CHAPTER

5

Different Types of Tastes

While Chapter 4 introduced us to a general way of thinking about tastes, Chapter 5 gets much more specific and introduces particular dimensions along which we might differentiate between tastes. In particular, we differentiate tastes based on

1. The curvature of individual indifference curves — or how quickly the MRS changes *along an indifference curve*;

2. The relationships between indifference curves — or how the MRS changes *across indifference curves within an indifference map*; and

3. Whether or not indifference curves *cross horizontal or vertical axes* or whether they *converge to the axes*.

The first of these in turn determines the degree to which consumers are willing to substitute between goods (and will lead to what we call the "substitution effect" in Chapter 7) while the second of these determines how consumer behavior responds to changes in income (and will lead to what we call the "income effect" in Chapter 7). Finally, the third category of taste differences becomes important in Chapter 6 where we will see how corner versus interior optimal solutions for a consumer emerge.

Chapter Highlights

The main points of the chapter are:

1. The **degree of substitutability** or, in part B language, the **elasticity of substitution** for a consumer at a particular consumption bundle arises from the **curvature** of the indifference curve at that bundle. There may be no substitutability (as in perfect complements) or perfect substitutability (perfect substitutes) or an infinite number of cases in between these extremes.

2. **Quasilinearity** and **Homotheticity** of tastes represent special cases that describe how indifference curves from the same map relate to one another. These properties have no direct relationship to the concept of substitutability. Tastes are quasilinear in a good x if the MRS only depends on the level of x consumption (and not the level of other goods' consumption. Tastes are homothetic when the MRS depends only on the relative levels of the goods in a bundle.

3. Sometimes it is reasonable to assume that indifference curves only converge to the axes without ever crossing them; other times we assume that they cross the axes. When an indifference curve crosses an axis, it means that we can gain utility beyond what we have by not consuming even if we consume none of one of the goods. When indifference curves only converge to the axes, then some consumption of all goods is necessary in order for a consumer to experience utility above what she would experience by not consuming at all.

4. If you are reading part B of the chapter, you should begin to understand the family of **constant elasticity of substitution utility functions** — with perfect complements, perfect substitutes and Cobb-Douglas tastes as special cases. You should also be able to demonstrate whether a utility function is homothetic or quasilinear. (Most utility functions we use in this text tend to be one or the other.)

5A Solutions to Within-Chapter-Exercises for Part A

Exercise 5A.1

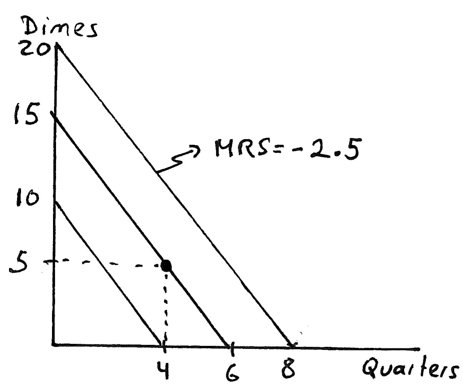
How would the graph of indifference curves change if Coke came in 8 ounce cans and Pepsi came in 4 ounce cans?

Answer: The indifference curves would then have slope of −2 instead of −1 because you would be willing to trade 2 four ounce cans of Pepsi for 1 eight ounce can of Coke. For instance, the indifference curve that contains 1 can of Coke on the horizontal axis would also contain 2 cans of Pepsi on the vertical as well as half a can of Coke and 1 can of Pepsi. All those combinations contain 16 ounces of soft drink.

Exercise 5A.2

On a graph with "quarters" on the horizontal axis and "dimes" on the vertical, what might your indifference curves look like? Use the same method we just employed to graph my indifference curves for Coke and Pepsi — by beginning with one arbitrary bundle of quarters and dimes (say 4 quarters and 5 dimes) and then asking which other bundles might be just as good.

Answer: Dimes are worth 10 cents while quarters are worth 25 cents. Thus, you are willing to trade 2.5 dimes for 1 quarter. At 4 quarters and 5 dimes, you have \$1.50. Any other combination of dimes and quarters should be equally desirable. For instance, 15 dimes also make \$1.50, as do 6 quarters. Thus, the indifference curve through the bundle (4,5) has intercept 6 on the horizontal (quarters) axis and 15 on the vertical (dimes) axis. This gives it a slope of −2.5 which is in fact the rate at which we are willing to trade dimes for quarters. This (and two other) indifference curves are depicted in Exercise Graph 5A.2.



