CHAPTER

2

Choice Sets and Budget Constraints

Consumers are people who try to do the "best they can" given their "budget circumstances" or what we will call their **budget constraints**. This chapter develops a model for these budget constraints that simply specify which **bundles of goods and services** are affordable for a consumer.

Chapter Highlights

The main points of the chapter are:

1. Constraints arise from "what we bring to the table" — whether that is in the form of an **exogenous income** or an **endowment** — and from the **opportunity costs** that arise through prices.

2. Changes in "what we bring to the table" do not alter opportunity costs — and thus **shift budgets without changing slopes**.

3. Changes in prices result in changes in opportunity costs — and thus alter the **slopes of budgets**.

4. With three goods, budget constraints become planes and choice sets are 3-dimensional — or they can be treated mathematically instead of graphically.

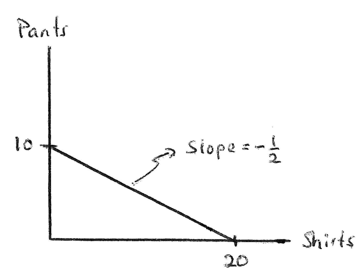
5. A **composite good** represents a way of indexing consumption other than the good of interest — and allows us to make the 2-good model more general.

2A Solutions to Within-Chapter-Exercises for Part A

Exercise 2A.1

Instead of putting pants on the horizontal axis and shirts on the vertical, put pants on the vertical and shirts on the horizontal. Show how the budget constraint looks and read from the slope what the opportunity cost of shirts (in terms of pants) and pants (in terms of shirts) is.

Answer: This is illustrated in Exercise Graph 2A.1. The slope of the budget would now be $-1/2$. Since the slope of the budget is the opportunity cost of the good on the horizontal axis in terms of the good on the vertical axis, this implies that the opportunity cost of shirts in terms of pants is 1/2. The inverse of the slope of the budget is the opportunity cost of the good on the vertical axis in terms of the good on the horizontal axis. Therefore the opportunity cost of pants in terms of shirts is 2.

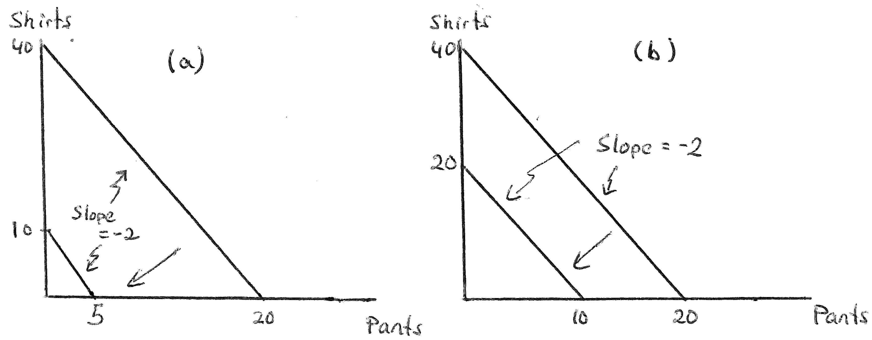


Exercise Graph 2A.1 : Graph for Within-Chapter-Exercise 2A.1

Exercise 2A.2

Demonstrate how my budget constraint would change if, on the way into the store, I had lost \$300 of the \$400 my wife had given to me. Does my opportunity cost of pants (in terms of shirts) or shirts (in terms of pants) change? What if instead the prices of pants and shirts had doubled while I was driving to the store?

Answer: The budgets would shift parallel as shown in Exercise Graph 2A.2. The slopes of the budget constraints do not change in either case — implying that the opportunity cost of one good in terms of the other does not change.

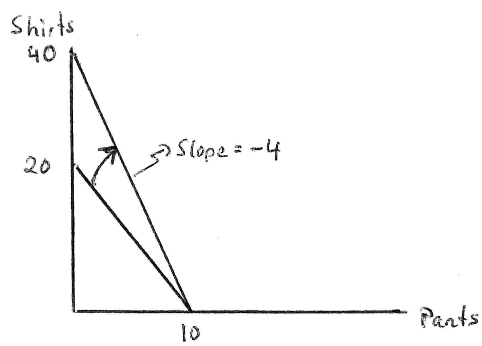


Exercise Graph 2A.2 : (a) $300 lost and (b) both prices doubled

Exercise 2A.3

How would my budget constraint change if, instead of a 50% off coupon for pants, my wife had given me a 50% off coupon for shirts? What would the opportunity cost of pants (in terms of shirts) be?

Answer: The budget constraint would change as depicted in Exercise Graph 2A.3. The new opportunity cost of pants in terms of shirts would be 4 — i.e. for every pair of pants you now buy, you would be giving up 4 (rather than 2) shirts.

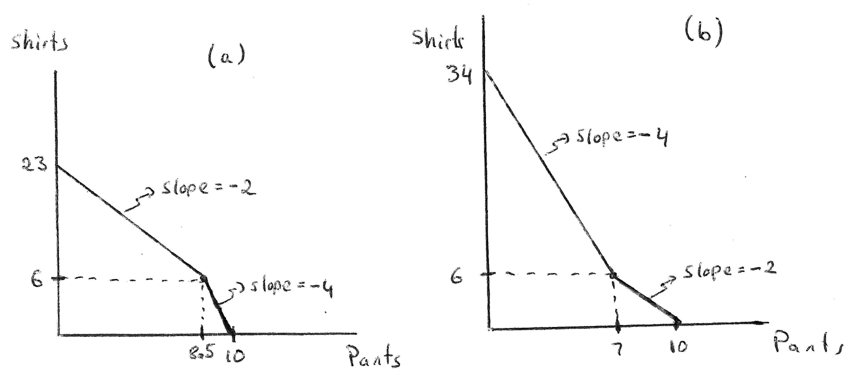


Exercise Graph 2A.3 : 50% off coupon for shirts (instead of pants)

Exercise 2A.4

Suppose that the two coupons analyzed above were for shirts instead of pants. What would the budget constraints look like?

Answer: Panel (a) of Exercise Graph 2A.4 depicts the constraint for a 50% off coupon that applies only to the first 6 shirts bought. If you buy 6 shirts with this coupon, you will have spent \$30 and will therefore have given up 1.5 pants. Thus, over this range, the opportunity cost of pants is $6/1.5 = 4$. After spending \$30 on the first 6 shirts, you could spend up to another \$170 on shirts. If you spent all of it on shirts, you could therefore afford an additional 17 shirts for a total of 23.



Exercise Graph 2A.4 : 2 coupons for shirts (instead of pants)

Panel (b) depicts the constraint for a coupon that gives 50% off for all shirts after the first 6. If you buy 6 shirts, you therefore spend \$60 (because you buy the first 6 at full price) – thus giving up the equivalent of 3 shirts. At that point, you have up to \$140 left to spend, and if you spend all of it on shirts at 50% off, you can afford to get 28 more – for a total of 34.

Exercise 2A.5

Revisit the coupons we discussed in Section 2A.3 and illustrate how these would alter the choice set when defined over pants and a composite good.

Answer: The graphs would look exactly the same as the kinked budget constraint in Graph 2.4 of the text – except that the vertical axis would be denominated in "dollars of other good consumption" with values 10 times what they are in Graph 2.4. This would also have the effect of increasing the slopes 10-fold.

