CHAPTER

4

Tastes and Indifference Curves

This chapter begins a 2-chapter treatment of tastes — or what we also call "preferences". In the first of these chapters, we simply investigate the basic logic behind modeling tastes and the most fundamental assumptions we make. In the next chapter, we then turn to what specific types of tastes look like. Call me a geek — but I think it's pretty cool that we have found ways of systematically modeling something as abstract as people's tastes!

Chapter Highlights

The main points of the chapter are:

1. **Tastes have nothing to do with budgets** — they are conceptually distinct. Budgets are all about what is feasible — and they arise objectively from "what we bring to the table" and the prices we face. Tastes are all about what we desire and have nothing to do with incomes, endowments or prices.

2. While our model of tastes respects the fact that there is a **great diversity of tastes across people**, we assume that **some aspects of tastes are constant across individuals**. The most basic of these are completeness and transitivity, but the assumptions of monotonicity, convexity and continuity are also intuitively appealing in most circumstances. When we say tastes are "rational" we only mean that individuals with those tastes are capable of making decisions.

3. **Maps of indifference curves are a way of describing tastes**, with the usual shapes and orderings arising from our assumptions of monotonicity and convexity. For those covering part B, indifference curves are simply levels of utility functions — and these levels can no more cross than the elevations of mountains on geography maps can cross.

4. We typically assume that **utility cannot be measured objectively** — which is why the labels on indifference curves do not matter beyond indicating the **ordering** of what is better and what is worse.

5. The slope of an indifference curve — or the **marginal rate of substitution** — at a particular bundle tells us how an individual feels about different goods *at the margin* — i.e. how much the individual is willing to trade one good for another *given that she currently has this particular bundle*.

4A Solutions to Within-Chapter-Exercises for Part A

Exercise 4A.1

Do we know from the monotonicity assumption how E relates to D, A and B? Do we know how A relates to D?

Answer: E must be preferred to D because it contains more of everything (i.e. more pants and more shirts). Monotonicity does not tell us anything about the relationship between A and E — A has more shirts but fewer pants and E has more pants but fewer shirts. For analogous reasons, monotonicity does not tell us anything about how E and B are ranked. A has more shirts and the same number of pants as D — so we know that A is at least as good as D (and probably better).

Exercise 4A.2

What other goods are such that we would prefer to have fewer of them rather than many? How can we re-conceptualize choices over such goods so that it becomes reasonable to assume "more is better"?

Answer: Examples might include pollution, bugs in our houses, weeds in our yard and disease in our bodies. In each case, however, we can re-conceptualize the "bad" by redefining it into a "good" that we want more of. We want less pollution but more clean air and water; fewer bugs in our houses or more "bug-free" square feet of housing; fewer weeds in our yard but more square feet of weed-less grass; less disease and more health.

Exercise 4A.3

Combining the convexity and monotonicity assumptions, can you now conclude something about the relationship between the pairs E and A and E and B if you do not know how A and B are related? What if you know that I am indifferent between A and B?

Answer: We can only apply the convexity assumption if we know some pair of bundles we are indifferent between — because convexity says that, when faced with bundles we are indifferent between, we prefer averages of such bundles (or at the very least like averages just as much). So, without knowing more, I can't use monotonicity and convexity to say anything about how A and E (or B and E) are related to one another. If we know that I am indifferent between A and B, on the other hand, then I know that C is at least as good as A and B because C is the average between A and B. Since E has more of everything than C, we also know from monotonicity that E is better than C. So E is better than C which is at least as good as A and B. By transitivity, that implies that E is better than A and B.

Exercise 4A.4

Knowing that I am indifferent between A and B, can you now conclude something about how B and D are ranked by me? In order to reach this conclusion, do you have to invoke the convexity assumption?

Answer: By just invoking the monotonicity assumption, I know that A is at least as good as D since it has more of one good and the same of the other. If A is indifferent to B, I then also know (by transitivity) that B is at least as good as D. Invoking convexity won't actually allow me to say anything beyond that since indifference between A, B and D is consistent with convexity. (It is not consistent with a *strict* notion of convexity — where by "strict" we mean that averages are strictly better than (indifferent) extremes. In that case, A and B are definitely preferred to D if we are indifferent between A and B.)

Exercise 4A.5

Illustrate the area in Graph 4.2b in which F must lie — keeping in mind the monotonicity assumption.

Answer: By monotonicity, F must have less than C and must therefore lie to the southwest of C. Thus, it must have no more than 5 shirts and no more than 6 pants. But it also cannot have fewer than 4 pants because then it would contain fewer pants and shirts than A and would therefore be worse than A. And it cannot have fewer than 2 shirts because it would then have less of everything than B and could no longer be indifferent to B. F must therefore lie in the area illustrated in Exercise Graph 4A.5.

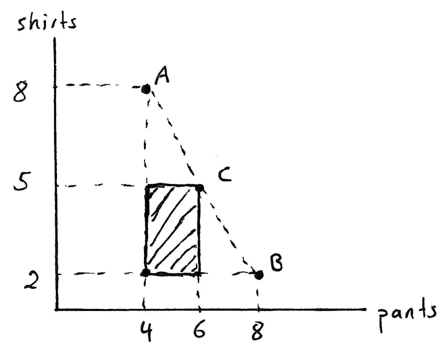
Exercise 4A.6

Suppose our tastes satisfy *weak* convexity in the sense that averages are just as good (rather than strictly better than) extremes. Where does F lie in relation to C in that case?

Answer: In that case F is the same bundle as C — because C is the average of the more extreme bundles A and B.

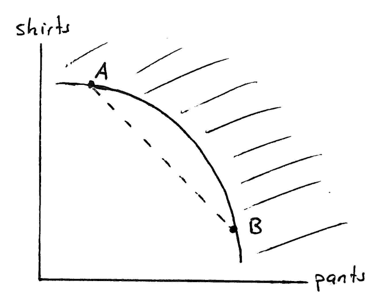
Exercise 4A.7

Suppose extremes are better than averages. What would an indifference curve look like? Would it still imply diminishing marginal rates of substitution?



Exercise Graph 4A.5 : Graph for Within-Chapter-Exercise 4A.5

Answer: The indifference curve would bend away from instead of toward the origin, as illustrated in Exercise Graph 4A.7. The shaded area to the northeast of the indifference curve would contain all the better bundles (because of monotonicity). But the line connecting A and B — which contains averages between A and B — does not lie in this "better" region. Therefore, averages are worse than extremes. The slope of this indifference curve is then shallow at A and becomes steeper as we move along the indifference curve to B. Thus, the marginal rate of substitution is no longer diminishing along the indifference curve — and the indifference curve exhibits increasing marginal rates of substitution.



Exercise Graph 4A.7 : Non-concex tastes

Exercise 4A.8

Suppose averages are just as good as extremes? Would it still imply diminishing marginal rates of substitution?

Answer: If averages are just as good as extremes, then indifference curves are straight lines. As a result, the slope would be the same along an indifference curve —

implying constant rather than diminishing marginal rates of substitution. This is the borderline case between strictly convex tastes that have diminishing marginal rates of substitution and strictly non-convex tastes that have strictly increasing marginal rates of substitution.

Exercise 4A.9

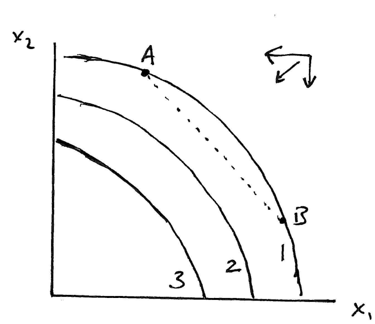
Show how you can prove the last sentence in the previous paragraph by appealing to the transitivity of tastes.

Answer: Pick any bundle that lies on the bold portion of the indifference curve to the southwest of *E* and call it *B*. As noted in the text, we know from monotonicity that *E* is better than *B*. Because *A* and *B* lie on the same indifference curve, you are indifferent between them. Thus, *E* is better than *B* which is indifferent to *A*. Transitivity then implies that *E* is better than *A*.

Exercise 4A.10

Suppose less is better than more and averages are better than extremes. Draw three indifference curves (with numerical labels) that would be consistent with this.

Answer: Exercise Graph 4A.10 illustrates three such curves. Since less is better, the consumer becomes better off in the direction of the arrows at the top right of the graph. Thus, if I take *A* and *B* that lie on the same indifference curve, the line connecting them (which contains averages of the two) lies fully in the region that is more preferred. Thus, averages are indeed better than extremes. Since the consumer becomes better off as she moves southwest, the numbers accompanying the indifference curves must be increasing as we approach the origin.



Exercise Graph 4A.10 : Convex tastes over "bads"

4B Solutions to Within-Chapter-Exercises in Part B

Exercise 4B.1

True or False: If only one of the statements in (4.6) is true for a given set of bundles, then that statement's "$\succsim$" can be replaced by "$\succ$".

Answer: True. If both statements are true, then the consumer is indifferent between the A and the B bundles (because that is the only way that the A bundle can be at least as good as B and the B bundle can be at least as good as A at the same time). If only one of the statements is true, then the consumer is not indifferent between the bundles. That must mean that one of the bundles is strictly preferred to the other, which means we can indeed replace "$\succsim$" with "$\succ$".

Exercise 4B.2

Does transitivity also imply that (4.8) implies (4.9) when "$\succsim$" is replaced by "$\succ$"?

Answer: Yes. If A is strictly preferred to B and B is strictly preferred to C, transitivity implies that A must be strictly preferred to C.

