CHAPTER

# 3

# Choice Sets in Labor and Financial Markets

This chapter is a straightforward extension of Chapter 2 where we had shown that budget constraints can arise from someone owning an **endowment** that he can sell to generate the income needed for purchasing a different consumption bundle. That's exactly what workers and savers do: Workers own their time and can sell it to earn income, and savers own some investment that they can sell and turn into consumption. (Borrowers, on the other hand, own some future asset — such as the income they can earn in the future — that they can sell.) Once you understand how budgets can arise from stuff we own, it becomes straightforward to think about workers and savers/borrowers.

## Chapter Highlights

The main points of the chapter are:

1. **Wages** and **interest rates** are prices in particular markets — and therefore give rise to the opportunity costs we face when making choices in those markets as leisure time or investments are sold.

2. When budgets arise from the sale of endowments, price increases no longer unambiguously shrink budgets nor do price decreases unambiguously increase budgets **as budget lines rotate through the endowment bundle**.

3. **Endowment bundles** are those that can always be consumed regardless of what prices (or wages or interest rates) emerge in the economy.

4. **Government policies** can change the economic incentives faced by workers and savers by changing the choice sets they face.

5. An amount $X in the future has a **present value** less than $X — because borrowing on that amount to consume more now entails paying interest.

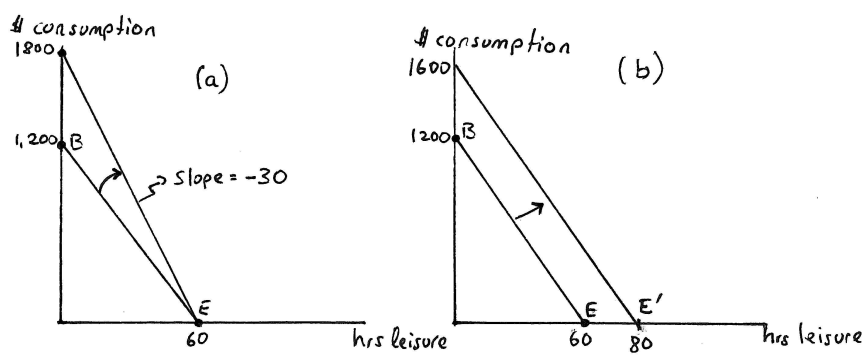
6. The 2-dimensional models of leisure/consumption and intertemporal budgets are really just **"slices" of higher dimensional budgets** along which some things are held fixed.

# 3A Solutions to Within-Chapter Exercises for Part A

**Exercise 3A.1**

Illustrate what happens to the original budget constraint if your wage increases to $30 per hour. What if your friend instead introduces you to caffeine which allows you to sleep less and thus take up to 80 hours of leisure time per week?

Answer: If the wage goes up to $30 per hour, you could earn as much as $1,800 per week if you spent all 60 hours working. Thus, the consumption-intercept goes to 1,800, but the endowment point *E* has not changed. The resulting budget constraint (graphed in panel (a) of Exercise Graph 3A.1) therefore rotates clockwise through *E* and has a new slope equal to $-w = -30$. If, on the other hand, the endowment of leisure goes up to 80, the budget shifts parallel as in panel (b) of the graph. The slope remains at $-w = -20$ since the wage has not changed.




Exercise Graph 3A.1 : (a) An increase in $w$; (b) An increase in Leisure

**Exercise 3A.2**

Verify the dollar quantities on the axes in Graph 3.3a-c.

Answer: In panel (a), you have $10,000 now, which places your endowment point on the horizontal axis at $10,000. At a 10% interest rate, you would have earned $1,000 in interest if you consumed nothing today and you saved everything.

That would leave you with $11,000 of consumption next year at $B$. When the interest rate falls to 5%, the most in interest you can earn is $500, leaving you with $10,500 next year if you consume nothing this year. This is the relevant intercept at $B'$.

In panel (b), you earn $11,000 next year but nothing now — thus placing your endowment point on the vertical axis at $11,000. When you borrow on next year's income in order to consume this year, the most the bank will lend you is an amount that, when paid back with interest, will be equal to what you earn next year. When the interest rate is 10%, the bank would then be willing to lend you $\$11,000/(1+0.1) = \$10,000$. If you ended up borrowing $10,000, you would then owe the bank $10,000 plus interest of $1,000 for a total of $11,000. Thus, at a 10% interest rate, the most you can consume this year is $10,000 if you are willing to not consume at all next year (bundle $A$). When the interest rate falls to 5%, the bank would be willing to lend you up to $11,000/(1+1.05)=$10,476.19 (bundle $A'$).

In panel (c), you earn $5,000 now and $5,500 next year — making that your endowment point. Were you to save all your current income at a 10% interest rate, you could have $5,500 in the bank next year — which, together with your $5,500 income next year, would allow you total consumption of $11,000 (bundle $B$). If, on the other hand, you decide to do all your consumption this year, you can borrow $5,500/(1+0.1)=$5,000 — which, together with this year's income of $5,000, leaves you with $10,000 in consumption this year (bundle $A$). At a 5% interest rate, on the other hand, you would accumulate $5,000(1+0.05)=$5,250 in your savings account by saving all your current income, leaving you with a total of $10,750 (bundle $B'$) next year when next year's income of $5,500 is added. Or you can consume all now, with the bank lending you $5,500/(1+0.05)=$5,238.10 that, together with this year's income of $5,000, lets you consume $10,238 (bundle $A'$) now.

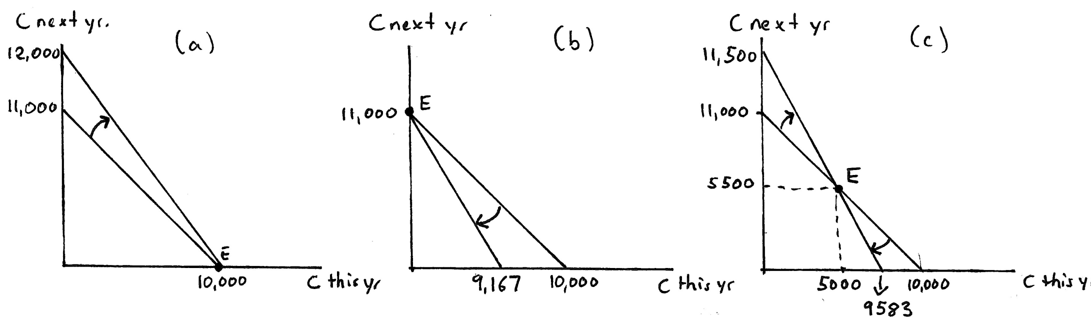
**Exercise 3A.3**

In each of the panels of Graph 3.3, how would the choice set change if the interest rate went to 20%?

Answer: Since the interest rate is higher, the slopes would all become steeper — with the constraints rotating through the relevant endowment bundle. These changes are depicted in panels (a) through (c) of Exercise Graph 3A.3. For the reasoning behind the intercepts, see the previous within-chapter-exercise.

**Exercise 3A.4**

So far, we have implicitly assumed that interest compounds yearly —- i.e. you begin to earn interest on interest only at the end of each year. Often, interest compounds more frequently. Suppose that you put $10,000 in the bank now at an annual interest rate of 10% but that interest compounds monthly rather than yearly. Your monthly interest rate is then 10/12 or 0.833%. Defining $n$ as the number of months and using the information in the previous paragraph, how much would




Exercise Graph 3A.3 : An increase in the interest rate to 20%

you have in the bank after 1 year? Compare this to the amount we calculated you would have when interest compounds annually.

Answer: You would have $10000(1+r)^n = 10000(1.00833)^{12} = 11,047.13$. Thus, over a 1 year period, by putting $10,000 into a savings account at 10% annual interest that compounds monthly, you get $47.13 more in interest than when putting the same amount into a savings account at the same interest rate but compounded annually.

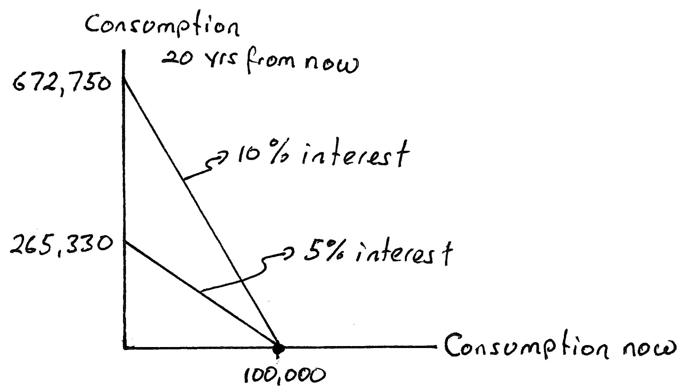
**Exercise 3A.5**

Suppose you just inherited $100,000 and you are trying to choose how much of this to consume now and how much of it to save for retirement 20 years from now. Illustrate your choice set with "dollars of consumption now" and "dollars of consumption 20 years from now" assuming an interest rate of 5% (compounded annually). What happens if the interest rate suddenly jumps to 10% (compounded annually)?

