CHAPTER

17

Choice and Markets in the Presence of Risk

In this chapter, we expand the consumer model to include the presence of risk. In the process, we are able to think about certain types of markets that insure against risk, markets that play important roles in modern life. And we can use the general equilibrium approach developed in Chapter 16 to investigate the market forces that arise when individuals attempt to insure against risk. In many classes, the primary emphasis will be on the material in the first section of the chapter, a section that deals with basic models of risk aversion when gambles are primarily about money. In the second section, we then see how this model is actually a special case of a model in which there are other things about the different "states of the world" that matter, and in the third section we introduce risk into the general equilibrium model.

Chapter Highlights

The main points of the chapter are:

1. When money is all that matters, we can model attitudes over risk in a straightforward way using a consumption/utility relationship whose shape determines the degree of **risk aversion**. Within this context, we can develop concepts like **certainty equivalence** and **risk premium**, both of which are related to the degree of risk aversion.

2. **Actuarily fair** insurance contracts have the feature that the expected value of consumption remains unchanged when the individual buys insurance — implying that the insurance company makes zero profits on average. In the simplest model of risk aversion, any risk averse individual will **fully insure** against risk when faced with a full menu of actuarily fair insurance contracts.

3. When tastes are **state-dependent**, the model becomes richer in that it allows for risk averse individuals to rationally choose to over- or under-insure.

The **state-independent** model is a special case of the more general state-dependent model.

4. In terms of the underlying math, the concept of a **von-Neumann Morgenstern expected utility function** can typically be used to model consumer tastes. Such a function has the feature that the utility over a risky gamble can be expressed as the probability-weighted average — or expected — utility. This expected utility is less than the utility of the expected value of the gamble whenever individuals are risk averse. (Expected utility theory, however, only holds as long as the *independence axiom* (developed in an appendix of the chapter) holds — and there exist well-known anomalies in consumer behavior that are not consistent with this axiom.)

5. In a general equilibrium setting, actuarily fair insurance contracts often cannot arise. The presence of **aggregate risk** in an economy, for instance, implies that there are not enough individuals willing to sell "recession insurance" on terms that would be actuarily fair. As a result, risk averse individuals in such an economy will not fully insure because of the equilibrium pricing of such insurance.

17A Solutions to Within-Chapter-Exercises for Part A

Exercise 17A.1

If the relationship depicted in Graph 17.1a were a single input production function, would it have increasing, decreasing or constant returns to scale?

Answer: It would have decreasing returns to scale — doubling inputs results in less than doubling of the "output".

Exercise 17A.2

Verify that my wife's expected household consumption is \$190,000.

Answer: The expected value of household consumption is

$$0.75(250,000) + 0.25(10,000) = \$190,000. \tag{17A.2}$$

Exercise 17A.3

What is the relationship between increasing, constant and decreasing marginal utility of consumption to risk loving, risk neutral and risk averse tastes?

Answer: Decreasing marginal utility of consumption implies risk averse tastes; constant marginal utility of consumption implies risk neutral tastes; and increasing marginal utility of consumption implies risk loving tastes.

Exercise 17A.4

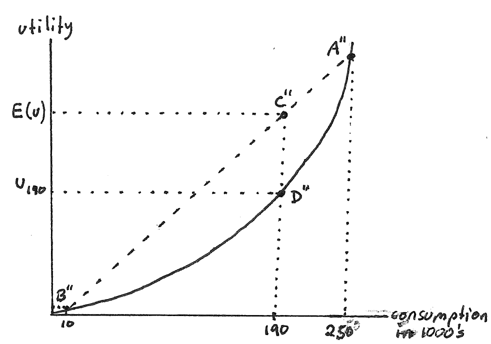
We said above that "a risk averse person's utility of the expected value of a gamble is always higher than the expected utility of the gamble." How does this statement change for risk neutral and risk loving tastes?

Answer: A risk neutral person's utility of the expected value of a gamble is always the same as the expected utility of the gamble; and a risk loving person's utility of the expected value of a gamble is always less than the expected utility of the gamble.

Exercise 17A.5

Illustrate that, if tastes are as described in panel (c), my wife prefers the "risky gamble" (of getting $250,000 with probability 0.75 and $10,000 with probability 0.25) over the "sure thing" ($190,000 with certainty) that has the same expected value.

Answer: This is illustrated in Exercise Graph 17A.5 where the expected utility of the gamble is read off from point C'' as $E(u)$ and the utility of 190 for sure is read off from point D'' as u_{190}. Since $E(u) > u_{190}$, the expected utility from taking the gamble exceeds the utility from getting the expected value of the gamble for sure — i.e. the individual prefers more risk for the same expected outcome.



Exercise Graph 17A.5 : Risk Loving Tastes

Exercise 17A.6

What is the certainty equivalent and the risk premium for my wife if she had tastes that can be summarized as in panel (b) of Graph 17.2?

Answer: The certainty equivalent would be \$190,000 — because, in this case, she does not care about risk and simply cares about the expected value of the gamble. This implies that the risk premium — the difference between the expected value of the gamble and the certainty equivalent — is zero.

Exercise 17A.7

In panel (c) of Graph 17.2, is the risk premium positive or negative? Can you reconcile this with the fact that the tastes in this graph represent those of a risk lover?

Answer: In this case, the certainty equivalent is greater than the expected value of the gamble — which means that the risk premium is negative. Put differently, in this case the person is willing to pay to play the risky game rather than receive the expected value with certainty.

Exercise 17A.8

True or False: As an individual becomes more risk averse, the certainty equivalent for a risky gamble will fall and the risk premium will rise.

Answer: This is true. More risk averse tastes imply the person is willing to accept less for sure in order to step away from the gamble because she dislikes risk more — which implies the certainty equivalent falls with increasing risk aversion. Since the risk premium is the difference between the expected value of the gamble and the certainty equivalent, this implies that the risk premium increases with the degree of risk aversion.

Exercise 17A.9

Verify that the zero-profit relationship between b and p is as described in the previous sentence.

Answer: If the probability of the "bad" outcome is δ, the insurance company will incur costs that average δb per person. Since it collects premiums from everyone regardless of which outcome he/she faces, the average revenue per person is p. When profits are zero, revenues are equal to costs — i.e. $\delta b = p$ or, dividing both sides by δ, $b = p/\delta$.

Exercise 17A.10

Verify that my wife's expected income is still \$190,000 under this insurance policy.

Answer: Her expected income is

$$0.75(230,000) + 0.25(70,000) = \$190,000. \tag{17A.10}$$

Exercise 17A.11

What are some examples of other actuarily fair insurance contracts that do not provide full insurance? Would each of these also earn zero profit for insurance companies? Can you see why none of them would ever be preferred to full insurance by my wife?

Answer: With $\delta = 0.25$, any insurance contract that satisfies $b = 4p$ is actuarily fair; for instance $(b, p) = (10, 40)$, $(b, p) = (25, 100)$, etc. Each of these would indeed result in zero profit for the insurance company (assuming the insurance companies incur no other costs). But full insurance would be preferred (by someone with risk averse tastes) to any insurance policy that has a benefit level less than the one that insures fully (as well as any policy that has an insurance benefit greater than the full insurance amount.) This is because any actuarily fair insurance policy that does not fully insure has more risk than full insurance but the same expected consumption level.

Exercise 17A.12

Referring back to what you learned in Graph 17.3, what is my wife's consumer surplus if she fully insures in actuarily fair insurance markets?

Answer: Consumer surplus is the amount a consumer is willing to pay to participate in a market as opposed to not participating. We found earlier that my wife's certainty equivalent was equal to \$115,000 — i.e. she is indifferent between getting \$115,000 for sure as opposed to taking her chances without insurance. Getting rid of the risk is therefore worth \$75,000 — her risk premium. In other words, she would have been willing to pay up to \$75,000 more for the full insurance contract that has premium \$60,000 and benefit \$240,000. If she were to pay this maximum premium she is willing to pay, she would be buying a policy with benefit of \$240,000 and premium of \$135,000. This would result in a "good" outcome of $\$250,000 - \$135,000 = \$115,000$ and a "bad" outcome of $\$10,000 + \$240,000 - \$135,000 = \$115,000$.

Exercise 17A.13

What actuarily fair insurance policy would a risk loving consumer purchase? Can you illustrate your answer within the context of a graph that begins as in Graph 17.2c? (*Hint*: The benefit and premium levels will be negative.)

Answer: A risk loving consumer will want to transfer consumption from the "bad" state to the "good" state (rather than the other way around, as was the case for a risk averse consumer). In our case, my wife has \$10,000 available to transfer into the good state. This will leave her with zero consumption in the bad state as a result of receiving benefit b and paying premium p that, in order to be actuarily fair, have to satisfy the equation $b = 4p$. Thus,

$$0 = 10,000 + b - p = 10,000 + 4p - p = 10,000 + 3p. \quad (17A.13)$$

Solving this for p, we get $p = -\$3,333.33$, and substituting this back into $b = 4p$, we get $b = -\$13,333.33$. Thus, the risk lover would want to pay $(b-p) = 13,333.33 - 3,333.33 = \$10,000$ in the bad state in order to increase consumption in the good state to $250,000 - p = 250,000 - (-3,333,33) = \$253,333.33$. All policy-holders for this insurance contract would therefore *receive* \$3,333.33 regardless of what state they end up in, the 25% that end up in the bad state would pay \$13,333.33. On average, the insurance company therefore has revenues of \$3,333.33 per customer and costs of 0.25(13,333.33)=\$3,333.33 — thus making zero profit.

Exercise 17A.14

True or False: A risk neutral consumer will be indifferent between all actuarily fair insurance contracts.

Answer: This is true. Actuarily fair insurance contracts reduce risk while keeping expected consumption levels the same. Since risk neutral consumers only care about the expected consumption level and are indifferent to different levels of risk, they are indifferent between insurance policies which keep the expected consumption levels the same while changing risk.

Exercise 17A.15

Verify the numbers on the horizontal axis of Graph 17.5.

Answer: The first policy is $(b_1, p_1) = (65, 20)$; the second is $(b_2, p_2) = (100, 40)$; and the third is $(b_3, p_3) = (122, 60)$. Letting x denote the "bad" outcome and y the "good" outcome, we get outcomes $(x_1, y_1) = (55, 230)$, $(x_2, y_2) = (70, 210)$ and $(x_3, y_3) = (72, 190)$ under the three policies. The expected consumption levels c_1, c_2 and c_3 are then

$$c_1 = 0.25(55) + 0.75(230) = 186.25, \quad c_2 = 0.25(70) + 0.75(210) = 175 \text{ and}$$
$$c_3 = 0.25(72) + 0.75(190) = 160.5. \tag{17A.15}$$

Exercise 17A.16

True or False: If firms in a perfectly competitive insurance industry face recurring fixed costs and marginal administration costs that are increasing, risk averse individuals will not fully insure in equilibrium.

Answer: This is true. The combination of recurring fixed costs and upward sloping marginal administrative costs creates U-shaped average cost curves for firms. In equilibrium, firms will have to cover these costs (that are in addition to the cost of honoring insurance claims) — and thus cannot price insurance at "actuarily fair" rates. Risk averse individuals will fully insure if insurance is actuarily fair but not when it is actuarily unfair (as it has to be in order for firms that face these additional costs to make zero profit).

Exercise 17A.17

Suppose only full insurance contracts were offered by the insurance industry — i.e. only contracts that insure that my wife will be equally well off financially regardless of what happens to me. What is the most actuarially unfair insurance contract that my wife would agree to buy? (*Hint*: Refer back to Graph 17.3.)

Answer: This relates to what we already did in answering within-chapter exercise 17A.12 where we calculated that my wife would be indifferent between not insuring and fully insuring with an actuarially unfair policy that has a benefit of $240,000 and a premium of $135,000. This is "full insurance" because the outcome in the "good" and "bad" states is the same for my wife if she carries this insurance: In the "bad" state, she pays $135,000 but receives $240,000 in addition to the $10,000 she starts with — leaving her with $115,000. In the "good" state, on the other hand, she just pays $135,000 from her initial $250,000 — again leaving her with $115,000. And since $115,000 is the certainty equivalent for her, we know that she is indifferent between getting $115,000 for sure or taking the uninsured gamble.

Exercise 17A.18

Why does consumption in the bad state rise only by 3 times the premium amount when actuarially fair insurance benefits are 4 times as high as the premium?

Answer: This is because even in the bad state, my wife has to pay the premium. Thus, when $b = 4p$ (with b the benefit in the bad state and p the premium of the policy), my wife gets $b - p = 4p - p = 3p$ once we take into account the fact that she still pays the premium.

Exercise 17A.19

Why is the slope of the budget constraint $-(1-\delta)/\delta$?

Answer: We concluded before that actuarially fair insurance implies that $b = p/\delta$ (where b is the benefit paid by the insurance company in the bad state, p is the insurance premium and δ is the probability of the bad state occurring.) Thus, if the consumer gives up $1 in the good state, she receives $b - 1 = (1/\delta) - 1 = (1-\delta)/\delta$ in the bad state (where she gets the benefit b but still has to pay the premium p).

Exercise 17A.20

What would indifference curves look like for a risk-neutral consumer? What insurance policy would she purchase?

Answer: The risk neutral consumer would have linear indifference curves with slope $-(1-\delta)/\delta$. Thus, there is an indifference curve that lies on the actuarially fair insurance contract budget line — which implies that the consumer is indifferent between all actuarially fair insurance policies. This makes sense — the expected value is the same all along the actuarially fair budget line, and all that risk neutral consumers care about is the expected value, not the risk.

Exercise 17A.21

What would indifference curves look like for a risk-loving consumer? What insurance policy would she purchase?

Answer: Risk loving consumers would have non-convex indifference curves that bend outward — implying that they will choose a corner solution with 0 consumption in the bad state.

Exercise 17A.22

We concluded previously that, when the two states are the same (aside from the income level associated with each state), $MRS = -(1-\delta)/\delta$ along the 45-degree line. In the case we just discussed, can you tell whether the MRS is greater or less than this along the 45-degree line?

Answer: The indifference curves along the 45 degree line are now steeper — which implies the MRS is larger in absolute value — implying we are willing to trade more consumption in the bad state for additional consumption in the good state when we are along the 45 degree line (compared to the case of state-independent tastes).

Exercise 17A.23

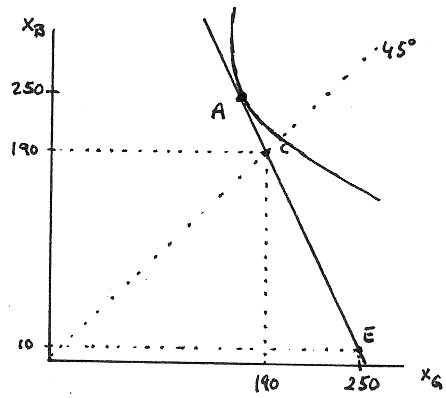
Suppose my wife was actually depressed by my presence and tolerates me solely for the paycheck I bring. Due to this depression, consumption is not very meaningful in the "good" state when I am around, but if I were not around, she would be able to travel the world and truly enjoy life. Might this cause her to purchase more than "full" life insurance on me? How would you illustrate this in a graph?

Answer: Yes, she would now "over-insure" in the sense that consumption in the bad state will be larger than consumption in the good state once she buys the optimal insurance policy. This is illustrated in Exercise Graph 17A.23 where indifference curves along the 45-degree line are shallower than would be the case for the optimal outcome bundle to lie on the 45 degree line (where full insurance would happen at C). Thus, my wife would choose A — an outcome bundle where she ends up consuming more in the bad state than in the good state.

Exercise 17A.24

Can you think of a different scenario in which it makes sense for the sports fan to bet against her own team?

Answer: Some sports fans might enter a near-full state of bliss when their team wins — requiring little additional consumption to gain further utility — i.e. their marginal utility from additional consumption is low when their team wins; but when their team loses, they need to consume various expensive libations to drown out their sorrows — i.e. the marginal utility of consumption is high for the same level of overall consumption as in the good state. In that case, the sports fan would



Exercise Graph 17A.23 : Over-Insurance

want to bet against her own team if she is risk averse — either her team wins and she is happy, or her team loses and she gets to consume a lot to forget about the loss.

Exercise 17A.25

Which assumption in our example results in the square shape of this Edgeworth Box?

Answer: The assumption of "no aggregate risk" — because that assumption implies that the *total* number of bananas on the island will be the same regardless of whether we are in the rainy or the dry state.