Exercise Graph 5A.2 : Tastes over Dimes and Quarters

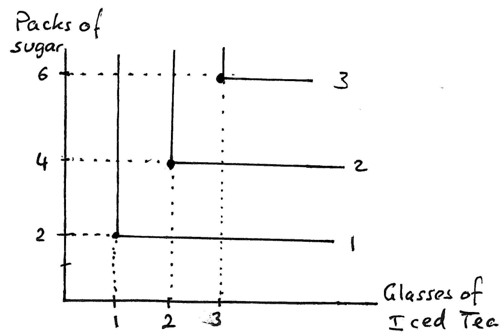
Exercise 5A.3

What would my wife's indifference curves for packs of sugar and glasses of iced tea look like if she required 2 instead of one packs of sugar for each glass of iced tea?

Answer: The corners of the indifference curves would now occur at bundles with twice as much sugar as iced tea. For instance, 1 glass of iced tea and 2 packets of sugar make a complete beverage, and no additional sugar and no additional iced tea will make her better off unless she gets more of both. This gives us the indifference curve labeled "1" in Exercise Graph 5A.3. Similarly, 2 glasses of iced tea and 4 packets of sugar make 2 complete beverages, giving the corner of the indifference curve labeled "2".

Exercise 5A.4

Suppose I told you that each of the indifference maps graphed in Graph 5.3 corresponded to my tastes for one of the following sets of goods, which pair would



Exercise Graph 5A.3 : 2 sugars for each iced tea

you think corresponds to which map? Pair 1: Levi Jeans and Wrangler Jeans; Pair 2: Pants and Shirts; Pair 3: Jeans and Dockers pants.

Answer: To answer this, we should ask which of the pairs represents goods that seem most substitutable for one another. I would think that would be Pair 1 since that includes two different types of jeans (which many of us probably can't even tell apart easily). Thus, I would think that panel (a) represents Pair 1. We could then ask which of the three pairs represent goods that are most complementary (or least substitutable). Of the remaining pairs, pants and shirts seems less substitutable than Jeans and Dockers pants. Thus Pair 2 — pants and shirts — would correspond to panel (c) where there is the least substitutability between the goods. This leaves panel (b) for Pair 3 — Jeans and Dockers pants.

Exercise 5A.5

Are my tastes over Coke and Pepsi as described in Section 5A.1 homothetic? Are my wife's tastes over iced tea and sugar homothetic? Why or why not?

Answer: Yes, both are homothetic. Homothetic tastes are tastes such that the *MRS* is the same along any ray from the origin. For perfect substitutes like Coke and Pepsi, the *MRS* is the same everywhere — which means it is certainly the same along any ray from the origin. For perfect complements like sugar and iced tea, it is easy to also see that the slope of the indifference curves does not change along any ray from the origin. Below the 45 degree line (when one pack of sugar goes with one iced tea), the indifference curve is flat along any ray from the origin; above the 45 degree line, the indifference curve is vertical along any ray from the origin. On the 45 degree line, there is no slope since this is where all the corners of the indifference curve lie. (Since the slope is technically undefined for parts of the indifference map for perfect complements, you can think of this instead as the limit of a sequence of indifference maps that graphs increasingly complementary goods — with each of the maps in the sequence having the characteristic that the *MRS* is unchanged along any ray from the origin.)

Exercise 5A.6

Are my tastes over Coke and Pepsi as described in Section 5A.1 quasilinear? Are my wife's tastes over iced tea and sugar quasilinear? Why or why not?

Answer: Tastes are quasilinear in the good on the horizontal axis if the *MRS* is unchanged along any vertical line emanating from the horizontal axis. (Alternatively, tastes are quasilinear in the good on the vertical axis if the *MRS* is unchanged along any horizontal line emanating from the vertical axis.) For perfect substitutes like Coke and Pepsi, the *MRS* is the same everywhere — which means it is certainly the same along any vertical or horizontal line. Thus, perfect substitutes are quasilinear in both goods. Perfect complements like tea and sugar, on the other hand, are not quasilinear in either good. Along any vertical line emanating from the horizontal axis, the indifference curve at some point changes from being horizontal to vertical. (The reverse is true for any horizontal line emanating from the vertical axis). You can also again think of the indifference maps that come closer and closer to those of perfect complements and treat perfect complements as the limiting case. For all maps that approach those of perfect complements, the slopes of indifference curves change along vertical and horizontal lines. Thus neither of the goods is quasilinear.