Exercise 4B.3

True or False: Assuming tastes are transitive, the third line in expression (4.11) is logically implied by the first and second lines.

Answer: True. Suppose we call the averaged bundle C. Then the first two lines say that the consumer being indifferent between A and B implies that she thinks C is at least as good as A. Thus, $C \succsim A \sim B$ implies by transitivity that $C \succsim B$, which is what the third line says.

Exercise 4B.4

If you were searching for the steepest possible straight route up the last 2,000 feet of Mount "Nechyba" (in Graph 4.9), from what direction would you approach the mountain?

Answer: It looks like you would approach it from the northwest (heading up the mountain toward the southeast) — because that is where the levels in the graph are closest to one another (which is where the mountain must be steepest).

Exercise 4B.5

In Political Science models, politicians are sometimes assumed to choose between bundles of spending on various issues — say military and domestic spending. Since they have to impose taxes to fund this spending, more is not necessarily better than less, and thus most politicians have some ideal bundle of domestic and military spending. How would such tastes be similar to the geographic mountain analogy?

Answer: Such tastes would be similar in that the "utility mountain" would have a peak just like geographic mountains do. This is not usually the case for our "utility mountains" because usually we make the assumption that more is better — which means we can always climb higher up a mountain without peak. (More on this in end-of-chapter exercise 4.11.)

Exercise 4B.6

How does the expression for the marginal rate of substitution change if tastes could instead be summarized by the utility function $u(x_1,x_2)=x_1^{1/4}x_2^{3/4}$

Answer: We would calculate this as

$$MRS=-\frac{(1/4)(x_1^{-3/4}x_2^{3/4})}{(3/4)(x_1^{1/4}x_2^{-1/4})}=-\frac{x_2}{3x_1}. \qquad (4B.6)$$

Exercise 4B.7

Can you verify that squaring the utility function in exercise 4B.6 also does not change the underlying indifference curves?

Answer: Squaring the utility function from the previous exercise results in $v(x_1,x_2)=(u(x_1,x_2))^2=\left(x_1^{1/4}x_2^{3/4}\right)^2=x_1^{1/2}x_2^{3/2}$. This will give rise to the same indifference curves so long as the MRS everywhere remains unchanged. The MRS is

$$MRS=-\frac{(1/2)(x_1^{-1/2}x_2^{3/2})}{(3/2)(x_1^{1/2}x_2^{1/2})}=-\frac{x_2}{3x_1}, \qquad (4B.7)$$

exactly as it was before. Thus, the shape of the indifference curves is unaffected.

Exercise 4B.8

Illustrate that the same conclusion we reached with respect to u and v representing the same indifference curves also holds when we take the square root of u — i.e. when we consider the function $w(x_1,x_2)=(x_1^{1/2}x_2^{1/2})^{1/2}=x_1^{1/4}x_2^{1/4}$.

Answer: The MRS is then

$$MRS=-\frac{(1/4)(x_1^{-3/4}x_2^{1/4})}{(1/4)(x_1^{1/4}x_2^{-3/4})}=-\frac{x_2}{x_1}, \qquad (4B.8)$$

exactly as it was when we calculated the MRS for $u(x_1,x_2)=x_1^{1/2}x_2^{1/2}$ in the text.

Exercise 4B.9

Consider the utility function $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. Take natural logs of this function and calculate the MRS of the new function. Is the natural log transformation one that can be applied to utility functions such that the new utility function represents the same underlying tastes?

Answer: Taking logs, we get: $\ln u(x_1, x_2) = \ln\left(x_1^{1/2} x_2^{1/2}\right) = (1/2)\ln x_1 + (1/2)\ln x_2$. Note that the derivative of this with respect to x_1 is $1/(2x_1)$ and the derivative with respect to x_2 is $1/(2x_2)$. The MRS is then

$$MRS = -\frac{1/(2x_1)}{1/(2x_2)} = -\frac{x_2}{x_1}, \qquad (4B.9)$$

exactly as it was before the log transformation. Thus, taking logs does not change the shape of indifference curves. Logs also do not change the ordering of the labels on indifference curves. Thus, when we take the log of a utility function, the new utility function represents the same tastes.

Exercise 4B.10

Consider the utility function $u(x_1, x_2, x_3) = x_1^{1/2} x_2^{1/2} x_3^{1/2}$. Take natural logs of this function and calculate the marginal rates of substitution of each pair of goods. Is the natural log transformation one that can be applied to utility functions of three goods such that the new utility function represents the same underlying tastes?

Answer: Taking logs, we get a new function $v(x_1, x_2, x_3) = (1/2)\ln x_1 + (1/2)\ln x_2 + (1/2)\ln x_3$. Taking any pair of good x_i and x_j (where i, and j can take values of 1, 2, and 3 but $i \neq j$), we get

$$MRS = -\frac{1/(2x_i)}{1/(2x_j)} = -\frac{x_j}{x_i}. \qquad (4B.10.i)$$

If we instead work with the original utility function $u(x_1, x_2, x_3) = x_1^{1/2} x_2^{1/2} x_3^{1/2}$, we can similarly calculate the MRS between x_i and x_j while denoting the third good as x_k:

$$MRS = -\frac{(1/2)x_i^{-1/2} x_j^{1/2} x_k^{1/2}}{(1/2)x_i^{1/2} x_j^{-1/2} x_k^{1/2}} = -\frac{x_j}{x_i}. \qquad (4B.10.ii)$$

We therefore again get the same expressions for the MRS between any two goods after we take logs of the utility function as we do before. Logs are general transformations that can always be applied to a utility function (regardless of how many goods the function is over) to get a new utility function that represents the same underlying tastes.

Exercise 4B.11

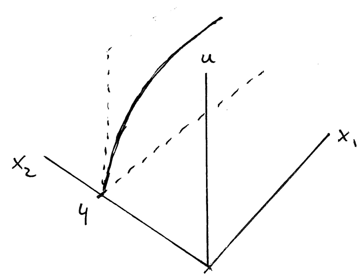
What would be the expression of the slope of the slice of the utility function $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$ when x_1 is fixed at 9? What is the slope of that slice when $x_2 = 4$?

Answer: When $x_1 = 9$, the expression reduces to $(1/2)(9)^{1/2} x_2^{-1/2} = (3/2)x_2^{-1/2}$. This is the expression of the slope of the slice holding $x_1 = 9$. When $x_2 = 4$, that slope is $(3/2)(4)^{-1/2} = 3/4$.

Exercise 4B.12

Calculate $\partial u/\partial x_1$ for $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. What does this reduce to when x_2 is fixed at 4? Where in Graph 4.12 does the slice along which this partial derivative represents the slope lie?

Answer: $\partial u/\partial x_1 = (1/2)x_1^{-1/2} x_2^{1/2}$ reduces to $x_1^{-1/2}$ when $x_2 = 4$. The relevant slice is depicted in Exercise Graph 4B.12.



Exercise Graph 4B.12 : Slice holding x_2 constant at 4

Exercise 4B.13

Calculate $\partial u/\partial x_1$ for the function $u(x_1, x_2) = 10 \ln x_1 + 5 \ln x_2$.

Answer: $\partial u/\partial x_1 = 10/x_1$.

Exercise 4B.14

Calculate $\partial u/\partial x_1$ for the function $u(x_1, x_2) = (2x_1 + 3x_2)^3$. (Remember to use the Chain Rule.)

Answer: $\partial u/\partial x_1 = 3(2x_1 + 3x_2)^2 (2) = 6(2x_1 + 3x_2)^2$.

Exercise 4B.15

Verify that equation (4.28) is correct.

Answer: The partial derivatives are

$$\frac{\partial u}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^{1/2} = \frac{x_2^{1/2}}{2x_1^{1/2}} \tag{4B.15.i}$$

and

$$\frac{\partial u}{\partial x_2} = \frac{1}{2} x_1^{1/2} x_2^{-1/2} = \frac{x_1^{1/2}}{2x_2^{1/2}}. \tag{4B.15.ii}$$

When substituted into the equation, it verifies what is in the text.

Exercise 4B.16

Calculate the total differential du of $u(x_1, x_2) = 10 \ln x_1 + 5 \ln x_2$.
Answer: This is

$$du = \frac{10}{x_1} dx_1 + \frac{5}{x_2} dx_2. \tag{4B.16}$$

4C Solutions to Odd Numbered End-of-Chapter Exercises

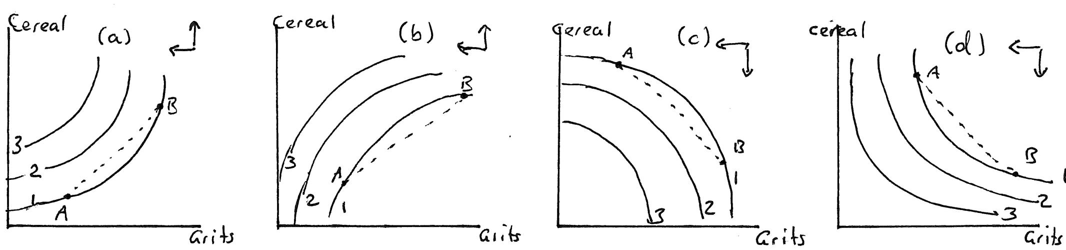
Exercise 4.1

I hate grits so much that the very idea of owning grits makes me sick. I do, on the other hand, enjoy a good breakfast of Coco Puffs Cereal.