Exercise 17A.26

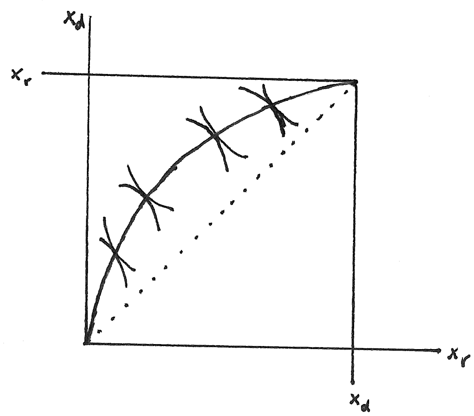
True or False: The 45-degree line is, in this case, the Contract Curve.

Answer: This is true. We know that, if tastes are not state-dependent, both individuals will have $MRS = -(1-\delta)/\delta$ along the 45-degree line — i.e. their indifference curves have the same slopes along the 45 degree line and are therefore tangent to one another. That is the defining characteristic of the contract curve, or the set of Pareto efficient outcomes.

Exercise 17A.27

What would the contract curve look like in this case?

Answer: This is depicted Exercise Graph 17A.27.



Exercise Graph 17A.27 : Contract Curve when you like Bananas more in the Rain

Exercise 17A.28

Suppose you liked bananas more when it rains than when it shines. Where would the equilibrium be?

Answer: The equilibrium would now lie below the 45-degree line, with $p_r^*/p_d^* < (1-\delta)/\delta$ — i.e. terms that are less favorable to me. This implies that the budget constraint is shallower than the actuarily fair budget constraint.

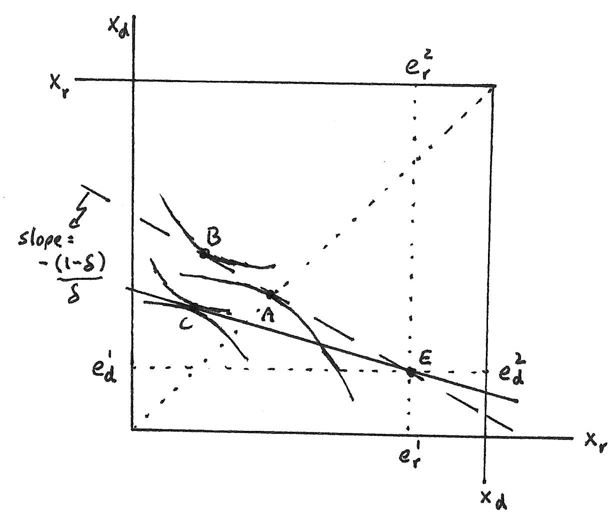
Exercise 17A.29

Suppose you were the one who had state-independent tastes and I was the one who values consuming bananas more when it shines than when it rains. Where would the equilibrium be?

Answer: This is depicted in Exercise Graph 17A.29. Since your tastes are state independent, we know that your indifference curves will have slope $-(1-\delta)/\delta$ along the 45 degree line. If the terms of trade were "actuarily fair", we would both be facing the dashed budget line, with you optimizing at A and me optimizing at B (since I value bananas more in the drought state). Since I want more bananas in the drought state than are supplied, the price for bananas in the drought state should rise relative to the price of bananas in the rainy state — leading to the solid budget line for both of us. In equilibrium, we will both optimize at some point like C.

Exercise 17A.30

Given that there are more bananas in the aggregate in the rainy state of the world, consider an endowment that has relatively more bananas in the dry state and another that has relatively more bananas in the rainy state. If you could choose your endowment, which endowment would you be more likely to want (assuming



Exercise Graph 17A.29 : Equilibrium when I like Bananas more in the Sunshine

we both have state-independent tastes and the overall endowments are not too different)?

Answer: We can see in the textbook graph that the equilibrium budget (in panel (b)) is shallower than the "actuarily fair" budget (in panel (a)). That means I will end up on a lower indifference curve in panel (b) than in panel (a), and you will end up on a higher indifference curve. You would therefore prefer to stick with your own endowment rather than switch with me. This makes intuitive sense: Since bananas are more scarce in the dry state, they are more valuable — which means, all else equal, you would want to have an endowment that has more bananas in the dry state and fewer in the wet state. (Of course you might still want the endowment that has relatively more bananas in the rainy state if that endowment is overall sufficiently larger than the other endowment option.)

Exercise 17A.31

In modeling equilibrium terms of trade that might emerge in financial markets, would you likely assume state-dependent or state-independent tastes?

Answer: In financial markets, investors care about the financial return to their investments — and there is no particular reason to expect the way they evaluate consumption to differ across different "states of the world." Thus, state-independent tastes might be a reasonable assumption for the model.

Exercise 17A.32

Suppose the two "states" of the world are recessions and economic booms. If you put consumption in economic booms on the horizontal axis, will the height of the Edgeworth box be larger or smaller than its width.

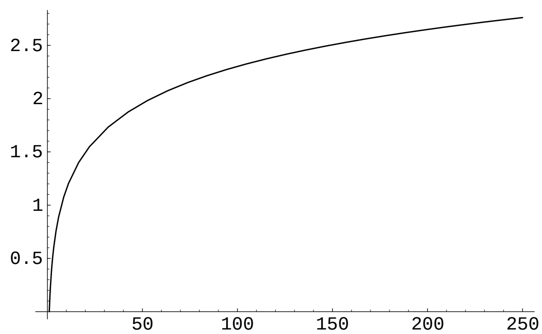
Answer: The overall level of consumption is larger during economic booms than during recessions — so you would expect the width of the box to be larger than its height.

17B Solutions to Within-Chapter-Exercises for Part B

Exercise 17B.1

Letting x denote consumption measured in thousands of dollars, illustrate the approximate shape of my wife's consumption/utility relationship in the range from 1 to 250 (interpreted as the range from $1,000 to $250,000.)

Answer: This is graphed in Exercise Graph 17B.1, with the function attaining $u(x) = 0$ when $x = 1$ (because $\ln(1) = 0$).



Exercise Graph 17B.1 : Graph of $u(x) = 0.5\ln(x)$ from x=1 through x=250

Exercise 17B.2

What does the graph of the utility function look like in the range of consumption between 0 and 1 (corresponding to 0 to $1,000)?

Answer: Between 0 and 1, the function gives negative values that approach negative infinity as x approaches 0.

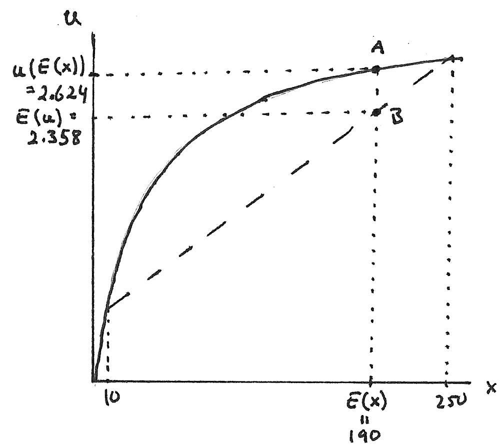
Exercise 17B.3

What is the utility of receiving the expected income, denoted $u(E(x))$? Illustrate $E(x)$, $E(u)$ and $u(E(x))$ on a graph of equation (17.2).

Answer: The utility of receiving the expected income $E(x)$ is

$$u(E(x)) = u(190) = 0.5\ln(190) \approx 0.5(5.247) = 2.6235. \quad (17B.3)$$

This is illustrated on the vertical axis in Exercise Graph 17B.3, together with $E(x) = 190$ on the horizontal axis and $E(u) = 2.358$ on the vertical.



Exercise Graph 17B.3 : $E(x)$, $E(u)$ and $E(u(x))$

Exercise 17B.4

True or False: If u is a concave function, then $u(E(x))$ is larger than $E(u)$, and if u is a convex function, then $u(E(x))$ is smaller than $E(u)$.

Answer: This is true. The first part is illustrated in Exercise Graph 17B.3, and the second part is easily seen by drawing the same graph with a convex function.

Exercise 17B.5

What would $E(u)$ and $u(E(x))$ be for my wife if her utility of consumption were given instead by the convex function $u(x) = x^2$? Illustrate your answer in a graph.

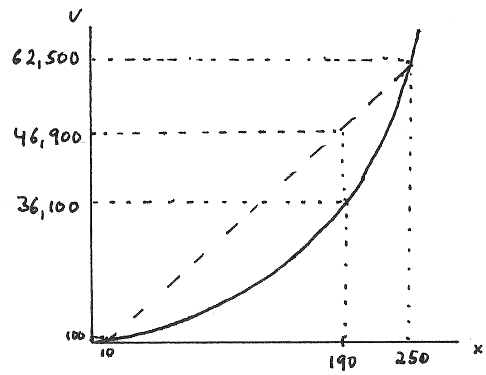
Answer: $E(x)$ would still be 190 as before. However, the utility of the outcomes in the good and bad states would now be

$$u(x_B) = u(10) = 10^2 = 100 \text{ and } u(x_G) = u(250) = 250^2 = 62,500. \quad (17B.5.i)$$

We thus get

$$u(E(x)) = u(190) = 36,100 \text{ and}$$
$$E(u) = 0.25u(x_B) + 0.75u(x_G) = 0.25(100) + 0.75(62500) = 46,900. \quad (17B.5.ii)$$

which is illustrated in Exercise Graph 17B.5.



Exercise Graph 17B.5 : Risk Loving Tastes

Exercise 17B.6

The convexity of a function f is defined analogously to concavity, with the inequality in equation (17.7) reversed. Can you show that tastes which exhibit risk loving (as opposed to risk aversion) necessarily imply that any $u(x)$ used to define an expected utility function must be convex?

Answer: A function f is convex if and only if

$$\delta f(x_1) + (1-\delta)x_2 > f(\delta x_1 + (1-\delta)x_2). \quad (17B.6.i)$$

For an individual to be risk loving, it must be the case that the expected utility of the gamble is greater than the utility of the expected value of the gamble; i.e. $U(x_G, x_B) > u(E(x))$. This can be expanded to read

$$\delta u(x_B) + (1-\delta)u(x_G) = U(x_G, x_B) > u(E(x)) = u(\delta x_B + (1-\delta)x_G). \quad (17B.6.ii)$$

Thus, the definition of risk loving in equation (17B.6.ii) implies that $u(x)$ must be convex as defined in equation (17B.6.i).

Exercise 17B.7

Can you show in analogous steps that convexity of $u(x)$ must imply non-convexity of the indifference curves over outcome pairs (x_G, x_B)?

Answer: Consider again an average bundle (x_G^3, x_B^3) of two more extreme bundles such that

$$x_G^3 = \alpha x_G^2 + (1-\alpha)x_G^1 \text{ and } x_B^3 = \alpha x_B^2 + (1-\alpha)x_B^1, \quad (17B.7.i)$$

with the more extreme bundles chosen to lie on the same indifference curve $\overline{U}$; i.e.

$$U(x_G^1, x_B^1) = U(x_G^2, x_B^2) = \overline{U}. \tag{17B.7.ii}$$

We can then again use the definition of the expected utility function and the convexity of $u(x)$ to show that

$$\begin{aligned}
U(x_G^3, x_B^3) &= \delta u(x_B^3) + (1-\delta)u(x_G^3) \\
&= \delta u\left(\alpha x_B^2 + (1-\alpha)x_B^1\right) + (1-\delta)u\left(\alpha x_G^2 + (1-\alpha)x_G^1\right) \\
&< \delta\left[\alpha u(x_B^2) + (1-\alpha)u(x_B^1)\right] + (1-\delta)\left[\alpha u(x_G^2) + (1-\alpha)u(x_G^1)\right] \\
&= \alpha\left[\delta u(x_B^2) + (1-\delta)u(x_G^2)\right] + (1-\alpha)\left[\delta u(x_B^1) + (1-\delta)u(x_G^1)\right] \\
&= \alpha U(x_G^2, x_B^2) + (1-\alpha)U(x_G^1, x_B^1) \\
&= \alpha\overline{U} + (1-\alpha)\overline{U} = \overline{U}.
\end{aligned} \tag{17B.7.iii}$$

Thus, we can conclude that

$$U(x_G^3, x_B^3) < \overline{U} = U(x_G^2, x_B^2) = U(x_G^1, x_B^1); \tag{17B.7.iv}$$

i.e. "averages are worse than extremes", implying non-convex indifference curves (that bend away from the origin). Put differently, we have now shown that risk loving tastes imply that any $u(x)$ used to construct an expected utility function that represents such tastes must be convex, and the convexity of $u(x)$ in turn implies that the indifference map is non-convex.

Exercise 17B.8

Illustrate x_{ce} and the risk premium on a graph with my wife's utility function $u(x) = 0.5\ln x$.

Answer: This is illustrated in Exercise Graph 17B.8.

Exercise 17B.9

Verify the expressions for p^* and b^*.

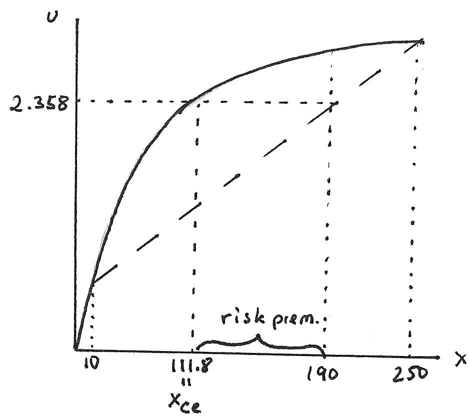
Answer: The problem

$$\max_p \ \delta\alpha\ln\left(x_B + \frac{(1-\delta)p}{\delta}\right) + (1-\delta)\alpha\ln(x_G - p), \tag{17B.9.i}$$

is an unconstrained optimization problem. So all we have to do is set the first derivative of the objective function with respect to the choice variable p to zero; i.e.

$$\alpha\delta\left(\frac{(1-\delta)}{\delta}\right)\left(\frac{\delta}{\delta x_B + (1-\delta)p}\right) - (1-\delta)\alpha\left(\frac{1}{x_G - p}\right) = 0. \tag{17B.9.ii}$$

Adding the second term to both sides and simplifying, we get



Exercise Graph 17B.8 : Certainty Equivalent and Risk Premium

$$\left(\frac{\delta}{\delta x_B + (1-\delta)p}\right) = \left(\frac{1}{x_G - p}\right) \quad (17B.9.iii)$$

or, cross-multiplying,

$$\delta x_B + (1-\delta)p = \delta(x_G - p). \quad (17B.9.iv)$$

Adding δp to both sides, and subtracting δx_B from both sides, we end up with

$$p^* = \delta(x_G - x_B). \quad (17B.9.v)$$

Substituting into the actuarily fair relationship between b and p — i.e. $b = p/\delta$ — we then also get $b^* = x_G - x_B$.

Exercise 17B.10

Even though we did not use the same underlying utility function as the one used to plot graphs in Section A, we have gotten the same result for the optimal actuarily fair insurance policy. Why is this?

Answer: This is because it is optimal for *all* risk averse individuals to fully insure — i.e. for all individuals with concave utility functions regardless of what precise shape the concavity takes. Full insurance just means a combination of premiums and benefits that results in the same consumption regardless of what state occurs — which does not depend on what utility functions are but just on what the good and bad outcomes as well as the associated probabilities are.

Exercise 17B.11

Derive the expression for the marginal rate of substitution for equation (17.20). Now suppose $\alpha=\beta$. What is the MRS along the 45 degree line on which $x_1 = x_2$? Compare this to the result we derived graphically in Graph 17.7.

Answer: The MRS is

$$MRS = -\frac{\partial U/\partial x_G}{\partial U/\partial x_B} = -\left(\frac{(1-\delta)\beta/x_G}{\delta\alpha/x_B}\right) = -\frac{(1-\delta)\beta x_B}{\delta\alpha x_G}. \quad (17B.11)$$

On the 45-degree line, $x_B = x_G$ and thus cancel in the equation — and, if $\alpha = \beta$, these terms cancel as well — leaving us with $MRS = -(1-\delta)/\delta$. This implies a slope of $(-(1-\delta)/\delta)$ along the 45 degree line — precisely the result we derived graphically in Section A.

Exercise 17B.12

Can you see that the indifference curves generated by $U(x_1, x_2)$ in equation (17.20) are Cobb-Douglas? Write the function as a Cobb-Douglas function and derive the MRS. Does the property that must hold along the 45 degree line when tastes are not state-dependent hold?

