w\ell + \pi(w,p) - px) \quad (16B.16.i)$$

giving rise to first order conditions

$$\frac{\partial \mathcal{L}}{\partial x} = \alpha x^{(\alpha-1)}(L-\ell)^{(1-\alpha)} - \lambda p = 0 \text{ and}$$
$$\frac{\partial \mathcal{L}}{\partial \ell} = -(1-\alpha)x^{\alpha}(L-\ell)^{-\alpha} + \lambda w = 0. \tag{16B.16.ii}$$

Rearranging these with the λ terms on one side, dividing the equations by each other and solving for x, we get

$$x = \frac{\alpha(L-\ell)w}{(1-\alpha)p}. \tag{16B.16.iii}$$

Substituting this into the constraint $w\ell + \pi(w,p) = px$ and solving for ℓ, we then get

$$\ell = \alpha L - \frac{(1-\alpha)\pi(w,p)}{w}, \tag{16B.16.iv}$$

and substituting this back into equation (16B.16.iii), we get

$$x = \frac{\alpha}{p}\left(wL + \pi(w,p)\right). \tag{16B.16.v}$$

These labor supply and banana demand equations can then be expanded by simply replacing $\pi(w,p)$ with the profit function

$$\pi(w,p) = (1-\beta)(Ap)^{1/(1-\beta)}\left(\frac{\beta}{w}\right)^{\beta/(1-\beta)} \tag{16B.16.vi}$$

to give us

$$\ell = \alpha L - \frac{(1-\alpha)(1-\beta)}{\beta}\left(\frac{\beta p A}{w}\right)^{1/(1-\beta)}$$
$$x = \frac{\alpha w}{p}\left(L + \frac{(1-\beta)}{\beta}\left(\frac{\beta p A}{w}\right)^{1/(1-\beta)}\right). \tag{16B.16.vii}$$

Exercise 16B.17

Verify that the same equilibrium relationship between prices and wages arises by solving $x^S = x^D$.

Answer: We need to solve

$$x^S(w,p) = A\left(\frac{\beta p A}{w}\right)^{\beta/(1-\beta)} = \frac{\alpha w}{p}\left(L + \frac{(1-\beta)}{\beta}\left(\frac{\beta p A}{w}\right)^{1/(1-\beta)}\right) = x^D(w,p). \tag{16B.17.i}$$

The problem is simplified if we re-write the right hand side as

$$x^D(w,p) = \frac{\alpha w}{p}\left(L + \frac{(1-\beta)}{\beta}\left(\frac{\beta p A}{w}\right)^{1/(1-\beta)}\right) = \\ = \frac{\alpha w}{p}L + \alpha(1-\beta)A\left(\frac{\beta p A}{w}\right)^{\beta/(1-\beta)}. \qquad (16B.17.ii)$$

Equation (16B.17.i) then becomes

$$A\left(\frac{\beta p A}{w}\right)^{\beta/(1-\beta)} = \frac{\alpha w}{p}L + \alpha(1-\beta)A\left(\frac{\beta p A}{w}\right)^{\beta/(1-\beta)} \qquad (16B.17.iii)$$

which can be re-written as

$$w^{1/(1-\beta)} = \frac{A(\beta p A)^{\beta/(1-\beta)}(1-\alpha(1-\beta))p}{\alpha L} = \\ = \frac{(A\beta)^{1/(1-\beta)}(1-\alpha(1-\beta))p^{1/(1-\beta)}}{\alpha\beta L}. \qquad (16B.17.iv)$$

Taking both the left and right hand sides to the power $(1-\beta)$ and re-arranging terms slightly, we then arrive at

$$w = A\beta\left(\frac{1-\alpha(1-\beta)}{\alpha\beta L}\right)^{(1-\beta)} p. \qquad (16B.17.v)$$

Exercise 16B.18

Can you tell from the graph of an equilibrium in Graph 16.10 that any combination of w and p that satisfies a particular ratio will generate the same equilibrium in the labor and banana markets?

Answer: In panel (b) of the graph we see that the equilibrium point C occurs at the tangency of the indifference curve u^* and the production frontier. The input and output prices that support this as an equilibrium are those that result in the isoprofit/budget line with slope w^*/p^* such that the line is tangent to both the indifference curve and the production frontier at C — and what matters for this is the *ratio* of the prices. (Economic theorists will often call the grey line in the graph a "separating hyperplane" — i.e. the "hyperplane" that separates the production frontier and indifference curve at precisely one point.)

Exercise 16B.19

Can you tell from the graph of an equilibrium in Graph 16.10 whether profit will be affected by different choices of w and p that satisfy the equilibrium ratio? Verify whether your intuition holds mathematically.

Answer: Suppose w^* and p^* satisfy the equilibrium ratio — resulting in profit π^*. The intercept of the grey isoprofit/budget line in panel (b) of the graph is π^*/p^*.

Now suppose that we increase both the wage and the output price by a factor t, resulting in profit π'. The intercept of the grey line that separates the indifference curve and production frontier in the graph does not change, which implies $\pi'/(tp^*) = \pi^*/p^*$ — which can only hold if $\pi' = t\pi^*$. Thus, as the input and output prices are scaled up and down, profit is scaled with it at the same rate. We can see this from the profit function by illustrating that $\pi(tw, tp) = t\pi(w, p)$ — i.e. by showing that the profit function is homogeneous of degree 1:

$$\begin{aligned}\pi(tw, tp) &= (1-\beta)(Atp)^{1/(1-\beta)}\left(\frac{\beta}{tw}\right)^{\beta/(1-\beta)} = \\ &= t(1-\beta)(Ap)^{1/(1-\beta)}\left(\frac{\beta}{w}\right)^{\beta/(1-\beta)} = t\pi(w,p).\end{aligned} \tag{16B.19}$$

Exercise 16B.20

Demonstrate that the equilibrium banana consumption (and production) is equal to the optimal level of banana consumption.

Answer: When first considering the optimal decision for Robinson Crusoe in the text, we derived the pareto optimal consumption level for bananas as

$$x = A\left(\frac{\alpha\beta L}{1-\alpha(1-\beta)}\right)^{\beta}. \tag{16B.20.i}$$

The *equilibrium* level can be determined by plugging our expression for the equilibrium wage w^* (in terms of p) into either the output supply or demand equation in the text. Using the output supply equation, we get

$$\begin{aligned} x^S &= A\left(\frac{\beta pA}{w^*}\right)^{\beta/(1-\beta)} = A\left(\beta pA\right)^{\beta/(1-\beta)}\left(\frac{1}{\beta Ap}\right)^{\beta/(1-\beta)}\left(\frac{\alpha\beta L}{1-\alpha(1-\beta)}\right)^{\beta} = \\ &= A\left(\frac{\alpha\beta L}{1-\alpha(1-\beta)}\right)^{\beta}.\end{aligned} \tag{16B.20.ii}$$

Substituting our expression for w^* into x^D gives the same answer.

Exercise 16B.21

Why is there no coalition to block A in the 2-person version of this economy?

Answer: In the 2-person economy, there are only three coalitions: (1) a coalition of both (and thus *all*) the individuals in the economy; (2) a coalition composed of a single individual of type 1; and (3) a coalition of a single individual of type 2. At A, "coalition" (2) is just as well off as at E while "coalition" (3) is strictly better off. Thus, neither "coalition" (2) nor "coalition" (3) would block the allocation A because neither can do better by splitting away from the economy. That leaves only coalition (1) — composed of both type 1 and type 2. At A, their indifference curves are tangent to one another — which implies there is no way we can make either of them better off without making the other worse off. Thus, coalition (1) has no reason to block A. As a result, none of the coalitions in the economy can do better by splitting off than the individuals in the coalitions do at A. The difference in the 4-person economy is that there are now more coalitions to consider — including 3-person coalitions composed of two of one type and one of the other.

Exercise 16B.22

Can you demonstrate that a coalition of two of the "type 2" individuals with on "type 1" individual will block the allocation B?

Answer: Such a coalition will block B for exactly the same reasons that the coalition of two type 1 individuals with 1 type 2 individual blocks A. Suppose B is proposed as the allocation for the economy — with each type getting what the axes for that type indicate at B. Now suppose the coalition of two type 2 and one type 1 individual splits off with their endowments. In the Edgeworth Box, draw a line that goes through E and B. With both types starting at E, a trade in which the two type 2 consumers give up 1 unit of x_2 results in the one type 1 individual getting *two* units of x_2. Thus, if the individuals in the coalition trade along the line that passes through E and B, the one type 1 individual moves down the line twice as quickly as the two type 2 individuals do. Thus, the coalition can trade among itself and end up with an allocation for the type 1 consumer that lies to the southeast of B and an allocation for the type 2 consumers that lies to the northwest of B — with both consumer types ending up on an indifference curve that is higher than the one that passes through B. Thus, the coalition can do better by splitting off the economy with its endowment than their members do under the allocation B — which implies that the coalition will "block" B from being implemented.

Exercise 16B.23

Why must the distance between E and D' be twice the distance from E to C'?

Answer: This is because there are two type 1 individuals and one type 2 individual — which means that everything a type 1 individual gives up is received twice by the single type 2 — and anything that the type 2 individual gives up in exchange is split between the two type 1 people.

Exercise 16B.24

Demonstrate that the competitive equilibrium allocation must lie in the core of the replicated exchange economy.

Answer: An equilibrium allocation X has the feature that the indifference curves for type 1 and type 2 consumers are tangent at X in the Edgeworth Box — with the line that passes through E and X "separating" the two indifference curves at X. If any coalition splits off, it will trade along a line that passes through E — and this line is either shallower or steeper than the one that passes through E and X. Suppose it is shallower. In that case all points on that line lie "below" the equilibrium indifference curve for type 2 consumers — and thus there is no coalition that includes a type 2 individual which could in fact do better for type 2. Suppose instead that the line is steeper. Then it lies entirely "below" the equilibrium indifference curve for type 1 consumers — implying that no type 1 consumer could be part of a blocking coalition. Thus, neither consumer could be part of a blocking coalition — which implies that the equilibrium allocation X lies in the core.