A: *In each of the following, put boxes of grits on the horizontal axis and boxes of cereal on the vertical. Then graph three indifference curves and number them.*

(a) *Assume that my tastes satisfy the convexity and continuity assumptions and otherwise satisfy the description above.*

Answer: Panel (a) of Exercise Graph 4.1 graphs an example of such tastes. In the top right corner, arrows indicate the directions in which I become better off. As you will see in this exercise, convexity always implies that indifference curves bend toward the origin that is created by arrows such as these that indicate the directions in which a consumer becomes better off. In the graph, A and B appear on the same indifference curve — and the line connecting them lies "above" the curve in the sense that it contains only bundles to the northwest that are more preferred.



Exercise Graph 4.1 : Grits and Cereal

(b) *How would your answer change if my tastes were "non-convex" — i.e. if averages were worse than extremes.*

Answer: Panel (b) graphs an example of such tastes. I still become better off moving toward the northwest where there are fewer grits and more cereal. But now the indifference curves bend in the other direction. A and B again lie on the same indifference curve, but the line connecting them now lies "below" the indifference curve in the sense that all bundles on that line segment lie to the southeast where I become worse off.

(c) *How would your answer to (a) change if I hated both Coco Puffs and grits but we again assumed my tastes satisfy the convexity assumption.*

Answer: An example of such tastes is graphed in panel (c) of Exercise Graph 4.1. Now the arrows at the top right of the graph point down and left, with better points lying to the southwest as we move toward the origin of the graph. Convexity again implies that indifference curves bend toward the origin that is created by the arrows that indicate which direction makes us better off. Bundles A and B again lie on the same indifference curve, and the line connecting them lies "above" the indifference curve in the sense that it lies to the southwest where I become better off.

(d) *What if I hated both goods and my tastes were non-convex?*

Answer: An example of such tastes is graphed in panel (d) of the graph. As in panel (c), the consumer becomes better off moving toward the southwest. But because tastes are non-convex, the indifference curves now bend in the other direction (and away from the origin that is created by the arrows in the top right corner of the graph). A and B are once again on the same indifference curve, but the line connecting them now lies "below" the indifference curve in the sense that it lies to the northeast where the consumer becomes worse off.

B: *Now suppose you like both grits and Coco Puffs, that your tastes satisfy our five basic assumptions and that they can be represented by the utility function* $u(x_1, x_2) = x_1 x_2$.

(a) *Consider two bundles, A=(1,20) and B=(10,2). Which one do you prefer?*

Answer: You would be indifferent between the two because, when you plug these into the utility function, you get the same utility value; i.e. $u(1,20) = 1(20) = 20$ and $u(10,2) = 10(2) = 20$.

(b) *Use bundles A and B to illustrate that these tastes are in fact convex.*

Answer: Suppose I construct a new bundle C that is the average of A and B — i.e. take half of A and mix it with half of B. This would give 5.5 boxes of grits and 11 boxes of cereal; i.e. C=(5.5,11). Plugging this into the utility function, we get $u(5.5, 11) = 5.5(11) = 60.5$. Thus, utility of the average is higher than utility of the extremes.

(c) *What is the MRS at bundle A? What is it at bundle B?*

Answer: The MRS for this utility functions is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{x_2}{x_1}. \quad (4.1.i)$$

Plugging in the values for x_1 and x_2 at A and B, we then get $MRS^A = -20$ and $MRS^B = -2/10 = -1/5$.

(d) *What is the simplest possible transformation of this function that would represent tastes consistent with those described in A(d)?*

Answer: The simplest possible transformation would be to multiply the function by a negative 1. This would leave the shape of the indifference curves unchanged because the MRS would be the same. (The negative would cancel in the calculation of MRS.) But the ordering of the numbers accompanying the indifference curves would change because each number would now be multiplied by minus 1. This means that, rather than numbers going up as we move toward the northeast of the graph, numbers will go up as we go to the southwest of the graph. The indifference map would therefore look like the one we graphed in panel (d) of Exercise Graph 4.1.

(e) *Now consider tastes that are instead defined by the function $u(x_1,x_2) = x_1^2 + x_2^2$. What is the MRS of this function?*

Answer: The MRS is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{2x_1}{2x_2} = -\frac{x_1}{x_2}. \tag{4.1.ii}$$

(f) *Do these tastes have diminishing marginal rates of substitution? Are they convex?*

Answer: Notice that the MRS is the inverse of what we calculated for the Cobb-Douglas utility function x_1x_2. Consider, for instance, the bundles (1,5) and (5,1) which both lie on the same indifference curve (that gets utility 26). At (1,5), $MRS = -1/5$ while at (5,1), $MRS = -5/1 = -5$. Thus, the MRS is shallow toward the left of the indifference curve and gets steeper toward the right — we have increasing marginal rates of substitution rather than diminishing marginal rates of substitution. Put differently, these indifference curves bend away from rather than toward the origin. Since more is better, this implies that tastes are not convex.

(g) *How could you most easily turn this utility function into one that represents tastes like those described in A(c)?*

Answer: In A(c), the two goods are "bads" and tastes are convex. The tastes represented by the utility function $u(x_1,x_2) = x_1^2 + x_2^2$ in the previous part give rise to indifference curve with the shape needed for those in A(c) — but the direction of the labeling is one that assigns higher labels to bundles that contain more rather than fewer goods. By simply multiplying the function by -1, however, we reverse the labels and thus have indifference curves with the right shapes and labels increasing in the right direction. Thus, $v(x_1,x_2) = -x_1^2 - x_2^2$ would represent tastes such as those in A(c).

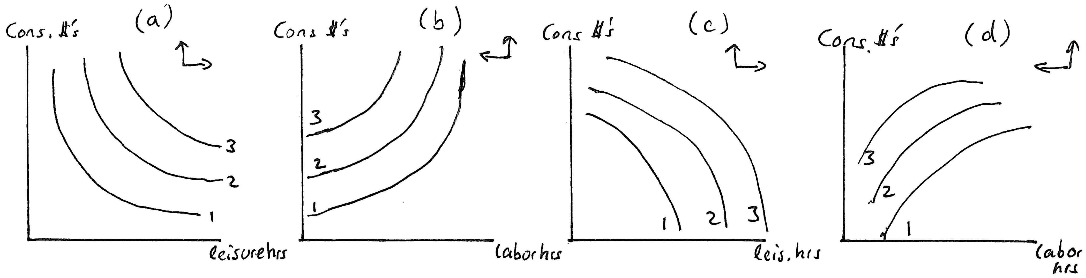
Exercise 4.3

Consider my tastes for consumption and leisure.

A: *Begin by assuming that my tastes over consumption and leisure satisfy our 5 basic assumptions.*

(a) *On a graph with leisure hours per week on the horizontal axis and consumption dollars per week on the vertical, give an example of three indifference curves (with associated utility numbers) from an indifference map that satisfies our assumptions.*

Answer: Panel (a) of Exercise Graph 4.3 graphs an example of three indifference curves that satisfy these assumptions.



Exercise Graph 4.3 : Tastes over Consumption and Labor/Leisure

(b) *Now redefine the good on the horizontal axis as "labor hours" rather than "leisure hours". How would the same tastes look in this graph?*

Answer: Panel (b) illustrates these indifference curves with the good on the horizontal axis redefined. I now become better off going to the northwest in the graph since I prefer less labor.

(c) *How would both of your graphs change if tastes over leisure and consumption were non-convex — i.e. if averages were worse than extremes.*

Answer: Panels (c) and (d) illustrate examples of indifference curves with leisure (panel (c)) and labor (panel (d)) when tastes are non-convex. The line connecting any two points on any of these indifference curves contains only bundles that lie in the "worse" region — implying that averages are worse than extremes.

B: *Suppose your tastes over consumption and leisure could be described by the utility function $u(\ell, c) = \ell^{1/2} c^{1/2}$.*

(a) *Do these tastes satisfy our 5 basic assumptions?*

Answer: Yes. The utility function is one that has been used a number of times in the chapter. It is clearly a continuous function that assigns higher value to bundles that have more consumption and leisure (i.e. it represents monotonic tastes). The *MRS* for this utility function is given by $MRS = -\ell/c$. When ℓ is low and c is high (i.e. to the left in our graph), the *MRS* is therefore large in absolute value, and when ℓ is low and c is high (i.e. to the right in our graph), the *MRS* is small in absolute value. Thus, we have indifference curves that have the property of diminishing marginal rates of substitution — which is the case only when convexity is satisfied. Thus, continuity, monotonicity and convexity are all satisfied. (And the function clearly assigns utility values to all bundles — thus representing complete tastes — and any mathematical function automatically satisfies transitivity.)

(b) *Can you find a utility function that would describe the same tastes when the second good is defined as labor hours instead of leisure hours? (*Hint*:*

Suppose your weekly endowment of leisure time is 60 hours. How do labor hours relate to leisure hours?)

Answer: Let l represent labor hours and assume that I have a total of 60 hours per week in possible leisure time. Then, since $\ell = 60 - l$ (because the leisure hours we actually consume are just those during which we do not work), we can write the utility function in terms of l instead of ℓ by replacing ℓ with $(60 - l)$. Our new function is then $v(c, l) = c^{1/2}(60 - l)^{1/2}$.

(c) *What is the marginal rate of substitution for the function you just derived? How does that relate to the sign of the slopes of indifference curves you graphed in part A(b)?*

Answer: The marginal rate of substitution is

$$MRS = -\frac{\partial u/\partial l}{\partial u/\partial c} = -\frac{(-1/2)c^{1/2}(60-l)^{-1/2}}{(1/2)c^{-1/2}(60-l)^{1/2}} = \frac{c}{60-l}. \quad (4.3.i)$$

Note the minus sign that appears in the denominator (because of the Chain Rule), which cancels the minus sign in front of the fraction to give a positive MRS. This is exactly what we graphed in panel (b) of Exercise Graph 4.3 where the slope of indifference curves is positive. (The expression above also implies that slopes start shallow and become steeper as they do in our graph — see the answer to the next part for an explanation to this.)