Answer: The Cobb-Douglas utility function that gives the same indifference curves is

$$\overline{U}(x_G, x_B) = x_B^{\delta\alpha} x_G^{(1-\delta)\beta}. \quad (17B.12.i)$$

(You can verify this by simply taking the natural log of this equation — which gives you back the original utility function with the expected utility form.) The MRS is

$$MRS = -\frac{\partial U/\partial x_G}{\partial U/\partial x_B} = -\left(\frac{(1-\delta)\beta x_B^{\delta\alpha} x_G^{-\delta\beta}}{\delta\alpha x_B^{(\delta\alpha-1)} x_G^{(1-\delta)\beta}}\right) = -\frac{(1-\delta)\beta x_B}{\delta\alpha x_G}. \quad (17B.12.ii)$$

This is the same MRS as the one calculated in the previous exercise for the original utility function (showing once again that the two give rise to the same indifference curves.) On the 45 degree line, $x_B = x_G$ and, when $\alpha = \beta$ (as it must if tastes are not state-dependent), this reduces to $MRS = -(1-\delta)/\delta$. This is the condition that in fact must hold along the 45 degree line. (In this case, the utility function can in fact be further simplified as $U(x_1, x_2) = x_1^{\delta} x_2^{(1-\delta)}$.)

Exercise 17B.13

True or False: The expected utility function $U(x_G, x_B)$ can be transformed in all the ways that utility functions in consumer theory can usually be transformed without changing the underlying indifference curves, but such transformations will imply a loss of the expected utility form.

Answer: This is true. In transforming our original expected utility function $U(x_B, x_G) = \delta\alpha \ln x_B + (1-\delta)\beta \ln x_G$ to one that takes the usual Cobb-Douglas form in exercise (17B.12), for instance, we have preserved the shape of indifference curves but the new utility function is no longer the probability weighted sum of the utilities associated with each outcome as measured by a function $u(x)$.

Exercise 17B.14

On a graph with x_G on the horizontal and x_B on the vertical axis, illustrate this budget constraint using values derived from the example of my wife's choices over insurance contracts. Compare it to Graph 17.6b.

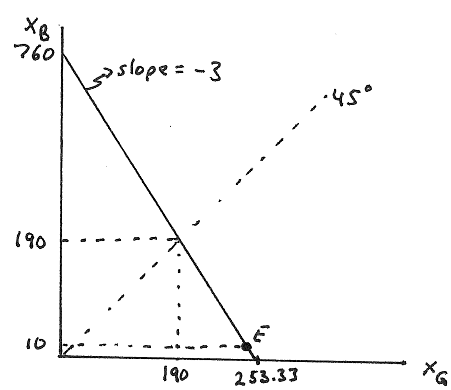
Answer: The budget line equation

$$x_B = \frac{\delta e_B + (1-\delta)e_G}{\delta} - \frac{(1-\delta)}{\delta}x_G \qquad \text{(17B.14.i)}$$

becomes

$$x_B = \frac{0.25(10) + (1-0.25)(250)}{0.25} - \frac{(1-0.25)}{0.25}x_G = 760 - 3x_G. \qquad \text{(17B.14.ii)}$$

The budget line therefore has vertical intercept of 760 and slope of −3 (giving us horizontal intercept of 253.33.) This is depicted in Exercise Graph 17B.14.



Exercise Graph 17B.14 : Actuarily Fair Budget Constraint

Exercise 17B.15

Verify the result in equation (17.25).

Answer: The Lagrange function for this problem is

$$\mathcal{L} = \delta\alpha \ln x_B + (1-\delta)\beta \ln x_G + \lambda(\delta e_B + (1-\delta)e_G - \delta x_B - (1-\delta)x_G) \quad (17B.15.i)$$

giving us first order conditions

$$\frac{\partial \mathcal{L}}{\partial x_B} = \frac{\delta\alpha}{x_B} - \lambda\delta = 0 \text{ and } \frac{\partial \mathcal{L}}{\partial x_G} = \frac{(1-\delta)\beta}{x_G} - \lambda(1-\delta) = 0. \quad (17B.15.ii)$$

These can be solved for $x_G = \beta x_B/\alpha$. When plugged into the budget constraint $\delta e_B + (1-\delta)e_G = \delta x_B + (1-\delta)x_G$, we can then solve for

$$x_B^* = \frac{\alpha(\delta e_B + (1-\delta)e_G)}{\delta\alpha + (1-\delta)\beta}. \quad (17B.15.iii)$$

Plugging this back into $x_G = \beta x_B/\alpha$, we also get

$$x_G^* = \frac{\beta(\delta e_B + (1-\delta)e_G)}{\delta\alpha + (1-\delta)\beta}. \quad (17B.15.iv)$$

Exercise 17B.16

Using the values of \$10 and \$250 as the consumption level my wife gets in state B and state G in the absence of insurance, what level of consumption does she get in each state when she chooses her optimal actuarily fair insurance policy (assuming, as before, that state B occurs with probability 0.25 and state G occurs with probability 0.75)?

Answer: She gets

$$x = \delta e_B + (1-\delta)e_G = 0.25(10) + (1-0.25)(250) = 190. \quad (17B.16)$$

Exercise 17B.17

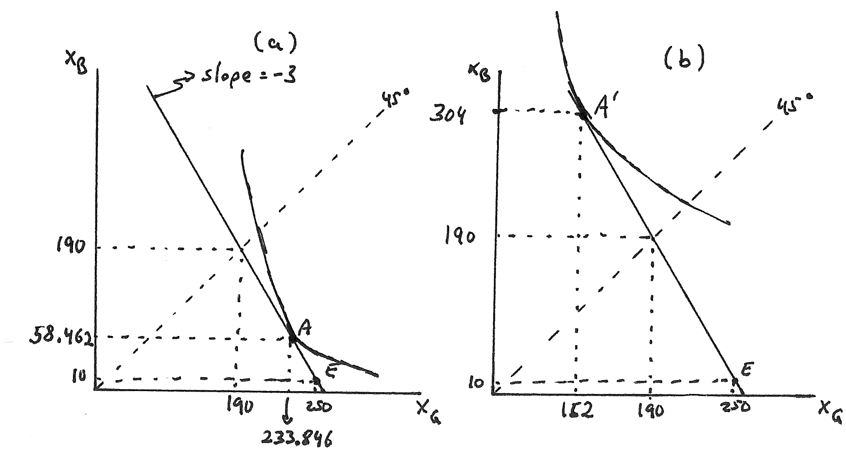
Using an Excel spreadsheet, can you verify the numbers in Table 17.1?

Answer: You can do this by simply specifying fields for e_B, e_G, δ and α while setting $\beta = 1$. Then program the formula for x_B^* and x_G^* derived in the text to calculate x_B and x_G in the table. The premium can then be derived simply as $p = e_G - x^G$, and the benefit is $b = p/\delta$.

Exercise 17B.18

Using a graph similar to Graph 17.8, illustrate the case of $\alpha/\beta = 1/4$ (row 2 in the table).

Answer: This is done in panel (a) of Exercise Graph 17B.18.



Exercise Graph 17B.18 : Two state-dependent tastes and actuarily fair insurance

Exercise 17B.19

Using a graph similar to Graph 17.8, illustrate the case of $\alpha/\beta = 2/1$ (row 7 in the table).

Answer: This is done in panel (b) of Exercise Graph 17B.18 in the answer to within-chapter exercise 17B.18.

Exercise 17B.20

For what values of α and β is utility state-independent for each of these consumers?

Answer: For $\alpha = \beta$.

Exercise 17B.21

Are we imposing any real restrictions by assuming that the utility weights placed on log consumption in the two states sum to 1 for each of the two consumers?

Answer: No. To see this, we can first derive the *MRS* for the expected utility functions as written, which are

$$\begin{aligned} MRS^1 &= -\frac{\partial U^1/\partial x_1}{\partial U^1/\partial x_2} = \frac{\delta \alpha x_2}{(1-\delta)(1-\alpha)x_1} \quad \text{and} \\ MRS^2 &= -\frac{\partial U^2/\partial x_1}{\partial U^2/\partial x_2} = \frac{\gamma \beta x_2}{(1-\gamma)(1-\beta)x_1}. \end{aligned} \qquad (17B.21.i)$$

Now suppose we multiply α and $(1-\alpha)$ by k, and we multiply β and $(1-\beta)$ by t. We would then get the expected utility functions

$$V^1(x_1,x_2) = \delta k\alpha \ln x_1 + (1-\delta)k(1-\alpha)\ln x_2 \tag{17B.21.ii}$$

and

$$V^2(x_1,x_2) = \gamma t\beta \ln x_1 + (1-\gamma)t(1-\beta)\ln x_2. \tag{17B.21.iii}$$

The marginal rates of substitution for these utility functions are then identical to the ones above because the k and the t appears in both numerator and denominator and therefore cancels. Thus, while the labeling of the indifference curves changes, the shapes of the indifference curves do not. This is generally true for any *linear* transformation of the u functions — i.e. if we have identified u functions for each state that allow us to represent the indifference curves with an expected utility function, then any linear transformation of the u's will also allow us to represent the same indifference curves with an expected utility function using these transformed u's.

Exercise 17B.22

How would you write the analogous optimization problem for individual 2?
Answer: You would write it as

$$\max_{x_1^2,x_2^2} \gamma\beta \ln x_1^2 + (1-\gamma)(1-\beta)\ln x_2^2 \text{ subject to } p_1 e_1^2 + p_2 e_2^2 = p_1 x_1^2 + p_2 x_2^2. \tag{17B.22}$$

Exercise 17B.23

Suppose that the overall endowment in the economy is the same in each of the two states — i.e. $e_1^1 + e_1^2 = e_2^1 + e_2^2$; suppose that each consumer has state-independent utility (i.e. $\alpha = (1-\alpha)$ and $\beta = (1-\beta)$), and suppose that both consumers evaluate risk in the same way (i.e. $\delta = \gamma$). Can you then demonstrate that equilibrium terms of trade will be actuarily fair — i.e. $p_2^*/p_1^* = (1-\delta)/\delta$?

Answer: Using the equation derived in the text for p_2^* (when p_1^* is normalized to 1) and substituting α for $(1-\alpha)$, β for $(1-\beta)$ and δ for γ, we get

$$\begin{aligned}
p_2^* &= \frac{\alpha(1-\delta)\left(\beta\delta+\beta(1-\delta)\right)e_1^1 + \beta(1-\delta)\left(\alpha\delta+\alpha(1-\delta)\right)e_1^2}{\alpha\delta\left(\beta\delta+\beta(1-\delta)\right)e_2^1 + \beta\delta\left(\alpha\delta+\alpha(1-\delta)\right)e_2^2} \\
&= \frac{\alpha(1-\delta)\beta e_1^1 + \beta(1-\delta)\alpha e_1^2}{\alpha\delta\beta e_2^1 + \beta\delta\alpha e_2^2} \\
&= \frac{\alpha\beta(1-\delta)\left(e_1^1+e_1^2\right)}{\alpha\beta\delta\left(e_2^1+e_2^2\right)} \\
&= \frac{(1-\delta)}{\delta}.
\end{aligned} \tag{17B.23}$$

Exercise 17B.24

For the scenario described in the previous exercise, can you use individual demand functions to illustrate that each consumer will choose to equalize consumption across the two states? Where in the Edgeworth Box does this imply the equilibrium falls?

Answer: The demand functions derived in the text are

$$x_1^1(p_1,p_2)=\frac{\alpha\delta(p_1e_1^1+p_2e_2^1)}{(\alpha\delta+(1-\alpha)(1-\delta))\,p_1} \text{ and}$$
$$x_1^2(p_1,p_2)=\frac{\beta\gamma(p_1e_1^2+p_2e_2^2)}{\big(\beta\gamma+(1-\beta)(1-\gamma)\big)\,p_1}. \tag{17B.24.i}$$

Substituting $p_1=1$, α for $(1-\alpha)$, β for $(1-\beta)$ and δ for γ, these become

$$x_1^1(p_2)=\frac{\alpha\delta(e_1^1+p_2e_2^1)}{(\alpha\delta+\alpha(1-\delta))}=\frac{\alpha\delta(e_1^1+p_2e_2^1)}{\alpha}=\delta(e_1^1+p_2e_2^1) \tag{17B.24.ii}$$
$$x_1^2(p_2)=\frac{\beta\delta(e_1^2+p_2e_2^2)}{\big(\beta\delta+\beta(1-\delta)\big)}=\frac{\beta\delta(e_1^2+p_2e_2^2)}{\beta}=\delta(e_1^2+p_2e_2^2). \tag{17B.24.iii}$$

Plugging in the equilibrium price $p_2^*=(1-\delta)/\delta$, we then get

$$x_1^1=\delta\left(e_1^1+\frac{(1-\delta)}{\delta}e_2^1\right)=\delta e_1^1+(1-\delta)e_2^1 \text{ and} \tag{17B.24.iv}$$
$$x_1^2=\delta\left(e_1^2+\frac{(1-\delta)}{\delta}e_2^2\right)=\delta e_1^2+(1-\delta)e_2^2. \tag{17B.24.v}$$

For each consumer, the budget constraint has to bind. Consider, for instance, consumer 1. His budget constraint is $p_1e_1^1+p_2e_2^1=p_1x_1^1+p_2x_2^1$. Setting p_1 equal to its normalized value of 1, plugging in our equilibrium price for p_2 (i.e. $p_2=(1-\delta)/\delta$), and substituting our consumption level $x_1^1=\delta e_1^1+(1-\delta)e_2^1$, this becomes

$$e_1^1+\frac{(1-\delta)}{\delta}e_2^1=\left(\delta e_1^1+(1-\delta)e_2^1\right)+\frac{(1-\delta)}{\delta}x_2^1 \tag{17B.24.vi}$$

which solves to

$$x_2^1=\delta e_1^1+(1-\delta)e_2^1. \tag{17B.24.vii}$$

Using the same method, we can also show that $x_2^2=\delta e_1^2+(1-\delta)e_2^2$. Thus,

$$x_1^1=x_2^1 \text{ and } x_1^2=x_2^2; \tag{17B.24.viii}$$

i.e. consumer 1 consumes the same quantity in each state, as does consumer 2. This equilibrium allocation therefore appears on the 45-degree line of the Edgeworth box — which, because we are assuming no aggregate risk, goes through the origins of both consumers' axes.

Exercise 17B.25

What is the shape of the Edgeworth Box representing an economy in which $e_1^1 + e_1^2 = e_2^1 + e_2^2$?

Answer: It is a square.

Exercise 17B.26

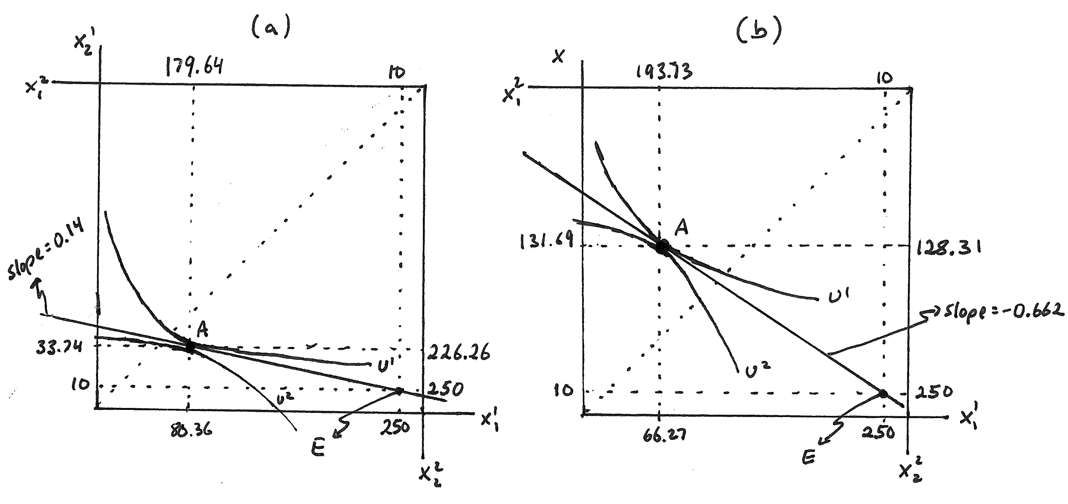
Why do you think individual 1 ends up with less consumption than individual 2 once they fully insure?

Answer: Individual 1 begins with an endowment of 250 in state 1 and 10 in state 2, but state 1 occurs only with probability 0.25 while state 2 occurs with probability 0.75. Individual 1's "expected endowment" is therefore only $0.25(250) + 0.75(10) = 70$. Individual 2, on the other hand, begins with an endowment of 10 in state 1 and 250 in state 2 — giving him an "expected endowment" of $0.25(10) + 0.75(250) = 190$. Thus, individual 1 is substantially richer. Even though the endowments appear to be symmetric, they are not because the different states do not arise with equal probability.