16C Solutions to Odd Numbered End-of-Chapter Exercises

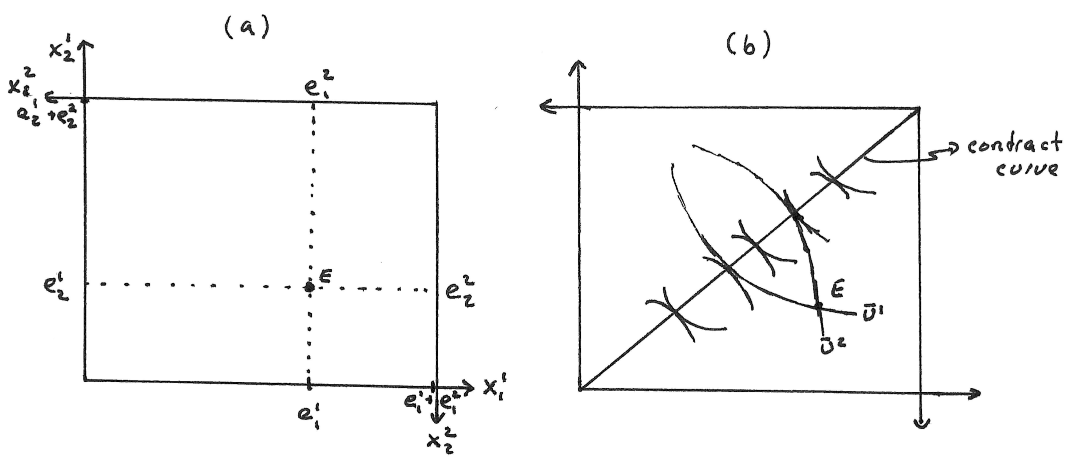
Exercise 16.1

Consider a 2-person/2-good exchange economy in which person 1 is endowed with (e_1^1, e_2^1) and person 2 is endowed with (e_1^2, e_2^2) of the goods x_1 and x_2.

A: *Suppose that tastes are homothetic for both individuals.*

(a) *Draw the Edgeworth Box for this economy, indicating on each axis the dimensions of the box.*

Answer: This is illustrated in panel (a) of Exercise Graph 16.1(1) where the width of the box is $(e_1^1 + e_1^2)$ and the height is $(e_2^1 + e_2^2)$.



Exercise Graph 16.1(1) : Contract Curve with Homothetic Tastes

(b) *Suppose that the two individuals have identical tastes. Illustrate the contract curve — i.e. the set of all efficient allocations of the two goods.*

Answer: Homothetic tastes have the characteristic that the *MRS* is the same along any ray from the origin. Consider the ray that passes from the lower left to the upper right corners of the box — i.e. from the origin for person 1 to the origin for person 2. If tastes are homothetic for each of the two individuals, this means that, for each individual, it is the case that the *MRS* is constant along this ray. And if their tastes are identical, then their *MRS*'s are the same along that ray — i.e. on each point of the ray, the indifference curves that pass through that point are tangent to one another. Since the contract curve is the set of allocations where the indifference curves are tangent, this ray is then the contract curve. It is depicted in panel (b) of Exercise Graph 16.1(1).

(c) True or False*: Identical tastes in the Edgeworth Box imply that there are no mutually beneficial trades.*

Answer: This is false. In panel (b) of Exercise Graph 16.1(1), for instance, the indifference curves $\overline{u}^1$ and $\overline{u}^2$ contain the endowment bundle E — with allocations inside the lens created by these indifference curves representing mutually beneficial trades. The only way that there are no mutually beneficial trades when both individuals have identical homothetic tastes is if the endowment bundle falls on the contract curve — i.e. on the line connecting the origins for the two individuals.

(d) *Now suppose that the two individuals have different (but still homothetic) tastes.* True or False*: The contract curve will lie to one side of the line that connects the lower left and upper right corners of the Edgeworth Box — i.e. it will never cross this line inside the Edgeworth Box.*

Answer: This is true (almost). If the two individuals' tastes are not identical, then their indifference curves are not likely to be tangent on the line connecting the lower left and upper right corners of the box. Take one point on that line — it is likely the case that the indifference curve for person 1 is steeper or shallower than that for individual 2 at this point. Suppose first that individual 1's indifference curve is shallower. Then the two indifference curves form a lens shape — with the entire area of the lens lying *above* the line connecting the corners of the box. Since the slopes of the indifference curves are constant along this line, this same lens shape will appear above the line for *any* allocation on the line. This implies that the tangencies of indifference curves (which form the contract curve) must also lie *above* the line (because these tangencies will be found within the lens shapes formed from indifference curves that cross on the line.) The reverse will be true if individual 1's indifference curve is steeper along the line than indifference curves for individual 2 — with the entire contract curve now lying *below* the line. The reason the answer is true (*almost*) is that it is still possible that the marginal rates of substitution for the two individuals are in fact equal along the line connecting the lower left and upper right corners of the box. For instance, it might be that the tastes have different degrees of substitutability (and are therefore different) but still have the same marginal rates of substitution on that line. In that case, the contract curve lies on the line connecting the lower left and upper right corners. Thus, homothetic tastes imply that the contract curve lies either on the line connecting the corners or all to one side of that line.

B: *Suppose that the tastes for individuals 1 and 2 can be described by the utility functions $u^1 = x_1^{\alpha}x_2^{(1-\alpha)}$ and $u^2 = x_1^{\beta}x_2^{(1-\beta)}$ (where α and β both lie between 0 and 1). Some of the questions below are notationally a little easier to keep track off if you also denote $E_1 = e_1^1 + e_1^2$ as the economy's endowment of x_1 and $E_2 = e_2^1 + e_2^2$ as the economy's endowment of x_2.*

(a) *Let $\overline{x}_1$ denote the allocation of x_1 to individual 1, and let $\overline{x}_2$ denote the allocation of x_2 to individual 1. Then use the fact that the remainder of the economy's endowment is allocated to individual 2 to denote individual 2's allocation as $(E_1 - \overline{x}_1)$ and $(E_2 - \overline{x}_2)$ for x_1 and x_2 respectively. Derive the contract curve in the form $\overline{x}_2 = x_2(\overline{x}_1)$ — i.e. with the allocation of x_2 to person 1 as a function of the allocation of x_1 to person 1.*

<u>Answer</u>: You can derive this either by setting the MRS for individual 1 equal to the MRS for individual 2 — or you can solve the problem

$$\max_{\overline{x}_1,\overline{x}_2} \overline{x}_1^{\alpha}\overline{x}_2^{(1-\alpha)} \text{ subject to } u^2 = (E_1 - \overline{x}_1)^{\beta}(E_2 - \overline{x}_2)^{(1-\beta)} \tag{16.1.i}$$

where person 1's utility is maximized subject to getting person 2 to a particular indifference curve u^2. Either way, you will get to the point where you have an expression that sets the marginal rates of substitution equal to one another — i.e.

$$\begin{aligned}\frac{\partial u^1(\overline{x}_1,\overline{x}_2)/\partial x_1}{\partial u^1(\overline{x}_1,\overline{x}_2)/\partial x_2} = \frac{\alpha\overline{x}_2}{(1-\alpha)\overline{x}_1} = \frac{\beta(E_2-\overline{x}_2)}{(1-\beta)(E_1-\overline{x}_1)} = \\ = \frac{\partial u^2((E_1-\overline{x}_1),(E_2-\overline{x}_2))/\partial x_1}{\partial u^2((E_1-\overline{x}_1),(E_2-\overline{x}_2))/\partial x_2}.\end{aligned} \tag{16.1.ii}$$

Solving the middle of this expression for $\overline{x}_2$, we then get the contract curve

$$x_2(\overline{x}_1) = \frac{(1-\alpha)\beta E_2\overline{x}_1}{\alpha(1-\beta)E_1 + (\beta-\alpha)\overline{x}_1}. \tag{16.1.iii}$$

(b) *Simplify your expression under the assumption that tastes are identical — i.e. $\alpha = \beta$. What shape and location of the contract curve in the Edgeworth Box does this imply?*

<u>Answer</u>: Replacing β with α, the expression then simplifies to

$$x_2(\overline{x}_1) = \frac{(1-\alpha)\alpha E_2\overline{x}_1}{\alpha(1-\alpha)E_1 + (\alpha-\alpha)\overline{x}_1} = \frac{E_2}{E_1}\overline{x}_1. \tag{16.1.iv}$$

This is simply the equation of a line with zero vertical intercept and slope E_2/E_1 — which is the slope of the ray that passes from the lower left to the upper right corner of the Edgeworth Box. Thus, when tastes are identical, we get that the contract curve is the line that connects the origins for the two individuals in the Edgeworth Box — exactly as we did for homothetic tastes in part A of the question (and as depicted in panel (b) of Exercise Graph 16.1(1).)

(c) *Next, suppose that $\alpha \neq \beta$. Verify that the contract curve extends from the lower left to the upper right corner of the Edgeworth Box.*

Answer: Evaluating the contract curve equation (16.1.iii) at $\overline{x}_1 = 0$, we get $x_2(0) = 0$ — i.e. the contract curve passes through the lower left hand corner of the Edgeworth Box. Evaluating the contract curve at $\overline{x}_1 = E_1$, we get

$$x_2(E_1) = \frac{(1-\alpha)\beta E_2 E_1}{\alpha(1-\beta)E_1 + (\beta-\alpha)E_1} = \frac{(1-\alpha)\beta E_2 E_1}{(1-\alpha)\beta E_1} = E_2; \qquad (16.1.v)$$

i.e. the contract curve passes through the upper right corner of the box where individual 1 gets all the goods in the economy.