Answer: Regardless of the interest rate, you can choose to consume the entire $100,000 now (which therefore is your endowment point that lies on the horizontal axis). If you save all of it, you will collect $100000(1+r)^{20}$ where $r$ is 0.05 when the interest rate is 5% and 0.1 when the interest rate is 10%. This results in intercepts on the vertical axis of $265,330 if the interest rate is 5% and $672,750 if the interest rate is 10%. This is depicted in Exercise Graph 3A.5.

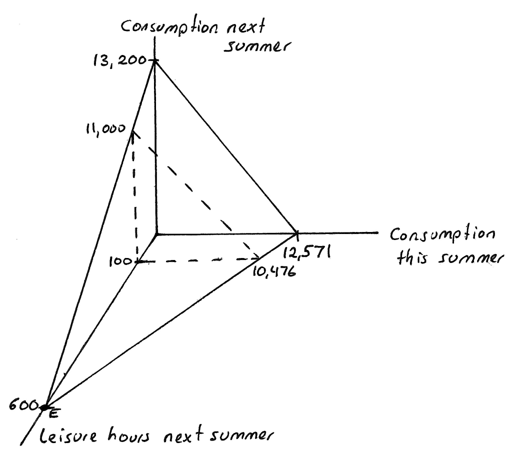
**Exercise 3A.6**

Draw a budget constraint similar to Graph 3.5 assuming you do not work this summer but rather next summer at a wage of $22 per hour (with a total possible number of leisure hours of 600 next summer) and assuming that the interest rate is 5%. Where is the 5% interest rate budget line from Graph 3.3b in the graph you have just drawn?




Exercise Graph 3A.5 : Consume now or 20 years from now

Answer: The graph is depicted in Exercise Graph 3A.6, with the dashed slice equivalent to the budget line in the 2-dimensional graph earlier in the text. The endowment point is once again leisure — only this time 600 leisure hours *next* summer. At \$22 per hour, this translates into \$13,200 of consumption next summer if you choose to work all 600 hours. If you were to work all 600 hours but you wanted to borrow and consume all the resulting income now, your could consume $\$13,200/(1+0.05) = \$12,571$. The 2-dimensional graph earlier on took \$11,000 of income next summer as the starting point, thus implicitly assuming that you have chosen to work for 500 hours next summer, leaving you with 100 hours of leisure. Thus, the slice at 100 hours of leisure next summer represents the budget line in Graph 3.3b in the textbook.




Exercise Graph 3A.6 : Consuming over 2 summers but working only next summer

# 3B Solutions to Within-Chapter Exercises for Part B

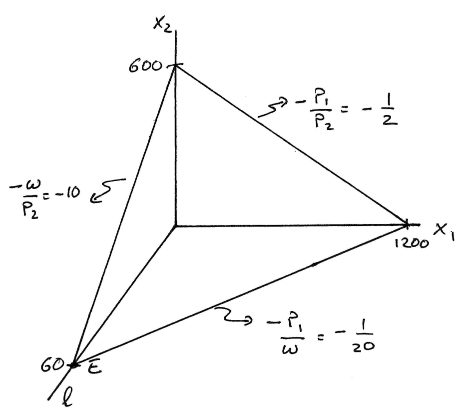
**Exercise 3B.1**

Graph the choice set in equation (3.5) when $n$=2, $p_1 = 1$, $p_2 = 2$, $w = 20$ and $L = 60$.

Answer: Using the values given in the exercise, the choice set is defined as

$$\left\{(x_1, x_2, \ell) \in \mathbb{R}^3_+ \mid x_1 + 2x_2 \le 20(60 - \ell)\right\}. \tag{3B.1}$$

This is depicted in Exercise Graph 3B.1. The endowment point is leisure of 60 hours with no consumption. If this worker works all the time, she will earn \$1,200 given that she earns a wage of \$20 per hour. With that, she could buy as many as 1,200 units of $x_1$ if that was all she bought (as $p_1 = 1$), or as many as 600 units of $x_2$ if that is all she bought (at $p_2 = 2$).




Exercise Graph 3B.1 : Leisure and 2 goods

**Exercise 3B.2**

Translate the choice sets graphed in Graph 3.2 into mathematical notation defining the choice sets.

Answer: The choice set in panel (a) of the textbook Graph 3.2 is

$$\{\, (\ell, c) \in \mathbb{R}^2_+ \mid \quad c = 1000 - 10\ell \quad \text{for } \ell \le 20 \text{ and} \quad c = 1200 - 20\ell \quad \text{for } \ell > 20 \,\}, \tag{3B.2.i}$$

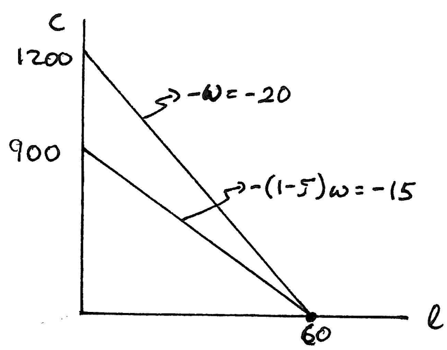
while the choice set in panel (b) is

$$\{ (\ell, c) \in \mathbb{R}^2_+ \mid \quad c = 1400 - 30\ell \quad \text{for } \ell \leq 20 \text{ and}$$
$$c = 1200 - 20\ell \quad \text{for } \ell > 20 \}. \qquad (3B.2.ii)$$

**Exercise 3B.3**

Suppose $w = 20$ and $L = 60$. Graph the budget constraint in the absence of taxes. Then suppose a wage tax $t = 0.25$ is introduced. Illustrate how this changes your equation and the graph.

Answer: The budget equation is given by $c = (1-t)w(L-\ell)$ which is $c = 20(60-\ell) = 1200-20\ell$ when $t = 0$ and $c = (1-0.25)20(60-\ell) = 900-15\ell$ when $t = 0.25$. These are depicted in Exercise Graph 3B.3 below.




Exercise Graph 3B.3 : 25% tax on wages

**Exercise 3B.4**

How would the budget line equation change if, instead of a tax on wages, the government imposed a tax on all consumption goods such that the tax paid by consumers equaled 25% of consumption. Show how this changes the equation and the corresponding graph of the budget line.

Answer: Since all income earned through wages is by definition consumed in this model, a tax equivalent to 25% of consumption is equivalent to a 25% wage tax. So nothing would change in the equation or graph.

**Exercise 3B.5**

Suppose $(e_1 - c_1)$ is negative — i.e. suppose you are borrowing rather than saving in period 1. Can you still make intuitive sense of the equation?

Answer: When $(e_1 - c_1)$ is negative, you have consumed your period 1 endowment $e_1$ plus an amount $(c_1 - e_1)$ on top of it. The only way you could consume more than you had in period 1 is to borrow from period 2 — thus you must have borrowed the amount $(c_1 - e_1)$. One year later, you have to pay back that amount plus interest for a total of $(1+r)(c_1 - e_1)$. We therefore have to subtract that amount from $e_2$ to determine how much you will have left after paying back what you owe to the bank. Thus, consumption $c_2$ in period 2 must be no more than $e_2 - (1+r)(c_1 - e_1)$, which can be re-written as $c_2 \leq (1+r)(e_1 - c_1) + e_2$. In the case of borrowing, the quantity $(1+r)(e_1 - c_1)$ is therefore negative and equal to what you owe the bank in period 2.

**Exercise 3B.6**

Use the information behind each of the scenarios graphed in Graph 3.3 to plug into equation (3.8) that scenario's relevant values for $e_1$, $e_2$ and $r$. Then demonstrate that the budget lines graphed are consistent with the underlying mathematics of equation (3.8), and more generally, make intuitive sense of the intercept and slope terms as they appear in equation (3.8).

Answer: In panel (a) of the textbook Graph 3.3, $(e_1, e_2) = (10000, 0)$ and the interest rate is initially $r = 0.1$ and then falls to $r = 0.05$. Plugging these into the equation, we get $c_2 \leq 10000(1+r) - (1+r)c_1$. The intercept term is then 11,000 when $r = 0.1$ and 10,500 when $r = 0.05$, and the slopes are analogously either 1.10 or 1.05. This is precisely what is graphed in the textbook.

In panel (b) of the textbook Graph 3.3, $(e_1, e_2) = (0, 11000)$. The equation then becomes $c_2 \leq 11000 - (1+r)c_1$. Thus, regardless of the interest rate, the intercept is 11,000, but the slope is 1.10 when $r = 0.1$ and 1.05 when $r = 0.05$, all as depicted in the graphs in the text.