Exercise 17B.27

Can you draw out the equilibrium in rows 1 and 3 in Edgeworth Boxes?

Answer: These are illustrated in panels (a) and (b) where E is the initial endowment outcome bundle and A is the equilibrium. The slope of the budgets are equal to $-1/p_2^*$ since p_1 was normalized to 1.



Exercise Graph 17B.27 : Two Equilibria

Exercise 17B.28

Can you offer a similar intuitive explanation for the third set of results in Table 17.2?

Answer: In this set of simulations, individual 1's tastes are held constant, placing relatively little weight on state 1 consumption. In the first row, individual 2 places similarly little weight on state 1 consumption. As a result, the equilibrium price for buying state 2 consumption is high, which implies individual 2 who has most of the state 2 endowment is quite rich compared to individual 1 who has relatively little state 2 endowment. Both individuals end up fully insuring, but, because of the effective wealth disparity, individual 1 ends up with much less consumption than individual 2. As individual 2's β increases over the next three rows, he places increasingly more weight on consumption in state 1. As a result, demand for state 2 consumption falls, causing p_2^* to fall. Individual 1's tastes remain fixed throughout these simulations — so the only impact on him comes from the falling price of state 2 consumption — leading him to consume more in state 2 and less in state 1. Individual 2 ends up lowering state 2 consumption (despite the fact that it is becoming cheaper) because he is placing increasingly more weight on consumption in state 1 as β increases.

Exercise 17B.29

Suppose $\alpha = \beta = 0.5$. For what values of δ and γ will the equilibrium be the same as the one in the first row of Table 17.2? (*Hint*: This is harder than it appears. In row one of the table, $\beta\gamma = 1/16$ and $(1-\beta)(1-\gamma) = 9/16$. Thus, the overall weight placed on state 2 is 9 times the weight placed on state 1. When you now change β from 0.25 to 0.5, you need to make sure when you change γ that the overall weight placed on state 2 is again 9 times the weight placed on state 1.)

Answer: When β is raised to 0.5, we want γ to satisfy the condition that 0.5γ is 9 times as high as $0.5(1-\gamma)$ — i.e. $9(0.5)\gamma = 0.5(1-\gamma)$. Solving for γ, we get $\gamma = 0.10$. Thus, when $\beta = 0.5$ and person 2's belief about γ is equal to 0.10, the expected utility function for individual 2 will give rise to the same indifference curves as the expected utility function when $\beta = 0.25$ and $\gamma = 0.25$.

Exercise 17B.30

Can you see from the demand equations why consumption in the rainy season remains unchanged?

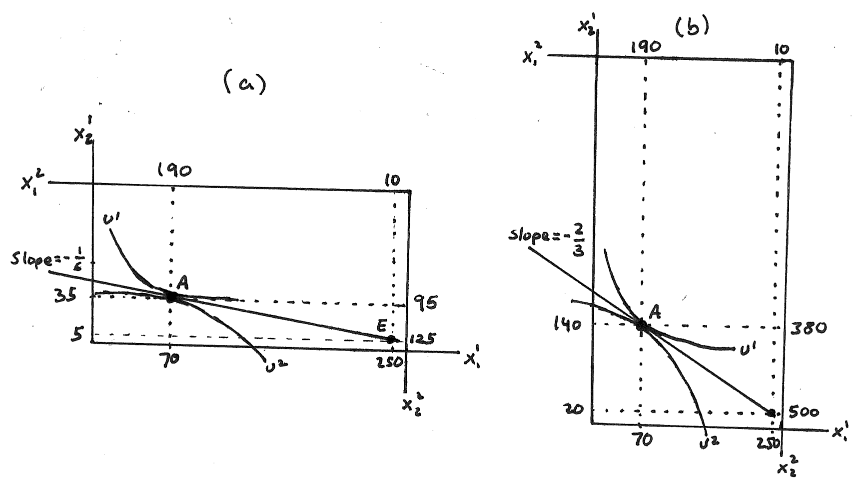
Answer: The demand equations for consumption in state 1 are derived in the text as

$$x_1^1(p_1,p_2) = \frac{\alpha\delta(p_1e_1^1 + p_2e_2^1)}{(\alpha\delta + (1-\alpha)(1-\delta))\,p_1} \text{ and}$$
$$x_1^2(p_1,p_2) = \frac{\beta\gamma(p_1e_1^2 + p_2e_2^2)}{\big(\beta\gamma + (1-\beta)(1-\gamma)\big)\,p_1}. \qquad (17B.30)$$

The only variables in the equation for $x_1^1(p_1, p_2)$ that are affected are e_2^1 which is cut in half and p_2 which has doubled. Thus, the two changes exactly offset one another. Similarly, the only variables in the equation for $x_1^2(p_1, p_2)$ that are affected are e_2^2 which is cut in half and p_2 which doubles — thus again offsetting one another.

Exercise 17B.31

Can you depict the equilibria in rows 1 and 3 in two Edgeworth Boxes?
Answer: These are depicted in panels (a) and (b) of Exercise Graph 17B.31



Exercise Graph 17B.31 : Different types of aggregate risk

Exercise 17B.32

In the third set of results of Table 17.4, we hold your land productivity constant while varyng mine. Can you make sense of the results?

Answer: In the first row, bananas in the rainy season (state 1) are relatively scarce — giving us a relatively low price for buying state contingent consumption in the drought season (state 2). This change in price is the only factor that is changing for individual 2 — and, as a result, she consumes somewhat more bananas in state 2 and less in state 1. In fact, from her perspective, the big factor is the increase in the relative price of consumption in state 1 which she is attempting to purchase given that she has very little endowment in state 1. Individual 1, on the other hand, suffers a large cut in his production (relative to row 2 of the table) — leading him to consume less in both states. In row 3, on the other hand, bananas in the rainy season (state 1) are relatively abundant — causing the price of state-contingent consumption in the drought season to rise (and the relative price of state contingent consumption in state 1 to fall). For individual 2, this change in relative prices is again the only factor that changes (relative to row 2) — making her endowment more valuable and lowering the price of the consumption she needs to buy (in state 1). As a result, her consumption in state 1 increases. Individual 1 is richer in that his endowment has doubled (relative to row 2), but the price of his state 1 endowment has fallen while the price of state 2 consumption (which he needs given his low endowment in state 2) has increased. We therefore see a modest increase in his consumption in state 2 because of his increased wealth and in spite of the increased price, and a much larger increase in his consumption in state 1. Once again, we see the terms of trade more favorable for whoever is buying in the state where bananas are more abundant: In row 1, bananas are more abundant in state 2 — and thus the terms of trade are more favorable for individual 1 who has little endowment in that state. In row 3, bananas are more abundant in state 1, and we thus see more favorable terms of trade for individual 2 who needs to purchase in state 1 given her low endowment there.

Exercise 17B.33

What is the probability of reaching outcome 2 if we play Gamble 1 half the time and Gamble 2 half the time?

Answer: The probability of reaching outcome 2 is

$$0.5(0.4)+0.5(0.8)=0.60. \tag{17B.33}$$

Exercise 17B.34

What weights would I have to put on Gambles 1 and 2 in order for the mixed gamble to result in a 0.50 probability of reaching outcome 1 and a 0.50 probability of reaching outcome 2?

Answer: Denoting the weight we would place on Gamble 1 as α, we would like to choose α such that

$$\alpha(0.60)+(1-\alpha)(0.2)=0.5 \text{ and } \alpha(0.40)+(1-\alpha)(0.80)=0.5. \tag{17B.34}$$

Solving either one of these, we get $\alpha=0.75$.

Exercise 17B.35

Does the paradox still hold if people's tastes are state-dependent? (*Hint*: The answer is yes.)

Answer: Suppose that tastes are state-dependent but can still be represented by an expected utility function. All this means is that there now exist three different

u functions — $u_A(x)$, $u_B(x)$ and $u_C(x)$, corresponding the states of A="winning \$5 million", B="winning \$1 million" and C="winning nothing", that allow us to write the expected utility of each gamble as a probability weighted average of u_5, u_1 and u_0 — i.e. of $u_5 = u_A(5,000,000)$, $u_1 = u_B(1,000,000)$ and $u_0 = u_c(0)$. The rest of the paradox then unfolds exactly the same way.

17C Solutions to Odd Numbered End-of-Chapter Exercises

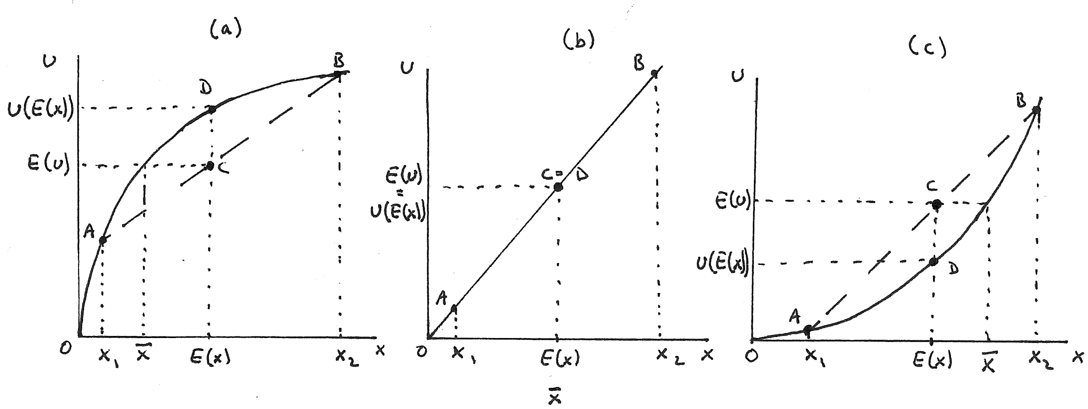
Exercise 17.1

In this exercise we review some basics of attitudes toward risk when tastes are state-independent and, in part B, we also verify some of the numbers that appear in the graphs of part A of the chapter.

A: *Suppose that there are two possible outcomes of a gamble: Under outcome A, you get* $\$x_1$ *and under outcome B you get* $\$x_2$ *where* $x_2 > x_1$*. Outcome A happens with probability* $\delta = 0.5$ *and outcome B happens with probability* $(1 - \delta) = 0.5$.

(a) *Illustrate three different consumption/utility relationships — one that can be used to model risk averse tastes over gambles, one for risk neutral tastes and one for risk loving tastes.*

Answer: This is done in panels (a) through (c) of Exercise Graph 17.1.



Exercise Graph 17.1 : Different Attitudes about Risk

(b) *On each graph illustrate your expected consumption on the horizontal axis and your expected utility of facing the gamble on the vertical. Which of these — expected consumption or expected utility — does not depend on whether your degree of risk aversion*

Answer: The expected consumption level is simply $E(x) = 0.5x_1 + 0.5x_2$ and is illustrated in each panel as lying halfway between x_1 and x_2 on the horizontal axis. It does not depend on attitudes toward risk because it is simply a probability-weighted average of the two consumption levels that might happen. The expected utility $E(u)$ is the probability-weighted average of the utilities associated with each of the two possible outcomes — and is read off the line connecting A and B in each panel.

(c) *How does the expected utility of the gamble differ from the utility of the expected consumption level of the gamble in each graph?*

Answer: The expected consumption level of the gamble is $E(x)$. The utility associated with that level of consumption is read off the consumption/utility relationship itself — and is indicated as $u(E(x))$ in each panel of the graph. It is the utility the person gets from getting the expected consumption level without risk. It differs from the utility of the gamble because, although the gamble has the same expected consumption value, it involves risk. If you don't like risk, then the utility of the gamble will be less than the utility of the expected value of the gamble (as in panel (a)). But if you like risk, the utility of the gamble will be greater than the utility of the expected value of the gamble (as in panel (c)). The two will be the same in the case of risk neutrality (panel (b)) where the individual does not care one way or another about risk.

(d) *Suppose I offer you* $\$\overline{x}$ *to not face this gamble. Illustrate in each of your graphs where* $\overline{x}$ *would lie if it makes you just indifferent between taking* $\overline{x}$ *and staying to face the gamble.*

Answer: This is illustrated in each panel as the quantity that, if obtained without risk, will provide the same utility as the expected utility $E(u)$ of the gamble. It is what we called in the text the certainty equivalent.

(e) *Suppose I come to offer you some insurance — for every dollar you agree to give me if outcome B happens, I will agree to give you y dollars if outcome A happens. What's y if the deal I am offering you does not change the expected value of consumption for you?*

Answer: If the expected value of consumption is to remain unchanged, it must mean the expected value of what you are getting is the same as the expected value of what you are paying. When you agree to pay me \$1 if B happens, you agree to give me \$1 with probability 0.5 (since B happens with probability 0.5). Thus, the expected value of what you are giving me is 0.5. In return I give you \$y if A happens — which means the expected value of what I am giving you is 0.5y because A happens with probability 0.5. For the expected value of consumption to remain the same, it must therefore be the case that 0.5y=0.5 — i.e. y=\$1.

(f) *What changes in your 3 graphs if you buy insurance of this kind — and how does it impact your expected consumption level on the horizontal axis and the expected utility of the remaining gamble on the vertical?*

Answer: In each graph, x_1 increases by the same amount that x_2 decreases as I buy such insurance — thus reducing the risk of the gamble. However, the expected value $E(x)$ remains the same. In panel (a), however, the line on which expected utility is measured shifts up as a result of insurance — implying that $E(u)$ increases with insurance (as the expected value of the gamble remains unchanged but risk falls). But in panel (c), the line on which $E(u)$ is measured falls with insurance — implying the expected utility of the gamble falls as risk is decreased by insurance (while the expected value of consumption remains unchanged). This should make sense: In panel (a), you dislike risk — while in panel (c) you like it. Insurance that keeps the expected value of the gamble unchanged will therefore makes you better off in panel (a) and worse off in panel (c) — because such insurance reduces risk. In panel (b), on the other hand, we don't care about risk one way or another — which implies insurance that lowers risk without changing the expected consumption value of the gamble leaves you indifferent.

B: *Suppose we can use the function $u(x) = x^\alpha$ for the consumption/utility relationship that allows us to represent your indifference curves over risky outcomes using an expected utility function. Assume the rest of the set-up as described in A.*

(a) *What value can α take if you are risk averse? What if you are risk neutral? What if you are risk loving?*

Answer: When $0 < \alpha < 1$, we get the concave shape required for risk aversion; when $\alpha = 1$, we simply get the equation of a line $u(x) = x$ and thus get the shape required for risk neutrality; and if $\alpha > 1$, we get the convex shape required for risk loving. These correspond to the cases graphed in panels (a) through (c) of Exercise Graph 17.1.

(b) *Write down the equations for the expected consumption level as well as the expected utility from the gamble. Which one depends on α and why?*

Answer: The expected consumption value of the gamble is given by

$$E(x) = \delta x_1 + (1-\delta)x_2 = 0.5x_1 + 0.5x_2 \tag{17.1.i}$$

which does not depend on α because it has nothing to do with tastes. The expected utility is given by

$$U = E(u) = \delta u(x_1) + (1-\delta)u(x_2) = 0.5x_1^\alpha + 0.5x_2^\alpha. \tag{17.1.ii}$$

(c) *What's the equation for the utility of the expected consumption level?*

Answer: This is

$$u(0.5x_1 + 0.5x_2) = (0.5x_1 + 0.5x_2)^\alpha. \tag{17.1.iii}$$

(d) *Consider $\overline{x}$ as defined in A(d). What equation would you have to solve to find $\overline{x}$?*

Answer: It has to be the case that $u(\overline{x}) = E(u)$; i.e.

$$\overline{x}^\alpha = 0.5x_1^\alpha + 0.5x_2^\alpha. \tag{17.1.iv}$$

(e) *Suppose $\alpha = 1$. Solve for $\overline{x}$ and explain your result intuitively.*

Answer: In this case, equation (17.1.iv) simply becomes

$$\overline{x} = 0.5x_1 + 0.5x_2 \tag{17.1.v}$$

where the right hand side is simply $E(x)$. This is reflected in panel (b) of Exercise Graph 17.1 where tastes are risk neutral and the certainty equivalent of a gamble is simply equal to the expected consumption value of the gamble (since risk neutral individuals don't care one way or another about the risk of the gamble).