(d) *Consider the slopes of the contract curve when $\overline{x}_1 = 0$ and when $\overline{x}_1 = E_1$. How do they compare to the slope of the line connecting the lower left and upper right corners of the Edgeworth Box if $\alpha > \beta$? What if $\alpha < \beta$?*

Answer: The slope of the contract curve is the derivative of equation (16.1.iii) with respect to x_1 —

$$\begin{aligned}\frac{\partial x_2(\overline{x}_1)}{\partial x_1} &= \frac{(1-\alpha)\beta E_2}{\alpha(1-\beta)E_1 + (\beta-\alpha)\overline{x}_1} - \frac{(\beta-\alpha)(1-\alpha)\beta E_2 \overline{x}_1}{\left[\alpha(1-\beta)E_1 + (\beta-\alpha)\overline{x}_1\right]^2} = \\ &= \frac{\alpha\beta(1-\alpha)(1-\beta)E_1 E_2}{\left[\alpha(1-\beta)E_1 + (\beta-\alpha)\overline{x}_1\right]^2}.\end{aligned} \qquad (16.1.vi)$$

Evaluated at $\overline{x}_1 = 0$ and at $\overline{x}_1 = E_1$, we get

$$\frac{\partial x_2(0)}{\partial x_1} = \frac{\beta(1-\alpha)E_2}{\alpha(1-\beta)E_1} \text{ and } \frac{\partial x_2(E_1)}{\partial x_1} = \frac{\alpha(1-\beta)E_2}{\beta(1-\alpha)E_1}. \qquad (16.1.vii)$$

The slope of the line connecting the two corners of the Edgeworth box is E_2/E_1. If $\alpha = \beta$, both derivatives in expression (16.1.vii) are equal to E_2/E_1 — i.e. the slope of the contract curve is exactly the slope of the line connecting the corners (as we already concluded previously). If $\alpha > \beta$, then

$$\frac{\beta(1-\alpha)}{\alpha(1-\beta)} < 1 \text{ and } \frac{\alpha(1-\beta)}{\beta(1-\alpha)} > 1 \qquad (16.1.viii)$$

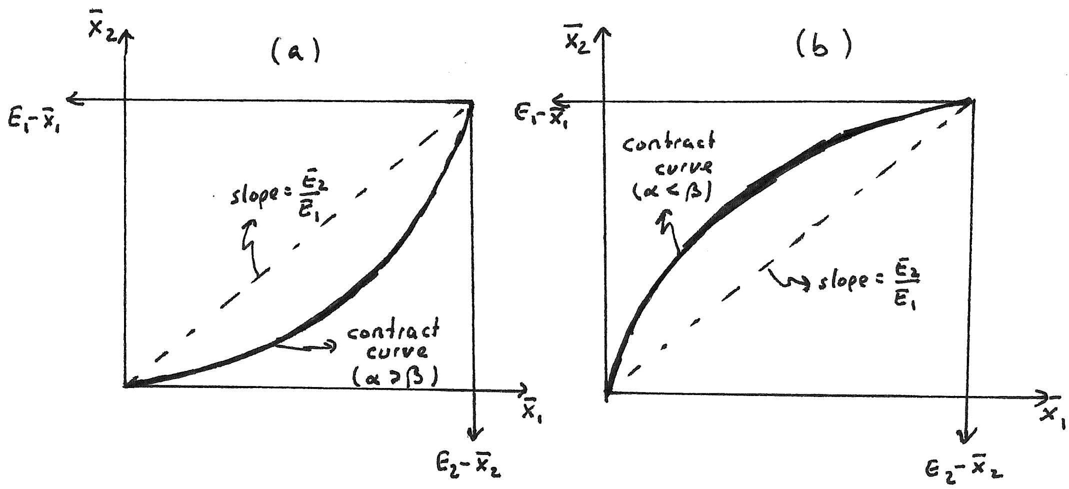
implying that

$$\frac{\partial x_2(0)}{\partial x_1} < \frac{E_2}{E_1} \text{ and } \frac{\partial x_2(E_1)}{\partial x_1} > \frac{E_2}{E_1}. \tag{16.1.ix}$$

The reverse relationship holds when $\alpha < \beta$.

(e) *Using what you have concluded, graph the shape of the contract curve for the case $\alpha > \beta$ and for the case when $\alpha < \beta$?*

The contract curves consistent with these relationships are graphed in Exercise Graph 16.1(2).



Exercise Graph 16.1(2) : Contract Curves when (a) $\alpha > \beta$ and when (b) $\alpha < \beta$

(f) *Suppose that the utility function for the two individuals instead took the more general constant elasticity of substitution form $u = (\alpha x_1 + (1-\alpha)x_2)^{-1/\rho}$. If the tastes for the two individuals are identical, does your answer to part (b) change?*

Answer: No, the answer does not change. The *MRS* for this utility function (derived in Chapter 5) is

$$MRS = -\left(\frac{\alpha}{(1-\alpha)}\right)\left(\frac{x_2}{x_1}\right)^{\rho+1}. \tag{16.1.x}$$

Using our notation and setting the *MRS*'s equal to each other for the two individuals, we then get

$$\left(\frac{\alpha}{(1-\alpha)}\right)\left(\frac{\overline{x}_2}{\overline{x}_1}\right)^{\rho+1} = \left(\frac{\alpha}{(1-\alpha)}\right)\left(\frac{(E_2-\overline{x}_2)}{(E_1-\overline{x}_1)}\right)^{\rho+1} \tag{16.1.xi}$$

which we can solve for $\overline{x}_2$ to get the contract curve

$$x_2(\overline{x}_1) = \left(\frac{E_2}{E_1}\right)\overline{x}_1; \tag{16.1.xii}$$

i.e. the contract curve again has zero vertical intercept and slope E_2/E_1, the slope of the ray that connects the two corners of the Edgeworth Box.

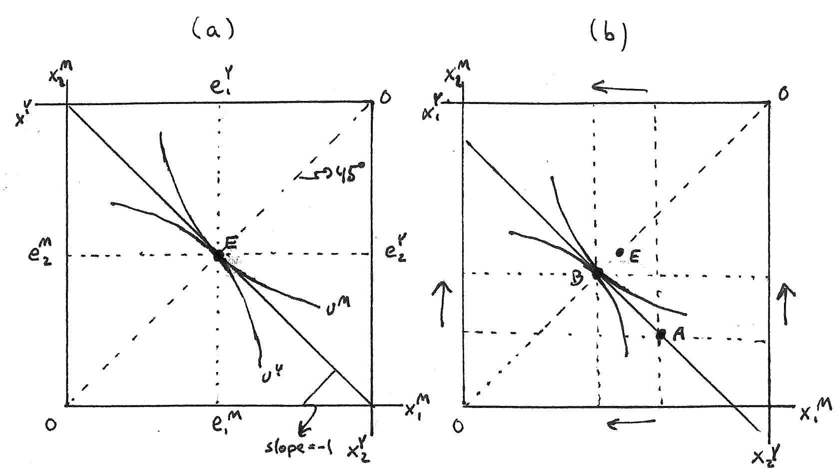
Exercise 16.3

Suppose you and I have the same homothetic tastes over x_1 and x_2, and our endowments of the two goods are $E^M = (e_1^M, e_2^M)$ for me and $E^Y = (e_1^Y, e_2^Y)$ for you.

A: *Suppose throughout that, when $x_1 = x_2$, our MRS is equal to -1.*

(a) *Assume that $e_1^M = e_2^M = e_1^Y = e_2^Y$. Draw the Edgeworth box for this case and indicate where the endowment point $E = (E^M, E^Y)$ lies.*

Answer: This is done in panel (a) of Exercise Graph 16.3 where the Edgeworth Box is drawn as a square (because the overall endowments of x_1 are equal to those of x_2), with the endowment bundle E located in the center (since we are endowed with equal amounts of everything.)



Exercise Graph 16.3 : Equal Endowments and Same Tastes

(b) *Draw the indifference curves for both of us through E. Is the endowment allocation efficient?*

Answer: This is also done in panel (a). Since our endowment lies on the 45-degree line and our *MRS* along the 45-degree line is always -1, the indifference curves through E are tangent to one another. This implies that the endowment allocation is efficient — because there is no lens shape between our indifference curves that would give us room to make both of us better off (or at least one better off without making the other worse off).

(c) *Normalize the price of x_2 to 1 and let p be the price of x_1. What is the equilibrium price p^*?*

Answer: The equilibrium price must pass through E and induce budget constraints for me and you such that both of us optimize at the same point within the Edgeworth Box. In this case, the only way to do this is to let $p^* = 1$ — resulting in a budget with slope -1 through E. Since the MRS at E is -1 for both of us, we will both choose to remain at our endowment bundle at this price. This is also illustrated in panel (a) of Exercise Graph 16.3.

(d) *Where in the Edgeworth Box is the set of all efficient allocations?*

Answer: The set of all efficient allocations lies on the 45-degree line — because along the 45 degree line, our MRS's are equal to 1 and thus equal to one another, implying indifference curves that are tangent to one another. This is the contract curve.

(e) *Pick another efficient allocation and demonstrate a possible way to reallocate the endowment among us such that the new efficient allocation becomes an equilibrium allocation supported by an equilibrium price. Is this equilibrium price the same as p^* calculated in (c)?*

Answer: This is illustrated in panel (b) of Exercise Graph 16.3 where we consider the equilibrium if the endowment is redistributed so that it moves from E to A. The only place where the indifference curves are tangent to one another is on the 45-degree line where their slope is -1. Thus, the new equilibrium price must again be 1 — and the new budget must pass through the new endowment A as drawn. This will cause us to trade from A to B along the budget line with slope $p = 1$ — with me giving up x_1 to get more x_2 and you giving up an equal amount of x_2 to get more x_1 (as indicated by the arrows on the axes.)