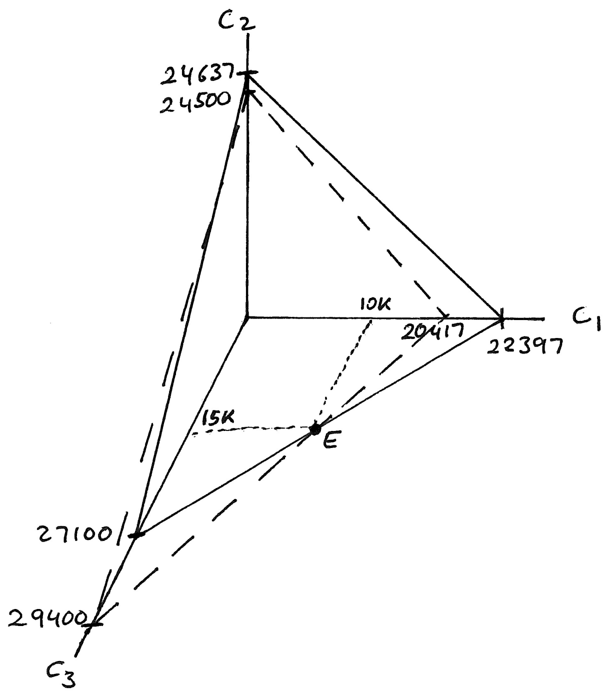
Finally, in panel (c) of the textbook Graph 3.3, $(e_1, e_2) = (5000, 5500)$. When plugged into the equation, this becomes $c_2 \leq (1+r)5000 + 5500 - (1+r)c_1$. This gives an intercept of $1.1(5000) + 5500 = 11000$ when $r = 0.10$ and $1.05(5000) + 5500 = 10750$ when $r = 0.05$, with slopes of 1.10 and 1.05 — all as depicted in the graph in the text.

For an intuitive explanation of all this, see the answer to within-chapter exercise 3A.2.

**Exercise 3B.7**

Suppose you expect to earn \$10,000 this summer, \$0 next summer and \$15,000 two summers from now. Using $c_1$, $c_2$, and $c_3$ to denote consumption over these three summers, write down your budget constraint assuming an annual (and annually compounding) interest rate of 10%. Then illustrate this constraint on a three dimensional graph with $c_1$, $c_2$, and $c_3$ on the three axes. How does your equation and graph change if the interest rate increases to 20%?

Answer: Exercise Graph 3B.7 depicts the constraint when the interest rate is 10% in solid lines and the constraint when the interest rate is 20% in dashed lines.




Exercise Graph 3B.7 : Income now and 2 years from now

Both constraints pass through the endowment point $E$. The underlying equation for each budget plane is

$$c_3 + (1+r)c_2 + (1+r)^2 c_1 = 15000 + 10000(1+r)^2. \quad (3B.7)$$

Consider first how this relates to the solid budget constraint when the interest rate is $r = 0.10$. To determine the $c_3$ intercept, we set $c_1 = c_2 = 0$ and get that $c_3 = 15000 + 10000(1+0.1)^2) = 27100$. To determine the $c_2$ intercept, we set $c_1 = c_3 = 0$ and get $(1+0.1)c_2 = 27100$ or $c_2 = 27100/1.1 = 24637$. Finally, to get the $c_1$ intercept, we set $c_3 = c_2 = 0$ and get $(1+0.1)^2 c_1 = 27100$ or $c_3 = 22397$. Notice that, focusing simply on the slice of the graph that holds $c_3 = 0$, we see that the $c_2$ intercept (24,637) divided by the $c_1$ intercept (22,397) is 1.1 or $(1+r)$ — which is exactly the slope we would expect for a one-period intertemporal budget constraint. The same is true for the slope on the slice that holds $c_1 = 0$. And, for the bottom slice where $c_2 = 0$, the slope is 1.21 or $(1+r)^2$ — again what we would expect for a 2-period intertemporal budget constraint.

The intercepts on the dashed budget plane can be similarly calculated, this time substituting $r = 0.2$ rather than $r = 0.1$ into the equations.

**Exercise 3B.8**

When $L = 600$, $w = 20$ and $r = 0.1$, show how the equation above translates directly into Graph 3.5.

Answer: Plugging in these values, we get $1.1c_1 + c_2 = 1.1(20)(600 - \ell)$ which can also be written as

$$1.1c_1 + c_2 + 22\ell = 13200. \tag{3B.8}$$

To determine the intercept on the $c_1$ axis, we simply set $c_2 = \ell = 0$ and get $1.1c_1 = 13200$ or $c_1 = 12000$. To determine the intercept on the $c_2$ axis, we set $c_1 = \ell = 0$ and get $c_2 = 13200$. Finally, we can check that the $\ell$ intercept is equal to the leisure endowment — by plugging $c_1 = c_2 = 0$ in to get $22\ell = 13200$ or $\ell = 600$.

**Exercise 3B.9**

Define mathematically a generalized version of the choice set in expression (3.18) under the assumption that you have both a leisure endowment $L_1$ this summer and another leisure endowment $L_2$ next summer. What is the value of $L_2$ in order for Graph 3.5 to be the correct 3-dimensional "slice" of this 4-dimensional choice set?

Answer: The budget set would then be defined as

$$\begin{aligned} B(L_1, L_2, w, r) = \{&(c_1, c_2, \ell_1, \ell_2) \in \mathbb{R}^4_+ \mid \\ &(1+r)c_1 + c_2 = (1+r)w(L_1 - \ell_1) + w(L_2 - \ell_2)\}. \end{aligned} \tag{3B.9.i}$$

When $L_2 = \ell_2$, the equation inside the set becomes

$$(1+r)c_1 + c_2 = (1+r)w(L_1 - \ell_1), \tag{3B.9.ii}$$

which is exactly the 3-dimensional choice set referred to in the text (where it was implicitly assumed that you consume all your leisure next summer and thus derive no income next summer).

# 3C Solutions to Odd-Numbered End-of-Chapter Exercises

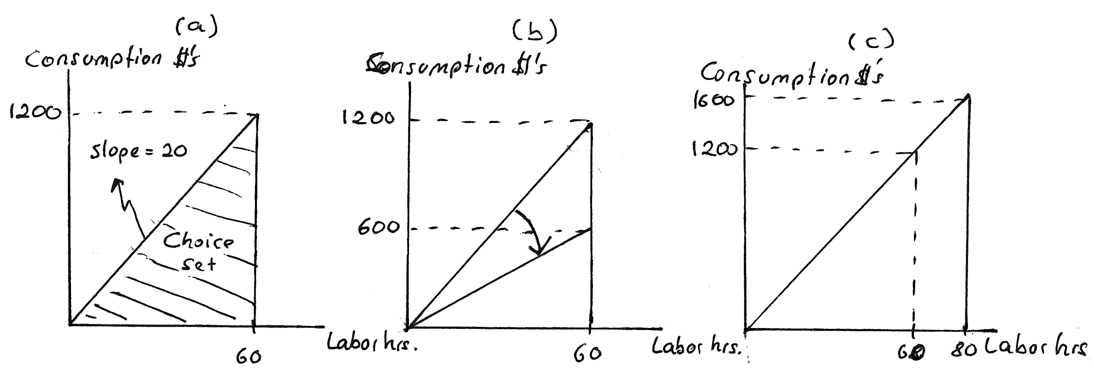
**Exercise 3.1**

*In this chapter, we graphed budget constraints illustrating the trade-off between consumption and leisure.*

**A:** *Suppose that your wage is $20 per hour and you have up to 60 hours per week that you could work.*

(a) *Now, instead of putting leisure hours on the horizontal axis (as we did in Graph 3.1), put labor hours on the horizontal axis (with consumption in dollars still on the vertical). What would your choice set and budget constraint look like now?*

Answer: Panel (a) of Exercise Graph 3.1 illustrates this choice set and budget constraint. It would begin at the origin (where no labor is provided and thus no income earned for consumption) and would rise by the wage rate (i.e. $20) for each hour of labor.




Exercise Graph 3.1 : Labor/Consumption Tradeoff

(b) *Where on your graph would the endowment point be?*

Answer: The endowment point is always the bundle that a consumer can consume regardless of market prices (or wages). In this case, the bundle (0,0) — i.e. no labor and no consumption, is always possible. This is equivalent to the endowment bundle (60,0) when we put leisure instead of labor on the horizontal axis.

(c) *What is the interpretation of the slope of the budget constraint you just graphed?*

Answer: The slope is equal to the wage rate (just as it is equal to the negative wage rate when leisure is graphed on the horizontal axis).

(d) *If wages fall to $10 per hour, how does your graph change?*

Answer: It changes as in panel (b) of Exercise Graph 3.1, with a new slope of 10 rather than 20.