(f) *Suppose that, instead of 2 outcomes, there are actually 3 possible outcomes: A, B and C, with associated consumption levels x_1, x_2 and x_3 occurring with probabilities δ_1, δ_2 and $(1-\delta_1-\delta_2)$. How would you write the expected utility of this gamble?*

Answer: You would then simply write it as

$$U = E(u) = \delta_1 u(x_1) + \delta_2 u(x_2) + (1-\delta_1-\delta_2)u(x_3) = \\ = \delta_1 x_1^{\alpha} + \delta_2 x_2^{\alpha} + (1-\delta_1-\delta_2)x_3^{\alpha}. \tag{17.1.vi}$$

(g) *Suppose that u took the form*

$$u(x) = 0.1x^{0.5} - \left(\frac{x}{100,000}\right)^{2.5} \tag{17.1.vii}$$

This is the equation that was used to arrive most of the graphs in part A of the chapter, where x is expressed in thousands but plugged into the equation as its full value; i.e. consumption of 200 in a graph represents $x = 200,000$. *Verify the numbers in Graphs 17.1 and 17.3. (Note that the numbers in the graphs are rounded.)*

Answer: For Graph 17.1, the utility levels associated with points A, B, C and D are

$$\text{For } A\text{:}\;\; u(250,000) = 0.1(250,000)^{0.5} - \left(\frac{250,000}{100,000}\right) = 40.1179 \approx 40 \tag{17.1.viii}$$

$$\text{For } B\text{:}\;\; u(10,000) = 0.1(10,000)^{0.5} - \left(\frac{10,000}{100,000}\right) = 9.9968 \approx 10 \tag{17.1.ix}$$

$$\text{For } C\text{:}\;\; E(u) = 0.25(10) + 0.75(40) = 32.5 \tag{17.1.x}$$

$$\text{For } D\text{:}\;\; u(190,000) = 0.1(190,000)^{0.5} - \left(\frac{190,000}{100,000}\right) = 38.613 \approx 38.5 \tag{17.1.xi}$$

In Graph 17.3 of the text, we also calculated the certainty equivalent. Since the expected utility of the gamble is 32.5, the certainty equivalent $\overline{x}$ must satisfy $u(\overline{x}) = 32.5$. Plugging 115,000 (which appears in the graph as 115 on the horizontal axis) into the consumption/utility relationship $u(x)$, we get

$$u(115,000) = 0.1(115,000)^{0.5} - \left(\frac{115,000}{100,000}\right) = 32.4934 \approx 32.5 \tag{17.1.xii}$$

which verifies that indeed this consumer is indifferent between the gamble and getting \$115,000 for sure.

Exercise 17.3

We have illustrated in several settings the role of actuarily fair insurance contracts (b, p) (where b is the insurance benefit in the "bad state" and p is the insurance premium that has to be paid in either state). In this problem we will discuss it in a slightly different way that we will later use in Chapter 22.

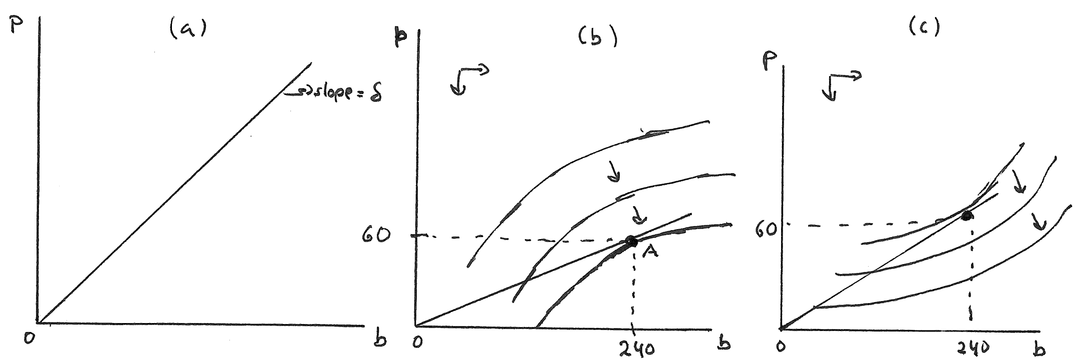
A: *Consider again the example, covered extensively in the chapter, of my wife and life insurance on me. The probability of me not making it is δ, and my wife's consumption if I don't make it will be 10 and her consumption if I do make it will be 250 in the absence of any life insurance.*

(a) *Now suppose that my wife is offered a full set of actuarily fair insurance contracts. What does this imply for how p is related to δ and b?*

Answer: Actuarial fairness implies that what my wife pays is equal to what she receives *in expectation.* She will receive $(b-p)$ with probability δ, and she will pay p with probability $(1-\delta)$. Thus, actuarial fairness implies that $\delta(b-p) = (1-\delta)p$ or simply $p = \delta b$.

(b) *On a graph with b on the horizontal axis and p on the vertical, illustrate the set of all actuarily fair insurance contracts.*

Answer: This is illustrated in panel (a) of Exercise Graph 17.3.



Exercise Graph 17.3 : Tastes over premiums p and benefits b

(c) *Now think of what indifference curves in this picture must look like. First, which way must they slope (given that my wife does not like to pay premiums but she does like benefits)?*

Answer: Indifference curves must slope up. Consider any initial bundle (b, p). We know that an increase in b to b' will make my wife unambiguously better off — which means that the bundle containing b' that is indifferent to (b,p) must have an offsetting increase in p which, by itself, would make my wife unambiguously worse off. You can thus think of this as indifference curves over two goods where one of the goods, namely the premium p, is really a "bad".

(d) *In which direction within the graph does my wife have to move in order to become unambiguously better off?*

Answer: She becomes unambiguously better off as p falls and b increases — thus, she becomes better off moving to the southeast in the graph.

(e) *We know my wife will fully insure if she is risk averse (and her tastes are state-independent). What policy does that imply she will buy if* $\delta = 0.25$*?*

Answer: As was shown in the text, this would imply buying a policy $(b,p) = (240,60)$ which satisfies the actuarily fair relationship derived in (a). Under this policy, she would have consumption of only 190 in the "good" state (where she has income of 250 but needs to pay the premium of 60) but she also has consumption of 190 in the "bad" state (where she has income of 10, has to pay the premium of 60 but also gets a benefit of 240).

(f) *Putting indifference curves into your graph from (b), what must they look like in order for my wife to choose the policy that you derived in (e).*

Answer: This is illustrated in panel (b) of Exercise Graph 17.3.

(g) *What would her indifference map look like if she were risk neutral? What if she were risk-loving?*

Answer: If her tastes were risk neutral, she should be indifferent between all the actuarily fair insurance policies along the budget line $p = \delta b$. Thus, her indifference curves must be straight lines with slope δ. If she were risk loving, then she would still become better off moving to the southeast in the graph, but her indifference curves would bow in the opposite direction from those involving risk aversion. This is pictured in panel (c) of Exercise Graph 17.3.

B: *Suppose* $u(x) = \ln(x)$ *allows us to write my wife's tastes over gambles using the expected utility function. Suppose again that my wife's income is 10 if I am not around and 250 if I am — and that the probability of me not being around is* δ.

(a) *Given her incomes in the good and bad state in the absence of insurance, can you use the expected utility function to arrive at her utility function over insurance policies* (b,p)*?*

Answer: Her expected utility is

$$U(x_B,x_G) - \delta u(x_B) + (1-\delta)u(x_G) = \delta \ln x_B + (1-\delta)\ln x_G \tag{17.3.i}$$

where x_B is her consumption in the event that I am not around and x_G is her consumption in the event that I am around. For any insurance policy (b,p), $x_B = (10+b-p)$ and $x_G = (250-p)$. We can therefore write her expected utility of the policy (b,p) as

$$U(b,p) = \delta \ln(10+b-p) + (1-\delta)\ln(250-p). \tag{17.3.ii}$$

(b) *Derive the expression for the slope of an indifference curve in a graph with b on the horizontal and p on the vertical axis.*

Answer: This is just the MRS which is

$$MRS = -\frac{\partial U(b,p)/\partial b}{\partial U(b,p)/\partial p} = -\frac{\delta/(10+b-p)}{\big(-\delta/(10+b-p)\big) - \big((1-\delta)/(250-p)\big)}$$
$$= \frac{\delta(250-p)}{\delta(250-p)+(1-\delta)(10+b-p)}. \tag{17.3.iii}$$

(c) *Suppose $\delta = 0.25$ and my wife has fully insured under policy $(b,p) = (240,60)$. What is her MRS now?*

Answer: Plugging $\delta = 0.25$, $b = 240$ and $p = 60$ into equation (17.3.iii) gives us

$$MRS = \frac{0.25(190)}{0.25(190)+0.75(190)} = 0.25. \tag{17.3.iv}$$

(d) *How does your answer to (c) compare to the slope of the budget formed by mapping out all actuarily fair insurance policies (as in A(b))? Explain in terms of a graph.*

Answer: We concluded in A(b) that the slope of the budget line is δ which is equal to 0.25 in our case. Now we concluded that, at the actuarily fair full insurance policy, the MRS of our indifference curve is also 0.25. Thus, the indifference curve is tangent to the budget line at the full insurance policy — implying that my wife is optimizing by fully insuring in the acuarily fair insurance market. We depicted this already in panel (b) of Exercise Graph 17.3 where tastes were assumed to be risk averse (as they are when we can use the concave function $u(x) = \ln x$ to represent tastes over gambles using the expected utility function.)

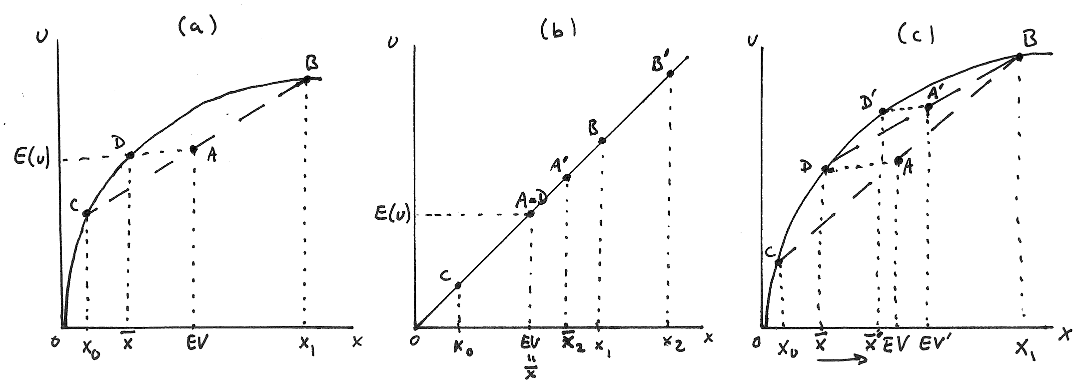
Exercise 17.5

Everyday Application: *Teenage Sex and Birth Control: Consider a teenager who evaluates whether she should engage in sexual activity with her partner of the opposite sex. She thinks ahead and expects to have a present discounted level of life-time consumption of x_1 in the absence of a pregnancy interrupting her educational progress. If she gets pregnant, however, she will have to interrupt her education and expects the present discounted value of her life-time consumption to decline to x_0 — considerably below x_1.*

A: *Suppose that the probability of a pregnancy in the absence of birth control is 0.5 and assume that our teenager does not expect to evaluate consumption any differently in the presence of a child.*

(a) *Putting the present discounted value of lifetime consumption x on the horizontal axis and utility on the vertical, illustrate the consumption/utility relationship assuming that she is risk averse. Indicate the expected utility of consumption if she chooses to have sex.*

Answer: This is done in panel (a) of Exercise Graph 17.5 where the concave shape of the relationship incorporates risk aversion. The expected utility of consumption if the teenager has sex is indicated as $E(u)$.



Exercise Graph 17.5 : Sex, Education and Birth Control

(b) *How much must the immediate satisfaction of having sex be worth in terms of lifetime consumption in order for her to choose to have sex?*

Answer: The expected utility of consumption if she has sex is $E(u)$ — which has a certainty equivalent of $\overline{x}$. Thus, she is giving up $(x_1 - \overline{x})$ by choosing to have sex — which is the least that the experience must be worth to rationalize the action.

(c) *Now consider the role of birth control which reduces the probability of a pregnancy. How does this alter your answers?*

Answer: Birth control reduces the probability of a pregnancy — and thus shifts EV in panel (a) up along the dotted line that connects B and C. Perfectly reliable birth control would imply that A shifts on top of B. As the probability of a pregnancy declines, $\overline{x}$ increases — implying that $(x_1 - \overline{x})$ falls. Thus, sex does not have to be as valued in order for the teenager to choose to engage in it.

(d) *Suppose her partner believes his future consumption paths will develop similarly to hers depending on whether or not there is a pregnancy — but he is risk neutral. For any particular birth control method (and associated probability of a pregnancy), who is more likely to want to have sex assuming no other differences in tastes?*

Answer: This is illustrated in panel (b) of Exercise Graph 17.5 where the consumption/utility relationship is graphed as linear to incorporate risk neutrality. The points A, B and C correspond to those in panel (a) — with $E(u)$ again representing the expected utility from consumption if sexual activity ensues. Note, however, that $\overline{x}$ — the certainty equivalent — is now equal to EV — which implies that $(x_1 - \overline{x})$ is lower in panel (b) than in panel (a). The risk neutral partner therefore requires less immediate satisfaction from sexual activity to rationalize it than the risk averse partner.

(e) *As the payoff to education increases in the sense that x_1 increases, what does the model predict about the degree of teenage sexual activity assuming that the effectiveness and availability of birth control remains unchanged and assuming risk neutrality?*

Answer: Consider the case where the probability of a pregnancy is 0.5. We have already shown in panel (b) of Exercise Graph 17.5 that a risk neutral partner would need to place value of at least $(x_1 - \overline{x})$ on sex in order to engage in it under these assumptions. Now suppose x_1 increases to x_2. This implies the expected value as well as the certainty equivalent increase to $\overline{x}_2$ — and the increase from $\overline{x}$ to $\overline{x}_2$ is half as much as the increase from x_1 to x_2. The new minimum value that this person must place on sex in order to justify it rationally is $(x_2 - \overline{x}_2)$ — as compared to the previous $(x_1 - \overline{x}_1)$. But, since the distance from x_1 to x_2 is twice the distance from $\overline{x}$ to $\overline{x}_2$, $(x_2 - \overline{x}_2) > (x_1 - \overline{x}_1)$ — meaning the value one must place on sex to engage in it has increased. Thus, fewer people will do so.

(f) *Do you think your answer to (e) also holds under risk aversion?*

Answer: Yes. Under risk aversion, the certainty equivalent changes more slowly as x_1 increases — which implies that the value that one must place on sex in order to engage in it (holding birth control constant) would increase more than in the case of risk neutrality.

(g) *Suppose that a government program makes daycare more affordable — thus raising x_0. What happens to the number of risk averse teenagers having sex according to this model?*

Answer: This is illustrated in panel (c) of Exercise Graph 17.5 where the original certainty equivalent is $\overline{x}$ and the original minimum value one must place on sex in order to engage in it is $(x_1 - \overline{x})$. As x_0 increases, the certainty equivalent increases (to $\overline{x}'$) but x_1 remains unchanged — which implies that $(x_1 - \overline{x}')$, the new minimum value one must place on sex, is less than the original $(x_1 - \overline{x})$. Thus, more teenagers will have sex according to this model (assuming teenagers vary in the value they place on having sex).