(d) *Do the tastes represented by the utility function in part (b) satisfy our 5 basic assumptions?*

Answer: They do not because l enters negatively — which implies more l reduces utility. Thus, monotonicity is violated because of the way we have redefined the goods. The other assumptions, however, still hold. We still have a continuous function that assigns values to all bundles (i.e. we have continuity, completeness and transitivity). Also, when l and c are both low (to the left of the graph), the denominator of our MRS is large while the numerator is small — leading to a small positive number. When l and c are both high (to the right of the graph), on the other hand, the denominator becomes small while the numerator is large — leading to a large positive number. Thus, the slope of indifference curves starts small (i.e. shallow) and becomes large (i.e. steep) — precisely as depicted in panel (b) of Exercise Graph 4.3 that mapped out indifference curves under the assumption of convexity.

Exercise 4.5

In this exercise, we explore the concept of marginal rates of substitution (and, in part B, its relation to utility functions) further.

A: *Suppose I own 3 bananas and 6 apples, and you own 5 bananas and 10 apples.*

(a) *With bananas on the horizontal axis and apples on the vertical, the slope of my indifference curve at my current bundle is −2, and the slope of your* *indifference curve through your current bundle is* -1. *Assume that our tastes satisfy our usual five assumptions. Can you suggest a trade to me that would make both of us better off? (Feel free to assume we can trade fractions of apples and bananas).*

Answer: The slope of my indifference curve at my bundle tells us that I am willing to trade as many as 2 apples to get one more banana. The slope of your indifference curve at your bundle tells us that you are willing to trade apples and bananas one for one. If you offer me 1 banana in exchange for 1.5 apples, you would be better off because you would have been willing to accept as little as 1 apple for 1 banana. I would also be better off because I would be willing to give you as many as 2 apples for 1 banana — only having to give you 1.5 apples is better than that. (If you are uncomfortable with fractions of apples being traded, you could also propose giving me 2 bananas for 3 apples.)

This is only one possible example of a trade that would make us both better off. You could propose to give me 1 banana for x apples, where x can lie between 1 and 2. Since I am willing to give up as many as 2 apples for one banana, any such trade would make me better off, and since you are willing to trade them one for one, the same would be true for you.

(b) *After we engage in the trade you suggested, will our MRS's have gone up or down (in absolute value)?*

Answer: Any trade that makes both of us better off moves me in the direction of more bananas and fewer apples — which, given diminishing marginal rates of substitution, should decrease the absolute value of my *MRS*; i.e. as I get more bananas and fewer apples, I will be willing to trade fewer apples to get one more banana than I was willing to originally. You, on the other hand, are giving up bananas and getting apples, which moves you in the opposite direction toward fewer bananas and more apples. Thus, you will become less willing to trade 1 banana for 1 apple and will in future trades demand more bananas in exchange for 1 apple. Thus, in absolute value, your *MRS* will get larger.

(c) *If the values for our MRS's at our current consumption bundles were reversed, how would your answers to (a) and (b) change?*

Answer: The trades would simply go in the other direction; i.e. I would be willing to trade 1 banana for x apples so long as x is at least 1, and you would be willing to accept such a trade so long as x is no more than 2. Thus, x again lies between 1 and 2 if both of us are to be better off from the trade, only now I am giving you bananas in exchange for apples rather than the other way around.

(d) *What would have to be true about our MRS's at our current bundles in order for you not to be able to come up with a mutually beneficial trade?*

Answer: In order for us not to be able to trade in a mutually beneficial way, your *MRS* at your current bundle would have to be identical to my *MRS* at my current bundle.

(e) True or False: *If we have different tastes, then we will always be able to trade with both of us benefitting.*

Answer: This statement is generally false. What matters is not that we have different tastes (i.e. different maps of indifference curves). What matters instead is that, at our current consumption bundle, we value goods differently — that at our current bundle, our MRS's are different. It is quite possible for us to have different tastes (i.e. different maps of indifference curves) but to also be at bundles where our MRS is the same. In that case, we would have the same tastes *at the margin* even though we have different tastes overall (i.e. different indifference maps.)

(f) True or False: *If we have the same tastes, then we will never be able to trade with both of us benefitting.*

Answer: False. People with the same tastes but different bundles of goods may well have different marginal rates of substitution at their current bundles — and this opens the possibility of trading with benefits for both sides.

B: *Consider the following five utility functions and assume that α and β are positive real numbers:*

$$\begin{aligned}
&1.\ u^A(x_1,x_2) = x_1^{\alpha}x_2^{\beta}\\
&2.\ u^B(x_1,x_2) = \alpha x_1 + \beta x_2\\
&3.\ u^C(x_1,x_2) = \alpha x_1 + \beta \ln x_2\\
&4.\ u^D(x_1,x_2) = \left(\frac{\alpha}{\beta}\right)\ln\ x_1 + \ln x_2\\
&5.\ u^E(x_1,x_2) = -\alpha \ln x_1 - \beta \ln x_2
\end{aligned} \tag{4.5}$$

(a) *Calculate the formula for MRS for each of these utility functions.*

Answer: These would be

$$\begin{aligned}
&1.\ MRS^A = -\frac{\alpha x_1^{\alpha-1}x_2^{\beta}}{\beta x_1^{\alpha}x_2^{\beta-1}} = -\frac{\alpha x_2}{\beta x_1}\\
&2.\ MRS^B = -\frac{\alpha}{\beta}\\
&3.\ MRS^C = -\frac{\alpha}{\beta/x_2} = -\frac{\alpha x_2}{\beta}\\
&4.\ MRS^D = -\frac{\alpha/(\beta x_1)}{1/x_2} = -\frac{\alpha x_2}{\beta x_1}\\
&5.\ MRS^E = -\frac{-\alpha/x_1}{-\beta/x_2} = -\frac{\alpha x_2}{\beta x_1}
\end{aligned} \tag{4.5.i}$$

(b) *Which utility functions represent tastes that have linear indifference curves?*

Answer: Linear indifference curves are indifference curves that have the same slope everywhere — i.e. indifference curves with constant rather than diminishing MRS. Thus, the MRS cannot depend on x_1 or x_2 for the indifference curve to be linear — which is the case only for $u^B(x_1, x_2)$.

(c) *Which of these utility functions represent the same underlying tastes?*

Answer: Two conditions have to be met for utility functions to represent the same tastes: (1) the indifference curves they give rise to must have the same shapes, and (2) the numbering on the indifference curves needs to have the same order (though not the same magnitude.) To check that indifference curves from two utility functions have the same shape, we have to check that the MRS for those utility functions are the same. This is true for u^A, u^D and u^E. To check that the ordering of the numbers associated with indifference curves goes in the same direction, we need to go back to the utility functions. In u^A, for instance, more of x_1 and/or x_2 means higher utility values. The same is true for u^D. Thus u^A and u^D represent the same underlying tastes because they give rise to the same shapes for all the indifference curves and both have increasing numbers associated with indifference curves as we move northeast in the graph of the indifference curves. But u^E is different: While it gives rise to indifference curves with the same shapes as u^A and u^D, the utility values associated with the indifference curves become increasingly negative — i.e. they decline — as we increase x_1 and/or x_2. Thus, higher numerical labels for indifference curves happen to the southwest rather than the northeast — indicating that less is better than more. So the only two utility functions in this problem that represent the same tastes are u^A and u^D.

(d) *Which of these utility functions represent tastes that do not satisfy the monotonicity assumption?*

Answer: As just discussed in the answer to B(c), u^E represents tastes for which less is better than more — because the labeling on the indifference curves gets increasingly negative as we move to the northeast (more of everything) and increasingly less negative as we move toward the origin. In all other cases, more x_1 and/or more x_2 creates greater utility as measured by the utility functions.

(e) *Which of these utility functions represent tastes that do not satisfy the convexity assumption?*

Answer: As we move to the right on an indifference curve, x_1 increases and x_2 decreases. We can then look at the formulae for MRS that we derived for each utility function to see what happens to the MRS as x_1 increases while x_2 decreases. In MRS^A, for instance, this would result in a decrease in the numerator and an increase in the denominator — i.e. we are dividing a smaller number by a larger number as x_1 increases and x_2 decreases. Thus, in absolute value, the MRS declines as we move to the right in our graph — which implies we have diminishing MRS and the usual shape for the indifference curves. Since they share the same MRS, the same holds for u^D and u^E. For u^C, it is similarly true that an increase in x_1 accompanied by a decrease in x_2 (i.e. a movement along the indifference curve toward the right in the graph) causes the MRS to fall — only this time x_1 plays no role and the drop is entirely due to the reduction in the numerator. For u^B, the MRS is constant — implying no change in the MRS as we move along an indifference curve to the right in the graph.

We can then conclude the following: u^B satisfies the convexity assumption but barely so — averages are the same as extremes (but not better). Furthermore, u^A, u^C and u^D all represent monotonic tastes with diminishing marginal rates of substitution along indifference curves. Thus, averages between extremes that lie on the same indifference curve will be preferred to the extremes because the averages lie to the northeast of some bundles on the indifference curves on which the extremes lie, and, since more is better, this implies the averages are better than the extremes. So u^A, u^C and u^D all satisfy the convexity assumption. That leaves only u^E which we concluded before does not satisfy the monotonicity assumption but its indifference curves look exactly like they do for u^A and u^D. If you pick any two bundles on an indifference curve, it will therefore again be true that the average of those bundles lies to the northeast of some of the bundles on that indifference curve — but now a movement to the northeast makes the individual worse off, not better off. Thus, averages are worse than extremes for the tastes represented by u^E — which implies that u^E represents tastes that are neither convex nor monotonic.