Exercise 2A.6

True or False: When we model the good on the vertical axis as "dollars of consumption of other goods," the slope of the budget constraint is $-p_1$, where p_1 denotes the price of the good on the horizontal axis.

Answer: True. The slope of the budget constraint is always $-p_1/p_2$. When x_2 is a composite good denominated in dollar units, its price is $p_2 = 1$ since "1 dollar of other good consumption" by definition costs exactly 1 dollar. Thus the slope $-p_1/p_2$ simply reduces to $-p_1$.

2B Solutions to Within-Chapter-Exercises for Part B

Exercise 2B.1

What points in Graph 2.1 satisfy the necessary but not the sufficient conditions in expression (2.1)?

Answer: The points to the northeast of the blue budget line – i.e. all the non-shaded points outside the budget line. These bundles satisfy the necessary condition that $(x_1, x_2) \in \mathbb{R}^2_+$, but they do not satisfy the sufficient condition that $20x_1 + 10x_2 \leq 200$.

Exercise 2B.2

Using equation (2.5), show that the exact same change in the budget line could happen if both prices fell by half at the same time while the dollar budget remained the same. Does this make intuitive sense?

Answer: Replacing p_1 with $0.5p_1$ and p_2 with $0.5p_2$ in the equation, we get

$$x_2 = \frac{I}{0.5p_2} - \frac{0.5p_1}{0.5p_2}x_1 = \frac{2I}{p_2} - \frac{p_1}{p_2}x_1. \tag{2B.2}$$

If the initial income is \$200, this implies the budget constraint when all prices fall by half is equivalent to one with the original prices and income equal to \$400. This makes intuitive sense: If all prices fall by half, then any given cash budget can buy twice as much. Thus, the simultaneous price drop is equivalent to an increase in (cash) income.

Exercise 2B.3

Using the mathematical formulation of a budget line (equation (2.5)), illustrate how the slope and intercept terms change when p_2 instead of p_1 changes. Relate this to what your intuition would tell you in a graphical model of budget lines.

Answer: When p_2 changes to p_2', the intercept changes from I/p_2 to I/p_2'. If $p_2' > p_2$, this implies that the intercept falls, while if $p_2' < p_2$ it implies that the intercept increases. This makes intuitive sense since an increase in the price of good 2 means that you can buy less of good 2 if that is all you spend your income on, and a decrease in the price of good 2 means that you can buy more of good 2 when that is all you spend your income on.

Looking at the slope term, an increase in p_2 causes $-p_1/p_2$ to fall in absolute value — implying a shallower budget line. Similarly, a decrease in p_2 causes $-p_1/p_2$ to rise in absolute value — implying a steeper budget. This also makes intuitive sense: When p_2 increases, the opportunity cost of x_1 falls (as illustrated by the shallower budget line), and when p_2 falls, the opportunity cost of x_1 increases (as illustrated by the steeper budget line).

Exercise 2B.4

Convert the two equations contained in the budget set (2.7) into a format that illustrates more clearly the intercept and slope terms (as in equation (2.5)). Then, using the numbers for prices and incomes from our example, plot the two lines on a graph. Finally, erase the portions of the lines that are not relevant given that each line applies only for some values of x_1 (as indicated in (2.7)). Compare your graph to Graph 2.4a.

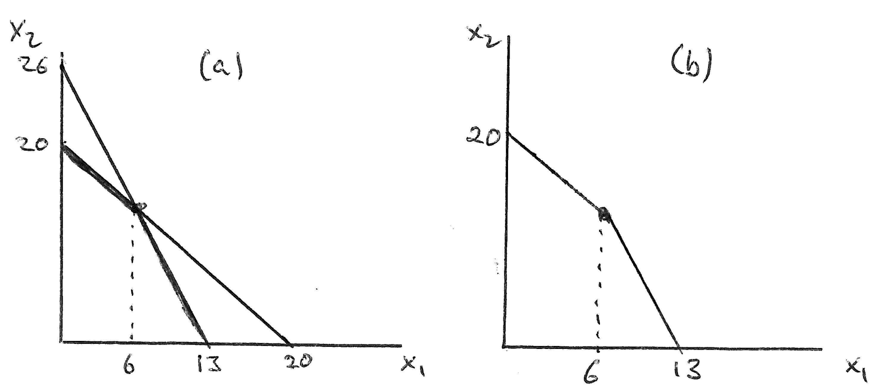
Answer: The two equations can be written as

$$x_2 = \frac{I}{p_2} - \frac{p_1}{2p_2}x_1 \text{ and } x_2 = \frac{I+3p_1}{p_2} - \frac{p_1}{p_2}x_1. \tag{2B.4.i}$$

Plugging in $I = 200$, $p_1 = 20$ and $p_2 = 10$ as in the example with pants (x_1) and shirts (x_2), this gives

$$x_2 = \frac{200}{10} - \frac{20}{2(10)}x_1 = 20 - x_1 \text{ and } x_2 = \frac{260}{10} - \frac{20}{10}x_1 = 26 - 2x_1. \tag{2B.4.ii}$$

Panel (a) in Exercise Graph 2B.4 plots these two lines, and panel (b) erases the portions that are not relevant given that the first equation applies only to values of x_1 less than or equal to 6 and the second equation applies only to values of x_1 greater than 6. The resulting graph is identical to the one we derived intuitively in the text.



Exercise Graph 2B.4 : Graphs of equations in exercise 2B.4

Exercise 2B.5

Now suppose that the 50% off coupon is applied to all pants purchased after you bought an initial 6 pants at regular price. Derive the mathematical formulation of the budget set (analogous to equation (2.7)) and then repeat the previous exercise. Compare your graph to Graph 2.4b.

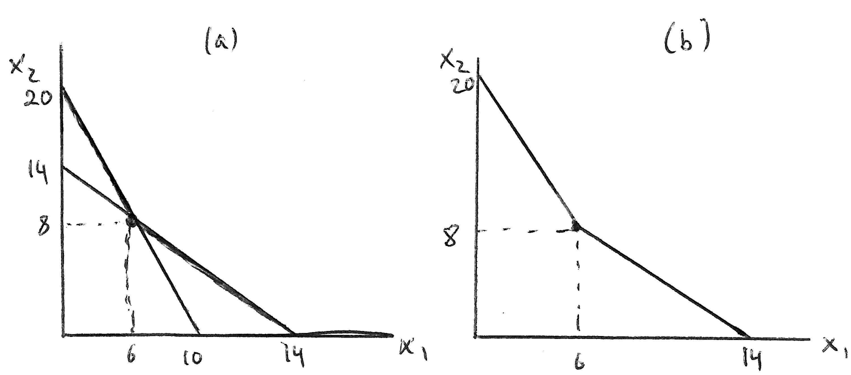
Answer: The normal budget constraint would apply to the initial range of pants (since the coupon does not kick in until 6). After that, the price of pants (p_1) falls by half. Furthermore, since we already spent \$120 to get to 6 pair of pants, we only have \$80 left — implying that the most we could buy is 8 more pants at the reduced price for a total of 14 pants. Were we to be able to buy 14 pants at a price of \$10 (which is assumed along this line segment), our total spending would be \$140 — implying that our effective income on this line segment is \$60 less than the $I = 200$ we started with. More generally, when the price falls to $0.5p_1$ after the 6th pair, the vertical intercept of the shallower budget falls to $(I - 0.5(6p_1)) = I - 3p_1$. This gives us the following definition of the budget line:

$$B(p_1, p_2, I) = \{ (x_1, x_2) \in \mathbb{R}^2_+ \mid \begin{array}{ll} p_1x_1 + p_2x_2 = I & \text{for } x_1 \leq 6 \text{ and} \\ 0.5p_1x_1 + p_2x_2 = I - 3p_1 & \text{for } x_1 > 6 \}. \end{array} \quad (2B.5.i)$$