B: *Now suppose that the function $u(x) = \ln(x)$ allows us to represent a teenager's tastes over gambles involving lifetime consumption using an expected utility function. Let δ represent the probability of a pregnancy occurring if the teenagers engage in sexual activity, and let x_0 and x_1 again represent the two lifetime consumption levels.*

(a) *Write down the expected utility function.*

Answer: The expected utility function is

$$U(x_0, x_1) = \delta \ln x_0 + (1-\delta) \ln x_1. \tag{17.5.i}$$

(b) *What equation defines the certainty equivalent? Using the mathematical fact that $\alpha \ln x + (1-\alpha)\ln y = \ln(x^{\alpha} y^{(1-\alpha)})$, can you express the certainty equivalent as a function x_0, x_1 and δ?*

Answer: The certainty equivalent $\overline{x}$ is the level of consumption whose utility is equal to the expected utility of the gamble; i.e. $\overline{x}$ is such that

$$\ln \overline{x} = \delta \ln x_0 + (1-\delta) \ln x_1. \tag{17.5.ii}$$

Using the mathematical fact pointed out in the question, this implies

$$\overline{x} = x_0^{\delta} x_1^{(1-\delta)}. \tag{17.5.iii}$$

(c) *Now derive an equation $y(x_0, x_1, \delta)$ that tells us the least value (in terms of consumption) that this teenager must place on sex in order to engage in it.*

Answer: This is

$$y(x_0, x_1, \delta) = x_1 - \overline{x} = x_1 - x_0^{\delta} x_1^{(1-\delta)}. \tag{17.5.iv}$$

(d) *What happens to y as the effectiveness of birth control increases? What does this imply about the fraction of teenagers having sex (as the effectiveness of birth control increases) assuming that all teenagers are identical except for the value they place on sex?*

Answer: To see this, we can take the partial derivative of y with respect to δ. This gives us

$$\frac{\partial y(x_0, x_1, \delta)}{\partial \delta} = -(\ln x_0) x_0^{\delta} x_1^{(1-\delta)} + (\ln x_1) x_0^{\delta} x_1^{(1-\delta)} = (\ln x_1 - \ln x_0) x_0^{\delta} x_1^{(1-\delta)} > 0. \tag{17.5.v}$$

Thus, as δ increases, y rises; and as δ decreases, y falls. Birth control becoming more effective implies δ decreases — which therefore implies that the consumption value placed on sex in order for a teenager to engage in it decreases. Put differently, as birth control becomes more effective, some teenagers for whom sex was not sufficiently valuable before will now find it worth it — and thus the fraction of teenagers having sex increases.

(e) *What happens to y as the payoff from education increases in the sense that x_1 increases? What does this imply for the fraction of teenagers having sex (all else equal)?*

Answer: Again, we take a partial derivative to find

$$\frac{\partial y(x_0, x_1, \delta)}{\partial x_1} = 1 - (1-\delta) x_0^{\delta} x_1^{-\delta} = 1 - (1-\delta)\left(\frac{x_0}{x_1}\right)^{0.5} > 0. \tag{17.5.vi}$$

The reason this expression is greater than zero is because $(x_0/x_1) < 1$ (since $x_0 < x_1$) and $(1-\delta) < 1$ — which implies the term that is being subtracted from 1 in the equation is the product of three numbers that are all below 1 (which must itself then be below 1). This then implies that an increase in x_1 results in an increase in y — i.e. the greater payoffs to education imply that the payoff from sex must increase in order for teenagers to be willing to engage in it. As a result, all else being equal, fewer teenagers will have sex.

(f) *What happens to y as the government makes it easier to continue going to school — i.e. as it raises x_0? What does this imply for the fraction of teenagers having sex?*

Answer: Again, taking the right partial derivative, we get

$$\frac{\partial y(x_0,x_1,\delta)}{\partial x_0} = -\delta x_0^{(\delta-1)} x_1^{(1-\delta)} = -\delta\left(\frac{x_1}{x_0}\right)^{(1-\delta)} < 0. \quad (17.5.vii)$$

Thus, as it gets easier to continue going to school despite a pregnancy, y falls — i.e. the value a teenager must place on sex in order to engage in it falls. This implies that more teenagers will have sex.

(g) *How do your answers change for a teenager with risk neutral tastes over gambles involving lifetime consumption that can be expressed using an expected utility function involving the function $u(x) = x$?*

Answer. The expected utility function would then be $U = \delta x_0 + (1-\delta)x_1$, and the certainty equivalent would be $\overline{x} = \delta x_0 + (1-\delta)x_1$. This implies that y is

$$y(x_0,x_1,\delta) = x_1 - (\delta x_0 + (1-\delta)x_1) = \delta(x_1 - x_0). \quad (17.5.viii)$$

Taking the three partial derivatives, we then get

$$\frac{\partial y}{\partial \delta} = (x_1 - x_0) > 0\,, \quad \frac{\partial y}{\partial x_1} = \delta > 0 \text{ and } \frac{\partial y}{\partial x_0} = -\delta < 0. \quad (17.5.ix)$$

The signs of these derivatives are the same as before — implying changes in the same direction as δ, x_1 and x_0 change.

(h) *How would your answers change if $u(x) = x^2$?*

Answer: The expected utility function would be $U = \delta x_0^2 + (1-\delta)x_1^2$, and the certainty equivalent $\overline{x}$ is defined by the equation $u(\overline{x}) = \overline{x}^2 = \delta x_0^2 + (1-\delta)x_1^2$ which solves to

$$\overline{x} = \left(\delta x_0^2 + (1-\delta)x_1^2\right)^{0.5} \quad (17.5.x)$$

which implies

$$y(x_0,x_1,\delta) = x_1 - \left(\delta x_0^2 + (1-\delta)x_1^2\right)^{0.5}. \quad (17.5.xi)$$

Taking the three partial derivatives, we then get

$$\frac{\partial y}{\partial \delta} = -\frac{1}{2}(x_0^2 - x_1^2)\left(\delta x_0^2 + (1-\delta)x_1^2\right)^{-0.5} = \frac{x_1^2 - x_0^2}{2\left(\delta x_0^2 + (1-\delta)x_1^2\right)^{0.5}} > 0 \quad (17.5.xii)$$

$$\frac{\partial y}{\partial x_1} = 1 - \frac{1}{2}(2(1-\delta)x_1)\left(\delta x_0^2 + (1-\delta)x_1^2\right)^{-0.5} =$$

$$= \frac{\left(\delta x_0^2 + (1-\delta)x_1^2\right)^{0.5} - (1-\delta)x_1}{\left(\delta x_0^2 + (1-\delta)x_1^2\right)^{0.5}} > 0 \quad (17.5.xiii)$$

$$\frac{\partial y}{\partial x_0} = -2\delta x_0\left(\delta x_0^2 + (1-\delta)x_1^2\right)^{-0.5} = \frac{-2\delta x_0}{\left(\delta x_0^2 + (1-\delta)x_1^2\right)^{0.5}} < 0. \quad (17.5.xiv)$$

Thus we again get the same inequalities — implying all the effects still operate in the same direction. The first inequality is straightforward to see since $x_1 > x_0$. The second inequality holds so long as

$$\left(\delta x_0^2 + (1-\delta)x_1^2\right)^{0.5} - (1-\delta)x_1 > 0. \quad (17.5.xv)$$

Adding the second term to both sides and squaring, we get

$$\delta x_0^2 + (1-\delta)x_1^2 > (1-\delta)^2 x_1^2 \quad (17.5.xvi)$$

which can be re-written as

$$\delta x_0^2 > x_1^2\left[(1-\delta)^2 - (1-\delta)\right] = -\delta(1-\delta)x_1^2. \quad (17.5.xvii)$$

This always holds given that $\delta x_0^2 > 0$ and the right hand side is less than zero.

Exercise 17.7

Everyday Application: *Venice and Regret: Suppose that you can choose to participate in one of two gambles: In Gamble 1 you have a 99% chance of winning a trip to Venice and a 1% chance of winning tickets to a movie about Venice; and in Gamble 2, you have a 99% of winning the same trip to Venice and a 1% chance of not winning anything.*

A: *Suppose you very much like Venice, and, were you to be asked to rank the three possible outcomes, you would rank the trip to Venice first, the tickets to the movie about Venice second, and having nothing third.*

(a) *Assume that you can create a consumption index such that getting nothing is denoted as 0 consumption, getting the tickets to the movie is $x_1 > 0$ and getting the trip is $x_2 > x_1$. Denote the expected value of Gamble 1 by $E(G_1)$ and the expected value of Gamble 2 by $E(G_2)$. Which is higher?*

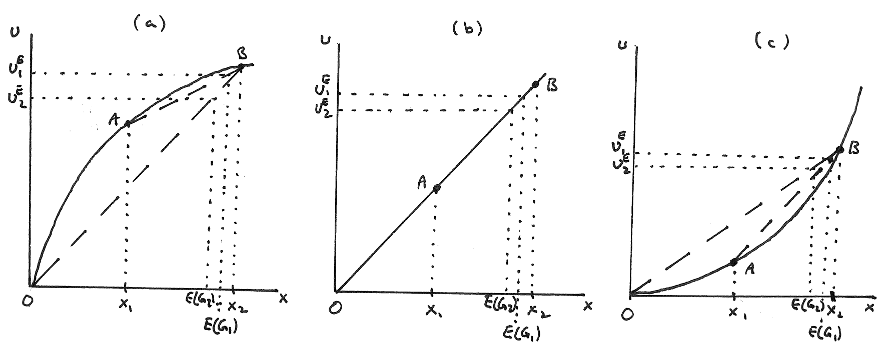
Answer: The expected value of Gamble 1, $E(G_1)$, is higher than the expected value of Gamble 2, $E(G_2)$ — because we take the same weighted average between x_2 and x_1 to get to $E(G_1)$ as we are between x_2 and 0 to get to $E(G_2)$. Thus

$$E(G_1) = 0.99x_2 + 0.01x_1 \text{ and } E(G_2) = 0.99x_2 + 0.01(0) = 0.99x_2. \quad (17.7.i)$$

Since $x_1 > 0$, it must therefore be the case that $E(G_1) > E(G_2)$.

(b) *On a graph with x on the horizontal axis and utility on the vertical, illustrate a consumption/utility relationship that exhibits risk aversion.*

Answer: This is illustrated in panel (a) of Exercise Graph 17.7 where the relationship takes on the concave shape necessary for risk aversion.



Exercise Graph 17.7 : Trips to and Movies about Venice

(c) *In your graph, illustrate the expected utility you receive from Gamble 1 and from Gamble 2. Which gamble will you choose to participate in?*

Answer: This is also illustrated in panel (a) of Exercise Graph 17.7. The expected utility of Gamble 1 is read off on the line connecting points A and B, and the expected utility of Gamble 2 is read off the line connecting (the origin) 0 to B. Thus, u_1^E is the expected utility of Gamble 1 and u_2^E is the expected utility of Gamble 2. We can see immediately that $u_1^E > u_2^E$ — thus you would choose to participate in Gamble 1 over Gamble 2.

(d) *Next, suppose tastes are risk neutral instead. Re-draw your graph and illustrate again which gamble you would choose. (*Hint*: Be careful to accurately differentiate between the expected values of the two gambles.)*

Answer: This is illustrated in panel (b) of Exercise Graph 17.7 where the shape of the consumption/utility relationship is now linear (as is required for risk neutrality). The expected utility of the gambles is again read off the lines that connect A and B (for Gamble 1) and 0 and B (for Gamble 2) — but these now lie on the consumption/utility relationship. Since $E(G_1) > E(G_2)$, we see that the expected utility of Gamble 1, u_1^E, is greater than the expected utility of Gamble 2, u_2^E. Again, you will choose Gamble 1 over Gamble 2.

(e) *It turns out (for reasons that become clearer in part B) that, risk aversion (or neutrality) is irrelevant for how individuals whose behavior is explained by expected utility theory will choose among these gambles. In a separate graph, illustrate the consumption/utility relationship again, but this time assume risk loving. Illustrate in the graph how your choice over the two gambles might still be the same as in parts (c) and (d). Can you think of why it in fact* has to be *the same?*

Answer: This is illustrated in panel (c) of Exercise Graph 17.7. Although the line connecting A and B now lies above the line connecting 0 and B, it is still the case that $E(G_1) > E(G_2)$. Thus, the graph can easily be drawn with the expected utility of Gamble 1 (u_1^E) greater than the expected utility of Gamble 2 (u_2^E). To see why this in fact *has to* be the case, denote the consumption/utility relationship $u(x)$. Thus, the utility of x_2 is given by $u(x_2)$, the utility of x_1 is given by $u(x_1)$ and the utility of 0 is simply $u(0) = 0$. The expected utility levels u_1^E and u_2^E (that lie on the lines connecting the outcomes) associated with Gambles 1 and 2 are then given by

$$u_1^E = 0.99u(x_2) + 0.01u(x_1) \text{ and } u_2^E = 0.99u(x_2) + 0.01u(0) = 0.99u(x_2). \tag{17.7.ii}$$

Since $u(x_1) > 0$, it must then be that $u_1^E > u_2^E$, and it is irrelevant whether the u function is concave or convex — so long as it slopes up and thus $u(x_1) > 0$. More intuitively, Gambles 1 and 2 place the same probability on winning the trip, but Gamble 1 places the remaining probability on winning a movie ticket while Gamble 2 does not. Thus, because Gamble 1 contains something "extra", it must be preferred to Gamble 2 under expected utility theory.

(f) *It turns out that many people, when faced with a choice of these two gambles, end up choosing Gamble 2. Assuming that such people would indeed rank the three outcomes the way we have, is there any way that such a choice can be explained using expected utility theory (taking as given that the choice implied by expected utility theory does not depend on risk aversion?)*

Answer: No, it cannot given the answer to (e). Put simply, the person gets the trip with probability 0.99 in both Gambles, but he gets something additional in Gamble 1 but not in Gamble 2. If that something additional — the movie ticket — is valuable, then Gamble 1 has to be better than Gamble 2 according to expected utility theory.

(g) *This example is known as* Machina's Paradox. *One explanation for it (i.e. for the fact that many people choose Gamble 2 over Gamble 1) is that expected utility theory does not take into account* regret. *Can you think of how this might explain people's paradoxical choice of Gamble 2 over Gamble 1?*

Answer: Having had such a high chance of actually winning the trip, not getting it might cause regret — and then watching a movie about Venice might make it worse. Thus, it is not that the person does not, all else equal, prefer the movie ticket to nothing. But the movie ticket — after coming so close to being able to get to Venice in person — might actually be worse than nothing because of the fact that the person is reminded of what he has lost. None of this fits into expected utility theory.

B: *Assume again, as in part A, that individuals prefer a trip to Venice to the movie ticket, and they prefer the movie ticket to getting nothing. Furthermore, suppose there exists a function u that assigns u_2 as the utility of getting the trip, u_1 as the utility of getting the movie ticket and u_0 as the utility of getting nothing, and suppose that this function u allows us to represent tastes over risky pairs of outcomes using an expected utility function.*

(a) *What inequality defines the relationship between u_1 and u_0?*

Answer: It must be that $u_1 > u_0$.

(b) *Now multiply both sides of your inequality from (a) by 0.01, and then add $0.99u_2$ to both sides. What inequality do you now have?*

Answer: Multiplying the two inequalities as instructed, we get $0.01u_1 > 0.01u_0$, and adding $0.99u_2$ to both sides, we get

$$0.99u_2 + 0.01u_1 > 0.99u_2 + 0.01u_0. \tag{17.7.iii}$$

(c) *Relate the inequality you derived in (b) to the expected utility of the two gambles in this example. What gamble does expected utility theory predict a person will choose (assuming the outcomes are ranked as we have ranked them)?*

Answer: The left hand side is the expected utility of Gamble 1 and the right hand side is the expected utility of Gamble 2. Since the left hand side is greater than the right hand side, expected utility theory implies that this person will choose Gamble 1 over Gamble 2.

(d) *When we typically think of a "gamble", we are thinking of different outcomes that will happen with different probabilities. But we can also think of "degenerate" gambles — i.e. gambles where one outcome happens with certainty. Define the following three such "gambles": Gamble A results in the trip to Venice with probability of 100%; Gamble B results in the movie ticket with probability of 100%; and Gamble C results in nothing with probability of 100%. How are these degenerate "gambles" ranked by someone who prefers the trip to the ticket to nothing?*

Answer: It must then be the case that $G_A \succ G_B \succ G_C$.

(e) *Using the notion of* mixed gambles *introduced in Appendix 1, define Gambles 1 and 2 as mixed gambles over the degenerate "gambles" we have just defined in (d). Explain how the* Independence Axiom *from Appendix 1 implies that Gamble 1 must be preferred to Gamble 2.*

Answer: Gamble 1 is simply Gamble A (which is equivalent to getting the trip) and Gamble B (which is equivalent to getting the movie ticket) mixed with weight 0.99 on G_A and 0.01 weight on G_B. Similarly, Gamble 2 is equivalent to mixing Gamble A with weight 0.99 and Gamble C with weight 0.01. We can thus write that

$$G_1 = 0.99G_A + 0.01G_B \text{ and } G_2 = 0.99G_A + 0.01G_C. \quad (17.7.iv)$$

(f) True or False*: When individuals who rank the outcomes the way we have assumed choose Gamble 2 over Gamble 1, expected utility theory fails because the independence axiom is violated.*

Answer: This is true. The independence axiom says that, if a Gamble B is preferred to a Gamble C, then the mixture of Gamble B with a third Gamble A must be preferred to the mixture of Gamble C with Gamble A so long as they are mixed with equal weights; i.e.

$$G_B \succ G_C \text{ implies } (\delta G_B + (1-\delta)G_A) \succ (\delta G_C + (1-\delta)G_A) \text{ for all } 0 < \delta < 1. \quad (17.7.v)$$

When $\delta = 0.01$, the left hand side of this implication becomes G_1 and the right hand side becomes G_2. Thus, the independence axiom implies that, if the movie ticket is worth more than nothing to the individual, then $G_1 \succ G_2$. Expected utility theory cannot predict that someone like this will choose Gamble 2 over Gamble 1 because such a prediction would imply a violation of the independence axiom on which expected utility theory is built.