Exercise 5A.7

Can you explain why tastes for perfect substitutes are the only tastes that are both quasilinear and homothetic?

Answer: Quasilinearity implies that the *MRS* does not change along any vertical line emanating from the horizontal axis (or along any horizontal line emanating from the vertical axis). Homotheticity implies that the *MRS* is constant along any ray from the origin. Consider any vertical line emanating from the horizontal axis. All rays emanating from the origin pass through that line at some point. So if the *MRS* has to be the same along the vertical line and it has to he the same along rays from the origin, it must be that the *MRS* is the same everywhere. (The same is true if we instead considered a horizontal line emanating from the vertical axis when the good on the vertical axis is quasilinear). And the only tastes for which the *MRS* is the same everywhere are those of perfect substitutes.

Exercise 5A.8

True or False: Quasilinear goods are never essential.

Answer: The idea of an "essential" good is meant to capture the following: Is the good such that if I were to consume none of it, I might as well not consume any goods at all? The good on the horizontal axis is then *not* essential if indifference curves cross there vertical axis. (If they do cross the vertical axis, then I can consume none of the good on the horizontal axis and still get utility greater than I would if I consumed at the origin of the graph.) Quasilinear goods are typically of this type; that is they typically have indifference maps with curves crossing the axes, in which case they are not essential. I say typically, however, because there are mathematical subtleties I am neglecting. For instance, the utility function $u(x_1, x_2) = \ln x_1 + x_2$ has indifference curves that converge to the vertical axis but never cross it – but they do cross a vertical line at $x_1 = 1$ (and other vertical lines closer to $x_1 = 0$). It is therefore possible for a quasilinear good to be, strictly speaking, essential.

5B Solutions to Within-Chapter-Exercises for Part B

Exercise 5B.1

Calculate the same approximate elasticity of substitution for the indifference curve in Graph 5.7b.

Answer: The ratio (x_2/x_1) changes from $10/2 = 5$ to $8/4 = 2$. The percentage change in this ratio is therefore $-3/5 = -0.6$. The percentage change in the MRS is again 0.5. Thus, the elasticity of substitution is $(0.6/0.5) = 1.2$.

Exercise 5B.2

What numerical labels would be attached to the 3 indifference curves in Graph 5.1 by the utility function in equation (5.2)?

Answer: Each indifference curve would have the label equal to its vertical (or horizontal) intercept; i.e. 1 for the lowest, 2 for the middle and 3 for the highest indifference curve in the graph.

Exercise 5B.3

Suppose you measured coke in 8 ounce cans and Pepsi in 4 ounce cans. Draw indifference curves and find the simplest possible utility function that would give rise to those indifference curves.

Answer: Such indifference curves are drawn in Exercise Graph 5B.3 where the consumer is willing to trade 2 (4 oz) cans of Pepsi for 1 (8 oz) can of Coke — leading to slopes of −2 when Coke is graphed on the horizontal axis. You therefore get twice as much happiness from a can of Coke as from a can of Pepsi, which implies one way of representing these tastes is

$$u(x_1, x_2) = 2x_1 + x_2. \quad (5B.3.i)$$

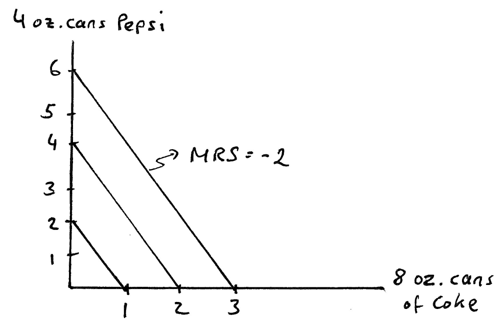
You can check that the MRS in this case is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{2}{1} = -2. \quad (5B.3.ii)$$

Exercise 5B.4

Can you use similar reasoning to determine the elasticity of substitution for the utility function you derived in exercise 5B.3?