Taking x_2 to one side in both of these equations, and substituting in $p_1 = 20$, $p_2 = 10$ and $I = 200$, we get

$$x_2 = \frac{200}{10} - \frac{20}{10}x_1 = 20 - 2x_1 \text{ and } x_2 = \frac{200-60}{10} - \frac{20}{2(10)}x_1 = 14 - x_1. \quad (2B.5.ii)$$

Panel (a) of Exercise Graph 2B.5 plots these two lines, and panel (b) erases the portions that are not relevant. The resulting graph is identical to the one for this coupon in the text.



Exercise Graph 2B.5 : Graphs of equations in exercise 2B.5

Exercise 2B.6

Using the equation in (2.19), derive the general equation of the budget line in terms of prices and endowments. Following steps analogous to those leading to equation (2.17), identify the intercept and slope terms. What would the budget line look like when my endowments are 10 shirts and 10 pants and when prices are $5 for pants and $10 for shirts? Relate this to both the equation you derived and to an intuitive derivation of the same budget line.

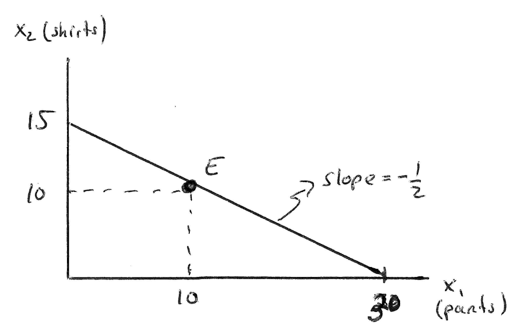
Answer: Changing the inequality to an equality and solving the equation for x_2, we get

$$x_2 = \frac{p_1 e_1 + p_2 e_2}{p_2} - \frac{p_1}{p_2} x_1. \tag{2B.6.i}$$

The slope is therefore $-p_1/p_2$ as it always is. The x_2 intercept is $(p_1 e_1 + p_2 e_2)/p_2$ — which is just the value of my endowment divided by price. When endowments are 10 shirts and 10 pants and when prices are $p_1 = 5$ and $p_2 = 10$, the equation becomes

$$x_2 = \frac{5(10) + 10(10)}{10} - \frac{5}{10} x_1 = 15 - \frac{1}{2} x_1. \tag{2B.6.ii}$$

We would intuitively derive this as follows: We would begin at the endowment point (10,10). Given the prices of pants and shirts, I could sell my 10 pants for $50 and with that I could buy 5 more shirts. Thus, the most shirts I could buy if I only bought shirts is 15 — the x_2 intercept. Since pants cost half what shirts cost, I could buy 30 pants. The resulting budget line, which is equivalent to the one derived mathematically above, is depicted in Exercise Graph 2B.6.



Exercise Graph 2B.6 : Graph of equation in exercise 2B.6

2C Solutions to Odd-Numbered End-of-Chapter Exercises

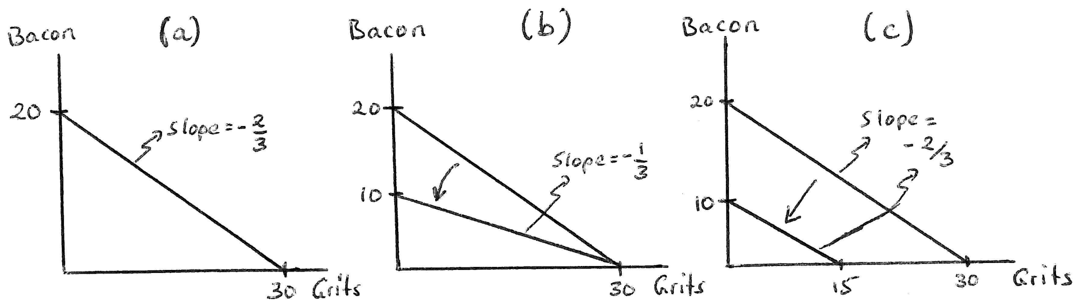
Exercise 2.1

Any good Southern breakfast includes grits (which my wife loves) and bacon (which I love). Suppose we allocate $60 per week to consumption of grits and bacon, that grits cost $2 per box and bacon costs $3 per package.

A: *Use a graph with boxes of grits on the horizontal axis and packages of bacon on the vertical to answer the following:*

(a) *Illustrate my family's weekly budget constraint and choice set.*

Answer: The graph is drawn in panel (a) of Exercise Graph 2.1.



Exercise Graph 2.1 : (a) Answer to (a); (b) Answer to (c); (c) Answer to (d)

(b) *Identify the opportunity cost of bacon and grits and relate these to concepts on your graph.*

Answer: The opportunity cost of grits is equal to 2/3 of a package of bacon (which is equal to the negative slope of the budget since grits appear on the horizontal axis). The opportunity cost of a package of bacon is 3/2 of a box of grits (which is equal to the inverse of the negative slope of the budget since bacon appears on the vertical axis).

(c) *How would your graph change if a sudden appearance of a rare hog disease caused the price of bacon to rise to $6 per package, and how does this change the opportunity cost of bacon and grits?*

Answer: This change is illustrated in panel (b) of Exercise Graph 2.1. This changes the opportunity cost of grits to 1/3 of a package of bacon, and it changes the opportunity cost of bacon to 3 boxes of grits. This makes sense: Bacon is now 3 times as expensive as grits — so you have to give up 3 boxes of grits for one package of bacon, or 1/3 of a package of bacon for 1 box of grits.

(d) *What happens in your graph if (instead of the change in (c)) the loss of my job caused us to decrease our weekly budget for Southern breakfasts from \$60 to \$30? How does this change the opportunity cost of bacon and grits?*

Answer: The change is illustrated in panel (c) of Exercise Graph 2.1. Since relative prices have not changed, opportunity costs have not changed. This is reflected in the fact that the slope stays unchanged.

B: *In the following, compare a mathematical approach to the graphical approach used in part A, using x_1 to represent boxes of grits and x_2 to represent packages of bacon:*

(a) *Write down the mathematical formulation of the budget line and choice set and identify elements in the budget equation that correspond to key features of your graph from part 2.1A(a).*

Answer: The budget equation is $p_1x_1 + p_2x_2 = I$ can also be written as

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}x_1. \tag{2.1.i}$$

With $I = 60$, $p_1 = 2$ and $p_2 = 3$, this becomes $x_2 = 20 - (2/3)x_1$ — an equation with intercept of 20 and slope of $-2/3$ as drawn in Exercise Graph 2.1(a).

(b) *How can you identify the opportunity cost of bacon and grits in your equation of a budget line, and how does this relate to your answer in 2.1A(b).*

Answer: The opportunity cost of x_1 (grits) is simply the negative of the slope term (in terms of units of x_2). The opportunity cost of x_2 (bacon) is the inverse of that.