(g) *Would the paradox disappear if we assumed state-dependent tastes? (*Hint*: As with the Allais paradox in Appendix 2, the answer is no.)*

Answer: The reason that assuming state-dependent tastes does not resolve the Machina Paradox is because it does not matter whether we choose one function u to assign utility to each outcome (as we would if tastes are state-independent) or we choose three separate functions u_2, u_1 and u_0 to assign utility to the outcomes. The question is whether we can assign utility values at all such that the expected utility of each gamble is a probability weighted average of the utilities associated with each outcome in the gamble. If we can find a way to assign such utility values, then expected utility theory can be applied, and it implies (as we have shown) that Gamble 1 is preferred to Gamble 2. Choosing Gamble 2 over Gamble 1 is then inconsistent with expected utility theory regardless of whether tastes are state dependent.

Exercise 17.9

Business Application: *Diversifying Risk along the Business Cycle*: *Suppose you own a business that does well during economic expansions but not so well during recessions which happen with probability δ. Let x_E denote your consumption level* *during expansions and let x_R denote your consumption level during recessions. Unless you do something to diversify risk, these consumption levels are $E = (e_E, e_R)$ where e_E is your income during expansions and e_R your income during recessions (with $e_E > e_R$). Your tastes over consumption are the same during recessions as during expansions and you are risk averse. For any asset purchases described below, assume that you pay for these assets from whatever income you have depending on whether the economy is in recessions or expansion.*

A: *Suppose I own a financial firm that manages asset portfolios. All I care about as I manage my business is expected returns, and any asset I sell is characterized by (p, b_R, b_E) where p is how much I charge for 1 unit of the asset, b_R is how much the asset will pay you (as, say, dividends) during recessions and b_E is how much the asset will pay you during expansions.*

(a) *Is someone like me — who only cares about expected returns — risk averse, risk loving or risk neutral?*

Answer: Someone who only cares about expected returns (but not risk) is risk neutral.

(b) *Suppose that all the assets I offer have the feature that those who buy these assets experience no change in their expected consumption levels as a result of buying my assets. Derive an equation that expresses the price p of my assets in terms of δ, b_R and b_E.*

Answer: In order for your expected consumption to remain unchanged, it must be that the expected change in consumption during recessions is exactly offset by the expected change in consumption during expansions — i.e.

$$\delta(-p + b_R) = -(1-\delta)(-p + b_E) \tag{17.9.i}$$

which solves to give us

$$p = \delta b_R + (1-\delta) b_E. \tag{17.9.ii}$$

(c) *What happens to my expected returns when I sell more or fewer of such assets?*

Answer: Just as your expected consumption is unchanged when you buy these assets, my expected returns are unchanged.

(d) *Suppose you buy 1 asset (p, b_R, b_E) that satisfies our equation from (b). How does your consumption during expansions and recessions change as a result?*

Answer: Your consumption during recessions will be

$$x_R = e_R - p + b_R = e_R - (\delta b_R + (1-\delta) b_E) + b_R = e_R + (1-\delta)(b_R - b_E), \tag{17.9.iii}$$

and your consumption during expansion would be

$$x_E = e_E - p + b_E = e_E - (\delta b_R + (1-\delta)b_E) + b_E = e_E + \delta(b_E - b_R). \quad (17.9.iv)$$

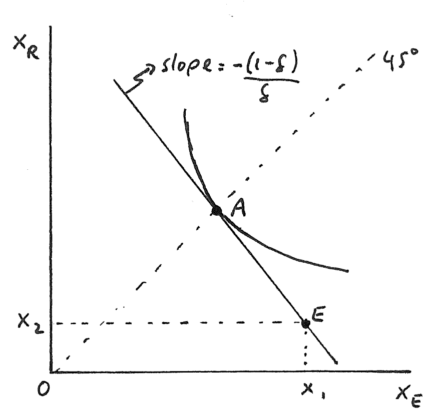
(e) *At what rate do assets of the kind I am offering allow you to transfer consumption opportunities from expansions to recessions? On a graph with x_E on the horizontal and x_R on the vertical axis, illustrate the "budget line" that the availability of such assets creates for you.*

Answer: In order to transfer consumption from expansions to recessions, you need to pick assets with $b_E < b_R$. Suppose you pick an asset with $b_R - b_E = 1$. Then, when you buy one unit of such an asset, your consumption in the two states becomes

$$x_R = e_R + (1-\delta) \text{ and } x_E = e_E - \delta. \quad (17.9.v)$$

Thus, you are trading δ in consumption during expansions for $(1-\delta)$ in consumption during recessions — or, put differently, for every \$1 in consumption you give up during expansions, you get $\$(1-\delta)/\delta$ during recessions.

This is then illustrated in a graph in Exercise Graph 17.9.



Exercise Graph 17.9 : Reducing Risk across Business Cycles

(f) *Illustrate in your graph your optimal choice of assets.*

Answer: This is also illustrated in Exercise Graph 17.9 where risk aversion and state-independence of tastes implies that you will "fully insure" because the terms of trade are "actuarily fair" in the sense that your expected consumption level does not change. As a result, you optimize at point A in the graph.

(g) *Overall output during recessions is smaller than during expansions. Suppose everyone is risk averse. Is it possible for us to all end up doing what you concluded you would do in (f)? (We will explore this further in exercise 17.10.)*

Answer: No — if the economy shrinks, it is not possible for everyone to fully insure in the sense of maintaining the same level of consumption regardless of the state of the economy.

B: *Suppose that the function $u(x) = x^{\alpha}$ is such that we can express your tastes over gambles using expected utility functions.*

(a) *If you have not already done so in part A, derive the expression $p(\delta, b_R, b_E)$ that relates the price of an asset to the probability of a recession δ, the dividend payment b_R during recessions and the dividend payment b_E during economic expansions assuming that purchase of such assets keeps expected consumption levels unchanged.*

Answer: Repeating our derivation from before: In order for your expected consumption to remain unchanged, it must be that the expected change in consumption during recessions is exactly offset by the expected change in consumption during expansions — i.e.

$$\delta(-p + b_R) = -(1-\delta)(-p + b_E) \tag{17.9.vi}$$

which solves to give us

$$p = \delta b_R + (1-\delta) b_E. \tag{17.9.vii}$$

(b) *Suppose you purchase k units of the same asset (b_E, b_R) which is priced as you derived in part (a) and for which $(b_R - b_E) = y > 0$. Derive an expression for x_R defined as your consumption level during recessions (given your recession income level of e_R) assuming you purchase these assets. Derive similarly an expression for your consumption level x_E during economic expansions.*

Answer: Your consumption during recessions will be equal to your recession income plus the dividends from your assets minus the price of the assets:

$$\begin{aligned} x_R &= e_R + kb_R - kp = e_R + kb_R - k(\delta b_R + (1-\delta) b_E) = \\ &= e_R + (1-\delta)k(b_R - b_E) = e_R + (1-\delta)ky. \end{aligned} \tag{17.9.viii}$$

Similarly,

$$\begin{aligned} x_E &= e_E + kb_E - kp = e_E + kb_E - k(\delta b_R + (1-\delta) b_E) = \\ &= e_E + \delta k(b_E - b_R) = e_E - \delta ky. \end{aligned} \tag{17.9.ix}$$

(c) *Set up an expected utility maximization problem where you choose k — the number of such assets that you purchase. Then solve for k.*

Answer: We have already determined the consumption levels x_R and x_B conditional on how many assets you buy subject to the pricing constraints. Thus, all we have to solve is the unconstrained optimization problem

$$\max_k \delta u(x_R) + (1-\delta)u(x_E) = \delta x_R^\alpha + (1-\delta)x_E^\alpha = \\ = \delta[e_R + (1-\delta)ky]^\alpha + (1-\delta)[e_E - \delta ky]^\alpha. \quad (17.9.x)$$

Taking the first derivative of the right-hand side and setting it to zero, we get

$$\alpha\delta(1-\delta)y[e_R + (1-\delta)ky]^{(\alpha-1)} = \alpha(1-\delta)\delta y[e_E - \delta ky]^{(\alpha-1)} \quad (17.9.xi)$$

which simplifies to

$$e_R + (1-\delta)ky = e_E - \delta ky \quad (17.9.xii)$$

which we can solve for

$$k = \frac{(e_E - e_R)}{y} = \frac{(e_E - e_R)}{(b_R - b_E)}. \quad (17.9.xiii)$$

(For the last equality, we simply substituted back in for $y = (b_R - b_E)$.)

(d) *How much will you consume during recessions and expansions?*

Answer: Substituting (17.9.xiii) into (17.9.viii) and (17.9.ix), we get

$$x_R = e_R + (1-\delta)\left(\frac{(e_E - e_R)}{y}\right)y = \delta e_R + (1-\delta)e_E \quad (17.9.xiv)$$

$$x_E = e_E + \delta\left(\frac{(e_E - e_R)}{y}\right)y = \delta e_R + (1-\delta)e_E. \quad (17.9.xv)$$

Thus, you will buy sufficient numbers of assets such that consumption in recessions and expansions is equalized.

(e) *For what values of α is your answer correct?*

Answer: The answer is correct for $\alpha < 1$ when tastes over gambles are risk averse. It is not correct for $\alpha > 1$ when tastes are risk-loving. The calculus still produces the same answer, but the indifference curves now bow out and, while they are tangent to the budget at the "full insurance" bundle, they are tangent from below and therefore a local minimum rather than a maximum. When tastes are risk loving, the true solution is a corner solution. And when $\alpha = 1$, all outcome bundles with the same expected consumption value are optimal — including the one derived.

(f) True or False*: So long as assets that pay more dividends during recessions than expansions are available at "actuarily fair" prices, you will be able to fully insure against consumption shocks from business cycles.*

Answer: This is true, as we have just shown. We assumed $(b_R > b_E)$ for the assets that we are buying — and equation (17.9.xiii) shows that the smaller the difference between the recession and expansion dividends, the more assets we will buy — always with the ultimate goal of equalizing consumption across the business cycle.

(g) *Could you accomplish the same outcome by instead creating and selling assets with $(b_E > b_R)$?*

Answer: Yes — you can do exactly the same thing if you price such assets according to our pricing formula (equation (17.9.vii)) that keeps expected consumption constant. In this case, you would be paying someone else $(p - b_E)$ during expansions, but you would receive $(p - b_R) > 0$ during recessions.

Exercise 17.11

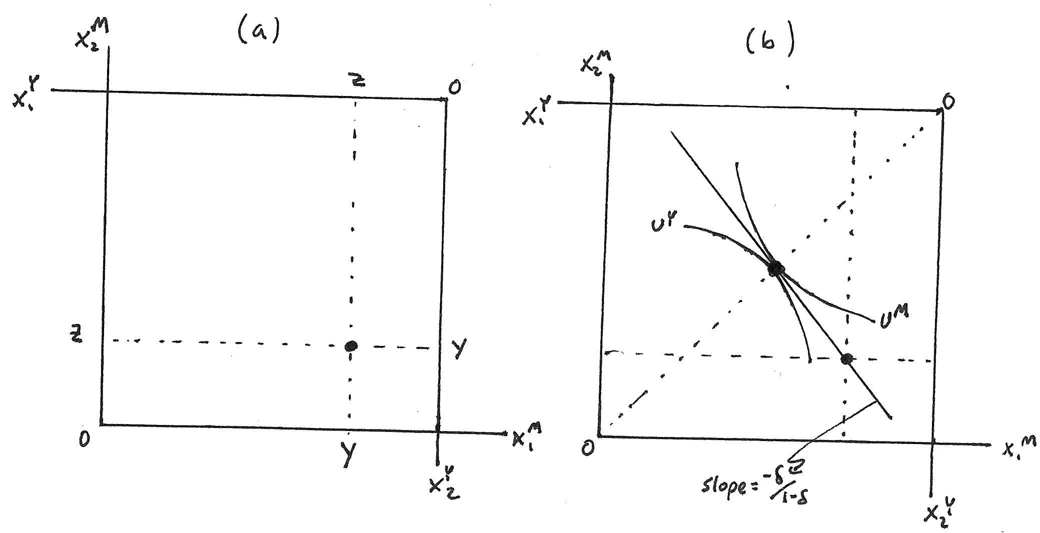
Business Application: *Local versus National Insurance: Natural disasters are local phenomena — impacting a city or a part of a state but rarely impacting the whole country, at least if the country is geographically large. To simplify the analysis, suppose there are two distinct regions that might experience local disasters.*

A: *Define "state 1" as region 1 experiencing a natural disaster, and define "state 2" as region 2 having a natural disaster. I live in region 2 while you live in region 1. Both of us have the same risk averse and state-independent tastes, and our consumption level falls from y to z when a natural disaster strikes. The probability of state 1 is δ and the probability of state 2 is $(1-\delta)$.*

(a) *Putting consumption x_1 in state 1 on the horizontal axis and consumption x_2 in state 2 on the vertical, illustrate an Edgeworth box assuming you and I are the only ones living in our respective regions. Illustrate our "endowment" bundle in this box.*

Answer: This is illustrated in panel (a) where my consumption is superscripted by M and yours by Y. Since I live in region 2, my consumption is high in state 1 when the disaster strikes in region 1; and since you live in region 1, your consumption is high in state 2 when the disaster strikes in region 2. The box is a square because, no matter where the disaster strikes, the overall level of consumption in the economy is $(y+z)$ — i.e. there is no aggregate risk.

(b) *Suppose an insurance company wanted to insure us against the risks of natural disasters. Under actuarily fair insurance, what is the opportunity cost of state 2 consumption in terms of state 1 consumption? What is the opportunity cost of state 1 consumption in terms of state 2 consumption? Which of these is the slope of the acuarily fair budget in your Edgeworth Box?*



Exercise Graph 17.11(1) : Disaster Insurance

Answer: Actuarily fair insurance means that our expected consumption remains unchanged from being insured. Thus, if I want insurance that gives me \$1 in state 2 that happens with probability $(1-\delta)$, I am asking for an expected net benefit of $(1-\delta)$. This needs to be offset by an expected net payment of δp that will be made to the insurance company in state 1 — and actuarial fairness implies $\delta p = (1-\delta)$ or $p = (1-\delta)/\delta$. Thus, the opportunity cost of \$1 in state 2 is $(1-\delta)/\delta$ in state 1. Alternatively, the opportunity cost of \$1 in state 1 is $\delta/(1-\delta)$ in state 2. The latter is the slope of the actuarily fair budget constraint when state 1 is on the horizontal axis.

(c) *Illustrate the budget line that arises from the set of all actuarily fair insurance contracts within the Edgeworth Box. Where would you and I choose to consume assuming we are risk averse?*

Answer: This is done in panel (b) of Exercise Graph 17.11(1). Under actuarily fair insurance terms, we would both choose to fully insure along the 45 degree line that connects the lower left to the upper right corners of the box.

(d) *How does this outcome compare to the equilibrium outcome if you and I were simply to trade state-contingent consumption across the two states?*

Answer: It is identical as can quickly be seen in panel (b) of Exercise Graph 17.11(1) where the budget line formed by the actuarily fair insurance terms causes us to optimize at the same point in the box.

(e) *Suppose there were two of me and two of you in this world. Would anything change?*

Answer: No, nothing would change as the same prices would still get us to optimize at the same point and, because there is no aggregate risk, there are enough resources in both states for the relevant trades to take place.

(f) *Now suppose that the two of me living in region 2 go to a local insurance company that operates only in region 2. Why might this company not offer us actuarily fair insurance policies?*

Answer: This insurance company may find it difficult to insure us because of the aggregate risk that the local economy faces. The insurance company needs to be able to write enough policies with risks that are offsetting so that it can in expectation meet the costs of all the benefits it has to pay with the premiums it is collecting in all those places where disaster does not strike. But if a local insurance company only sells local policies, it does not have people in other places where disaster won't strike to write offsetting policies.