Answer: The exact same reasoning holds for all indifference maps with linear indifference curves. Again, it is easiest to think of an indifference map that is close to linear everywhere — and then to think what happens as such an indifference



Exercise Graph 5B.3 : 8 oz Coke and 4 oz Pepsi

map approaches that of perfectly linear indifference curves. For indifference curves that are close to those with $MRS = -2$ everywhere, we can start at a bundle A with little x_1 and a lot x_2. Even a small change in the MRS will result in a large move down that indifference curve. Thus, the percentage change in the ratio of the goods (which is the numerator in the elasticity of substitution equation) is large for a small percentage change in the MRS (which is the denominator in the elasticity equation). In the limit, I can get larger and larger changes in this numerator with smaller and smaller changes in the denominator as the indifference curve gets closer and closer to being linear. Thus, in the limit the elasticity of substitution is ∞.

Exercise 5B.5

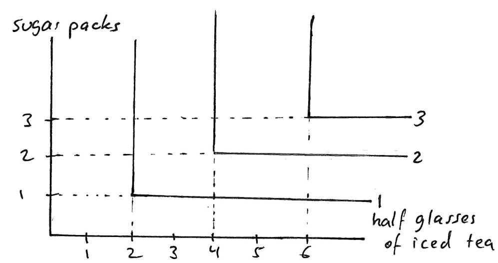
Plug the bundles $(3,1)$, $(2,1)$, $(1,1)$, $(1,2)$ and $(1,3)$ into this utility function and verify that each is shown to give the same "utility" — thus lying on the same indifference curve as plotted in Graph 5.2. What numerical labels does this indifference curve attach to each of the 3 indifference curves in Graph 5.2?

Answer: In each of these bundles, the utility function $u(x_1,x_2) = \min\{x_1,x_2\}$ picks the lower of the two quantities and assigns that number as the utility level of that bundle. Since the lower of the two values in each of these bundles is 1, each is assigned a utility value of 1. The values assigned to the three indifference curves are 1, 2 and 3 — the values of the vertical and horizontal coordinates at the corners of each indifference curve.

Exercise 5B.6

How would your graph and the corresponding utility function change if we measured iced tea in "half glasses" instead of glasses.

Answer: In that case, the perfect beverage requires 1 pack of sugar for every 2 units (half glasses) of iced tea. Any more sugar for 2 units of iced tea would add no further utility unless more tea was added as well, and more tea for 1 pack of sugar would not add more utility unless more sugar was added as well. Thus, the indifference curves representing the same tastes as before would look as in Exercise Graph 5B.6, with the corner points now lying on a ray from the origin that lies below the 45 degree line. A utility function that results in the labeling of the indifference curves that arises in this graph is $u(x_1,x_2) = \min\{0.5x_1, x_2\}$.



Exercise Graph 5B.6 : Half Glasses of tea and full packs of sugar

Exercise 5B.7

Can you determine intuitively what the elasticity of substitution is for the utility function you defined in exercise 5B.6?

Answer: It is again easiest to do this for tastes that are very close to those we graphed in Exercise Graph 5B.6 but without the sharp kink. Pick A a bit above the ray on which the corners of the indifference curves lie — with the ratio of x_1/x_2 just above 0.5. Then imagine moving to a shallower slope of the indifference curve that contains A. Because of the large curvature of the indifference curve around the ray that connects the corners of the indifference curves, even a relatively large change in the MRS will not cause us to have to slide very far along the indifference curve — implying a relatively modest change in the ratio x_1/x_2. Thus, for a large percentage change in the MRS (which is the denominator in the elasticity equation), we get a relatively small change in the ratio x_1/x_2 (which is the denominator in the elasticity equation.) As the indifference curve gets closer and closer to that of perfect complements, the percentage change in the consumption good ratio will fall for any percentage change in the MRS — and will approach 0 as the indifference curve approaches that of perfect complements. Thus, the numerator in the elasticity equation approaches zero — leaving us with an elasticity of substitution of zero in the limit.

Exercise 5B.8

Demonstrate that the functions u and v both give rise to indifference curves that exhibit the same shape by showing that the MRS for each function is the same.