(f) *Which of these utility functions represent tastes that are not rational (i.e. that do not satisfy the completeness and transitivity assumptions)?*

Answer: Each of these is a function that satisfies the mathematical properties of functions. In each case, you can plug in any bundle (x_1, x_2) and the function will assign a utility value. Thus, any two bundles can be compared — and completeness is satisfied. Furthermore, it is mathematically not possible for a function to assign a value to bundle A that is higher than the value it assigns to a different bundle B which in turn is higher than the value assigned to a third bundle C — without it also being true that the value assigned to C is lower than the value assigned to A. Thus, transitivity is satisfied.

(g) *Which of these utility functions represent tastes that are not continuous?*

Answer: All the functions are continuous without sudden jumps — and therefore represent tastes that are similarly continuous.

(h) *Consider the following statement: "Benefits from trade emerge because we have different tastes. If individuals had the same tastes, they would not be able to benefit from trading with one another." Is this statement ever true, and if so, are there any tastes represented by the utility functions in this problem for which the statement is true?*

Answer: What we found in our answers in part A is that, in order for individuals to be able to benefit from trading, it must be the case that their indifference curves through their current consumption bundle have different slopes. It does not matter whether their indifference maps are identical. So long as they are at different current bundles that have different *MRS*'s, mutually beneficial trades are possible. You and I, for instance, might have identical tastes over apples and bananas, but I might have mostly bananas and you might have mostly apples. Then you would probably be willing to trade lots of apples for more bananas, and I'd be willing to let go of bananas pretty easily to get more apples. The only way we cannot benefit from trading with one another is if our *MRS*'s through our current bundle are the same. This might be true for some bundles when we have identical tastes (such as when we currently own the same bundle), but it is not generally true just because we have the same tastes. The only utility function from this problem for which the statement generally holds is therefore u^B, the utility function that represents tastes with the same *MRS* at all bundles. If you and I shared those tastes, then we would have the same *MRS* regardless of which bundles we currently owned — and this makes it impossible for us to become better off through trade.

The statement in this problem could be re-phrased in a way that would make it universally true for all tastes: "Benefits from trade emerge because we have different tastes *at the margin*" — that is, when we have the same willingness to trade goods off for one another around the bundle we currently consume, then we have the same *MRS* and can't trade.

Exercise 4.7

Everyday Application: *Did 9/11 Change Tastes?*: *In another textbook, the argument is made that consumer tastes over "airline miles traveled" and "other goods" changed as a result of the tragic events of September 11, 2001.*

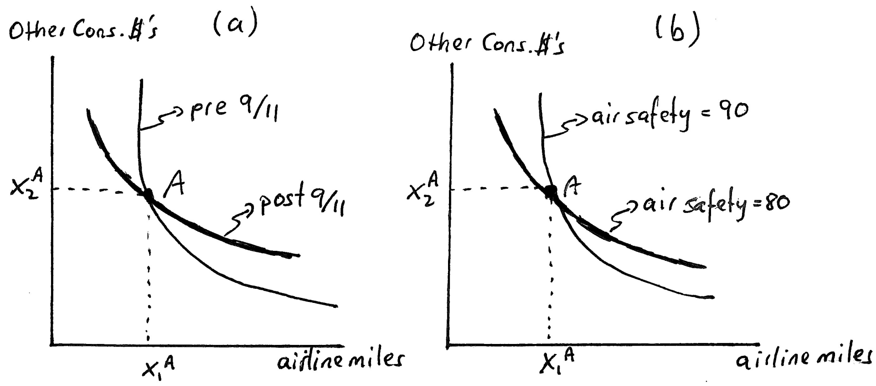
A: *Below we will see how you might think of that argument as true or false depending on how you model tastes.*

(a) *To see the reasoning behind the argument that tastes changed, draw a graph with "airline miles traveled" on the horizontal axis and "other goods" (denominated in dollars) on the vertical. Draw one indifference curve from the map of indifference curves that represent a typical consumer's tastes (and that satisfy our usual assumptions.)*

Answer: This is illustrated in panel (a) of Exercise Graph 4.7 with the indifference curve labeled "pre-9/11".

(b) *Pick a bundle on the indifference curve on your graph and denote it A. Given the perception of increased risk, what do you think happened to the typical consumer's MRS at this point after September 11, 2001?*

Answer: The *MRS* tells us how much in "dollars of other goods" a consumer is willing to give up to travel one more mile by air. After 9/11, it would stand to reason that the typical consumer would give up fewer dollars for additional air travel than before. Thus, the slope of the indiffer-



Exercise Graph 4.7 : Tastes before and after 9/11

ence curve at A should become shallower — which implies that the MRS is falling in absolute value.

(c) *For a consumer who perceives a greater risk of air travel after September 11, 2001, what is likely to be the relationship of the indifference curves from the old indifference map to the indifference curves from the new indifference map at every bundle?*

Answer: The reasoning from (b) holds not just at A but at all bundles. Thus, we would expect the new indifference map to have indifference curves with shallower slopes at every bundle.

(d) *Within the context of the model we have developed so far, does this imply that the typical consumer's tastes for air-travel have changed?*

Answer: Rationality (as we have defined it) rules out the possibility for indifference curves to cross. Thus, within the context of this model, it certainly seems that tastes must have changed.

(e) *Now suppose that we thought more comprehensively about the tastes of our consumer. In particular, suppose we add a third good that consumers care about — "air safety". Imagine a 3-dimensional graph, with "air miles traveled" on the horizontal axis and "other goods" on the vertical (as before) — and with "air safety" on the third axis coming out at you. Suppose "air safety" can be expressed as a value between 0 and 100, with 0 meaning certain death when one steps on an airplane and 100 meaning no risk at all. Suppose that before 9/11, consumers thought that air safety stood at 90. On the slice of your 3-dimensional graph that holds air safety constant at 90, illustrate the pre-9/11 indifference curve that passes through* (x_1^A, x_2^A)*, the level of air miles traveled* (x_1^A) *and other goods consumed* (x_2^A) *before 9/11.*

Answer: This is illustrated in panel (b) of Exercise Graph 4.7 as the indifference curve labeled "air safety = 90".

(f) *Suppose the events of 9/11 cause air safety to fall to 80. Illustrate your post-9/11 indifference curve through (x_1^A, x_2^A) on the slice that holds air safety constant at 80 but draw that slice on top of the one you just drew in (e).*

Answer: This is also done in panel (b) of the graph.

(g) *Explain that, while you could argue that our tastes changed in our original model, in a bigger sense you could also argue that our tastes did not change after 9/11, only our circumstances did.*

Answer: When we explicitly include air safety as something we value as consumers, we get indifference surfaces that lie in 3 dimensions. But since we don't get to choose the level of air safety, we effectively operate on a 2-dimensional slice of that 3-dimensional indifference surface — the slice that corresponds to the current level of air safety. That slice looks just like any ordinary indifference curve in a 2-good model even though it comes from a 3-good model. When 9/11 changes the perceptions of air safety, outside circumstances are shifting us to a different portion of our 3-dimensional indifference surface — with that slice once again giving rise to indifference curves that look like the ones we ordinarily graph in a 2-good model. But when viewed from this perspective, the fact that the indifference curve that corresponds to more air safety crosses the indifference curve that corresponds to less air safety merely arises because we are graphing two different slices of a 3-dimensional surface in the same 2-dimensional space. While both curves then contain the bundle (x_1^A, x_2^A), they occur at different levels of x_3. The pre-9/11 indifference curve really goes through bundle $(x_1^A, x_2^A, 90)$ while the post-9/11 indifference curve really goes through bundle $(x_1^A, x_2^A, 80)$ — and the two therefore do not cross. Thus, when viewed from this larger perspective, tastes have not changed, only circumstances have.

B: *Suppose an average traveler's tastes can be described by the utility function $u(x_1, x_2, x_3) = x_1 x_3 + x_2$, where x_1 is miles traveled by air, x_2 is "other consumption" and x_3 is an index of air safety that ranges from 0 to 100.*

(a) *Calculate the MRS of other goods for airline miles — i.e. the MRS that represents the slope of the indifference curves when x_1 is on the horizontal and x_2 is on the vertical axis.*

Answer: The MRS is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{x_3}{1} = -x_3. \tag{4.7.i}$$

(b) *What happens to the MRS when air safety (x_3) falls from 90 to 80?*

Answer: It changes from −90 to −80.

(c) *Is this consistent with your conclusions from part A? In the context of this model, have tastes changed?*

Answer: The change in the MRS as air safety falls is a decrease in absolute value — i.e. the slope of the indifference curve over x_1 and x_2 becomes shallower just as we concluded in part A. But we are representing tastes with exactly the same utility function as before — so tastes cannot have changed.

(d) *Suppose that $u(x_1, x_2, x_3) = x_1 x_2 x_3$ instead. Does the MRS of other consumption for air miles traveled still change as air safety changes? Is this likely to be a good model of tastes for analyzing what happened to consumer demand after 9/11?*

Answer: The MRS now is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{x_2 x_3}{x_1 x_3} = -\frac{x_2}{x_1}. \tag{4.7.ii}$$

Thus, the MRS for tastes represented by this utility function is unaffected by x_3 — the level of air safety. This would imply that the two indifference curves in panel (b) of Exercise Graph 4.7 would lie on top of one another. If we think consumers felt differently about air travel after 9/11 than before, then this utility function would not be a good one to choose for analyzing changes in consumer behavior.