(e) *If instead a new caffeine drink allows you to work up to 80 rather than 60 hours per week, how would your graph change?*

Answer: Since wages have not changed, the graph would be identical up to 60 hours of work. But now 20 additional potential hours of work are possible — causing the budget constraint to extend all the way to 80 hours of labor. This is depicted in panel (c).

**B:** *How would you write the choice set over consumption c and labor l as a function of the wage w and leisure endowment L?*

Answer: The choice set would be

$$C(w,L) = \left\{(c,l) \in \mathbb{R}^2_+ \mid c \leq wl \text{ and } l \leq L\right\}. \tag{3.1.i}$$

**Exercise 3.3**

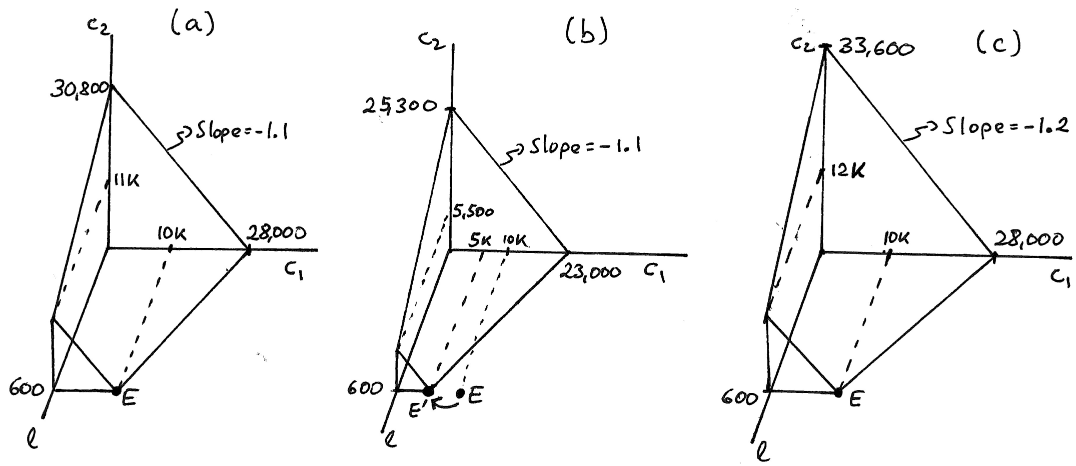
*You have $10,000 sitting in a savings account, 600 hours of leisure time this summer and an opportunity to work at a $30 hourly wage.*

**A:** *Next summer is the last summer before you start working for a living, and so you plan to take the whole summer off and relax. You need to decide how much to work this summer and how much to spend on consumption this summer and next summer. Any investments you make for the year will yield a 10% rate of return over the coming year.*

(a) *On a three dimensional graph with this summer's leisure ($\ell$), this summer's consumption ($c_1$) and next summer's consumption ($c_2$) on the axes, illustrate your endowment point as well as your budget constraint. Carefully label your graph and indicate where the endowment point is.*

Answer: This is graphed in panel (a) of Exercise Graph 3.3.

The endowment point — the point on the budget constraint that is always available regardless of prices — is $(\ell, c_1, c_2) = (600, 10000, 0)$ where no work is done and all $10,000 in the savings account is consumed now. No matter what the wage or the interest rate is, you can always consume this bundle. However, you can also work up to 600 hours this summer, which would earn you up to an additional $18,000 for consumption now. So the most you could consume this summer if you worked all the time and emptied your savings account is $28,000, the $c_1$ intercept. You can't consume any more than 600 hours of leisure — so 600 is the $\ell$ intercept. If you earned $18,000 this summer (by working all the time and thus consuming zero leisure), and if you consumed nothing this summer, then the most you could consume next summer is $28,000 times $(1+r)$ where $r = 0.1$ is the interest rate. This gives the $c_2$ intercept of $30,800. If you don't work (i.e. $\ell = 600$) and you consume nothing this summer




Exercise Graph 3.3 : Leisure/Consumption and Intertemporal Tradeoffs Combined

(i.e. $c_1 = 0$), then you simply have your $10,000 from your savings account plus $1,000 in interest next summer, for a total of $11,000 in $c_2$. The overall shape of the budget constraint then becomes the usual triangular shape but, because you can't buy leisure beyond 600 hours with your savings, the tip of the triangle is cut off.

(b) *How does your answer change if you suddenly realize you still need to pay $5,000 in tuition for next year, payable immediately?*

Answer: This is graphed in panel (b). Since $5,000 has to leave your savings account right now, this leaves you with only $5,000 in the account and thus shifts your endowment point in. The rest of the budget constraint is derived through similar logic to what was used above. The budget constraint is again a triangular plane with the tip cut off, only now the tip that's cut off is smaller. (Had the immediately-due tuition payment been $10,000, the cut-off tip would have disappeared.)

(c) *How does your answer change if instead the interest rate doubles to 20%?*

Answer: This is illustrated in panel (c) of Exercise Graph 3.3 where $E$ is back to what it was in part (a) but the amount of consumption next summer goes up because of the higher interest rate. The reasoning for the various intercepts is similar to that above.

(d) *In (b) and (c), which slopes are different than in (a)?*

Answer: Slopes are formed by ratios of prices — so the only way that slopes can change is if a price has changed. In the scenario in (b), no price has changed. Thus, the only thing that happens is that $E$ shifts in as indicated in the graph. This changes the various intercepts, but the slopes in each plane are parallel to those from panel (a). In the scenario in (c), on the other hand, the interest rate changes. The interest rate is a price that is reflected in any intertemporal budget constraint — i.e. any budget constraint that spans across time periods. In our graph in panel (c), this includes the constraint that lies in the plane that shows the tradeoff between $c_1$ and $c_2$, and the plane that shows the tradeoff between $\ell$ (which is leisure *this* summer) and $c_2$. Those slopes change as the interest rate changes, but the slope in the plane that illustrates the tradeoff between $\ell$ and $c_1$ does not — because that tradeoff happens within the same time period and thus does not involve interest payments.

**B:** *Derive the mathematical expression for your budget constraint in 3.3A and explain how elements of this expression relate to the slopes and intercepts you graphed.*

Answer: An intuitive way to construct this mathematical expression involves thinking about how much consumption $c_2$ is possible next summer. If you consume $\ell$ amount of leisure this summer, you will have a total of $10000+30(600-\ell)$ available for consumption *this* summer — the $10,000 in the savings account plus your earnings (at a wage of $30) from hours that you did not consume as leisure. When we take this amount and subtract from it the consumption $c_1$ you actually undertake this summer, we get the amount that will be in the savings account for the year to accumulate interest. Thus, next summer, you will have $10000+30(600-\ell)-c_1$ times $(1+r)$ (where $r=0.1$ is the interest rate); i.e.

$$c_2 = 1.1\,[10000+(30)(600-\ell)-c_1] = 30800-33\ell-1.1c_1\,, \tag{3.3.i}$$

or, written differently,

$$1.1c_1+c_2+33\ell = 30800. \tag{3.3.ii}$$

The only caveat to this when we define the budget plane is that we have to take into account that you cannot consume more than 600 hours of leisure (or negative quantities of consumption). We can incorporate this by defining the budget plane as

$$\left\{(c_1,c_2,\ell)\in\mathbb{R}^3_+ \mid 1.1c_1+c_2+33\ell = 30800 \text{ and } \ell\le 600\right\}. \tag{3.3.iii}$$

**Exercise 3.5**

*Suppose you are a carefree 20-year old bachelor whose lifestyle is supported by expected payments from a trust fund established by a relative who has since passed away. The trust fund will pay you $x when you turn 21 (a year from now), another $y when you turn 25 and $z when you turn 30. You plan to marry a rich heiress on your 30th birthday and therefore only have to support yourself for the next 10 years. The bank that maintains the trust account is willing to lend money to you at a 10% interest rate and pays 10% interest on savings. (Assume annual compounding.)*

**A:** *Suppose* $x = y = z = 100,000$.

(a) *What is the most that you could consume this year?*

Answer: For the initial $100,000 you get next year when you turn 21, the bank would be willing to lend you $100,000/(1+0.1)=$90,909.09. For the $100,000 you get 5 years from now when you turn 25, the bank would be willing to lend you $\$100,000/(1+0.1)^5$=$62,092.13. And the bank would lend you up to $\$100,000/(1+0.1)^{10}$=$38,554.33 for the $100,000 you get on your 30th birthday (10 years from now). That sums to $191,555.55.

(b) *What is the most you could spend at your bachelor party 10 years from now if you find a way to live without eating?*

Answer: If you saved your initial $100,000 for 9 years (until your 30th birthday around which you will have your bachelor's party), you would have accumulated $\$100,000(1+0.1)^9 = \$235,794.77$. If you save the $100,000 you get on your 25th birthday for 5 years, you would accumulate $\$100,000(1+0.1)^5$=$161,051. Add those two amounts to the $100,000 you get when you are 30, and you get that you could spend a total of $496,845.77 on your bachelor's party.