B: *Suppose our tastes can be represented by the CES utility function $u(x_1, x_2) = \left(0.5x_1^{-\rho} + 0.5x_2^{-\rho}\right)^{-1/\rho}$.*

(a) *Let p be defined as in A(c). Write down my and your budget constraint (assuming again endowments $E^M = (e_1^M, e_2^M)$ for me and $E^Y = (e_1^Y, e_2^Y)$.)*

Answer: The value of or endowments has to be equal to the value of what we consume. For me, this implies

$$pe_1^M + e_2^M = px_1^M + x_2^M, \tag{16.3.i}$$

and for you it means

$$pe_1^Y + e_2^Y = px_1^Y + x_2^Y. \tag{16.3.ii}$$

(b) *Write down my optimization problem and derive my demand for x_1 and x_2.*

Answer: My optimization problem is then

$$\max_{x_1,x_2} \left(0.5x_1^{-\rho} + 0.5x_2^{-\rho}\right)^{-1/\rho} \text{ subject to } pe_1^M + e_2^M = px_1 + x_2 \tag{16.3.iii}$$

where, for now, we suppress the M superscripts on the x variables. Setting up the Lagrangian and solving in the usual way, we get

$$x_1^M = \frac{pe_1^M + e_2^M}{p + p^{1/(\rho+1)}} \text{ and } x_2^M = \frac{p^{1/(\rho+1)}e_1^M + e_2^M}{p + p^{1/(\rho+1)}}. \tag{16.3.iv}$$

(c) *Similarly, derive your demand for x_1 and x_2.*

Answer: Repeating the steps in the previous part for you, we get

$$x_1^Y = \frac{pe_1^Y + e_2^Y}{p + p^{1/(\rho+1)}} \text{ and } x_2^Y = \frac{p^{1/(\rho+1)}e_1^Y + e_2^Y}{p + p^{1/(\rho+1)}}. \tag{16.3.v}$$

(d) *Derive the equilibrium price. What is that price if, as in part A, $e_1^M = e_2^M = e_1^Y = e_2^Y$?*

Answer: In equilibrium, the price has to be such that demand is equal to supply in both markets. Because of Walras' Law, we only have to solve for p in one of the markets though — and either one will work. Choosing the market for x_1, it must therefore be the case that $x_1^M + x_1^Y = e_1^M + e_1^Y$ or, plugging in our demands from the previous parts,

$$\frac{pe_1^M + e_2^M}{p + p^{1/(\rho+1)}} + \frac{pe_1^Y + e_2^Y}{p + p^{1/(\rho+1)}} = e_1^M + e_1^Y. \tag{16.3.vi}$$

Multiplying both sides by the denominators on the left hand side, we get

$$pe_1^M + e_2^M + pe_1^Y + e_2^Y = \left(e_1^M + e_1^Y\right)\left(p + p^{1/(\rho+1)}\right) \tag{16.3.vii}$$

and, rearranging terms,

$$p\left(e_1^M + e_1^Y\right) + \left(e_2^M + e_2^Y\right) = p\left(e_1^M + e_1^Y\right) + p^{1/(\rho+1)}\left(e_1^M + e_1^Y\right). \tag{16.3.viii}$$

Subtracting out the first term on each side and then solving for p, we get

$$p^* = \left(\frac{e_2^M + e_2^Y}{e_1^M + e_1^Y}\right)^{(\rho+1)}. \tag{16.3.ix}$$

When $e_1^M = e_2^M = e_1^Y = e_2^Y$, this simplifies to $p^* = 1$ — consistent with what we did in part A.

(e) *Derive the set of pareto efficient allocations assuming $e_1^M = e_2^M = e_1^Y = e_2^Y$. Can you see why, regardless of how we might redistribute endowments, the equilibrium price will always be $p = 1$?*

Answer: Let $e = e_1^M = e_2^M = e_1^Y = e_2^Y$. Then the economy is endowed with $2e$ of each good, which implies that, for any allocation (x_1^M, x_2^M) that I get, what's left over for you is $(2e - x_1^M), (2e - x_2^M)$. The pareto efficient set of (x_1^M, x_2^M) (with its implied consumption levels for you) is then defined as the set where our MRS's are equal to one another. The MRS for me at a bundle (x_1^M, x_2^M) is

$$MRS^M = -\frac{\partial u(x_1^M, x_2^M)/\partial x_1}{\partial u(x_1^M, x_2^M)/\partial x_2} = -\left(\frac{x_2^M}{x_1^M}\right)^{(\rho+1)} \tag{16.3.x}$$

and the MRS for you at the left-over bundle $((2e - x_1^M), (2e - x_2^M))$ is

$$MRS^Y = -\frac{\partial u((2e - x_1^M), (2e - x_2^M))/\partial x_1}{\partial u((2e - x_1^M), (2e - x_2^M))/\partial x_2} = -\left(\frac{2e - x_2^M}{2e - x_1^M}\right)^{(\rho+1)}. \tag{16.3.xi}$$

Setting MRS^M equal to MRS^Y and solving for x_2^M, we get

$$x_2^M = x_1^M; \tag{16.3.xii}$$

i.e. the contract curve is a straight line with slope 1 and intercept 0 — the 45-degree line in the Edgeworth Box. Since all efficient allocations happen on this line, and since equilibria are efficient, we know that any competitive equilibrium lies on the 45-degree line. This further implies that, when we plug $x_1^M = x_2^M$ and $2e - x_1^M = 2e - x_2^M$ into the equations for marginal rates of substitution, we get $MRS^M = -1 = MRS^Y$ in any equilibrium, which can only hold if the slope of the budget is -1. And that can only be true if $p = 1$.

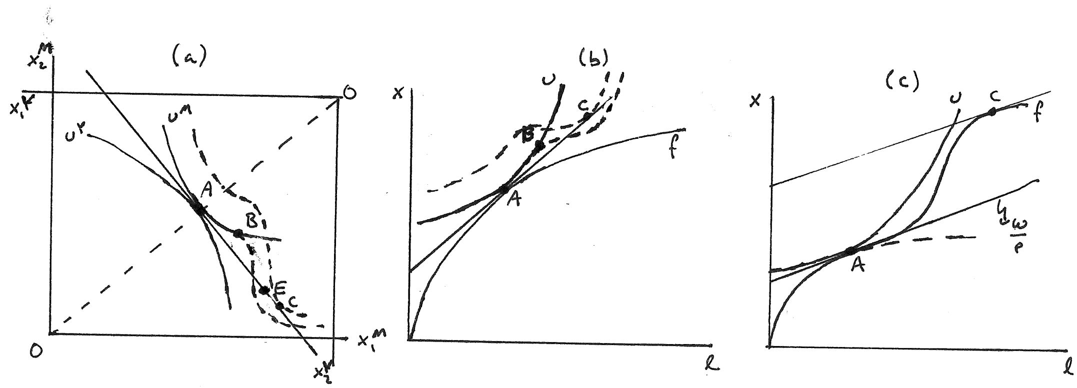
Exercise 16.5

In this exercise we explore some technical aspects of general equilibrium theory in exchange economies and Robinson Crusoe economies. Unlike in other problems, parts A and B are applicable to both those focused on A-Section material and those focused on B-Section material. Although the insights are developed in simple examples, they apply more generally in much more complex models.

A: The role of convexity in Exchange Economies: *In each part below, suppose you and I are the only individuals in the economy, and pick some arbitrary allocation E in the Edgeworth Box as our initial endowment. Assume throughout that your tastes are convex and that the contract curve is equal to the line connecting the lower left and upper right corners of the box.*

(a) *Begin with a depiction of an equilibrium. Can you introduce a non-convexity into my tastes such that the equilibrium disappears (despite the fact that the contract curve remains unchanged?)*

Answer: This is done in panel (a) of Exercise Graph 16.5 where the equilibrium budget passes through *E* and is tangent to both solid (and convex) indifference curves at *A*. Thus, *A* is an equilibrium allocation. However, if I permit my indifference curves to have non-convexities, I can maintain the tangency at *A* but lose the equilibrium at *A* by having my indifference curve continue along the dashed curve beginning at *B* and moving right. Notice that *A* is still efficient — but, when faced with the budget line that previously supported *A* as an equilibrium, I now no longer optimize at *A* but rather at *C* which lies on a higher dashed (and non-convex) indifference curve.



Exercise Graph 16.5 : Convexity Assumptions in General Equilibrium

(b) True or False*: Existence of a competitive equilibrium in an exchange economy cannot be guaranteed if tastes are allowed to be non-convex.*

Answer: This is true, as we have just shown.

(c) *Suppose an equilibrium does exist even though my tastes exhibit some non-convexity.* True or False*: The first welfare theorem holds even when tastes have non-convexities.*

Answer: The allocation *A* in panel (a) of Exercise Graph 16.5 would continue to be an equilibrium so long as the non-convexity that is introduced is not sufficiently pronounced so as to cause the indifference curve that is tangent at *A* to cross the budget line. Thus, had we drawn the non-convexity in a less pronounced manner, the budget line through *A* and *E* would still have been such that I optimize at *A* — and thus *A* would have continued to be an equilibrium. We can conclude that, *if an equilibrium exists in the presence of non-convex tastes*, then it will indeed still be efficient. The first welfare theorem therefore holds in the presence of non-convexities.