(g) *Instead of insurance against the consequences of natural disasters, suppose we instead considered insurance against non-communicable illness. Would a local insurance company face the same kind of problem offering actuarily fair insurance in this case?*

Answer: No, the same problem would not arise for a local insurance company — because the "disasters" are not striking randomly without being clustered in geographic areas.

(h) *How is the case of local insurance companies insuring against local natural disasters similar to the case of national insurance companies insuring against business cycle impacts on consumption? How might international credit markets that allow insurance companies to borrow and lend help resolve this?*

Answer: In both cases, the problem is aggregate risk that does not make sufficient resources available in one state to make it possible to pay the necessary obligations. If insurance companies have access to full international credit markets, though, they can resolve this problem. They would do so by borrowing in such markets during times when bad times hit all at once (due to aggregate risk) and lend in good times.

B: *Suppose that, as in exercise 17.10, the function* $u(x) = \ln x$ *allows us to represent our tastes over gambles as expected utilities. Assume the same set-up as the one described in A.*

(a) *Let* p_1 *be defined as the price of \$1 of consumption if state 1 occurs and let* p_2 *be the price of \$1 of consumption in the event that state 2 occurs. Set* $p_2 = 1$ *and then denote the price of \$1 of consumption in the event of state 1 occurring as* $p_1 = p$ *and write down your budget constraint.*

Answer. Your budget constraint is then $pz + y = px_1 + x_2$, where the left hand side is the value of your endowment and the right hand side is the value of your consumption opportunity bundle to which you trade.

(b) *Solve the expected utility maximization problem given this budget constraint to get your demand x_1 for state 1 consumption as well as your demand x_2 for state 2 consumption.*

Answer: Your expected utility function is $U(x_1, x_2) = \delta \ln x_1 + (1-\delta)\ln x_2$, and your expected utility maximization problem is

$$\max_{x_1,x_2} \delta \ln x_1 + (1-\delta)\ln x_2 \text{ subject to } pz + y = px_1 + x_2. \quad (17.11.i)$$

Solving this in the usual way, we get your demands

$$x_1 = \frac{\delta(pz+y)}{p} \text{ and } x_2 = (1-\delta)(pz+y). \quad (17.11.ii)$$

(c) *Repeat (a) and (b) for me.*

Answer: For me, the budget constraint is $py + z = px_1 + x_2$ (because my endowment is the symmetric opposite of yours.) Otherwise everything is the same — giving us the following demands for me:

$$x_1 = \frac{\delta(py+z)}{p} \text{ and } x_2 = (1-\delta)(py+z). \quad (17.11.iii)$$

(d) *Derive the equilibrium price. Is this acutarily fair?*

Answer: In equilibrium, the demand for x_1 must be equal to the economy's endowment $(y+z)$ (as must the demand for x_2 since the economy's endowment is the same in both states.) Equilibrium in state 1 therefore implies

$$x_1^M + x_1^Y = y + z, \quad (17.11.iv)$$

where M superscripts my demand and Y superscripts yours. Plugging in the demands we calculated before, this can be written as

$$\frac{\delta(pz+y)}{p} + \frac{\delta(py+z)}{p} = y + z. \quad (17.11.v)$$

Solving this, we get

$$p = \frac{\delta}{(1-\delta)}. \quad (17.11.vi)$$

(Note that this is the inverse of what we have often derived under similar conditions because the probability of the state 1 rather than the probability of state 2 is δ here.)

(e) *How much do we consume in each state?*

Answer: Plugging the equilibrium price back into our demands from before, we get

$$x_1^Y = \delta y + (1-\delta)z = x_2^Y \text{ and } x_1^M = \delta z + (1-\delta)y = x_2^M \quad (17.11.vii)$$

where Y again superscripts you and M superscripts me. Thus, we both fully insure.

(f) *Does the equlibrium price change if there are 2 of you and 2 of me?*

Answer: We would still need that the demand is equal to the available endowment in each state. For state 1, this implies

$$2x_1^M + 2x_1^Y = 2(y+z), \tag{17.11.viii}$$

which, once we cancel the 2's, is identical to the previous equilibrium equation (17.11.iv). Thus, the equilibrium does not change as we increase the number of parties on each side of the market.

(g) *Finally, suppose that the two of me attempt to trade state-contingent consumption just between us. What will be the equilibrium price?*

Answer: The equilibrium in state 1 now requires twice my demand to sum to twice my endowment — i.e.

$$2\left(\frac{\delta(py+z)}{p}\right) = 2y \tag{17.11.ix}$$

which we can solve for

$$p = \frac{\delta z}{(1-\delta)y}. \tag{17.11.x}$$

(h) *Will we manage to trade at all?*

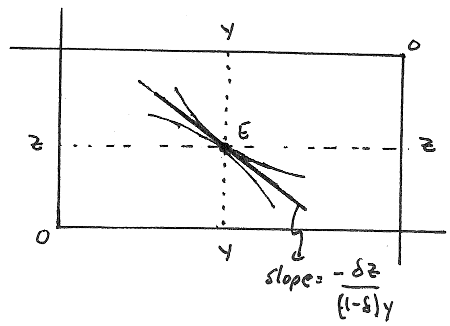
Answer: Plugging this back into my demand for x_1, we get

$$x_1 = \frac{\delta\left(\left(\frac{\delta z}{(1-\delta)y}y + z\right)\right)}{\frac{\delta z}{(1-\delta)y}} = y. \tag{17.11.xi}$$

Thus, at the equilibrium price, each of us simply consumes our endowment and no trade occurs.

(i) *Can you illustrate this in an Edgeworth Box? Is the equilibrium efficient?*

Answer: This is illustrated in Exercise Graph 17.11(2) where the Edgeworth Box is no longer a square since the economy's endowment in state 1 is now $2y$ and the endowment in state 1 is $2z$ (where $y > z$). Our individual endowment bundle is now in the center of the box, and the equilibrium price keeps both of us optimizing at that bundle. Since our indifference curves are tangent to one another, the equilibrium is efficient even though we both continue to face risk. The risk cannot be reduced because of the presence of aggregate risk.



Exercise Graph 17.11(2) : No Trade Equilibrium

Exercise 17.13

Policy Application: *More Police or More Teachers? Enforcement versus Education: Suppose again (as in exercise 17.12) that the payoff from engaging in a life of crime is x_1 if you don't get caught and x_0 (significantly below x_1) if you end up in jail, with δ representing the probability of getting caught. Suppose everyone has identical tastes but we differ in terms of the amount of income we can earn in the (legal) labor market — with (legal) incomes distributed uniformly (i.e. evenly) between x_0 and x_1.*

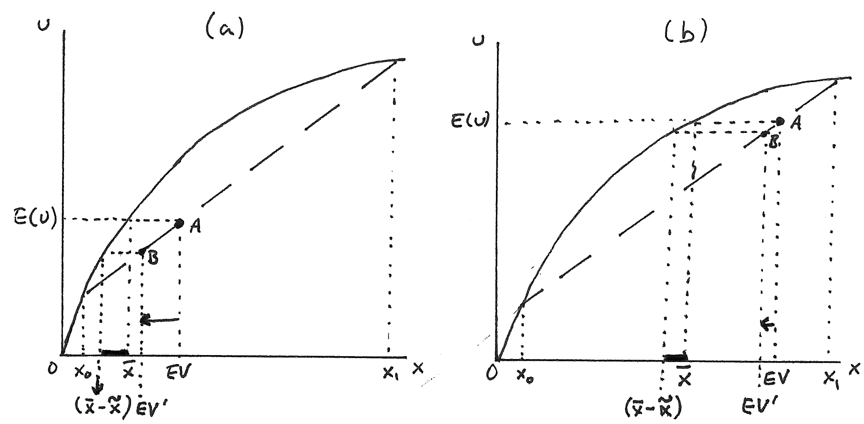
A: *Suppose there are two ways to lower crime rates: Spend more money on police officers so that we can make it more likely that those who commit crimes get caught, or spend more money on teachers so that we increase the honest income that potential criminals could make. The first policy raises δ; the second raises individual incomes through better education.*

(a) *Begin by drawing a risk averse individual's consumption/utility relationship and assume a high δ. Indicate the corresponding $\overline{x}$ that represents the (honest) income level at which a person is indifferent between an honest life and a life of crime.*

Answer: This is done in panel (a) of Exercise Graph 17.13 where the consumption/utility relationship is concave due to the assumption of risk aversion.

If δ is high, it means the probability of getting caught is high — which means the expected consumption value EV of a life of crime is relatively low. Thus, EV is drawn at a relatively low level — with the expected utility of a life of crime read off point A. The certainty equivalent is the level of consumption $\overline{x}$ that yields the same level of utility as the expected utility of the life of crime — and it is equal to the honest income level at which a person is indifferent between an honest life and a life of crime.

(b) *Consider a policy that invests in education and results in a uniform increase in all incomes by an amount $\tilde{x}$. On the horizontal axis of your graph,*



Exercise Graph 17.13 : More Police or More Teachers?

indicate which types of individuals (identified by their pre-policy income levels) will now switch from a life of crime to an honest life.

Answer: An individual who previously could make an honest living of $(\overline{x}-\tilde{x})$ will now end up earning $\overline{x}$ in the (legal) labor market. Thus, individuals whose pre-policy (honest) income falls in the darkened interval from $(\overline{x}-\tilde{x})$ to $\overline{x}$ will switch from a life of crime to an honest life as a result of the investments in education.

(c) *Next, consider the alternative policy of investing in more enforcement — thus increasing the probability of getting caught δ. Indicate in your graph how much the expected consumption level of a life of crime must be shifted in order for the policy to achieve the same reduction in crime as the policy in part (b).*

Answer: This is also illustrated in panel (a) of Exercise Graph 17.13. In order for a policy focused on raising δ to reduce crime by the same amount (without legal incomes rising), it must be that the expected consumption value of a life of crime falls sufficiently to make the expected *utility* of crime equal to the utility of an individual with (legal) income $(\overline{x}-\tilde{x})$. This would imply an increase in δ sufficiently high to move us to B — which requires a shift of the expected consumption value of crime from EV to EV'. Because the consumption/utility relationship is steep, $(EV-EV')$ is greater than $\overline{x}$.

(d) *If it costs the same to achieve a $1 increase in everyone's income through education investments as it costs to achieve a $1 reduction in the expected consumption level of a life of crime, which policy is more cost effective at reducing crime given we started with an already high δ.*

Answer: Since $(EV - EV')$ is greater than $\overline{x}$, it is more cost effective to reduce crime through investments in education in this case.

(e) *How does your answer change if δ is very low to begin with?*

Answer: This is pictured in panel (b) of Exercise Graph 17.13. Following the same steps as before, we now find that $(EV - EV')$ is less than $\overline{x}$ — implying it is more cost effective to reduce crime through increases in policing rather than investments in education.

(f) True or False*: Assuming people are risk averse, the following is an accurate policy conclusion from our model of expected utility: The higher current levels of law enforcement, the more likely it is that investments in education will cause greater reductions in crime than equivalent investments in additional law enforcement.*

Answer: This is true based on our analysis thus far. When δ is high, law enforcement levels are already high — in which case we found it is more likely to be cost effective to invest in education rather than additional law enforcement than when δ is low.

B: *Now suppose that, as in exercise 17.10, $x_0 = 20$ and $x_1 = 80$ (where we can think of these values as being expressed in terms of thousands of dollars).*

(a) *Suppose, again as in exercise 17.10, that expressing utility over consumption by $u(x) = \ln x$ allows us to express tastes over gambles using the expected utility function. If $\delta = 0.75$, what is the income level $\overline{x}$ at which an individual is indifferent between a life of crime and an honest life?*

Answer: When $\delta = 0.75$, the expected utility from a life of crime is given by

$$0.75\ln(20) + 0.25\ln(80) \approx 3.342. \quad (17.13.i)$$

The certainty equivalent $\overline{x}$ is the obtained by setting $\ln(\overline{x}) = 3.342$ which solves to $\overline{x} = e^{3.342} \approx 28.28$ — the value of an honest income that makes individuals indifferent between an honest life and a life of crime.

(b) *If an investment in eduction results in a uniform increase of income of 5, what are the pre-policy incomes of people who will now switch from a life of crime to an honest life?*

Answer: Since we concluded before that 28.28 is the cut-off (honest) income level above which the expected utility from a life of crime is below the utility of an honest life, it is those with pre-policy incomes between 23.28 and 28.28 that will switch from lives of crime to honest lives.

(c) *How much would δ have to increase in order to achieve an equivalent reduction in crime? How much would this change the expected consumption level under a life of crime?*

Answer: In order for an increase in δ to accomplish the same thing, it must be that δ is sufficiently high for the expected utility of crime to lie below the utility from consumption of 23.28; i.e.

$$\delta \ln(20) + (1-\delta)\ln(80) = \ln(23.28). \tag{17.13.ii}$$

Solving for δ, we get $\delta \approx 0.89$ — i.e. we have to increase enforcement from 0.75 to 0.89 in order to achieve the same reduction in crime as the education policy analyzed before.

(d) *If it is equally costly to raise incomes by \$1 through education investments as it is to reduce the expected value of consumption in a life of crime through an increase in δ, which policy is the more cost effective way to reduce crime?*

Answer: The education investment policy raises incomes for everyone by 5. The increased enforcement policy that raises δ to 0.89 results in a reduction in the expected consumption value of crime to

$$EV' = 0.89(20) + 0.11(80) = 26.60, \tag{17.13.iii}$$

down 8.4 from the initial expected consumption value of crime $EV = 35$. If it is equally costly to raise everyone's income as it is to lower EV, the increased enforcement policy is therefore a significantly more costly way of reducing crime than the education investment policy.

(e) *How do your answers change if $\delta = 0.25$ to begin with?*

Answer: When $\delta = 0.25$, we have an initial expected consumption level of crime equal to $EV = 65$ and an expected utility of crime of $E(u) \approx 4.035$ — with certainty equivalent of $\overline{x} \approx 56.57$. Thus, initially everyone whose honest income falls below 56.57 lives a life of crime. Under the education investment policy, all incomes rise by 5 — which implies that those who previously could earn between 51.57 and 56.57 in the (legal) labor market would switch from a life of crime to an honest life under this policy. In order to achieve an equivalent reduction in crime through lowering δ (using an increased enforcement policy), we need to find the δ for which the expected utility from a life of crime is equal to the utility of consuming 51.57. This gives us $\delta \approx 0.317$ — up from the initial 0.25. And, $\delta = 0.317$ implies an expected consumption value of crime equal to approximately 61 — down by only 4 from the initial 65. If it is equally costly to fund education investments that raise everyone's income by a dollar as it is to reduce the expected consumption value of crime by one dollar through increased enforcement, it now costs less to reduce crime through increased enforcement rather than investments in education.

Conclusion: Potentially Helpful Reminders

1. The basic model of risk aversion (and risk loving) from the first section of the Chapter requires that you get comfortable with the difference between the *expected utility* of a gamble and the *utility of the expected value* of the gamble. Once you know how to read these two concepts on a graph of a

consumption/utility graph, you will have come a long way toward mastering this model. The concepts of certainty equivalence and risk premiums emerge straightforwardly from this.

2. The expected utility of a gamble involving two outcomes is always read off the line connecting the utility of the individual outcomes (which in turn are read off the consumption/utility relationship). The utility of the expected value of the gamble, on the other hand, is read directly off the consumption/utility relationship. The former is lower than the latter for risk averse individuals and higher for the risk loving individuals.

3. When calculating the set of actuarily fair insurance contracts, keep in mind that the expected benefit from holding the insurance contract must be equal to the expected cost. This equivalence then gives rise to the relationship between the price of an actuarily fair insurance policy and the benefit level of that policy.

4. When working with indifference curves in models of state-dependent utility, keep in mind that the interpretation of these indifference curves is different from what we developed in the consumer model without risk even though the indifference map looks the same. In previous chapters, the bundles contained on indifference curves were actual bundles of goods that the consumer consumed; in our model here, the bundles are "outcome pairs" — with only one of the outcomes actually coming about. When consumers make choices, they do not know which outcome will happen — they only know the probability with which each of the outcomes is likely to happen.

5. Because the indifference curves in the state-dependent model arise from an expected utility relationship (or function), their shape is determined in part by the probabilities with which the outcomes are thought to occur. As the probabilities change, so does the indifference map (and the underlying expected utility function).

6. There is no reason to expect that insurance pricing in a general equilibrium setting will turn out to be actuarily fair, particularly in the presence of aggregate risk. This point is developed further in a number of end-of-chapter exercises.