**B:** *Define your 10 year intertemporal budget constraint mathematically in terms of x, y and z, letting* $c_1$ *denote this year's consumption,* $c_2$ *next year's consumption, etc. Let the annual interest rate be denoted by* $r$.

Answer: You can think of this in the following way: First, how much will you have in wealth on your 30th birthday if you spend nothing and save everything? You will have had $x$ in the bank for 9 years and $y$ for 5 years — in addition to just having received $z$ (on your 30th birthday). Thus, you will have

$$\text{Potential Wealth on 30th Birthday} = (1+r)^9 x + (1+r)^5 y + z. \tag{3.5.i}$$

Next, we can ask how much would be available for consumption in the year that starts with your 29th birthday (assuming you have to borrow whatever you intend to spend at the beginning of that year). Since we know how much wealth you would have on your 30th birthday if you did not spend anything leading up to it, we know that the most you can consume in the year before is how much you could borrow on that wealth; which means that the most you could consume in year 10, $c_{10}^{max}$, is andamount that would allow you to pay back $(1+r)^9 x + (1+r)^5 y + z$ one year from then — i.e.

$$(1+r)c_{10}^{max} = (1+r)^9 x + (1+r)^5 y + z. \tag{3.5.ii}$$

The most you can consume in year 9, $c_9^{max}$, is similarly what you could pay back on your 30th birthday (with 2 years of interest) minus what you actually consumed in year 10 ($c_{10}$) (plus one year interest) — i.e.

$$(1+r)^2 c_9^{max} + (1+r)c_{10} = (1+r)^9 x + (1+r)^5 y + z. \tag{3.5.iii}$$

Continuing this same logic backwards, we then get the 10-year intertemporal budget constraint

$$(1+r)^{10}c_1+(1+r)^9c_2+...+(1+r)^2c_9+(1+r)c_{10} \leq (1+r)^9x+(1+r)^5y+z. \quad (3.5.iv)$$

**Exercise 3.7**

Everyday Application: *Investing for Retirement: Suppose you were just told that you will receive an end-of-the-year bonus of $15,000 from your company. Suppose further that your marginal income tax rate is 33.33% — which means that you will have to pay $5,000 in income tax on this bonus. And suppose that you expect the average rate of return on an investment account you have set up with your broker to be 10% annually (and, for purposes of this example, assume interest compounds annually.)*

**A:** *Suppose you have decided to save all of this bonus for retirement 30 years from now.*

(a) *In a regular investment account, you will have to pay taxes on the interest you earn each year. Thus, even though you earn 10%, you have to pay a third in taxes — leaving you with an after-tax return of 6.67%. Under these circumstances, how much will you have accumulated in your account 30 years from now?*

Answer: You will have $\$10000(1+0.06667)^{30} = \$69,327$. Since you have already paid taxes on the initial bonus and on all interest income, no further taxes are due — so all $69,327 is available for consumption.

(b) *An alternative investment strategy is to place your bonus into a 401K "tax-advantaged" retirement account. The federal government has set these up to encourage greater savings for retirement. They work as follows: you do not have to pay taxes on any income that you put directly into such an account if you put it there as soon as you earn it, and you do not have to pay taxes on any interest you earn. Thus, you can put the full $15,000 bonus into the 401K account, and you can earn the full 10% return each year for the next 30 years. You do, however, have to pay taxes on any amount that you choose to withdraw after you retire. Suppose you plan to withdraw the entire accumulated balance as soon as you retire 30 years from now, and suppose that you expect you will still be paying 33.33% taxes at that time. How much will you have accumulated in your 401K account, and how much will you have after you pay taxes? Compare this to your answer to (a) — i.e. to the amount you would have at retirement if you saved outside the 401K plan.*

Answer: Your account will grow to $\$15000(1+0.1)^{30} = \$261,741$. But you still have to pay one third in taxes — leaving you with $(2/3)*(\$261,741) \approx \$174,503$. This is substantially larger than the amount of $69,327 we calculated in part (a).

(c) True or False: *By allowing individuals to defer paying taxes into the future, 401K accounts result in a higher rate of return for retirement savings.*

Answer: This is true. In both cases, you end up paying taxes on all your income — both the initial income as well as interest income. The only difference between the two investment strategies is that in one case income is taxed as it is made, in the other case it is taxed at the end when it is withdrawn for consumption. In the latter case, the investor benefits from accumulating more interest faster. In our example, for instance, you end up with $105,175 more with a 401K account than in a non-tax advantaged account.

**B:** *Suppose more generally that you earn an amount $I$ now, that you face (and will face in the future) a marginal tax rate of $t$ (expressed as a fraction between 0 and 1), that the interest rate now (and in the future) is $r$ and that you plan to invest for $n$ periods into the future.*

(a) *How much consumption will you be able to undertake $n$ years from now if you first pay your income tax on the amount $I$, then place the remainder in a savings account whose interest income is taxed each year. (Assume you add nothing further to the savings account between now and $n$ years from now).*

Answer: You would place $(1-t)I$ into the account, and it would earn an after-tax rate of return of $(1-t)r$. Over $n$ years, this results in $(1-t)I(1+(1-t)r)^n$.

(b) *Now suppose you put the entire amount $I$ into a tax-advantaged retirement account in which interest income can accumulate tax-free. Any amount that is taken out of the account is then taxed as regular income. Assume you plan to take the entire balance in the account out $n$ years from now (but nothing before then). How much consumption can you fund from this source $n$ years from now?*

Answer: You would then accumulate $I(1+r)^n$, but that amount would then be taxed at the end. So, what you are left with would be $(1-t)I(1+r)^n$.

(c) *Compare your answers to (a) and (b) and indicate whether you can tell which will be higher.*

Answer: In both cases, a quantity $(1-t)I$ is multiplied by another term in parentheses. In the case of no tax-advantaged treatment, this second term is $(1+(1-t)r)^n$; in the 401K case, it is $(1+r)^n$. So the latter is bigger so long as $(1+r)$ is larger than $(1+(1-t)r)$ which is equal to $(1+r-tr)$. When expressed this way, it is clear that the latter is smaller by $tr$ — tax advantaged savings accounts always result in larger future consumption for given levels of investment.

**Exercise 3.9**

Business Application: *Present Value of Winning Lottery Tickets*: *The introduction to intertemporal budgeting in this chapter can be applied to thinking about the* *pricing of basic financial assets. The assets we will consider will differ in terms of when they pay income to the owner of the asset. In order to know how much such assets are worth, we have to determine their* present value — *which is equal to how much* current *consumption such an asset would allow us to undertake.*

**A:** *Suppose you just won the lottery and your lottery ticket is transferable to someone else you designate — i.e. you can sell your ticket. In each case below, the lottery claims that you won \$100,000. Since you can sell your ticket, it is a financial asset, but depending on how exactly the holder of the ticket received the \$100,000, the asset is worth different amounts. Think about what you would be willing to actually sell this asset for by considering how much* current *consumption value the asset contains — assuming the annual interest rate is 10%.*

(a) *The holder of the ticket is given a \$100,000 government bond that "matures" in 10 years. This means that in 10 years the owner of this bond can cash it for \$100,000.*

Answer: To know how much this lottery ticket is worth, we have to determine how much the bank would be willing to lend us for current consumption. If we could get the \$100,000 one year from now, we know the bank would lend us up to \$100000/1.1=\$90,909.09. If we get the \$100,000 two years from now, however, the most the bank would be willing to lend us is $\$100000/(1.1^2) = \$82,644.63$. And if we can only get to the \$100,000 ten years from now, the most we can get for it now is $(\$100000/(1.1^{10}) = \$38,554.33$. Thus, that's the least you would be willing to sell the bond (and thus your ticket) for — and the most anyone else who faces a 10% interest rate should be willing to pay.

(b) *The holder of the ticket will be awarded \$50,000 now and \$50,000 ten years from now.*

Answer: The most you can borrow on an amount \$50,000 ten years from now is the amount $\$50000/(1.1^{10}) = \$19,277.16$. Thus, together with the \$50,000 the lottery awards you now, the most you could consume now is \$69,277.16. Thus, that is the least you would be willing to sell your ticket for.

(c) *The holder of the ticket will receive 10 checks for \$10,000 — one now, and one on the next 9 anniversaries of the day he/she won the lottery.*

Answer: For a check $n$ years from now, I can borrow $\$10000/(1.1^n)$. Calculating this for each of the next 9 years, the checks will be worth \$9,090.91, \$8,264.46, \$7,513.15, \$6,830.13, \$6,209.21, \$5,644.74, \$5,131.58, \$4,665.07 and \$4,240.98. Summing these and adding the value of my current \$10,000 check, the total possible consumption I can undertake is then \$67,590.24 — which is the current value of the ticket.