(c) *Illustrate how the budget line equation changes under the scenario of 2.1A(c) and identify the change in opportunity costs.*

Answer: Substituting the new price $p_2 = 6$ into equation (2.1.i), we get $x_2 = 10 - (1/3)x_1$ — an equation with intercept of 10 and slope of $-1/3$ as depicted in panel (b) of Exercise Graph 2.1.

(d) *Repeat (c) for the scenario in 2.1A(d).*

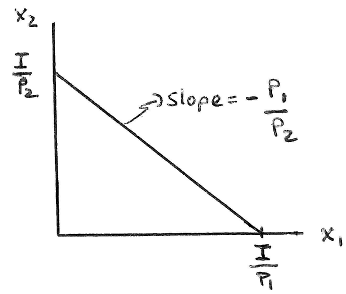
Answer: Substituting the new income $I = 30$ into equation (2.1.i) (holding prices at $p_1 = 2$ and $p_2 = 3$, we get $x_2 = 10 - (2/3)x_1$ — an equation with intercept of 10 and slope of $-2/3$ as depicted in panel (c) of Exercise Graph 2.1.

Exercise 2.3

Consider a budget for good x_1 (on the horizontal axis) and x_2 (on the vertical axis) when your economic circumstances are characterized by prices p_1 and p_2 and an exogenous income level I.

A: *Draw a budget line that represents these economic circumstances and carefully label the intercepts and slope.*

Answer: The sketch of this budget line is given in Exercise Graph 2.3.



Exercise Graph 2.3 : A budget constraint with exogenous income I

The vertical intercept is equal to how much of x_2 one could by with I if that is all one bought — which is just I/p_2. The analogous is true for x_1 on the horizontal intercept. One way to verify the slope is to recognize it is the "rise" (I/p_2) divided by the "run" (I/p_1) — which gives p_1/p_2 — and that it is negative since the budget constraint is downward sloping.

(a) *Illustrate how this line can shift parallel to itself without a change in I.*

Answer: In order for the line to shift in a parallel way, it must be that the slope $-p_1/p_2$ remains unchanged. Since we can't change I, the only values we can change are p_1 and p_2 — but since p_1/p_2 can't change, it means the only thing we can do is to multiply both prices by the same constant. So, for instance, if we multiply both prices by 2, the ratio of the new prices is $2p_1/(2p_2) = p_1/p_2$ since the 2's cancel. We therefore have not changed the slope. But we have changed the vertical intercept from I/p_2 to $I/(2p_2)$. We have therefore shifted in the line without changing its slope.

This should make intuitive sense: If our money income does not change but all prices double, then I can by half as much of everything. This is equivalent to prices staying the same and my money income dropping by half.

(b) *Illustrate how this line can rotate clockwise on its horizontal intercept without a change in p_2.*

Answer: To keep the horizontal intercept constant, we need to keep I/p_1 constant. But to rotate the line clockwise, we need to increase the vertical intercept I/p_2. Since we can't change p_2 (which would be the easiest way to do this), that leaves us only I and p_1 to change. But since we can't change I/p_1, we can only change these by multiplying them by the same constant. For instance, if we multiply both by 2, we don't change the horizontal intercept since $2I/(2p_1) = I/p_1$. But we do increase the vertical intercept from I/p_2 to $2I/p_2$. So, multiplying both I and p_1 by the same constant (greater than 1) will accomplish our goal.

This again should make intuitive sense: If you double my income and the price of good 1, I can still afford exactly as much of good 1 if that is all

I buy with my income. (Thus the unchanged horizontal intercept). But, if I only buy good 2, then a doubling of my income without a change in the price of good 2 lets me buy twice as much of good 2. The scenario is exactly the same as if p_2 had fallen by half (and I and p_1 had remained unchanged.)

B: *Write the equation of a budget line that corresponds to your graph in 2.3A.*

Answer: $p_1x_1 + p_2x_2 = I$, which can also be written as

$$x_2 = \frac{I}{p_2} - \frac{p_1}{p_2}x_1. \tag{2.3.i}$$

(a) *Use this equation to demonstrate how the change derived in 2.3A(a) can happen.*

Answer: If I replace p_1 with αp_1 and p_2 with αp_2 (where α is just a constant), I get

$$x_2 = \frac{I}{\alpha p_2} - \frac{\alpha p_1}{\alpha p_2}x_1 = \frac{(1/\alpha)I}{p_2} - \frac{p_1}{p_2}x_1. \tag{2.3.ii}$$

Thus, multiplying both prices by α is equivalent to multiplying income by $1/\alpha$ (and leaving prices unchanged).

(b) *Use the same equation to illustrate how the change derived in 2.3A(b) can happen.*

Answer: If I replace p_1 with βp_1 and I with βI, I get

$$x_2 = \frac{\beta I}{p_2} - \frac{\beta p_1}{p_2}x_1 = \frac{I}{(1/\beta)p_2} - \frac{p_1}{(1/\beta)p_2}x_1. \tag{2.3.iii}$$

Thus, this is equivalent to multiplying p_2 by $1/\beta$. So long as $\beta > 1$, it is therefore equivalent to reducing the price of good 2 (without changing the other price or income).

Exercise 2.5

Everyday Application: *Watching a Bad Movie: On one of my first dates with my wife, we went to see the movie "Spaceballs" and paid $5 per ticket.*

A: *Halfway through the movie, my wife said: "What on earth were you thinking? This movie sucks! I don't know why I let you pick movies. Let's leave."*

(a) *In trying to decide whether to stay or leave, what is the opportunity cost of staying to watch the rest of the movie?*

Answer: The opportunity cost of any activity is what we give up by undertaking that activity. The opportunity cost of staying in the movie is whatever we would choose to do with our time if we were not there. The price of the movie tickets that got us into the movie theater is NOT a part of this opportunity cost — because, whether we stay or leave, we do not get that money back.

(b) *Suppose we had read a sign on the way into theater stating "Satisfaction Guaranteed! Don't like the movie half way through — see the manager and get your money back!" How does this change your answer to part (a)?*

Answer: Now, in addition to giving up whatever it is we would be doing if we weren't watching the movie, we are also giving up the price of the movie tickets. Put differently, by staying in the movie theater, we are giving up the opportunity to get a refund — and so the cost of the tickets is a real opportunity cost of staying.