Answer: The MRS of $v = x_1^{\alpha}x_2^{1-\alpha}$ is

$$MRS^v = -\frac{\partial v/\partial x_1}{\partial v/\partial x_2} = -\frac{\alpha x_1^{\alpha-1} x_2^{1-\alpha}}{(1-\alpha)x_1^{\alpha} x_2^{-\alpha}} = -\frac{\alpha x_2}{(1-\alpha)x_1}, \qquad (5B.8.i)$$

and, since $\alpha = \gamma/(\gamma+\delta)$, this can also be written as

$$MRS^v = -\frac{\alpha x_2}{(1-\alpha)x_1} = -\frac{(\gamma/(\gamma+\delta))x_2}{(1-\gamma/(\gamma+\delta))x_1} = -\frac{(\gamma/(\gamma+\delta))x_2}{(\delta/(\gamma+\delta))x_1} = -\frac{\gamma x_2}{\delta x_1}. \qquad (5B.8.ii)$$

The MRS of the function $u(x_1, x_2) = x_1^{\gamma} x_2^{\delta}$ is

$$MRS^u = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{\gamma x_1^{\gamma-1} x_2^{\delta}}{\delta x_1^{\gamma} x_2^{\delta-1}} = -\frac{\gamma x_2}{\delta x_1}. \qquad (5B.8.iii)$$

Thus, $MRS^v = MRS^u$, which implies the indifference curves arising from the two utility functions are identical.

Exercise 5B.9

Derive the MRS for the Cobb-Douglas utility function and use it to show what happens to the slope of indifference curves along the 45-degree line as α changes.

Answer: The MRS for the Cobb-Douglas function which is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{\alpha x_1^{(\alpha-1)} x_2^{(1-\alpha)}}{(1-\alpha)x_1^{\alpha} x_2^{-\alpha}} = -\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{x_2}{x_1}\right). \qquad (5B.9)$$

Along the 45-degree line, $x_1 = x_2$ — which implies $x_2/x_1 = 1$ and the MRS along the 45 degree line is simply $-\alpha/(1-\alpha)$. Thus, when $\alpha = 0.5$, the MRS along the 45 degree line is exactly -1. When $\alpha > 0.5$, the MRS on the 45 degree line is greater than 1 in absolute value, and when $\alpha < 0.5$, the MRS is less than 1 in absolute value along the 45 degree line.

Exercise 5B.10

What is the elasticity of substitution in each panel of Graph 5.10?

Answer: The elasticity of substitution for CES utility functions is $\sigma = 1/(1+\rho)$. Thus, the $\rho = -0.8$ in panel (a) translates to $\sigma = 5$; the $\rho = -0.2$ in panel (b) translates to $\sigma = 1.25$; and the $\rho = 2$ in panel (c) translates to $\sigma = 0.33$.

Exercise 5B.11

Can you describe what happens to the slopes of the indifference curves on the 45 degree line, above the 45 degree line and below the 45 degee line as ρ becomes large (and as the elasticity of substitution therefore becomes small)?

Answer: The slopes of the indifference curves are described by the MRS which is given by

$$MRS = -\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{x_2}{x_1}\right)^{\rho+1}. \qquad (5B.11)$$

First, consider bundles on the 45-degree line where $x_1 = x_2$ and thus $x_2/x_1 = 1$. In this case, the second term in the equation remains 1 as ρ gets large — and the MRS therefore stays constant at $-\alpha/(1-\alpha)$.

Next, consider a bundle above the 45 degree line — i.e. a bundle such that $x_1 < x_2$. This implies that $x_2/x_1 > 1$ — which means the second term in the MRS equation increases as ρ gets large. Thus, as ρ gets large, the slope of indifference curves above the 45-degree line become steeper (approaching vertical lines as ρ approaches infinity.)

Finally, suppose we consider a bundle below the 45 degree line — i.e. a bundle such that $x_1 > x_2$. This implies $x_2/x_1 < 1$ — which implies that the second term in the MRS equation decreases as ρ gets large. Thus, the slopes of indifference curves get shallower below the 45 degree line (approaching horizontal lines as ρ approaches infinity).