(d) True or False*: The second welfare theorem holds even when tastes have non-convexities.*

Answer: The second welfare theorem says that any efficient allocation can be an equilibrium allocation so long as endowments can be appropriately redistributed. We have just shown in panel (a) of Exercise Graph 16.5 an example of an efficient allocation A that cannot be supported as an equilibrium no matter where we move the endowment. This is because, in order to support A as an equilibrium, the budget line *has to be* the line that is draw in the graph — because that is the only budget that will cause *you* to optimize at A. But that line crosses the dashed extension of my indifference curve that is tangent at A — implying that I will not optimize at A if my tastes are the non-convex kind in the graph. Thus, we have identified a case where an efficient allocation cannot become an equilibrium allocation regardless of where we put the endowment. The statement is therefore false — the second welfare theorem may not hold when tastes have non-convexities.

B: The role of convexity in Robinson Crusoe Economies: *Consider a Robinson Crusoe economy. Suppose throughout that there is a tangency between the worker's indifference curve and the production technology at some bundle A.*

(a) *Suppose first that the production technology gives rise to a convex production choice set. Illustrate an equilibrium when tastes are convex. Then show that A may no longer be an equilibrium if you allow tastes to have non-convexities even if the indifference curve is still tangent to the production choice set at A.*

Answer: This is illustrated in panel (b) of Exercise Graph 16.5. The solid indifference curve is tangent to the convex production choice set at A, with both tangent to the isoprofit/budget line (that has slope w/p). When viewed as a budget line, the worker is doing the best he can by choosing A, and when viewed as an isoprofit line, the firm is doing the best it can at A, with the wage/price ration w/p supporting A as an equilibrium. But we can take the same indifference curve, keep it tangent to the budget at A, but then change its shape from B on to take the shape of the dashed curve. When we do this, we introduce a non-convexity — and, as a result, the worker is no longer doing the best he can by choosing A when confronting the budget formed by the former equilibrium wage/price ratio. In particular, the worker would now be better off optimizing at C — but that lies outside the production frontier and is therefore not an equilibrium. Thus, by introducing the non-convexity, A ceases to be a competitive equilibrium in this economy.

(b) *Next, suppose again that tastes are convex but now let the production choice set have non-convexities. Show again that A might no longer be an equilibrium (even though the indifference curve and production choice set are tangent at A).*

Answer: This is shown in panel (c) of Exercise Graph 16.5 where the production frontier f is tangent to the indifference curve u — thus making

A an efficient production plan. The budget that is tangent to both the production frontier and the indifference curve at A — with slope w/p — causes the worker to optimize at A where his indifference curve is tangent. However, the firm would not be optimizing at A — because it can reach a higher isoprofit curve and would maximize profit at C instead. The production plan A would be optimal for the firm (and would thus be an equilibrium) if the production frontier took on the dashed shape following A — i.e. if the production choice set were convex. But A is lost as an equilibrium because of the non-convexity of the solid production choice set.

(c) True or False*: A competitive equilibrium may not exist in a Robinson Crusoe economy that has non-convexities in either tastes or production.*

Answer: This is true as shown in the previous two parts.

(d) True or False*: The first welfare theorem holds even if there are non-convexities in tastes and/or production technologies.*

Answer: This is true. The non-convexities may cause there to be no equilibrium, but *if there is an equilibrium*, it will again have the feature that the indifference curve is tangent to the production frontier at that point — which will make it efficient. You can see this in panels (b) and (c) if you imagine the non-convexity that was introduced as being less pronounced. In panel (b), A would remain an equilibrium so long as the dashed portion of the indifference curve does not cross the budget line to the right of B — which is certainly possible even if there were a less pronounced non-convexity. And that equilibrium would be efficient. Similarly, in panel (c) you can imagine a non-convexity in the production choice set either to the left of A or some distance to the right of A — and you can imagine such a non-convexity to not be sufficiently pronounced so as to cross the isoprofit line that is tangent at A. In that case, A would remain as an equilibrium — and it would be efficient. Thus, the first welfare theorem holds — every equilibrium (that exists) is indeed efficient.

(e) True or False*: The second welfare theorem holds regardless of whether there are non-convexities in tastes or production.*

Answer: This is false. In panel (b) of the graph, we have shown an efficient point A that cannot be an equilibrium because the budget line that must support it crosses the indifference curve that is tangent at A. In panel (c) we have shown another efficient point A that cannot be supported as an equilibrium because the isoprofit line that is needed to support it as an equilibrium crosses the production frontier because of a non-convexity. We have therefore shown that, when there are non-convexities, there may be efficient outcomes that cannot be supported as equilibria.

(f) *Based on what you have done in parts A and B, evaluate the following: "Non-convexities may cause a non-existence of competitive equilibria in general equilibrium economies, but if an equilibrium exists, it results in an efficient allocation of resources. However, only in the absence of non-convexities can we conclude that there always exists some lump-sum re-*

distribution such that any efficient allocation can also be an equilibrium allocation." (Note: Your conclusion on this holds well beyond the examples in this problem — for reasons that are quite similar to the intuition developed here.)

Answer: The statement is fully consistent with everything we have done in this exercise. We have shown — in both exchange and Robinson Crusoe economies — that non-convexities may lead to a non-existence of equilibria; that *if* equilibria exist, they will be efficient (i.e. the first welfare theorem holds); but not all efficient outcomes can be supported as equilibria (i.e. the second welfare theorem fails in the presence of non-convexities).

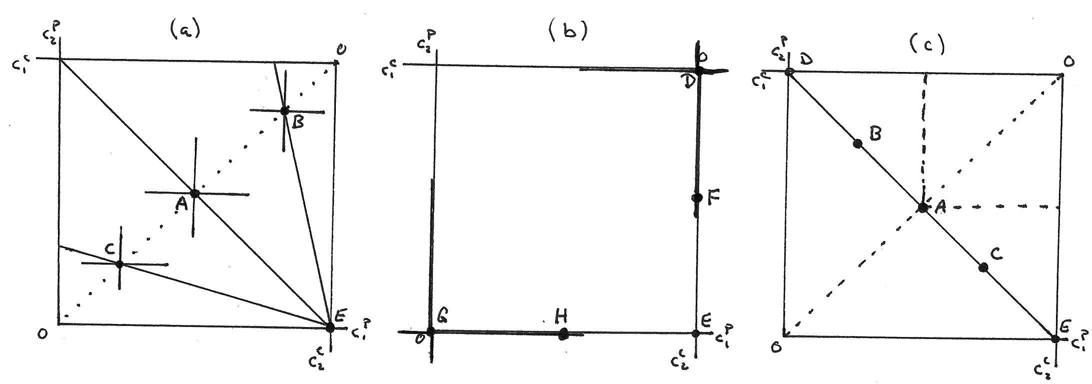
Exercise 16.7

Everyday Application: *Parents, Children and the Degree of Substitutability across Time: Consider again exactly the same scenario as in exercise 16.6.*

A: *This time, however, suppose that parent and child tastes treat consumption now and consumption in the future as perfect complements.*

(a) *Illustrate in an Edgeworth Box an equilibrium with a single parent and a single child.*

Answer: Perhaps the most obvious equilibrium is the one with price equal to 1 and thus a budget line that runs from E in the lower right corner to the upper left corner of the box — with the equilibrium allocation at A, pictured in panel (a) of Exercise Graph 16.7.



Exercise Graph 16.7 : Parents and Children: Part 1

(b) *Is the equilibrium you pictured in (a) the only equilibrium? If not, can you identify the set of all equilibrium allocations?*

Answer: It is not the only equilibrium — in fact, panel (a) of Exercise Graph 16.7 picture two others, with allocations at B and at C. Because of the sharp corners on indifference curves for perfect complements, any budget line with negative slope can be fit to any "tangency" of the two indifference curves on the 45 degree line. Thus, all allocations on the 45 degree line have some budget line that passes through the endowment allocation *E and* is "tangent" to both indifference curves on that point of the 45 degree line. The entire 45-degree line in the box is therefore the set of possible equilibrium allocations.

(c) *Now suppose that there were two children and one parent. Keep the Edgeworth Box with the same dimensions but model this by recognizing that, on any equilibrium budget line, it must now be the case that the parent moves twice as far from the endowment E as the child (since there are two children and thus any equilibrium action by a child must be half the equilibrium action by the parent). Are any of the equilibrium allocations for parent and child that you identified in (b) still equilibrium allocations? (*Hint*: Consider the corners of the box.)*

Answer: For any budget line that intersects the 45-degree line inside the box, both parent and child will optimize on the 45 degree line. But with two children and one parent, that cannot be an equilibrium — because the parent's action must be twice the children's in the opposite direction in order for demand to equal supply. Thus, none of the efficient allocations on the 45 degree line inside the box can be an equilibrium allocation. However, suppose that $p = \infty$. Then the budget line becomes vertical and passes through E. The parent will optimize at the top corner (point D in panel (b) of Exercise Graph 16.7), and the children don't care where on the budget they optimize because all the bundles on that budget lie on the same indifference curve. Thus, it is not inconsistent with optimization to assume that the children will choose F — halfway up the budget and halfway to D where the parent optimizes. Thus, children consume nothing now and give half of what they earn in the future to the parent, and parents consume everything now and half of everything (i.e. half of what each of the two children earns) in the future.

(d) *Suppose instead that there are two parents and one child. How does your answer change?*

Answer: No equilibrium allocation can lie on the 45 degree line for the same reason as in the previous case — and now we end up with the child optimizing at G in panel (b) of Exercise Graph 16.7 and the two parents optimizing at H, with $p = 0$. Thus, parents consume half their income now and nothing in the future, while children consume half of each parents' income now and everything in the future.