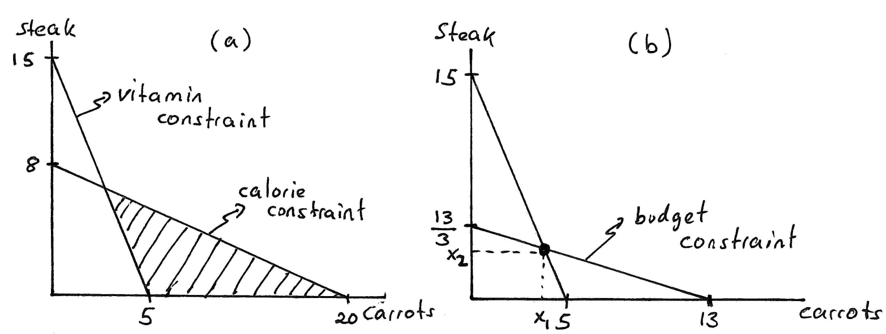
Exercise 2.7

Everyday Application: *Dieting and Nutrition: On a recent doctor's visit, you have been told that you must watch your calorie intake and must make sure you get enough vitamin E in your diet.*

A: *You have decided that, to make life simple, you will from now on eat only steak and carrots. A nice steak has 250 calories and 10 units of vitamins, and a serving of carrots has 100 calories and 30 units of vitamins Your doctor's instructions are that you must eat* no more *than 2000 calories and consume* at least *150 units of vitamins per day.*

(a) *In a graph with "servings of carrots" on the horizontal and steak on the vertical axis, illustrate all combinations of carrots and steaks that make up a 2000 calorie a day diet.*

Answer: This is illustrated as the "calorie constraint" in panel (a) of Exercise Graph 2.7. You can get 2000 calories only from steak if you eat 8 steaks and only from carrots if you eat 20 servings of carrots. These form the intercepts of the calorie constraint.



Exercise Graph 2.7 : (a) Calories and Vitamins; (b) Budget Constraint

(b) *On the same graph, illustrate all the combinations of carrots and steaks that provide exactly 150 units of vitamins.*

Answer: This is also illustrated in panel (a) of Exercise Graph 2.7. You can get 150 units of vitamins from steak if you eat 15 steaks only or if you eat

5 servings of carrots only. This results in the intercepts for the "vitamin constraint".

(c) *On this graph, shade in the bundles of carrots and steaks that satisfy both of your doctor's requirements.*

Answer: Your doctor wants you to eat no more than 2000 calories — which means you need to stay underneath the calorie constraint. Your doctor also wants you to get at least 150 units of vitamin E — which means you must choose a bundle *above* the vitamin constraint. This leaves you with the shaded area to choose from if you are going to satisfy both requirements.

(d) *Now suppose you can buy a serving of carrots for $2 and a steak for $6. You have $26 per day in your food budget. In your graph, illustrate your budget constraint. If you love steak and don't mind eating or not eating carrots, what bundle will you choose (assuming you take your doctor's instructions seriously)?*

Answer: With $26 you can buy 13/3 steaks if that is all you buy, or you can buy 13 servings of carrots if that is all you buy. This forms the two intercepts on your budget constraint which has a slope of $-p_1/p_2 = -1/3$ and is depicted in panel (b) of the graph. If you really like steak and don't mind eating carrots one way or another, you would want to get as much steak as possible given the constraints your doctor gave you and given your budget constraint. This leads you to consume the bundle at the intersection of the vitamin and the budget constraint in panel (b) — indicated by (x_1, x_2) in the graph. It seems from the two panels that this bundle also satisfies the calorie constraint and lies inside the shaded region.

B: *Continue with the scenario as described in part A.*

(a) *Define the line you drew in A(a) mathematically.*

Answer: This is given by $100x_1 + 250x_2 = 2000$ which can be written as

$$x_2 = 8 - \frac{2}{5}x_1. \tag{2.7.i}$$

(b) *Define the line you drew in A(b) mathematically.*

Answer: This is given by $30x_1 + 10x_2 = 150$ which can be written as

$$x_2 = 15 - 3x_1. \tag{2.7.ii}$$

(c) *In formal set notation, write down the expression that is equivalent to the shaded area in A(c).*

Answer:

$$\left\{(x_1, x_2) \in \mathbb{R}^2_+ \mid 100x_1 + 250x_2 \leq 2000 \text{ and } 30x_1 + 10x_2 \geq 150\right\} \tag{2.7.iii}$$

(d) *Derive the exact bundle you indicated on your graph in A(d).*

Answer: We would like to find the most amount of steak we can afford in the shaded region. Our budget constraint is $2x_1+6x_2=26$. Our graph suggests that this budget constraint intersects the vitamin constraint (from equation (2.7.ii)) within the shaded region (in which case that intersection gives us the most steak we can afford in the shaded region). To find this intersection, we can plug equation (2.7.ii) into the budget constraint $2x_1+6x_2=26$ to get

$$2x_1+6(15-3x_1)=26, \tag{2.7.iv}$$

and then solve for x_1 to get $x_1=4$. Plugging this back into either the budget constraint or the vitamin constraint, we can get $x_2=3$. We know this lies on the vitamin constraint as well as the budget constraint. To check to make sure it lies in the shaded region, we just have to make sure it also satisfies the doctor's orders that you consume fewer than 2000 calories. The bundle (x_1,x_2)=(4,3) results in calories of $4(100)+3(250)=1150$, well within doctor's orders.

Exercise 2.9

Business Application: *Pricing and Quantity Discounts*: *Businesses often give quantity discounts. Below, you will analyze how such discounts can impact choice sets.*

A: *I recently discovered that a local copy service charges our economics department \$0.05 per page (or \$5 per 100 pages) for the first 10,000 copies in any given month but then reduces the price per page to \$0.035 for each additional page up to 100,000 copies and to \$0.02 per each page beyond 100,000. Suppose our department has a monthly overall budget of \$5,000.*

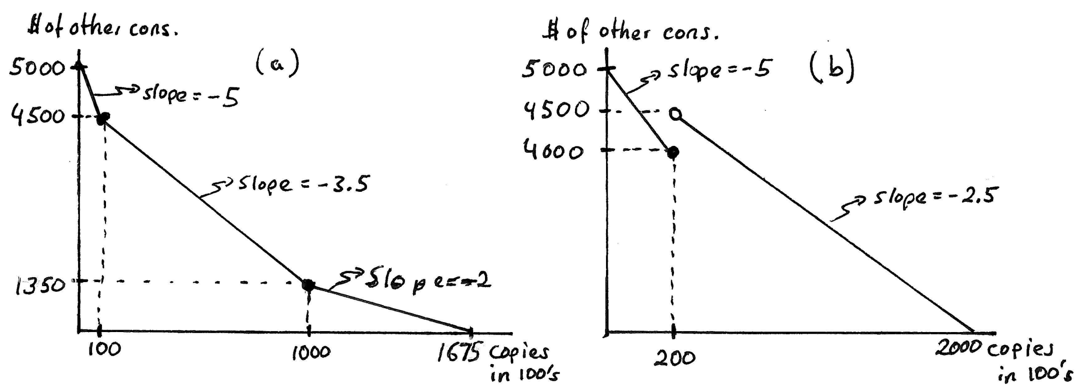
(a) *Putting "pages copied in units of 100" on the horizontal axis and "dollars spent on other goods" on the vertical, illustrate this budget constraint. Carefully label all intercepts and slopes.*

Answer: Panel (a) of Exercise Graph 2.9 traces out this budget constraint and labels the relevant slopes and kink points.

(b) *Suppose the copy service changes its pricing policy to \$0.05 per page for monthly copying up to 20,000 and \$0.025 per page for* all *pages if copying exceeds 20,000 per month. (*Hint*: Your budget line will contain a jump.)*

Answer: Panel (b) of Exercise Graph 2.9 depicts this budget. The first portion (beginning at the x_2 intercept) is relatively straightforward. The second part arises for the following reason: The problem says that, if you copy more than 2000 pages, *all* pages cost only \$0.025 per page — including the first 2000. Thus, when you copy 20,000 pages per month, you total bill is \$1,000. But when you copy 2001 pages, your total bill is \$500.025.