(e) *What if $u(x_1, x_2, x_3) = x_2 x_3 + x_1$?*

Answer: In this case, the MRS is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{1}{x_3}. \tag{4.7.iii}$$

This would imply that as x_3 — air safety — falls, the MRS increases in absolute value; i.e. it would mean that a decrease in air safety would make us willing to spend more on additional air travel than what we were willing to spend before. It would thus result in a steeper rather than a shallower slope for indifference curves post-9/11. It seems unlikely that a typical consumer would respond in this way to changes in air safety.

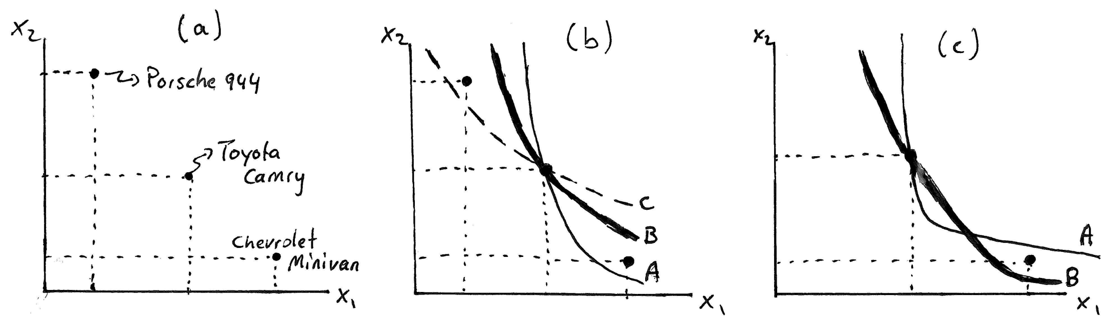
Exercise 4.9

Business Application: *Tastes for Cars and Product Characteristics: People buy all sorts of different cars depending on their income levels as well as their tastes. Industrial organization economists who study product characteristic choices (and advise firms like car manufacturers) often model consumer tastes as tastes over product characteristics (rather than as tastes over different types of products). We explore this concept below.*

A: *Suppose people cared about two different aspects of cars: the size of the interior passenger cabin and the quality of handling of the car on the road.*

(a) *Putting x_1 = "cubic feet of interior space" on the horizontal axis and x_2 = "speed at which the car can handle a curved mountain road" on the vertical, where would you generally locate the following types of cars assuming that they will fall on one line in your graph: a Chevrolet Minivan, a Porsche 944, and a Toyota Camry.*

Answer: Panel (a) of Exercise Graph 4.9 illustrates where the product characteristics of these cars would place them in a graph with interior space on the horizontal axis and speed on the vertical. Porsche's do not have much space in the interior but they handle well at high speeds. Minivans have tons of interior space but don't handle that well at high speeds. And Toyota Camrys are somewhere in between — with more space than Prosche's but not as much as minivans, and with better handling at high speeds than minivans but not as good as Porsches.



Exercise Graph 4.9 : Porsche, Toyota and Chevy

(b) *Suppose we considered three different individuals whose tastes satisfy our 5 basic assumptions, and suppose each person owns one of the three types of cars. Suppose further that each indifference curve from one person's indifference map crosses any indifference curve from another person's indifference map at most once. (When two indifference maps satisfy this condition, we often say that they satisfy the* single crossing property.*) Now suppose you know person A's MRS at the Toyota Camry is larger (in absolute value) than person B's, and person B's MRS at the Toyota Camry is larger (in absolute value) than person C's. Who owns which car?*

Answer: The indifference curves (through the Toyota Camry) for the 3 individuals are depicted in panel (b) of Exercise Graph 4.9. In order for one of these cars to be the most preferred for one and only one of the individuals, it must be that the Porsche lies above one person's indifference curve through the Camry and the minivan lies above another person's indifference curve through the Camry. If indifference curves from different indifference maps cross only once, it logically has to follow that the steepest indifference curve through the Camry lies below the minivan and the shallowest indifference curve through the Camry falls below the Porsche. Since person A's MRS is largest in absolute value, person A's indifference curve through the Camry has the steepest slope. By the same reasoning, person C has the shallowest slope going through the Camry. Thus, person A owns the minivan, person B owns the Camry and person C owns the Porsche.

(c) *Suppose we had not assumed the "single crossing property" in part (a). Would you have been able to answer the question "Who owns which car" assuming everything else remained the same?*

Answer: No, you would not have been able to answer the question. The ambiguity that arises when indifference curves from different indifference maps can cross more than once is depicted in panel (c) of Exercise Graph 4.9. Here, person B's (bold) indifference curve is shallower at the Camry than person A's just as described in the problem. However, person A's indifference curve takes a sharp turn at some point to the right of the Camry while person B's continues at roughly the same slope. Thus, B's indifference curve ends up below the minivan (making the minivan better for B than the Camry) while person A's indifference curve ends up above the minivan (making the Camry better for him than the minivan). Thus, once we allow multiple crossing of indifference curves from different indifference maps, it becomes ambiguous who is driving which car.

(d) *Suppose you are currently person B and you just found out that your uncle has passed away and bequeathed to you his 3 children, aged 4, 6 and 8 (and nothing else). This results in a change in how you value space and maneuverability. Is your new MRS at the Toyota Camry now larger or smaller (in absolute value)?*

Answer: You would now be wiling to sacrifice more speed and maneuverability for an increase in interior cabin space — which means the slope of your indifference curve at the Camry should get steeper. Thus, the MRS will increase in absolute value.

(e) *What are some other features of cars that might matter to consumers but that you could not fit easily into a 2-dimensional graphical model?*

Answer: You could think of many other car features: the quality of the upholstery, the shape of the seats, the color of the exterior and interior, whether there is a sun-roof, the quality of the speakers on the stereo system, the degree to which each passenger can control air temperature, the size of the engine, etc.

B: *Let x_1 denote cubic feet of interior space and let x_2 denote maneuverability as defined in part A. Suppose that the tastes of persons A, B and C can be represented by the utility functions $u^A(x_1,x_2) = x_1^{\alpha}x_2$, $u^B(x_1,x_2) = x_1^{\beta}x_2$ and $u^C(x_1,x_2) = x_1^{\gamma}x_2$ respectively.*

(a) *Calculate the MRS for each person.*

Answer: The MRS for person A is

$$MRS^A = -\frac{\partial u^A/\partial x_1}{\partial u^A/\partial x_2} = -\frac{\alpha x_1^{\alpha-1}x_2}{x_1^{\alpha}} = -\alpha\frac{x_2}{x_1}. \quad (4.9.i)$$

Similarly, $MRS^B = -\beta x_2/x_1$ and $MRS^C = -\gamma x_2/x_1$.

(b) *Assuming α, β and γ take on different values, is the "single crossing property" defined in part A(b) satisfied?*

Answer: Pick any product characteristic bundle $(\overline{x}_1, \overline{x}_2)$. Consider individual A and individual B and how their MRS's are related to one another at that bundle by dividing one MRS by the other; i.e.

$$\frac{MRS^A}{MRS^B} = \frac{-\alpha\overline{x}_2/\overline{x}_1}{-\beta\overline{x}_2/\overline{x}_1} = \frac{\alpha}{\beta}. \tag{4.9.ii}$$

Now, it does not matter what bundle $(\overline{x}_1, \overline{x}_2)$ I use, the above equation tells me that the MRS^A is always equal to α/β times the MRS^B. Thus, any indifference curve from A's indifference map can cross any indifference curve from B's indifference map only once. If that were not the case, (as in panel (c) of the graph), the relationship between the slopes of the indifference curves would have to be different at the second crossing — but we have just concluded that this relationship is the same everywhere. The same of course holds for any other pair of individuals from our group of persons A, B and C.

(c) *Given the description of the three persons in part A(b), what is the relationship between α, β and γ?*

Answer: Since A's indifference curve at any product characteristic bundle is steeper than B's and B's is steeper than C's, it must be that $\alpha > \beta > \gamma$.

(d) *How could you turn your graphical model into a mathematical model that includes factors you raised in part A(e)?*

Answer: All that's required is that the utility function includes more product characteristics. So, if we identify n different product characteristics that matter to consumers, we would model their tastes as represented by a utility function $u(x_1, x_2, ..., x_n)$ where x_i is the ith product characteristic.

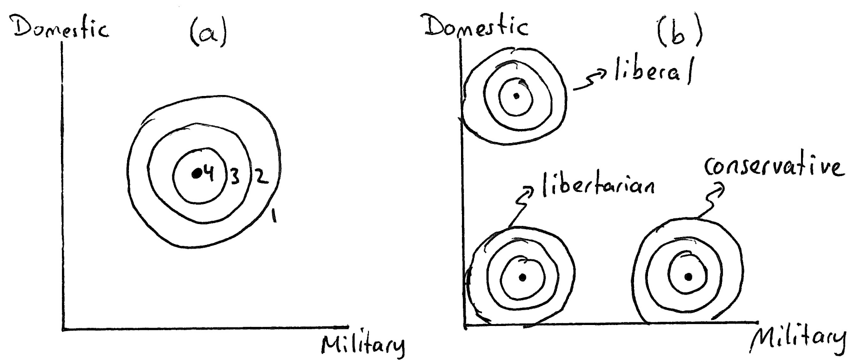
Exercise 4.11

Policy Application: *Ideology and Preferences of Politicians*: *Political scientists often assume that politicians have tastes that can be thought of in the following way: Suppose that the two issues a politician cares about are domestic spending and military spending. Put military spending on the horizontal axis and domestic spending on the vertical axis. Then each politician has some "ideal point" — some combination of military and domestic spending that makes him/her happiest.*

A: *Suppose that a politician cares only about how far the actual policy bundle is from his ideal point, not the direction in which it deviates from his ideal point.*

(a) *On a graph, pick any arbitrary "ideal point" and illustrate what 3 indifference "curves" would look like for such a politician. Put numerical labels on these to indicate which represent more preferred policy bundles.*

Answer: The first panel in Exercise Graph 4.11(1) illustrates an example of such indifference curves. The ideal point is at the center of concentric circles, with circles farther way from the ideal point representing policy bundles with less and less utility. Since distance from the ideal point is all that matters, the indifference "curves" should be circles with the ideal point at their center.