(e) *Repeat (a) through (d) for the case where consumption now and consumption in the future are perfect substitutes for both parent and child.*

Answer: When consumption across time is perfectly substitutable, the indifference curves have slope -1 at every allocation in the Edgeworth Box. Thus any equilibrium allocation inside the box must lie on the line connecting the upper left to the lower right corners of the box — the line pictured in panel (c) of Exercise Graph 16.7. Neither parent nor child cares where on that line they consume — and thus any split of the economy's endowment that falls on this line will be an equilibrium allocation with $p = 1$. For instance, when there is one child and one parent, A is a possible equilibrium allocation, as is C and B. When there are two children and 1 parent, any allocation that has the parent's bundle twice as far from E as the children's works — for instance A for the parent and C for the children. When there are two parents and one child, then any allocation that has the child twice as far as the parents from E works. In all cases, the equilibrium price continues to be $p = 1$ — because it makes no sense for individuals to trade on other terms when consumption now is the same as consumption in the future.

(f) *Repeat for the case where consumption now and consumption in the future are perfect complements for parents and perfect substitutes for children.*

Answer: Consider first the case of one parent and one child. For any budget with positive slope (not equal to infinity), the parent will optimize on the 45-degree line. For any price not equal to 1, the child will choose a corner solution (since consumption now and in the future are the same for her). Thus, the only way the child will trade to permit the parent to get to the 45 degree line is if $p = 1$ and the budget line takes the shape graphed in panel (c) of Exercise Graph 16.7. The equilibrium allocation is then A — where the parent's indifference curve is drawn as a dotted L-shape. Next, suppose there are two children. Nothing has changed in terms of the children's willingness to trade to an interior solution only at $p = 1$ and in terms of the parent's optimal bundle falling on the 45 degree line for any positive price. Thus, p will remain 1, the parent will optimize at A and the children will each optimize at C — halfway between A and E. Finally, suppose there are two parents and one child. Again, for the same reasons as before, price has to remain 1, and the parents' optimization has to lead to A. Thus, parents end up at A and the child ends up at the top left corner D — twice as far from E as the two parents.

(g) True or False*: The more consumption is complementary for the parent relative to the child, and the more children there are per parent, the more gains from trade will accrue to the parent.*

Answer: This is roughly true, as illustrated in the previous parts of the question. For instance, when parent viewed consumption as perfectly complementary across time while children viewed it as substitutable (in panel (c) of Exercise Graph 16.7), the children gain no utility from trading while the parent(s) get all gains from trade. Similarly, we saw in this and the previous exercise that more gains typically accrue to the party that is in control of the goods that are scarcer. Parents are in control of consumption now — which is relatively more scarce the more children there are per parent.

B: *Suppose that parent and child tastes can be represented by the CES utility function* $u(c_1, c_2) = \left(0.5c_1^{-\rho} + 0.5c_2^{-\rho}\right)^{-1/\rho}$. *Assume that the income earned by parents in period 1 and by children in period 2 is 100.*

(a) *Letting p denote the price of consumption now with price of future consumption normalized to 1, derive parent and child demands for current and future consumption as a function of ρ and p.*

Answer: We want to maximize utility (which is the same for parents and children) subject to the budget constraint — which is $100p = pc_1 + c_2$ for parents (who are endowed with 100 now) and $100 = pc_1 + c_2$ for children (who are endowed with 100 in the future). Solving this in the usual way, we get

$$c_1^P = \frac{100p^{1/(\rho+1)}}{p^{\rho/(\rho+1)}+1} \text{ and } c_2^P = \frac{100p}{p^{\rho/(\rho+1)}+1} \text{ for parents, and} \tag{16.7.i}$$

$$c_1^C = \frac{100}{p+p^{1/(\rho+1)}} \text{ and } c_2^C = \frac{100}{p^{\rho/(\rho+1)}+1} \text{ for children.} \tag{16.7.ii}$$

(b) *What is the equilibrium price — and what does this imply for equilibrium allocations of consumption between parent and child across time. Does any of your answer depend on the elasticity of substitution?*

Answer: This solves slightly more easily if we set demand and supply in the c_2 market equal to one another (rather than setting it equal to one another in the c_1 market. Of course the latter would give the same answer even if it is slightly more burdensome to get there.) Thus, we need to solve

$$\frac{100p}{p^{\rho/(\rho+1)}+1} + \frac{100}{p^{\rho/(\rho+1)}+1} = 100. \tag{16.7.iii}$$

Dividing by 100, multiplying by the denominator on the left hand side, and simplifying, we get

$$p = p^{\rho/(\rho+1)} \text{ or } 1 = \rho^{-1/(\rho+1)} \tag{16.7.iv}$$

which solves to $p = 1$. The answer therefore does not depend on ρ and thus is independent of the elasticity of substitution. (This is because the indifference curves for the utility function always have $MRS = -1$ along the 45 degree line no matter what elasticity of substitution is assumed.)

(c) *Next, suppose there are 2 children and only 1 parent. How does your answer change?*

Answer: We now have to sum twice the child demands with the parent demand for c_2 and set it equal to overall consumption in the future — which is 200 when there are two children. This implies we need to solve

$$\frac{100p}{p^{\rho/(\rho+1)}+1} + 2\left(\frac{100}{p^{\rho/(\rho+1)}+1}\right) = 200 \tag{16.7.v}$$

which solves to

$$p = 2^{\rho+1}. \tag{16.7.vi}$$

The equilibrium price now depends on ρ and thus on the elasticity of substitution. As ρ increases — which implies the elasticity of substitution falls — price increases. In the limit, as ρ approaches infinity — and consumption becomes perfectly complementary across time — price rises to infinity. This is exactly what we concluded in part A for perfect complements. As ρ falls to -1 — and consumption becomes perfectly substitutable across time, on the other hand, price becomes 1 — again exactly as we concluded in part A.

(d) *Next, suppose there are 2 parents and only 1 child. How does your answer change?*

Answer: We now have to sum twice the parent demands with the child demand for c_2 and set it equal to overall consumption in the future — which is 100 when there is only one child. This implies we need to solve

$$2\left(\frac{100p}{p^{\rho/(\rho+1)}+1}\right)+\frac{100}{p^{\rho/(\rho+1)}+1}=100 \qquad (16.7.\text{vii})$$

which solves to

$$p=\left(\frac{1}{2}\right)^{\rho+1}. \qquad (16.7.\text{viii})$$

The equilibrium price again depends on ρ and thus on the elasticity of substitution. As ρ increases — which implies the elasticity of substitution falls — price falls. In the limit, as ρ approaches infinity — and consumption becomes perfectly complementary across time — price falls to zero. This is exactly what we concluded in part A for perfect complements. As ρ falls to -1 — and consumption becomes perfectly substitutable across time, on the other hand, price becomes 1 — again exactly as we concluded in part A.

(e) *Explain how your answers relate to the graphs you drew for the extreme cases of both parent and child preferences treating consumption as perfect complements over time.*

Answer: We already did this. We showed that, as tastes become perfectly complementary, then p approaches infinity if there are two children and one parent and to zero if there are two parents and one child. We illustrated precisely this extreme case in panel (b) of Exercise Graph 16.7.

(f) *Explain how your answers relate to your graphs for the case where consumption was perfectly substitutable across time for both parents and children.*

Answer: Again, we already did this. We showed that, when consumption is perfectly substitutable across time, then price will be 1 regardless of the number of children relative to parents.

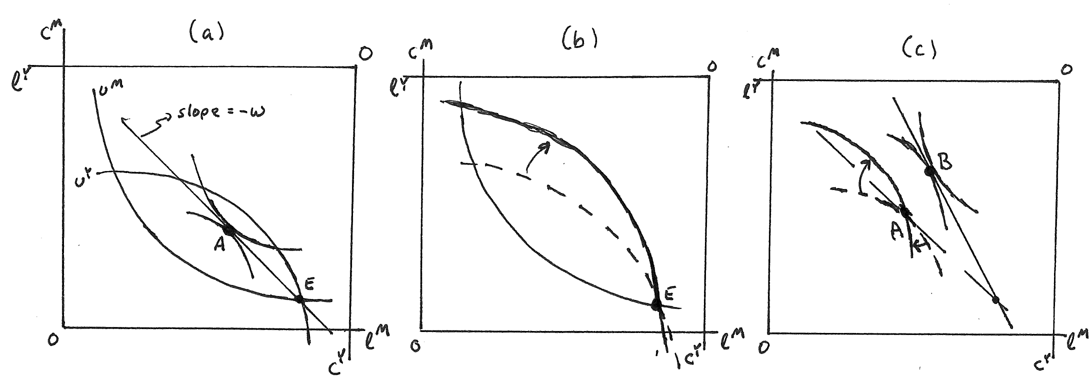
Exercise 16.9

Business Application: *Hiring an Assistant: Suppose you are a busy CEO — with lots of consumption but relatively little leisure. I, on the other hand, have only a part-time job and therefore lots of leisure with relatively little consumption.*

A: *You decide that the time has come to hire a personal assistant — someone who can do some of the basics in your life so that you can have a bit more leisure time.*

(a) *Illustrate our current situation in an Edgeworth Box with leisure on the horizontal and consumption on the vertical axis. Indicate an endowment bundle that fits the description of the problem and use indifference curves to illustrate a region in the graph where both of us would benefit from me working for you as an assistant.*

Answer: This is illustrated in panel (a) of Exercise Graph 16.9 where the endowment allocation E has me (on the lower axes) with lots of leisure but little consumption and you (on the upper axes) with the reverse. The mutually beneficial region is formed by the lens made from our indifference curves that pass through E. Both of us would prefer any allocation in that lens shape to the endowment bundle E.



Exercise Graph 16.9 : Cheerfulness in Office Assistants

(b) *Next, illustrate what an equilibrium would look like. Where in the graph would you see the wage that I am being paid?*

Answer: This is also illustrated in panel (a) of Exercise Graph 16.9 where the budget line that passes through A and E has slope $-w$ (where w is the wage when the price of consumption is normalized to 1).