(c) *What is the marginal (or "additional") cost of the first page copied after 20,000 in part (b)? What is the marginal cost of the first page copied after 20,001 in part (b)?*



Exercise Graph 2.9 : (a) Constraint from 2.9A(a); (b) Constraint from 2.9A(b)

Answer: The marginal cost of the first page after 20,000 is -$499.975, and the marginal cost of the next page after that is 2.5 cents. To see the difference between these, think of the marginal cost as the increase in the total photo-copy bill for each additional page. When going from 20,000 to 20,001, the total bill falls by $499.975. When going from 20,001 to 20,002, the total bill rises by 2.5 cents.

B: *Write down the mathematical expression for choice sets for each of the scenarios in 2.9A(a) and 2.9A(b) (using x_1 to denote "pages copied in units of 100" and x_2 to denote "dollars spent on other goods").*

Answer: The choice set in (a) is

$$\{(x_1,x_2) \in \mathbb{R}^2_+ \mid \begin{array}{ll} x_2 = 5000 - 5x_1 & \text{for } x_1 \leq 100 \text{ and} \\ x_2 = 4850 - 3.5x_1 & \text{for } 100 < x_1 \leq 1000 \text{ and} \\ x_2 = 3350 - 2x_1 & \text{for } x_1 > 1000\,\}. \end{array} \tag{2.9.i}$$

The choice set in (b) is

$$\{(x_1,x_2) \in \mathbb{R}^2_+ \mid \begin{array}{ll} x_2 = 5000 - 5x_1 & \text{for } x_1 \leq 200 \text{ and} \\ x_2 = 5000 - 2.5x_1 & \text{for } x_1 > 200\,\}. \end{array} \tag{2.9.ii}$$

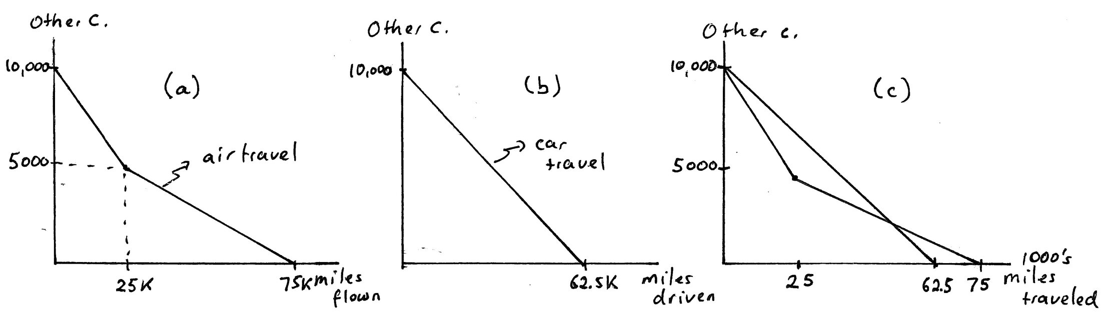
Exercise 2.11

Business Application: *Frequent Flyer Perks*: *Airlines offer frequent flyers different kinds of perks that we will model here as reductions in average prices per mile flown.*

A: *Suppose that an airline charges 20 cents per mile flown. However, once a customer reaches 25,000 miles in a given year, the price drops to 10 cents per mile flown for each additional mile. The alternate way to travel is to drive by car which costs 16 cents per mile.*

(a) *Consider a consumer who has a travel budget of $10,000 per year, a budget which can be spent on the cost of getting to places as well as "other consumption" while traveling. On a graph with "miles flown" on the horizontal axis and "other consumption" on the vertical, illustrate the budget constraint for someone who only considers flying (and not driving) to travel destinations.*

Answer: Panel (a) of Exercise Graph 2.11 illustrates this budget constraint.



Exercise Graph 2.11 : (a) Air travel; (b) Car travel; (c) Comparison

(b) *On a similar graph with "miles driven" on the horizontal axis, illustrate the budget constraint for someone that considers only driving (and not flying) as a means of travel.*

Answer: This is illustrated in panel (b) of the graph.

(c) *By overlaying these two budget constraints (changing the good on the horizontal axis simply to "miles traveled"), can you explain how frequent flyer perks might persuade some to fly a lot more than they otherwise would?*

Answer: Panel (c) of the graph overlays the two budget constraints. If it were not for frequent flyer miles, this consumer would never fly — because driving would be cheaper. With the frequent flyer perks, driving is cheaper initially but becomes more expensive per additional miles traveled if a traveler flies more than 25,000 miles. This particular consumer would therefore either not fly at all (and just drive), or she would fly a lot because it can only make sense to fly if she reaches the portion of the air-travel budget that crosses the car budget. (Once we learn more about how to model tastes, we will be able to say more about whether or not it makes sense for a traveler to fly under these circumstances.)

B: *Determine where the air-travel budget from A(a) intersects the car budget from A(b).*

Answer: The shallower portion of the air-travel budget (relevant for miles flown above 25,000) has equation $x_2 = 7500 - 0.1x_1$, where x_2 stands for other consumption and x_1 for miles traveled. The car budget, on the other hand, has equation $x_2 = 10000 - 0.16x_1$. To determine where they cross, we can set the two equations equal to one another and solve for x_1 — which gives $x_1 = 41,667$ miles traveled. Plugging this back into either equation gives $x_2 = \$3,333$.

Exercise 2.13

Policy Application: *Food Stamp Programs and other Types of Subsidies: The U.S. government has a food stamp program for families whose income falls below a certain poverty threshold. Food stamps have a dollar value that can be used at supermarkets for food purchases as if the stamps were cash, but the food stamps cannot be used for anything other than food.*

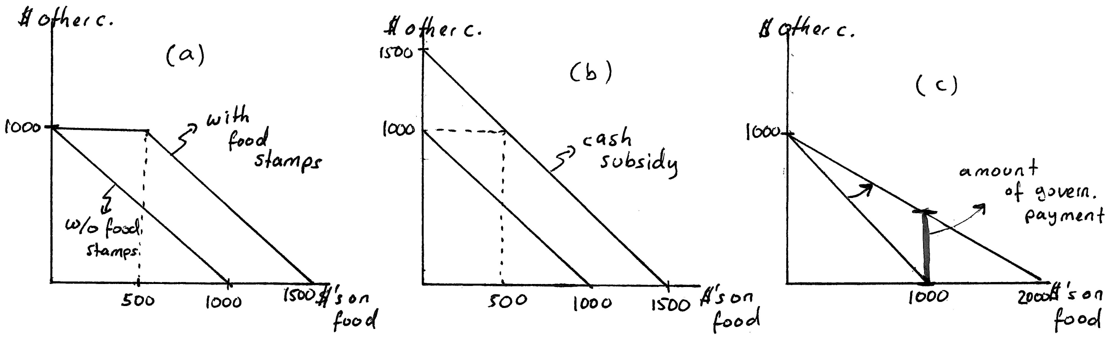
A: *Suppose the program provides $500 of food stamps per month to a particular family that has a fixed income of $1,000 per month.*

(a) *With "dollars spent on food" on the horizontal axis and "dollars spent on non-food items" on the vertical, illustrate this family's monthly budget constraint. How does the opportunity cost of food change along the budget constraint you have drawn?*

Answer: Panel (a) of Exercise Graph 2.13 illustrates the original budget — with intercept 1,000 on each axis. It then illustrates the new budget under the food stamp program. Since food stamps can only be spent on food, the "other goods" intercept does not change — owning some food stamps still only allows households to spend what they previously had on other goods. However, the family is now able to buy $1,000 in other goods even as it buys food — because it can use the food stamps on the first $500 worth of food and still have all its other income left for other consumption. Only after all the food stamps are spent — i.e. after the family has bought $500 worth of food — does the family give up other consumption when consuming additional food. As a result, the opportunity cost of food is zero until the food stamps are gone, and it is 1 after that. That is, after the food stamps are gone, the family gives up $1 in other consumption for every $1 of food it purchases.