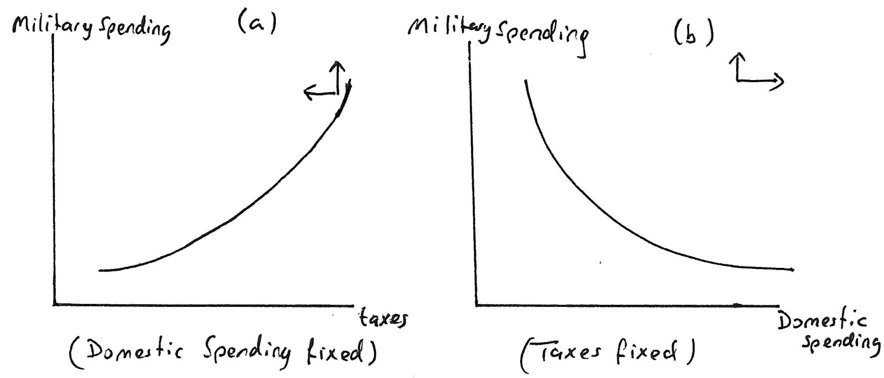
Exercise Graph 4.11(1) : Ideology and Political Tastes

(b) *On a separate graph, illustrate how tastes would be different for a political conservative (who likes a lot of military spending but is not as keen on domestic spending), a liberal (who likes domestic spending but is not as interested in military spending) and a libertarian (who does not like government spending in any direction to get very large).*

Answer: This is illustrated in the second panel of Exercise Graph 4.11(1). The politician's ideology determines the location of his ideal point, with ideal points lining up as described in the problem. Indifference "curves" will then again be concentric circles with each ideal point at the center of the circular indifference curves.

(c) *This way of graphing political preferences is a short-cut because it incorporates directly into tastes the fact that there are taxes that have to pay for government spending. Most politicians would love to spend increasingly more on everything, but they don't because of the increasing political cost of having to raise taxes to fund spending. Thus, there are really 3 goods we could be modeling: military spending, domestic spending and taxes, where a politician's tastes are monotone in the first two goods but not in the last. First, think of this as three goods over which tastes satisfy all our usual assumptions* — including monotonicity *and convexity — where we define the goods as spending on military, spending on domestic goods and the "relative absence of taxes". What would indifference "curves" for a politician look like in a 3-dimensional graph? Since it is difficult to draw this, can you describe it in words and show what a 2-dimensional slice looks like if it holds one of the goods fixed?*

Answer: The indifference "curves" in this 3-dimensional graph would be bowl-shaped, with the tip of the bowl facing the origin. Along any slice that holds one of the goods fixed, the shape would be the usual shape of an indifference curve in 2 dimensions as, for example, that depicted in panel (b) of Exercise Graph 4.11(2).



Exercise Graph 4.11(2) : Ideology and Political Tastes: Part 2

(d) *Now suppose you model the same tastes, but this time you let the third good be defined as "level of taxation" rather than "relative absence of taxes". Now monotonicity no longer holds in one dimension. Can you now graph what a slice of this 3-dimensional indifference surface would look like if it holds domestic spending fixed and has taxes on the horizontal and military spending on the vertical axis? What would a slice look like that holds taxes fixed and has domestic spending on the horizontal and military spending on the vertical axis?*

Answer: The indifference surface would still be bowl shaped but would now point toward the far end of the tax axis. The slice with military spending on the vertical and taxes on the horizontal is graphed in panel (a) of Exercise Graph 4.11(2) where the politician becomes better off with less taxes and more military spending. The slice with military spending on the vertical and domestic spending on the horizontal axis is illustrated in panel (b) — and looks like an ordinary indifference curve since taxes are fixed along the slice.

(e) *Pick a point on the slice that holds taxes fixed. How does the MRS at that point differ for a conservative from that of a liberal?*

Answer: The slope at that point would be shallower for a conservative than for a liberal because a conservative is willing to give up less military spending to get one more dollar of domestic spending. So, in absolute value, the conservative's MRS is smaller than the liberal's.

(f) *Pick a point on the slice that holds domestic spending fixed. How would the MRS at that point differ for a libertarian compared to a conservative?*

Answer: Libertarians would need to get a lot more military spending to justify one more unit of taxation while conservatives would need less.

Thus, libertarians would have a steeper slope — i.e. a higher MRS (in absolute value).

B: *Consider the following equation* $u(x_1, x_2) = P - \left((x_1 - a)^2 + (x_2 - b)^2\right)$.

(a) *Can you verify that this equation represents tastes such as those described in this problem (and graphed in part A(a))?*

Answer: Along any indifference curve, the utility level is constant. Consider one such indifference curve with utility constant at $\overline{u}$. This can then be written as

$$P - \overline{u} = (x_1 - a)^2 + (x_2 - b)^2. \qquad (4.11.i)$$

which is the equation of a circle with center (a, b) and radius $(P - \overline{u})^{1/2}$. At the ideal point $(x_1, x_2) = (a, b)$, utility is at its peak P. As x_1 deviates in either direction (with $x_2 = b$), utility declines by $(x_1 - a)^2$. For instance, if x_1 deviates in either direction by 1, utility declines to $(P - 1)$, and if x_1 deviates by 2 in either direction, utility declines to $(P-4)$. The same is true for deviations of x_2 in either direction (holding $x_1 = a$). And the same holds for any deviation from (a, b) in directions that involve changes in both x_1 and x_2. Thus, utility declines from its peak in relation to a policy bundle's distance from the ideal point (a, b).

(b) *What would change in this equation as you model conservative, liberal and libertarian politicians?*

Answer: Conservatives would have $a > b$ and liberals $b > a$. Libertarians would have low values of a relative to those of conservatives and low levels of b relative to liberals.

(c) *Do these tastes satisfy the convexity property?*

Answer: Yes, they do. To see this, take any two points on an indifference circle. The line connecting those two points lies in the region of policy bundles that are better than those on the indifference circle. Thus, averages of policy bundles that the politician is indifferent between are better than extremes.

(d) *Can you think of a way to write a utility function that represents the tastes you were asked to envision in A(c) and A(d)? Let t represent the tax rate with an upper bound of 1.*

Answer: To turn the tax from a "bad" to a "good", we can define it as the "relative absence of a tax" by writing it as $(1 - t)$. We can then treat $(1 - t)$ just like any other good, writing the utility function, for instance, as $u(x_1, x_2, t) = x_1^{\alpha} x_2^{\beta} (1 - t)^{\gamma}$ where α, β and γ are just numbers on the real line.

Exercise 4.13

In this exercise, we will explore some logical relationships between families of tastes that satisfy different assumptions.

A: *Suppose we define a strong and a weak version of convexity as follows: Tastes are said to be* strongly convex *if, whenever a person with those tastes is indifferent between A and B, she strictly prefers the average of A and B (to A and B). Tastes are said to be* weakly convex *if, whenever a person with those tastes is indifferent between A and B, the average of A and B is at least as good as A and B for that person.*

(a) *Let the set of all tastes that satisfy strong convexity be denoted as SC and the set of all tastes that satisfy weak convexity as WC. Which set is contained in the other? (We would, for instance, say that "WC is contained in SC" if any taste that satisfies weak convexity also automatically satisfies strong convexity.)*

Answer: Suppose your tastes satisfy the strong convexity condition. Then you always strictly prefer averages to extremes (where the extremes are such that you are indifferent between them). That automatically means that the average between such extremes is *at least as good as* the extremes — which means that your tastes automatically satisfy weak convexity. Thus, the set SC must be fully contained within the set WC.

(b) *Consider the set of tastes that are contained in one and only one of the two sets defined above. What must be true about some indifference curves on any indifference map from this newly defined set of tastes?*

Answer: We already concluded above that all strongly convex tastes are also weakly convex. So tastes that are strongly convex cannot be in the newly defined set because they appear in both SC and WC — and we are defining our new set to contain tastes that are only in one of these sets. The newly defined set therefore contains only tastes that satisfy weak convexity but not strong convexity. The only difference between weak and strong convexity is that the former permits averages to be just as good as extremes while the latter insists that averages are strictly better than extremes. When an average is just as good as two extremes from the same indifference curve, it must be that the line connecting the extremes is all part of the same indifference curve. Thus, some indifference curves in a weakly convex indifference map that lies outside SC must have "flat spots" that are line segments.