Thus, as ρ approaches infinity (and as the elasticity of substitution therefore approaches 0), the slopes of indifference curves along the 45 degree line remain unchanged while they flatten out below the 45 degree line and straighten up above the 45 degree line. In other words, as ρ gets large, the shape of the indifference curves approach those of perfect complements.

Exercise 5B.12

On the "Exploring Relationships" animation associated with Graph 5.10, develop an intuition for the role of the α parameter in CES utility functions and compare those to what emerges in Graph 5.9.

Answer: No particular answer here — the animated version should illustrate how changing α alters the shapes of indifference curves in ways that should seem familiar from our Cobb-Douglas example in the text.

Exercise 5B.13

Show that, when we normalize the exponents of the Cobb-Douglas utility function to sum to 1, the function is homogeneous of degree 1.

Answer: Using the utility function $u(x_1,x_2) = x_1^{\alpha}x_2^{(1-\alpha)}$,

$$u(tx_1,tx_2) = (tx_1)^{\alpha}(tx_2)^{(1-\alpha)} = t^{\alpha}x_1^{\alpha}t^{(1-\alpha)}x_2^{(1-\alpha)} = tx_1^{\alpha}x_2^{(1-\alpha)} = tu(x_1,x_2). \qquad (5B.13)$$

Exercise 5B.14

Consider the following variant of the CES function that will play an important role in producer theory: $f(x_1,x_2) = \left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-\beta/\rho}$. Show that this function is homogeneous of degree β.

Answer:

$$f(tx_1,tx_2) = \left(\alpha(tx_1)^{-\rho} + (1-\alpha)(tx_2)^{-\rho}\right)^{-\beta/\rho} =$$
$$= \left(t^{-\rho}(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho})\right)^{-\beta/\rho} = t^{\beta}\left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-\beta/\rho} =$$
$$= t^{\beta} f(x_1,x_2). \qquad (5B.14)$$

Exercise 5B.15

Can you demonstrate, using the definition of a homogeneous function, that it is generally possible to transform a function that is homogeneous of degree k to one that is homogeneous of degree 1 in the way suggested above?

Answer: Suppose a function $f(x_1,x_2)$ is homogeneous of degree k. Then this implies that $f(tx_1,tx_2) = t^k f(x_1,x_2)$ for any $t > 0$. Now consider the function

$$v(x_1,x_2) = \left(f(x_1,x_2)\right)^{1/k}. \qquad (5B.15.i)$$

Then

$$v(tx_1,tx_2) = \left(f(tx_1,tx_2)\right)^{1/k} = \left(t^k f(x_1,x_2)\right)^{1/k}$$
$$= t\left(f(x_1,x_2)\right)^{1/k} = tv(x_1,x_2). \qquad (5B.15.ii)$$

Thus, $v(tx_1,tx_2) = tv(x_1,x_2)$ which is the definition of a function that is homogeneous of degree 1.

Exercise 5B.16

Use the mathematical expression for quasilinear tastes to illustrate that neither good is essential if tastes are quasilinear in one of the goods.

Answer: If tastes are quasilinear in x_1, then we can represent them by a function

$$u(x_1,x_2) = v(x_1) + x_2. \qquad (5B.16)$$

At the bundle (0,0), this would result in utility of $u(0,0) = v(0)$. If the consumer consumes $(x_1,0)$ — i.e. if she consumes only x_1 but no x_2, her utility is $u(x_1,0) = v(x_1)$ which is greater than $v(0)$ which she gets by consuming nothing. Thus, the consumer can get more utility by consuming only x_1 than she could by consuming nothing — which implies that x_2 is not essential. Similarly, if she consumes a bundle $(0,x_2)$ — i.e. if she consumes only x_2 and no x_1, she gets utility $u(0,x_2) = v(0) + x_2$ which is also greater than $u(0,0) = v(0)$. Thus, x_1 is not essential.

Exercise 5B.17

Show that both goods are essential if tastes can be represented by Cobb-Douglas utility functions.

Answer: Suppose tastes can be represented by $u(x_1,x_2) = x_1^{\alpha}x_2^{(1-\alpha)}$. Then the utility from consuming (0,0) is $u(0,0) = 0$. Now consider the utility from a bundle $(x_1,0)$ — i.e. a bundle with no x_2 consumption. Utility from such a bundle is $u(x_1,0) = x_1^{\alpha}(0) = 0$ — exactly what it is when the consumer doesn't consume anything at all. Thus, x_2 is essential. By similar reasoning, x_1 is essential.