(b) *How would this family's budget constraint differ if the government replaced the food stamp program with a cash subsidy program that simply gave this family $500 in cash instead of $500 in food stamps? Which would the family prefer, and what does your answer depend on?*

Answer: In this case, the original budget would simply shift out by $500 as depicted in panel (b). If the family consumes more than $500 of food under the food stamp program, it would not seem like anything really changes under the cash subsidy. (We can show this more formally once we introduce a model of tastes). If, on the other hand, the family consumes $500 of food under the food stamps, it may well be that it would prefer to get cash instead so that it can consume more other goods instead.



Exercise Graph 2.13 : (a) Food Stamps; (b) Cash; (c) Re-imburse half

(c) *How would the budget constraint change if the government simply agreed to reimburse the family for half its food expenses?*

Answer: In this case, the government essentially reduces the price of $1 of food to 50 cents because whenever $1 is spent on food, the government reimburses the family 50 cents. The resulting change in the family budget is then depicted in panel (c) of the graph.

(d) *If the government spends the same amount for this family on the program described in (c) as it did on the food stamp program, how much food will the family consume? Illustrate the amount the government is spending as a vertical distance between the budget lines you have drawn.*

Answer: If the government spent $500 for this family under this program, then the family will be consuming $1,000 of food and $500 in other goods. You can illustrate the $500 the government is spending as the distance between the two budget constraints at $1,000 of food consumption. The reasoning for this is as follows: On the original budget line, you can see that consuming $1,000 of food implies nothing is left over for "other consumption". When the family consumes $1,000 of food under the new program, it is able to consume $500 in other goods because of the program — so the government must have made that possible by giving $500 to the family.

B: *Write down the mathematical expression for the choice set you drew in 2.13A(a), letting x_1 represent dollars spent on food and x_2 represent dollars spent on non-food consumption. How does this expression change in 2.13A(b) and 2.13A(c)?*

Answer: The original budget constraint (prior to any program) is just $x_2 = 1000 - x_1$, and the budget constraint with the $500 cash payment in A(b) is $x_2 = 1500 - x_1$. The choice set under food stamps (depicted in panel (a)) then is

$$\begin{aligned}\{(x_1,x_2)\in\mathbb{R}^2_+ \mid \quad & x_2 = 1000 && \text{for } x_1 \le 500 \text{ and}\\ & x_2 = 1500 - x_1 && \text{for } x_1 > 500\,\},\end{aligned} \qquad (2.13.i)$$

while the choice set in panel (b) under the cash subsidy is

$$\left\{(x_1,x_2)\in\mathbb{R}^2_+ \mid x_2 = 1500 - x_1\right\}. \qquad (2.13.ii)$$

Finally, the choice set under the re-imbursement plan from A(c) is

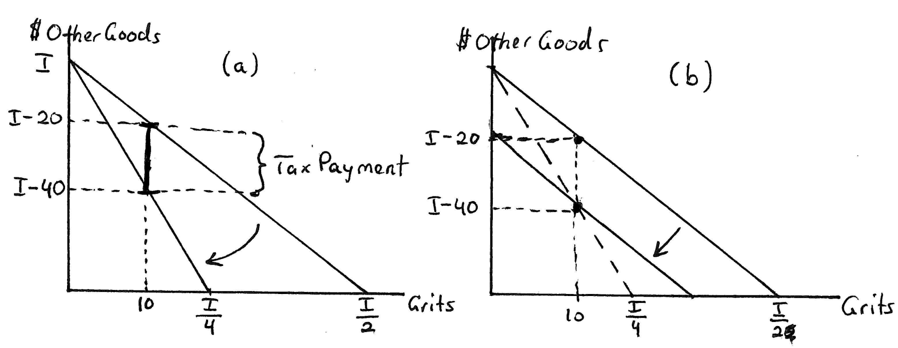
$$\left\{(x_1,x_2)\in\mathbb{R}^2_+ \mid x_2 = 1000 - \frac{1}{2}x_1\right\}. \qquad (2.13.iii)$$

Exercise 2.15

Policy Application: *Taxing Goods* versus *Lump Sum Taxes*: *I have finally convinced my local congressman that my wife's taste for grits are nuts and that the world should be protected from too much grits consumption. As a result, my congressman has agreed to sponsor new legislation to tax grits consumption which will raise the price of grits from \$2 per box to \$4 per box. We carefully observe my wife's shopping behavior and notice with pleasure that she now purchases 10 boxes of grits per month rather than her previous 15 boxes.*

A: *Putting "boxes of grits per month" on the horizontal and "dollars of other consumption" on the vertical, illustrate my wife's budget line before and after the tax is imposed. (You can simply denote income by I.)*

Answer: The tax raises the price, thus resulting in a rotation of the budget line as illustrated in panel (a) of Exercise Graph 2.15. Since no indication of an income level was given in the problem, income is simply denoted I.



Exercise Graph 2.15 : (a) Tax on Grits; (b) Lump Sum Rebate

(a) *How much tax revenue is the government collecting per month from my wife? Illustrate this as a vertical distance on your graph. (*Hint*: If you know how much she is consuming after the tax and how much in other consumption this leaves her with, and if you know how much in other consumption she would have had if she consumed that same quantity before the imposition of the tax, then the difference between these two "other consumption" quantities must be equal to how much she paid in tax.)*

Answer: When she consumes 10 boxes of grits after the tax, she pays $40 for grits. This leaves her with $(I-40)$ to spend on other goods. Had she bought 10 boxes of grits prior to the tax, she would have paid $20, leaving her with $(I-20)$. The difference between $(I-40)$ and $(I-20)$ is $20 — which is equal to the vertical distance in panel (a). You can verify that this is exactly how much she indeed must have paid — the tax is $2 per box and she bought 10 boxes, implying that she paid $2 times 10 or $20 in grits taxes.

(b) *Given that I live in the South, the grits tax turned out to be unpopular in my congressional district and has led to the defeat of my congressman. His replacement won on a pro-grits platform and has vowed to repeal the grits tax. However, new budget rules require him to include a new way to raise the same tax revenue that was yielded by the grits tax. He proposes to simply ask each grits consumer to pay exactly the amount he or she paid in grits taxes as a monthly lump sum payment. Ignoring for the moment the difficulty of gathering the necessary information for implementing this proposal, how would this change my wife's budget constraint?*

Answer: In panel (b) of Exercise Graph 2.15, the previous budget under the grits tax is illustrated as a dashed line. The grits tax changed the opportunity cost of grits — and thus the slope of the budget (as illustrated in panel (a)). The lump sum tax, on the other hand, does not alter opportunity costs but simply reduces income by $20, the amount of grits taxes my wife paid under the grits tax. This change is illustrated in panel (b).