(c) *Suppose you are told the following about 3 people: Person 1 strictly prefers bundle A to bundle B whenever A contains more of each and every good than bundle B. If only some goods are represented in greater quantity in A than in B while the remaining goods are represented in equal quantity, then A is at least as good as B for this person. Such tastes are often said to be* weakly monotonic. *Person 2 likes bundle A strictly better than B whenever at least some goods are represented in greater quantity in A than in B while others may be represented in equal quantity. Such tastes are said to be* strongly monotonic. *Finally, person 3's tastes are such that, for every bundle A, there always exists a bundle B very close to A that is strictly better than A. Such tastes are said to satisfy* local nonsatiation. *Call the set of tastes that satisfy strict monotonicity SM, the set of tastes that satisfy weak* *monotonicity WM, and the set of tastes that satisfy local non-satiation L. What is the relationship between these sets? Put differently, is any set contained in any other set?*

Answer: If your tastes satisfy strong monotonicity, it means that A is strictly preferred to B even if A and B are identical in every way except that A has more of one good than B. This means that your tastes would automatically satisfy weak monotonicity — because weak monotonicity only requires that A is at least as good under that condition and thus permits indifference between A and B unless all goods are more highly represented in A than in B. All strongly monotone tastes are weakly monotone, which means SM is fully contained in WM. Local non-satiation only requires that, for every bundle A, there exists some bundle B close to A such that B is preferred to A. If your tastes satisfy strong monotonicity, then we know such a bundle always exists: Begin at some A and then add a tiny bit of every good to A to form B. As long as we add a tiny bit to all goods, strong monotonicity says B is strictly better than A. The same works for weakly monotonic tastes. Thus, both SM and WM are fully contained in L. But there are also tastes in L such that these tastes are not in WM. Consider tastes where at some bundle A there are no bundles with more goods close to A that are preferred to A but there is a bundle with slightly fewer goods that is preferred to B. Then such tastes would satisfy local non-satiation but not weak (or strong) convexity.

(d) *What is true about tastes that fall in one and only one of these three sets?*

Answer: Since we have just concluded that SM is contained in WM which is contained in L, such tastes must satisfy local non-satiation but not weak monotonicity. Consider tastes over labor and consumption. We would generally like to expend less labor and have more consumption. Such tastes are not strongly or weakly monotonic because A is strictly less preferred to B if A contains the same amount of consumption but more labor. But they do satisfy local non-satiation because for every A, we can make the person better off through less labor or more consumption (or both).

(e) *What is true of tastes that are in one and only one of the two sets SM and WM?*

Answer: Since SM is contained in WM, such tastes must be weakly monotonic. (If they were strongly monotonic, they would be contained in both sets). Consider bundles A and B that are identical in every way except that A has more of one of the goods than B. For tastes to be weakly monotonic but not strongly monotonic, it must be that there exists such an A and B and that a person with such tastes is indifferent between A and B. (If such a person strictly preferred all such A bundles to all such B bundles, her tastes would be strongly monotonic.) Thus, tastes that fall in WM but not SM must have some indifference curves with either horizontal or vertical "flat spots".

B: *Below we will consider the logical implications of convexity for utility functions. For the following definitions,* $0 \le \alpha \le 1$*. A function* $f : \mathbb{R}^2_+ \to \mathbb{R}^1$ *is defined to be* quasiconcave *if and only if the following is true: Whenever* $f(x_1^A, x_2^A) \le f(x_1^B, x_2^B)$*, then* $f(x_1^A, x_2^A) \le f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right)$*. The same type of function is defined to be* concave *if and only if* $\alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) \le f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right)$*.*

(a) True or False*: All concave functions are quasiconcave but not all quasiconcave functions are concave.*

Answer: True. Suppose we start with a concave function f. Then

$$\alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) \le f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right). \tag{4.13.i}$$

Now suppose that $f(x_1^A, x_2^A) \le f(x_1^B, x_2^B)$. Then it must be true that

$$f(x_1^A, x_2^A) \le \alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B). \tag{4.13.ii}$$

But that implies that whenever $f(x_1^A, x_2^A) \le f(x_1^B, x_2^B)$, then

$$f(x_1^A, x_2^A) \le f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right) \tag{4.13.iii}$$

— which is the definition of a quasi-concave function. Thus, *concavity of a function implies quasi-concavity.*

But the reverse does not have to hold. Suppose that when $\alpha = 0.5$, $f(x_1^A, x_2^A) = 10$, $f(x_1^B, x_2^B) = 100$ and $f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right) = 20$. The condition for quasi-concavity is satisfied — so suppose f is in fact quasi-concave throughout. Notice, however, that $\alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) = 0.5(10) + (0.5)100 = 55$. Thus,

$$20 = f\left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right) < \alpha f(x_1^A, x_2^A) + (1-\alpha) f(x_1^B, x_2^B) = 55, \tag{4.13.iv}$$

which directly violates concavity.

An example of a function that is quasi-concave but not concave is $u(x_1, x_2) = x_1^2 x_2^2$.

(b) *Demonstrate that, if u is a quasiconcave utility function, the tastes represented by u are convex.*

Answer: Tastes are convex if averages of bundles over which we are indifferent are better than those bundles. Suppose tastes are represented by u and u is quasiconcave. Pick $A = (x_1^A, x_2^A)$ and $B = (x_1^B, x_2^B)$ such that $u(x_1^A, x_2^A) = u(x_1^B, x_2^B)$. Let bundle C be some weighted average between A and B; i.e.

$$C = (x_1^C, x_2^C) = \left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right). \tag{4.13.v}$$

Then quasiconcavity of u implies that

$$u(x_1^A, x_2^A) \leq u(x_1^C, x_2^C), \tag{4.13.vi}$$

which tells us that the average bundle C is at least as good as the extreme bundles A and B (since $u(x_1^A, x_2^A) = u(x_1^B, x_2^B)$) that the individual is indifferent between. Thus, *quasiconcavity of the utility function implies convexity of underlying tastes represented by that utility function.*

(c) *Do your conclusions above imply that, if u is a concave utility function, the tastes represented by u are convex?*

Answer: Since we concluded in (a) that all concave functions are quasiconcave, and since we concluded in (b) that all quasiconcave utility functions represent tastes that satisfy convexity, it must be that all concave utility functions also represent tastes that are convex.

(d) *Demonstrate that, if tastes over two goods are convex, any utility functions that represents those tastes must be quasiconcave.*

Answer: Suppose we consider bundle $A = (x_1^A, x_2^A)$ and $B = u(x_1^B, x_2^B)$ over which an individual with convex tastes is indifferent. Any utility function that represents these tastes must therefore be such that $u(x_1^A, x_2^A) = u(x_1^B, x_2^B)$ which makes the statement

$$u(x_1^A, x_2^A) \leq u(x_1^B, x_2^B) \tag{4.13.vii}$$

also true (since the inequality is weak). Now define a weighted average C of bundles A and B; i.e.

$$C = (x_1^C, x_2^C) = \left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right). \tag{4.13.viii}$$

Convexity of tastes implies that C is at least as good as A. Thus, any utility function that represents these tastes must be such that

$$u(x_1^A, x_2^A) \leq u(x_1^C, x_2^C). \tag{4.13.ix}$$

We have therefore concluded that the utility function representing convex tastes must be such that, whenever $u(x_1^A, x_2^A) \leq u(x_1^B, x_2^B)$, then

$$u(x_1^A, x_2^A) \leq \left(\alpha x_1^A + (1-\alpha)x_1^B, \alpha x_2^A + (1-\alpha)x_2^B\right), \tag{4.13.x}$$

which is the definition of a quasiconcave function. Thus, *convexity of tastes implies quasiconcavity of any utility function that represents those tastes.*(We actually showed that this statement holds when $u^A = u^B$ — but the same reasoning holds when $u^A < u^B$.)

(e) *Do your conclusions above imply that, if tastes over two goods are convex, any utility function that represents those tastes must be concave?*

Answer: No. We have concluded that convexity of tastes implies quasiconcavity of utility functions and we have shown in (a) that there are quasiconcave utility functions that are *not* concave. So the fact that convexity is represented by quasiconcave utility functions does not imply that convexity requires concave utility functions. In fact it does not — it only requires quasiconcavity.

(f) *Do the previous conclusions imply that utility functions which are not quasiconcave represent tastes that are not convex?*

Answer: Yes. In (d) we showed that convexity *necessarily* means that utility functions will be quasiconcave. Thus, when utility functions are *not* quasiconcave, they cannot represent convex tastes. They must therefore represent non-convex tastes.

Conclusion: Potentially Helpful Reminders

1. Convexity in tastes is easy to recognize when "more is better" — but might be a bit confusing otherwise. Here is a simple trick to check whether the tastes you have drawn are convex: Use two arrows that have the same starting point and indicate which horizontal and vertical direction is "better" for the consumer. (When tastes are monotonic, these point to the right and up.) Convexity then implies that the indifference curves bend toward the corner of the arrows you have drawn. (Graph 4.3 in the answer to within-chapter exercise 4A.10 has an example of this. Another example is in Graph 4.6 in the answer to end-of-chapter exercise 4.11.)

2. It should be reasonably clear that tastes — how we *subjectively feel* about stuff — should not typically depend on prices (which only affect what we can *objectively afford*). Put differently, our *circumstances* are different from our *tastes*. But sometimes that gets a little hazy when circumstances other than the usual budget parameters matter. An example of this is given in end-of-chapter exercise 4.7 where we think of "air safety" as one of the circumstances a consumer cannot himself change.

3. Exercise 4.5 is a good exercise to prepare for some of the ideas that are coming up in Chapter 6 as well as later on in Chapter 16.

4. But the last two end-of-chapter exercises are relatively abstract and probably beyond the level of most (but not all) courses that use this text.