(d) *How does you answer to part (c) change if the first of 10 checks arrived 1 year from now, with the second check arriving 2 years from now, the third 3 years from now, etc.?*

Answer: The value of the *first* 9 checks would then be the same as the value of the *last* 9 checks in the previous part. But you would lose the first check from the previous part (which was worth $10,000) to be replaced with a check 10 years from now, which is worth $\$10000/(1.1^{10}) = \$3,885.43$. Thus, the ticket would be worth $10,000−$3,885.43=$6,114.57 less, or $61,445.67 instead of $67,590.24.

(e) *The holder of the ticket gets $100,000 the moment he/she presents the ticket.*

Answer: This ticket is, of course, the only one that's worth $100,000 as claimed by the lottery.

**B:** *More generally, suppose the lottery winnings are paid out in installments of $x_1, x_2, ..., x_{10}$, with payment $x_i$ occurring $(i-1)$ years from now. Suppose the annual interest rate is $r$.*

(a) *Determine a formula for how valuable such a stream of income is in present day consumption — i.e. how much present consumption could you undertake given that the bank is willing to lend you money on future income?*

Answer: The present consumption $c$ that could be financed by such a stream of payments is

$$c = x_1 + \frac{x_2}{(1+r)} + \frac{x_3}{(1+r)^2} + \frac{x_4}{(1+r)^3} + ... + \frac{x_9}{(1+r)^8} + \frac{x_{10}}{(1+r)^9} \qquad (3.9.i)$$

which can also be written as

$$c = \sum_{i=1}^{10} \frac{x_i}{(1+r)^{i-1}}. \qquad (3.9.ii)$$

(b) *Check to make sure that your formula works for each of the scenarios in part A.*

Answer: Plugging in the appropriate values for each part, you should get the same answers as you did in part A.

(c) *The scenario described in part A(c) is an example of a $10,000 payment followed by an annual "annuity" payment. Consider an annuity that promises to pay out $10,000 every year starting 1 year from now for n years. How much would you be willing to pay for such an annuity?*

Answer: The present consumption that could be financed by such an annuity is

$$c = \frac{10000}{(1+r)} + \frac{10000}{(1+r)^2} + \frac{10000}{(1+r)^3} + ... + \frac{10000}{(1+r)^{n-1}} + \frac{10000}{(1+r)^n}, \qquad (3.9.iii)$$

which can also be written as

$$c = \sum_{i=1}^{n} \frac{10000}{(1+r)^i} = 10000 \sum_{i=1}^{n} \frac{1}{(1+r)^i}. \qquad (3.9.iv)$$

(d) *How does your answer change if the annuity starts with its first payment now?*

Answer: All that happens is that the expressions in the previous part get an additional \$10,000 added, which would allow us to write the second expression as

$$c = \sum_{i=0}^{n} \frac{10000}{(1+r)^i} = 10000 \sum_{i=0}^{n} \frac{1}{(1+r)^i}. \tag{3.9.v}$$

(e) *What if the annuity from (c) is one that never ends? (To give the cleanest possible answer to this, you should recall from your math classes that an infinite series of* $1/(1+x) + 1/(1+x)^2 + 1/(1+x)^3 + ... = 1/x$*.) How much would this annuity be worth if the interest rate is 10%?*

Answer: Using our equation (3.9.iv) from part (c), we can write this as an infinite series

$$c = 10000 \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} = \frac{10000}{r}. \tag{3.9.vi}$$

If the interest rate is $r = 0.1$, then this expression tells us the annuity would be worth \$100,000.

**Exercise 3.11**

Business Application: *Compound Interest over the Long Run: Uncle Vern has just come into some money — \$100,000 — and is thinking about putting this away into some investment accounts for a while.*

**A:** *Vern is a simple guy — so he goes to the bank and asks them what the easiest option for him is. They tell him he could put it into a savings account with a 10% interest rate (compounded annually).*

(a) *Vern quickly does some math to see how much money he'll have 1 year from now, 5 years from now, 10 years from now and 25 years from now assuming he never makes withdrawals. He doesn't know much about compounding — so he just guesses that if he leaves the money in for 1 year, he'll have 10% more; if he leaves it in 5 years at 10% per year he'll have 50% more; if he leaves it in for 10 years he'll have 100% more and if he leaves it in for 25 years he'll have 250% more. How much does he expect to have at these different times in the future?*

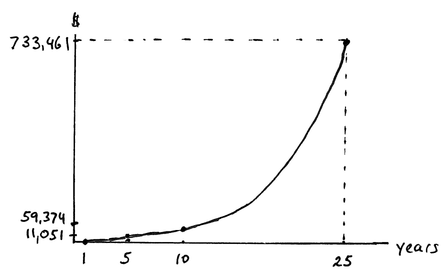
Answer: He expects to have \$110,000 1 year from now, \$150,000 five years from now, \$200,000 ten years from now and \$350,000 twenty-five years from now.

(b) *Taking the compounding of interest into account, how much will he really have?*

Answer: Using our usual formula, the actual balance $n$ years from now is $\$100000(1.1)^n$. This gives \$110,000 one year from now, \$161,051 five years from now, \$259,374.25 ten years from now, and \$1,083,460.59 twenty five years from now.

(c) *On a graph with years on the horizontal axis and dollars on the vertical, illustrate the size of Vern's error for the different time intervals for which he calculated the size of his savings account.*

Answer: The size of the error is \$0 one year from now, \$11,051 five years from now, \$59,374.25 ten years from now and \$733,460.59 twenty-five years from now. This is graphed in Exercise Graph 3.11.




Exercise Graph 3.11 : Error from not compounding over time

(d) *True/False: Errors made by not taking the compounding of interest into account expand at an increasing rate over time.*

Answer: The statement is clearly true based on the answers above.

**B:** *Suppose that the annual interest rate is $r$.*

(a) *Assuming you will put $x$ into an account now and leave it in for $n$ years, derive the implicit formula Vern used when he did not take into account interest compounding.*

Answer: Letting $y_n$ denote the amount he projected will be in the savings account $n$ years from now, he used the formula $y_n = x(1+nr)$.

(b) *What is the correct formula that includes compounding.*

Answer: Using $z_n$ to determine the actual amount in the savings account $n$ years from now, the correct formula is $z_n = x(1+r)^n$

(c) Define a new function that is the difference between these. Then take the first and second derivatives with respect to $n$ and interpret them.

Answer: The new function is $z_n - y_n = x(1+r)^n - x(1+nr)$. First, note that, when $n = 1$, this function reduces to zero — implying the difference between Vern's prediction and reality is zero 1 year from now (just as you determined earlier in the problem). The derivative of this function with respect to $n$ is

$$\frac{\partial(z_n - y_n)}{\partial n} = x(1+r)\ln(1+r) - xr. \qquad (3.11.i)$$

For any $n \geq 1$, this is clearly positive — which means the difference is increasing with time. The second derivative with respect to $n$ is $x(1+$

$r)^n[\ln(1+r)]^2$ which is also positive. Thus the rate at which the difference increases is increasing with time. This is exactly what we illustrated in Exercise Graph 3.11.

**Exercise 3.13**

Business Application: *Buying Houses with Annuities*: *Annuities are streams of payments that the owner of an annuity receives for some specified period of time. The holder of an annuity can sell it to someone else who then becomes the recipient of the remaining stream of payments that are still owed.*

**A:** *Some people who retire and own their own home finance their retirement by selling their house for an annuity: The buyer agrees to pay $x per year for n years in exchange for becoming the owner of the house after n years.*

(a) *Suppose you have your eye on a house down the street owned by someone who recently retired. You approach the owner and offer to pay her $100,000 each year (starting next year) for 5 years in exchange for getting the house in 5 years. What is the value of the annuity you are offering her assuming the interest rate is 10%?*

Answer: The value would be

$$\frac{\$100,000}{1.1}+\frac{\$100,000}{1.1^2}+\frac{\$100,000}{1.1^3}+\frac{\$100,000}{1.1^4}+\frac{\$100,000}{1.1^5}=\$379,078.68. \tag{3.13.i}$$

(b) *What if the interest rate is 5%?*

Answer: Now the value would be

$$\frac{\$100,000}{1.05}+\frac{\$100,000}{1.05^2}+\frac{\$100,000}{1.05^3}+\frac{\$100,000}{1.05^4}+\frac{\$100,000}{1.05^5}=\$432,947.67. \tag{3.13.ii}$$

(c) *The house's estimated current value is $400,000 (and your real estate agent assures you that homes are appreciating at the same rate as the interest rate.) Should the owner accept your deal if the interest rate is 10%? What if it is 5%?*

Answer: Since the house appreciates at the interest rate, we can use its current value and compare it to the current value of the annuity. Given what we calculated above, accepting the annuity is a good deal for the current owner at the 5% interest rate but not at the 10% interest rate.