B: *State the equations for the budget constraints you derived in A(a) and A(b), letting grits be denoted by x_1 and other consumption by x_2.*

Answer: The initial (before-tax) budget was $x_2 = I - 2x_1$ which becomes $x_2 = I - 4x_1$ after the imposition of the grits tax. The lump sum tax budget constraint, on the other hand, is $x_2 = I - 20 - 2x_1$.

Exercise 2.17

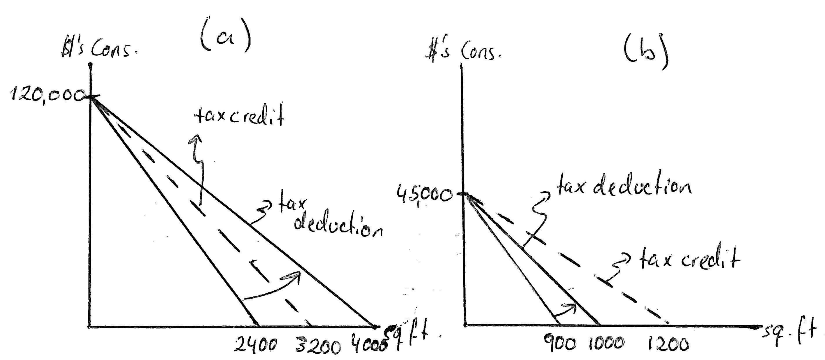
Policy Application: *Tax Deductions and Tax Credits*: *In the U.S. income tax code, a number of expenditures are "deductible". For most tax payers, the largest tax deduction comes from the portion of the income tax code that permits taxpayers to deduct home mortgage interest (on both a primary and a vacation home). This means that taxpayers who use this deduction do not have to pay income tax on the portion of their income that is spent on paying interest on their home mortgage(s). For purposes of this exercise, assume that the entire yearly price of housing is interest expense.*

A: True or False*: For someone whose marginal tax rate is 33%, this means that the government is subsidizing roughly one third of his interest/house payments.*

Answer: Consider someone who pays \$10,000 per year in mortgage interest. When this person deducts \$10,000, it means that he does not have to pay the 33% income tax on that amount. In other words, by deducting \$10,000 in mortgage interest, the person reduces his tax obligation by \$3,333.33. Thus, the government is returning 33 cents for every dollar in interest payments made — effectively causing the opportunity cost of paying \$1 in home mortgage interest to be equal to 66.67 cents. So the statement is true.

(a) *Consider a household with an income of \$200,000 who faces a tax rate of 40%, and suppose the price of a square foot of housing is \$50 per year. With square footage of housing on the horizontal axis and other consumption on the vertical, illustrate this household's budget constraint with and without tax deductibility. (Assume in this and the remaining parts of the question that the tax rate cited for a household applies to all of that household's income.)*

Answer: As just demonstrated, the tax deductibility of home mortgage interest lowers the price of owner-occupied housing, and it does so in proportion to the size of the marginal income tax rate one faces.



Exercise Graph 2.17 : Tax Deductions versus Tax Credits

Panel (a) of Exercise Graph 2.17 illustrates this graphically for the case described in this part. With a 40 percent tax rate, the household could consume as much as 0.6(200,000)=120,000 in other goods if it consumed no housing. With a price of housing of \$50 per square foot, the price falls to $(1-0.4)50 = 30$ under tax deductibility. Thus, the budget rotates out to the solid budget in panel (a) of the graph. Without deductibility, the consumer pays \$50 per square foot — which makes 120,000/50=2,400 the biggest possible house she can afford. But with deductibility, the biggest house she can afford is 120,000/30=4,000 square feet.

(b) *Repeat this for a household with income of $50,000 who faces a tax rate of 10%.*

Answer: This is illustrated in panel (b). The household could consume as much as $45,000 in other consumption after paying taxes, and the deductibility of house payments reduces the price of housing from $50 per square foot to (1 – 0.1)50 = $45 per square foot. This results in the indicated rotation of the budget from the lower to the higher solid line in the graph. The rotation is smaller in magnitude because the impact of deductibility on the after-tax price of housing is smaller. Without deductibility, the biggest affordable house is 45,000/50=900 square feet, while with deductibility the biggest possible house is 45,000/45=1,000 square feet.

(c) *An alternative way for the government to encourage home ownership would be to offer a tax* credit *instead of a tax* deduction. *A tax credit would allow all taxpayers to subtract a fraction k of their annual mortgage payments directly from the tax bill they would otherwise owe. (Note: Be careful — a tax credit is deducted from tax* payments *that are due, not from the taxable income.) For the households in (a) and (b), illustrate how this alters their budget if* $k = 0.25$.

Answer: This is illustrated in the two panels of Exercise Graph 2.17 — in panel (a) for the higher income household, and in panel (b) for the lower income household. By subsidizing housing through a credit rather than a deduction, the government has reduced the price of housing by the same amount (k) for everyone. In the case of deductibility, the government had made the price subsidy dependent on one's tax rate — with those facing higher tax rates also getting a higher subsidy. The price of housing how falls from $50 to (1 – 0.25)50 = $37.50 — which makes the largest affordable house for the wealthier household 120,000/37.5=3,200 square feet and, for the poorer household, 45,000/37.5=1,200 square feet. Thus, the poorer household benefits more from the credit when $k = 0.25$ while the richer household benefits more from the deduction.

(d) *Assuming that a tax deductibility program costs the same in lost tax revenues as a tax credit program, who would favor which program?*

Answer: People facing higher marginal tax rates would favor the deductibility program while people facing lower marginal tax rates would favor the tax credit.

B: *Let x_1 and x_2 represent square feet of housing and other consumption, and let the price of a square foot of housing be denoted p.*

(a) *Suppose a household faces a tax rate t for all income, and suppose the entire annual house payment a household makes is deductible. What is the household's budget constraint?*

Answer: The budget constraint would be $x_2 = (1-t)I - (1-t)px_1$.

(b) *Now write down the budget constraint under a tax credit as described above.*

Answer: The budget constraint would now be $x_2 = (1-t)I - (1-k)px_1$.

Conclusion: Potentially Helpful Reminders

1. When income I is exogenous, the intercepts of the budget line are I/p_1 (on the horizontal) and I/p_2 (on the vertical).

2. When income is endogenously derived from the sale of an endowment, you can calculate the person's cash budget I by simply multiplying each good's quantity in the endowment bundle by its price and adding up. (That's the value of the endowment bundle in the market). The vertical and horizontal intercepts of the budget line are then calculated just as in point 1 above.

3. The slope of budget lines — whether they emerge from exogenous incomes or endowments — is always $-p_1/p_2$, NOT $-p_2/p_1$. If good 2 is a composite good, the slope is just $-p_1$.

4. Remember that changes in the income or the endowment bundle cause parallel shifts; changes in prices cause rotations. And — if the income is exogenous, the rotation is through the intercept on the axis whose price has not changed; but if the income is endogenously derived from an endowment, the rotation is through the endowment bundle.

5. Be sure to do end-of-chapter exercises 2.6 and 2.15. Exercise 2.6 forms the basis for material introduced in Chapters 7, and Exercise 2.15 introduces a technique used repeatedly in Chapters 8 through 10.