(c) *Suppose that anyone can do the tasks you are asking of your assistant — but some will do it cheerfully and others will do it with attitude. You hate attitude — and therefore would prefer someone who is cheerful. Assuming* *you can read the level of cheerfulness in me, what changes in the Edgeworth box as your impression of me changes?*

Answer: As you think I am more cheerful, you will be willing to trade more of your consumption for an increase in your leisure. Thus, your indifference curves become steeper.

(d) *How do your impressions of me — i.e. how cheerful I am — affect the region of mutually beneficial trades?*

Answer: This is illustrated in panel (b) of Exercise Graph 16.9 where your original indifference curve through E is illustrated as a dashed indifference curve and your new indifference curve (that contains E) as my cheerfulness increases is illustrated as a bold curve. This increases the lens formed by our indifference curves through E — and thus the mutually beneficial region.

(e) *How does increased cheerfulness on my part change the equilibrium wage?*

Answer: This is illustrated in panel (c) of Exercise Graph 16.9 where A is the original equilibrium at low levels of cheerfulness and B is the new equilibrium at higher levels of cheerfulness. As my cheerfulness increases, your indifference curve through A becomes steeper — rotating from the dashed curve to the solid one. Thus, A can't be an equilibrium anymore because you now want more of me but I am not willing to offer any more at the original wage. Thus, the wage must increase in order to get me to offer more of myself and you to reduce your demand for me. This leads us to the steeper budget through B — with a higher wage. Cheerfulness is rewarded in the competitive market.

(f) *Your graph probably has the new equilibrium (with increased cheerfulness) occurring at an indifference curve for you that lies below (relative to your axes) the previous equilibrium (where I was less cheerful). Does this mean that you are worse off as a result of me becoming more cheerful?*

Answer: No, it does not. It is indeed true that your indifference curve through B in panel (c) of Exercise Graph 16.9 lies below A (relative to your axes). But this does not mean you are less happy — because my cheerfulness is what made your indifference curves get steeper. In terms of some of the earlier problems in our development of consumer theory, cheerfulness is a third good you care about — and as it changes in the problem, you switch to a different "slice" of your 3-dimensional indifference surfaces. Increased cheerfulness switches you to a slice where you are happier for any level of consumption and leisure than you were before — and so an indifference curve with more cheerfulness can lie below one with less cheerfulness and still be preferred.

B: *Suppose that my tastes can be represented by $u(c,\ell) = 200\ln\ell + c$ while yours can be represented by $u(c,\ell,x) = 100x\ln\ell + c$ where ℓ stands for leisure, c stands for consumption and x stands for cheerfulness of your assistant. Suppose that, in the absence of working for you, I have 50 leisure hours and 10 units of consumption while you have 10 leisure hours and 100 units of consumption.*

(a) *Normalize the price of c as 1. Derive our leisure demands as a function of the wage w.*

Answer: My budget constraint is $w\ell + c = 50w + 10$ while yours is $w\ell + c = 10w + 100$. Maximizing our utilities subject to these constraints, we get (by solving this in the usual way)

$$\ell^M = \frac{200}{w} \text{ for me and } \ell^Y = \frac{100x}{w} \text{ for you.} \tag{16.9.i}$$

(b) *Calculate the equilibrium wage as a function of x.*

Answer: The sum of our leisure demands has to be equal to the leisure supply of 60 in equilibrium — i.e.

$$\frac{200}{w} + \frac{100x}{w} = 60 \tag{16.9.ii}$$

which implies that the equilibrium wage is

$$w^* = \frac{10+5x}{3}. \tag{16.9.iii}$$

(c) *Suppose $x = 1$. What is the equilibrium wage, and how much will I be working for you?*

Answer: Substituting $x = 1$ into our equation for w^*, we get an equilibrium wage of 5. Plugging this wage into our leisure demand equations, we get that you will have 20 hours of leisure and I will have 40 — which is 10 less for me and 10 more for you than what we were endowed with. Thus, I'll be working for you for 10 hours.

(d) *How does your MRS change as my cheerfulness x increases?*

Answer: Your MRS is

$$MRS^Y = -\frac{\partial u(c,\ell,x)/\partial \ell}{\partial u(c,\ell,x)/\partial x} = -\frac{100x}{\ell}. \tag{16.9.iv}$$

Thus, for any bundle (ℓ, c), the MRS gets larger in absolute value as x increases — i.e your indifference curves become steeper as my cheerfulness increases.

(e) *What happens to the equilibrium wage as x increases to 1.2? What happens to the equilibrium number of hours I work for you? What if I get grumpy and x falls to 0.4?*

Answer: When x goes to 1.2, the equilibrium wage rises to 5.33 and the number of hours I work for you increases to 12.5. When x falls to 0.4, the equilibrium wage falls to 4 but you no longer hire me and we simply consume at our endowment bundles.

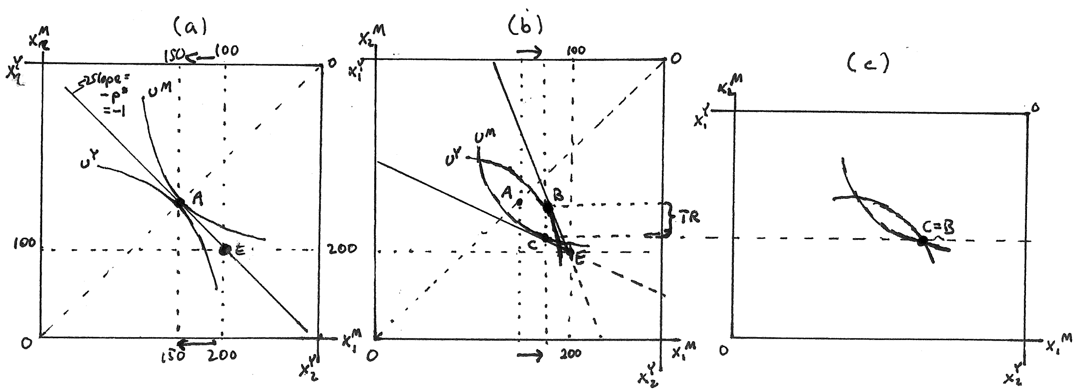
Exercise 16.11

Policy Application: *Distortionary Taxes in General Equilibrium: Consider, as in exercise 16.10, a 2-person exchange economy in which I own 200 units of x_1 and 100 units of x_2 while you own 100 units of x_1 and 200 units of x_2.*

A: *Suppose you and I have identical homothetic tastes.*

(a) *Draw the Edgeworth Box for this economy and indicate the endowment allocation E.*

Answer: This is illustrated in panel (a) of Exercise Graph 16.11 where the box takes on the shape of a square since the economy's endowment of both goods is 300.



Exercise Graph 16.11 : Distortionary Taxes

(b) *Normalize the price of good x_2 to 1. Illustrate the equilibrium price p^* for x_1 and the equilibrium allocation of goods in the absence of any taxes. Who buys and who sells x_1?*

Answer: This is also done in panel (a) of Exercise Graph 16.11 where A is the equilibrium allocation (which appears in the center of the box on the 45 degree line because of our identical homothetic tastes and endowments.) Thus, I sell 50 units of x_1 to you for the price of $p^* = 1$.

(c) *Suppose the government introduces a tax t levied on all transactions of x_1 (and paid in terms of x_2). For instance, if one unit of x_1 is sold from me to you at price p, I will only get to keep $(p - t)$. Explain how this creates a kink in our budget constraints.*

Answer: This implies that the price p paid by the buyer is greater than the price $(p - t)$ received by the seller. On my budget constraint, I am a seller to the left of E and a buyer to the right of E — implying that my budget has shallower slope $-(p - t)$ to the left of E and steeper slope $-p$ to the right of E, with a kink at E. The same is true for you — except that "right" and "left" are reversed when we flip your axes to create the Edgeworth Box. The portions along which I am a seller and you are a buyer of x_2 are illustrated as the solid lines in panel (b) of Exercise Graph 16.11, with the remaining portion of the constraints dashed to the right of E.

(d) *Suppose a post-tax equilibrium exists and that price increases for buyers and falls for sellers. In such an equilibrium, I will still be selling some quantity of x_1 to you. (Can you explain why?) How do the relevant portions of the budget constraints you and I face look in this new equilibrium, and where will we optimize?*

Answer: This is illustrated in panel (b) of Exercise Graph 16.11 where the steeper (solid) constraint is yours (with the higher post-tax price) and the shallower one is mine (with the lower pre-tax price). In equilibrium, it will still have to be the case that the amount of x_1 I sell to you is equal to the amount of x_1 you buy. Thus, in equilibrium, our two budgets have to be such that your optimum B lies right above my optimum C in the Edgeworth Box. We know that this will be to the right of the original equilibrium A — because your budget is steeper than before and mine is shallower than before. The fact that it is shallower for me means that I will be optimizing on a shallower ray from the origin (given that my tastes are homothetic), and the fact that it is steeper for you implies you will be optimizing on a steeper ray from your origin. Thus, the amount we trade will fall by the amount of the arrows in the graph. (The reason we know that I will still be selling (or at least not buying) x_1 under the tax is as follows: My budget under the tax has a kink at E — and becomes steeper to the right of E. Given that my tastes are homothetic, it cannot be that I optimize on that steeper portion — because the steeper parts of my indifference curves lie to the left of E).

(e) *When we discussed price changes with homothetic tastes in our development of consumer theory, we noted that there are often competing income (or wealth) and substitution effects. Are there such competing effects here relative to our consumption of x_1? If so, can we be sure that the quantity we trade* in equilibrium *will be less when t is introduced?*

Answer: Both of us experience a negative wealth effect — me because what I am selling has fallen in price, you because what you are buying has increased in price. Thus, the wealth effect says "consume less of x_1" for both of us. But the substitution effects operate in opposite directions for the two of us. For me, the price of x_1 falls as a result of the tax — which means the substitution effect will tell me to consume *more* of x_1. For you, on the other hand, the price of x_1 has increased — with the substitution effect therefore telling you to consume *less* of x_1. The wealth and substitution effects therefore point in opposite directions for me but not for you. This implies you will consume less x_1 under the tax, which means *in equilibrium* the prices have to adjust such that I will sell you less (and therefore consume more) even though the wealth effect tells me to consume less. (This implies that the equilibrium that we assume exists (with price increasing for buyers and falling for sellers) requires that the goods are sufficiently substitutable to create the necessary substitution effect.)