(d) True/False*: The value of an annuity increases as the interest rate increases.*

Answer: This is false, as we just demonstrated above. The value of an annuity increases as the interest rate falls. That should make sense — if I have a given amount to invest, I can invest it where it earns interest, or I can buy an annuity. When the interest rate falls, I will not be able to make as much by investing the money where it makes interest. So that should mean getting a fixed payment through an annuity becomes more valuable.

(e) *Suppose that, after making the second payment on the annuity, you fall in love with someone from a distant place and decide to move there. The house has appreciated in value (from its starting value of $400,000) by 10% each of the past two years. You no longer want the house and therefore would like to sell your right to the house in 3 years in exchange for having someone else make the last 3 annuity payments. How much will you be able to get paid to transfer this contract to someone else if the annual interest rate is always 10%?*

Answer: After two years, the house will be worth $\$400,000(1.1^2) = \$484,000$. The value of the annuity with 3 more payments is

$$\frac{\$100,000}{1.1} + \frac{\$100,000}{1.1^2} + \frac{\$100,000}{1.1^3} = \$248,685.20. \tag{3.13.iii}$$

Thus, you should be able to get $484,000−$248,685.20=$235,314.80 for selling your contract.

**B:** *In some countries, retirees are able to make contracts similar to those in part A except that they are entitled to annuity payments until they die and the house only transfers to the new owner after the retiree dies.*

(a) *Suppose you offer someone whose house is valued at $400,000 an annual annuity payment (beginning next year) of $50,000. Suppose the interest rate is 10% and housing appreciates in value at the interest rate. This will turn from a good deal to a bad deal for you when the person lives n number of years. What's n? (This might be easiest to answer if you open a spreadsheet and you program it to calculate the value of annuity payments into the future.)*

Answer: If the person lives for another 16 years, the value of the annuity is $391,185.43. If the person lives an addition year (for a total of 17 years), the value of the annuity becomes $401,077.67. Thus $n = 17$ because it is a good deal for you as long as the person lives fewer than 17 more years.

(b) *Recalling that the sum of the infinite series* $1/(1+x) + 1/(1+x)^2 + +1/(1+x)^3 + ...$ *is* $1/x$, *what is the most you would be willing to pay in an annual annuity if you want to be absolutely certain that you are not making a bad deal?*

Answer: If you approximate a long life by infinity, then you would want to pay an annual amount no greater than an amount $x$ that solves the equation $\$400,000 = x/0.1$. Solving this for $x$ gives $x = \$40,000$.

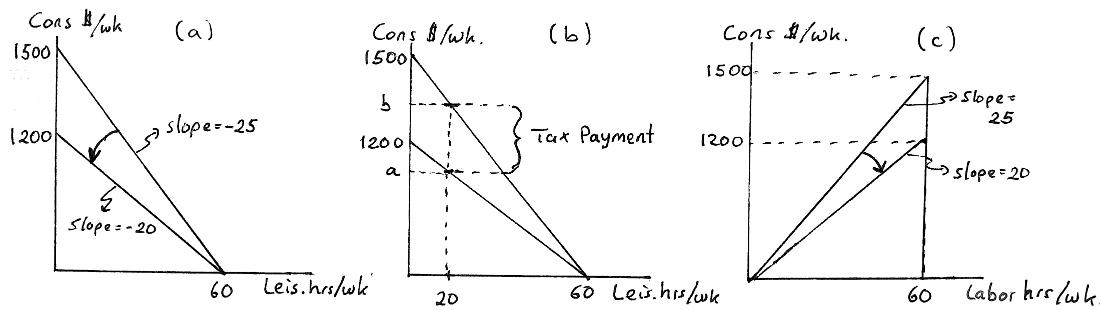
**Exercise 3.15**

Policy Application: *Wage Taxes and Budget Constraints*: *Suppose you have 60 hours of leisure that you could devote to work per week, and suppose that you can earn an hourly wage of $25.*

**A:** *Suppose the government imposes a 20% tax on all wage income.*

(a) *Illustrate your weekly budget constraint before and after the tax on a graph with weekly leisure hours on the horizontal and weekly consumption (measured in dollars) on the vertical axis. Carefully label all intercepts and slopes.*

Answer: The two budget constraints are illustrated in panel (a) of Exercise Graph 3.15. The after-tax wage is $w = 20$, 20% less than the before tax wage $w = 25$.




Exercise Graph 3.15 : Wage Taxes and Tax Payments

(b) *Suppose you decide to work 40 hours per week after the tax is imposed. How much wage tax do you pay per week? Can you illustrate this as a vertical distance in your graph? (*Hint*: Follow a method similar to that developed in end-of-chapter exercise 2.15)*

Answer: This is illustrated in panel (b) of the graph. When you work 40 hours a week, you consume 20 hours of leisure. On the after-tax budget, that leaves you with $a$ in consumption. On the before-tax budget, it leaves you with $b$ in consumption. The vertical difference $(b - a)$ must therefore be the total tax payment you made under the tax. Note that $a = 800$ and $b = 1000$ — with the vertical difference therefore equal to $(b - a) = 200$. This makes sense — when the worker is working for 40 hours a week, he is earning $1,000 before taxes and thus pays $200 in taxes at a 20% tax rate.

(c) *Suppose that instead of leisure hours on the horizontal axis, you put labor hours on this axis. Illustrate your budget constraints that have the same information as the ones you drew in (a). Assume again that the leisure endowment is 60 per week.*

Answer: This is illustrated in panel (c) of the graph.

**B:** Suppose the government imposes a tax rate $t$ (expressed as a rate between 0 and 1) on all wage income.

(a) *Write down the mathematical equations for the budget constraints and describe how they relate to the constraints you drew in A(a).*

Answer: For every hour of labor, a worker makes $w$ but pays $tw$ in taxes. Thus, his after-tax wage is $(1-t)w$. He will be able to consume as much as he earns, and how much he earns depends on how much leisure he does not consume. Letting $\ell$ be leisure hours consumed, this implies that $c=(1-t)w(60-\ell)$. For the case where $w=25$ and $t=0.2$, this becomes $c=0.8(25)(60-\ell)=1200-20\ell$ which is the after-tax equation we graphed in panel (a). The before-tax equation has $t=0$, with $c=25(60-\ell)=1500-25\ell$.

(b) *Use your equation to verify your answer to part A(b).*

Answer: Using the after-tax equation $c=1200-20\ell$, we can plug in $\ell=20$ which is when you choose to work 40 hours per week. This gives $c=1200-20(20)=800$ which is the consumption level denoted $a$ in panel (b) of the graph. Using the before-tax equation $c=1500-25\ell$, we get $c=1500-25(20)=1000$ which is the consumption level denoted $b$ in panel (b) of the graph. The difference between the two consumption levels is \$200 — which is the tax payment per week. This is intuitively correct — if you work for 40 hours at a wage of \$25, you earn \$1,000 per week, and if you pay taxes of 20% on that, you will pay \$200 in taxes.

(c) *Write down the mathematical equations for the budget constraints you derived in B(a) but now make consumption a function of labor, not leisure hours. Relate this to your graph in A(c).*

Answer: Let labor hours be denoted $l$. Then, with a tax $t$ and wage $w$, your consumption $c$ is simply the portion of your pay check that you get to keep — which is $(1-t)wl$. Thus, $c=(1-t)wl$. When $w=25$ and $t=0.2$, this becomes $c=0.8(25)l=20l$ which is the after tax budget constraint graphed in panel (c) of Exercise Graph 3.15.

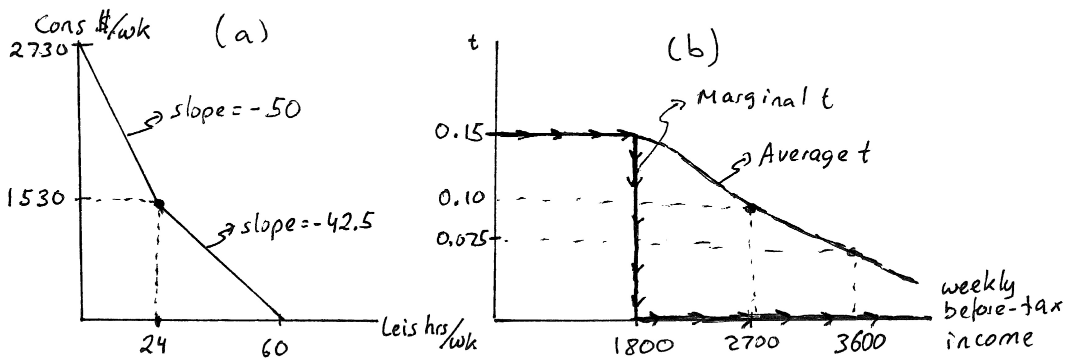
**Exercise 3.17**

Policy Application: *Social Security (or Payroll) Taxes*: *Social Security is funded through a payroll tax that is separate from the federal income tax. It works in a way similar to the following example: For the first \$1,800 in weekly earnings, the government charges a 15% wage tax but then charges no payroll tax for all earnings above \$1,800 per week.*

**A:** *Suppose that a worker has 60 hours of leisure time per week and can earn \$50 per hour.*

(a) *Draw this worker's budget constraint with weekly leisure hours on the horizontal axis and weekly consumption (in dollars) on the vertical.*

Answer: Panel (a) of Exercise Graph 3.17 traces out this budget constraint. The kink point happens at 24 hours of leisure — or 36 hours of labor. At that point, the worker earns \$1800 before taxes and pays 0.15(\$1800)=\$270 in taxes, leaving him with \$1,530 in consumption. For any lower levels of leisure (more work), the worker incurs no additional tax, causing his budget constraint to get steeper.