(f) *You should see that, in the new equilibrium, a portion of x_2 remains not allocated to anyone. This is the amount that is paid in taxes to the government. Draw a new Edgeworth box that is adjusted on the x_2 axes to reflect the fact that some portion of x_2 is no longer allocated between the two of us. Then locate the equilibrium allocation point that you derived in your previous graph. Why is this point not efficient?*

Answer: The portion of x_2 that remains not allocated in our tax-equilibrium in panel (b) of the graph is the vertical difference between B and C — labeled TR in the graph. Thus, the amount that gets allocated is TR less of x_2 than what is available — because the difference is collected by the government. If we shrink the Edgeworth Box by that vertical amount, we get the box illustrated in panel (c) of Exercise Graph 16.11. By shrinking the height of the box, we move B on top of C and now see even more clearly than in panel (b) that this allocation is not efficient. The reason it is inefficient is that both you and I would prefer to divide everything that was not taken by the government differently — with all the allocations in the lens shape between our indifference curves through $B = C$ all preferred by both of us. We could thus make everyone better off by moving the allocation into that lens shape without taking any of the tax revenue the government has raised back.

(g) True or False*: The deadweight loss from the distortionary tax on trades in x_1 results from the fact that our marginal rates of substitution are no longer equal to one another after the tax is imposed and* not *because the government raised revenues and thus lowered the amounts of x_2 consumed by us.*

Answer: This is true. The inefficiency we show in panel (c) arises from the fact that there is a lens shape between our indifference curves — and that lens shape arises from the fact that our marginal rates of substitution are not equal to one another (which is due to the fact that the prices we face as buyers and sellers is different when the government uses price-distorting taxes). The fact that the box has shrunk is not evidence of an inefficiency — because the government now has the difference and may well be doing some very useful things with the money. The problem is that what remains is not allocated efficiently due to distorted prices.

(h) True or False*: While the post-tax equilibrium is not efficient, it does lie in the region of mutually beneficial trades.*

Answer: This is true. In panel (b), the indifference curves through B and C still lie above E for both of us — i.e. trade is still making us better off than we would be without trade, just worse off than we would be if we could trade without price distortions. (Even if it is not obvious from the graph that our indifference curves through B and C lie above E, it should intuitively make sense that this has to be the case: After all, even in the presence of the distortionary tax, no one is forcing us to trade with one another — and we would not do so if trade made us worse off than we would be if we simply consumed our endowments.)

(i) *How would taxes that redistribute endowments (as envisioned by the Second Welfare Theorem) be different than the price distorting tax analyzed in this problem?*

Answer: Redistributions of endowments would involve lump sum taxes and subsidies that do not distort prices — because they would simply shift E around in the box. From the new E, markets could act as before — finding the competitive equilibrium price and causing the individuals to optimize where their indifference curves are tangent to one another and the resulting allocation is therefore efficient.

B: *Suppose our tastes can be represented by the utility function $u(x_1, x_2) = x_1 x_2$. Let our endowments be specified as at the beginning of the problem.*

(a) *Derive our demand functions for x_1 and x_2 (as functions of p — the price of x_1 when the price of x_2 is normalized to 1).*

Answer: My budget constraint is $px_1 + x_2 = 200p + 100$ while yours is $px_1 + x_2 = 100p + 200$. Solving our utility maximization problems subject to these constraints in the usual way, we get

$$x_1^M = \frac{100p + 50}{p} \text{ and } x_2^M = 100p + 50 \text{ for me, and} \tag{16.11.i}$$

$$x_1^Y = \frac{50p + 100}{p} \text{ and } x_2^Y = 50p + 100 \text{ for you.} \tag{16.11.ii}$$

(b) *Derive the equilibrium price p^* and the equilibrium allocation of goods.*

Answer: To derive the equilibrium price, we can sum the demands for x_1 and set them equal to 300 — the amount of x_1 that the economy is endowed with. Solving for p, we get $p^* = 1$. Substituting back into the demand equations, we get $x_1^M = x_2^M = x_1^Y = x_2^Y = 150$.

(c) *Now suppose the government introduces a tax t as specified in A(c). Given that I am the one that sells and you are the one that buys x_1, how can you now re-write our demand functions to account for t? (*Hint*: There are two ways of doing this — either define p as the pre-tax price and let the relevant price for the buyer be $(p + t)$ or let p be defined as the post-tax price and let the relevant price for the seller be $(p - t)$.)*

Answer: Letting p indicate the price paid by you and $(p - t)$ be equal to the price received by me (as the seller), we can substitute $(p - t)$ into my demand equations to get

$$x_1^M(t) = \frac{100(p - t) + 50}{(p - t)} \text{ and } x_2^M(t) = 100(p - t) + 50 \tag{16.11.iii}$$

Your demand functions would remain the same as before.

(d) *Derive the new equilibrium pre- and post-tax prices in terms of t. (*Hint*: You should get to a point where you need to solve a quadratic equation using the quadratic formula that gives two answers. Of these two, the larger one is the correct answer for this problem.)*

Answer: We again set demand for x_1 equal to supply to get the equation

$$x_1^M(t) + x_1^Y = \frac{100(p-t)+50}{(p-t)} + \frac{50p+100}{p} = 300. \tag{16.11.iv}$$

Multiplying both sides by $(p-t)p$, taking all terms to one side, summing like terms and dividing by 50, we get

$$3p^2 - 3(t+1)p + 2t = 0. \tag{16.11.v}$$

Applying the quadratic formula (and accepting the higher of the two solutions), we get

$$p = \frac{3(t+1) + \sqrt{9(t+1)^2 - 4(3)(2t)}}{6} = \frac{(t+1) + \sqrt{t^2 - \frac{2}{3}t + 1}}{2} \tag{16.11.vi}$$

which is the post-tax equilibrium price that buyers pay. The pre-tax price that sellers receive is then simply t less; i.e.

$$(p-t) = \frac{(1-t) + \sqrt{t^2 - \frac{2}{3}t + 1}}{2}. \tag{16.11.vii}$$

(e) *How much of each good do you and I consume if $t = 1$?*

Answer: Plugging $t = 1$ into our equations for p and $(p-t)$, we get $p \approx 1.5774$ and $(p-t) \approx 0.5774$. Plugging these into our demand equations, we get

$$x_1^M \approx 186.60, x_2^M \approx 107.74, x_1^Y \approx 113.40 \text{ and } x_2^Y \approx 178.87. \tag{16.11.viii}$$

(f) *How much revenue does the government raise if $t = 1$?*

Answer: The tax revenue must be the difference between the 300 units of x_2 that were available in the economy and the sum of our consumption levels of x_2; i.e. tax revenue must be $300-(107.74+178.87) = 13.39$. We can verify that this is the case by multiplying $t = 1$ times the quantity of x_1 that is sold by me to you in equilibrium — i.e. $(1)(200-186.60) = 13.40$. (The difference between the two values for tax revenue is rounding error.)

(g) *Show that the equilibrium allocation under the tax is inefficient.*

Answer: To show that the equilibrium allocation is inefficient, all we have to show is that our marginal rates of substitution at the equilibrium consumption bundles are not the same. For the utility function we are using, the MRS is given by

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{x_2}{x_1}. \qquad (16.11.ix)$$

Plugging in our consumption levels from equation (16.11.viii), we get $MRS^M \approx 0.5774$ and $MRS^Y \approx 1.5774$ for you — which are, of course, equal to the negative $(p-t)$ and p values we calculated earlier and that form the slopes of our two equilibrium budget constraints.

Conclusion: Potentially Helpful Reminders

1. Keep in mind that the dimensions of the Edgeworth Box are determined by the *overall* endowment of each of the goods in the economy. A point in the Edgeworth Box has four components — two measured on each of the axes that correspond to the two individuals in the economy.

2. The set of mutually beneficial trades can easily be found by drawing the indifference curves (for the two individuals) that pass through the endowment point in the Edgeworth Box. (This usually gives us a lens-shaped set of mutually beneficial trades.) Within this set, only some of the allocations are efficient — because only some of them have the characteristic that the marginal rates of substitution for the two individuals are equal to one another.

3. A competitive equilibrium in the Edgeworth Box always has the following features: It consists of prices that form a budget line passing through the endowment, with indifference curves for both individuals tangent to this budget at the equilibrium allocation. The allocation is efficient because this tangency implies that the marginal rates of substitution for the two individuals are the same at that allocation — with no further gains from trade possible.

4. A competitive equilibrium in the Robinson Crusoe economy has similar features: It consists of an isoprofit line that also doubles as a worker budget constraint, with this line tangent to both the production frontier and the worker's indifference curve.

5. Keep in mind that we are still assuming that individuals are all price takers — and so we do not have to think about relative bargaining power when we investigate competitive equilibria. This is sometimes hard to keep in mind because the simple economies we are dealing with in this chapter only have two individuals in them — and it is therefore artificial for us to treat them as if they were price takers. (It seems even sillier in the Robinson Crusoe economy where we treat a single individual as if he had a split personality!) But the point here is to illustrate the basic intuitions that continue to hold when the economies get much larger and the assumption becomes natural.

6. The mathematical steps in calculating an equilibrium in a general equilibrium economy follow straightforwardly from the Edgeworth Box (or Robinson Crusoe) pictures — so keep going back to the underlying pictures if you get lost in the math steps.