Exercise Graph 3.17 : Regressive Payroll Tax

(b) *Using the definitions given in exercise 3.16, what is the marginal and average tax rate for this worker assuming he works 30 hours per week? What if he works 40 hours per week? What if he works 50 hours per week?*

Answer: If he works 30 hours, his marginal and average tax rates are both 0.15 or 15%. If he works 40 or 50 hours, his marginal tax rate is zero. His before tax income at 40 hours is $2,000 and at 50 hours it is $2,500. In both cases, he pays $270 in weekly payroll taxes. Thus, his average tax rate at 40 hours of work is 270/2000=0.135 or 13.5%. His average tax rate at 50 hours of work is 270/2500=0.108 or 10.8%.

(c) *A wage tax is called* regressive *if the average tax rate falls as earnings increase. On a graph with weekly before-tax income on the horizontal axis and tax rates on the vertical, illustrate the marginal and average tax rates as income increases. Is this tax regressive?*

Answer: This is graphed in panel (b) of Exercise Graph 3.17. Taxes with declining average tax rates are regressive — so yes, this tax is regressive.

(d) True or False*: Budget constraints illustrating the tradeoffs between leisure and consumption will have no kinks if a wage tax is proportional. However, if the tax system is designed with different tax brackets for different incomes, budget constraints will have kinks that point inward when a wage tax is regressive and kinks that point outward when a wage tax is progressive.*

Answer: This is true as illustrated in this exercise and exercise 3.16.

**B:** *Consider the more general case of a tax that imposes a rate $t$ on income immediately but then falls to zero for income larger than $x$.*

(a) *Derive the average tax rate function $a(I,t,x)$ (where $I$ represents weekly income).*

Answer: The function is

$$a(I,t,x) = \begin{array}{ll} t & \text{if } I \leq x \text{ and} \\ (tx)/I & \text{if } x < I. \end{array} \qquad (3.17.i)$$

(b) *Derive the marginal tax rate function $m(I,t,x)$.*

Answer: The marginal tax rate function is

$$m(I,t,x) = \begin{array}{ll} t & \text{if } I < x \text{ and} \\ 0 & \text{if } x \geq I. \end{array} \qquad (3.17.ii)$$

(c) *Does the average tax rate reach the marginal tax rate for high enough income?*

Answer: No, it merely converges to 0 as income gets large but never reaches it because everyone always continues to pay $tx$ regardless of how high income gets.

**Exercise 3.19**

Policy Application: *The Earned Income Tax Credit: During the Clinton Administration, the EITC — or Earned Income Tax Credit, was expanded considerably. The program provides a wage subsidy to low income families through the tax code in a way similar to this example: Suppose, as in the previous exercise, that you can earn $5 per hour. Under the EITC, the government supplements your first $20 of daily earnings by 100% and the next $15 in daily earnings by 50%. For any daily income above $35, the government imposes a 20% tax.*

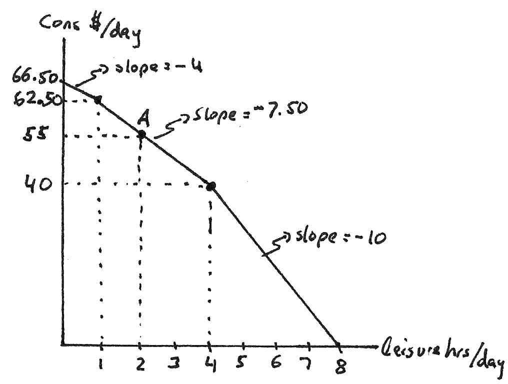
**A:** *Suppose you have at most 8 hours of leisure time per day.*

(a) *Illustrate your budget constraint (with daily leisure on the horizontal and daily consumption on the vertical axis) under this EITC.*

Answer: The budget constraint is graphed in Exercise Graph 3.19(1). For the first 4 hours of labor, the take-home wage is $10 per hour because of the 100% subsidy. For the next 3 hours of labor, the take-home wage is $7.50 because of the 50% subsidy. Finally, for any work beyond 7 hours, the take home wage is $4 because of the 20% tax.

(b) *Suppose the government ends up paying a total of $25 per day to a particular worker under this program and collects no tax revenue. Identify the point on the budget constraint this worker has chosen. How much is he working per day?*

Answer: The worker would work for 6 hours. At a wage of $5, this would mean making $30 per day. But, for the first $20, the government adds $20, and for the next $10 hours, the government adds $5 — for a total EITC supplement of $25. Thus, the worker will have $55 in income for other consumption. This gives us $A$ in the graph — leisure of 2 hours per day (because of 6 hours of work) and consumption of $55 per day.




Exercise Graph 3.19(1) : EITC Budget Constraint

(c) *Return to your graph of the same worker's budget constraint under the AFDC program in exercise 3.18. Suppose that the government paid a total of $25 in daily AFDC benefits to this worker. How much is he working?*

Answer: The worker is working at most 1 hour.

(d) *Discuss how the difference in trade-offs implicit in the EITC and AFDC programs could cause the same individual to make radically different choices in the labor market.*

Answer: Despite the government spending the same on the worker under AFDC and EITC, the worker might choose to not work much under AFDC and a lot under EITC. This is because of the implicit large tax rate imposed on the worker under AFDC but not under EITC.

**B:** *More generally, consider an EITC program in which the first $x$ dollars of income are subsidized at a rate $2s$; the next $x$ dollars are subsidized at a rate $s$; and any earnings above $2x$ are taxed at a rate $t$.*

(a) *Derive the marginal tax rate function $m(I,x,s,t)$ where $I$ stands for labor market income.*

Answer: This function is

$$m(I,x,s,t) = \begin{array}{ll} -2s & \text{if } I < x \text{ and} \\ -s & \text{if } x \leq I < 2x \text{ and} \\ t & \text{if } I \geq 2x. \end{array} \quad (3.19.\text{i})$$

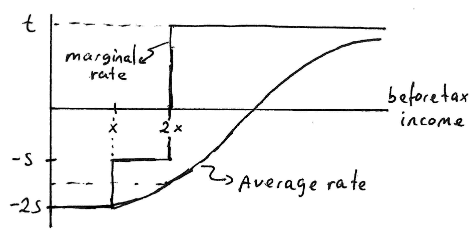
(b) *Derive the average tax rate function $a(I,x,s,t)$ where $I$ again stands for labor market income.*

Answer: This is

$$a(I,x,s,t) = \begin{cases} -2s & \text{if } I \leq x \text{ and} \\ -[2sx + s(I-x)]/I & \text{if } x < I \leq 2x \text{ and} \\ [-3sx + t(I-2x)]/I & \text{if } I > 2x. \end{cases} \quad (3.19.\text{ii})$$

(c) *Graph the average and marginal tax functions on a graph with before-tax income on the horizontal axis and tax rates on the vertical. Is the EITC progressive?*

Answer: This is graphed in Exercise Graph 3.19(2). Since average tax rates rise as income rises, the EITC is progressive.




Exercise Graph 3.19(2) : Average and Marginal Tax Rates under the EITC

# Conclusion: Potentially Helpful Reminders

1. When illustrating either worker or intertemporal budget constraints, always be sure you know where the endowment bundle lies — because the constraint will always rotate through that bundle as wages or interest rates change.

2. If you are unsure where the endowment bundle lies, just ask yourself: Which bundle can the worker (or saver or borrower) consumer *regardless* of what wages or interest rates are?

3. When leisure is modeled on the horizontal axis, then the slope of the worker's budget is $-w$ — or minus the wage. When current consumption is modeled on the horizontal axis of an intertemporal budget constraint, the slope of the budget line is $-(1+r)$

4. If you are interested in finance applications, check out in particular the end-of-chapter exercises 3.9 through 3.14.

5. End-of-chapter exercises 3.15 takes you through illustrating tax revenue in the worker budget graph — a skill that will show up repeatedly throughout the text, beginning in Chapter 8.