Exercise 5B.18

Can you demonstrate similarly that $\sigma = 1$ for the Cobb-Douglas utility function $u(x_1,x_2) = x_1^{\alpha}x_2^{(1-\alpha)}$?

Answer: We know from our previous work that the MRS of a Cobb-Douglas utility function of this type is $MRS = -(\alpha x_2)/((1-\alpha)x_1)$. Taking absolute values of both sides and solving for (x_2/x_1), we get

$$\frac{x_2}{x_1} = \frac{(1-\alpha)}{\alpha}|MRS|, \qquad (5B.18.i)$$

and taking logs,

$$\ln\frac{x_2}{x_1} = \ln|MRS| + \ln\frac{(1-\alpha)}{\alpha}. \qquad (5B.18.ii)$$

We can then apply the elasticity formula from the appendix to get

$$\sigma = \frac{d\ln(x_2/x_1)}{d\ln|MRS|} = 1. \qquad (5B.18.iii)$$

5C Solutions to Odd Numbered End-of-Chapter Exercises

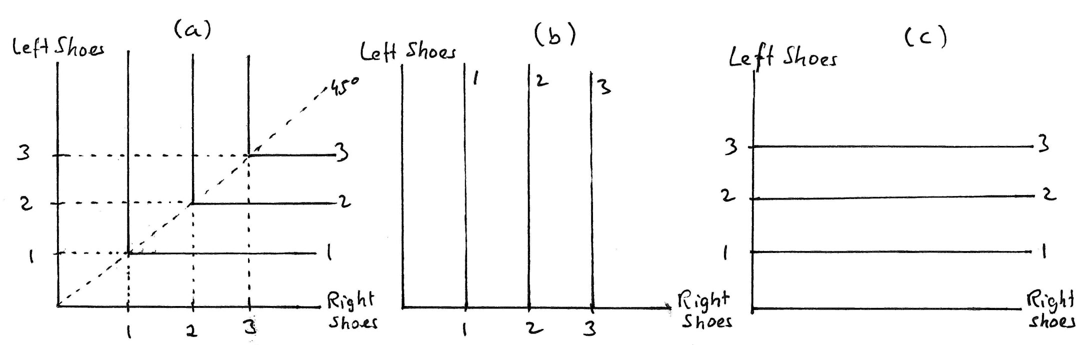
Exercise 5.1

Consider your tastes for right and left shoes.

A: *Suppose you, like most of us, are the kind of person that is rather picky about having the shoes you wear on your right foot be designed for right feet and the shoes you wear on your left foot be designed for left feet. In fact you are so picky that you would never wear a left shoe on your right foot or a right shoe on your left foot — nor would you ever choose (if you can help it) not to wear shoes on one of your feet.*

(a) *In a graph with the number of right shoes on the horizontal axis and the number of left shoes on the vertical, illustrate three indifference curves that are part of your indifference map.*

Answer: Panel (a) of Exercise Graph 5.1 illustrates the three indifference curves corresponding to the utility you get from 1 pair of shoes, 2 pair of shoes and 3 pair of shoes. Right and left shoes are perfect complements.



Exercise Graph 5.1 : Right Shoes and Left Shoes

(b) *Now suppose you hurt your left leg and have to wear a cast (which means you cannot wear shoes on your left foot) for 6 months. Illustrate how the indifference curves you have drawn would change for this period. Can you think of why goods such as left shows in this case are called* neutral goods*?*

Answer: Panel (b) of Exercise Graph 5.1 illustrates such indifference curves. For any given number of right shoes, utility would not change as you get more left shoes since you have no use for left shoes. The only way to get to higher utility is to increase right shoes. Goods like left shoes in this example are sometimes called *neutral goods* because you do not care one way or another if you have any of them.

(c) *Suppose you hurt your right foot instead. How would this change your answer to part (b).*

Answer: This is illustrated in panel (c) of Exercise Graph 5.1. Now you can only become better off by getting more left shoes, but getting more right shoes (for any level of left shoes) does nothing to change your utility.

(d) *Are any of the tastes you have graphed homothetic? Are any quasilinear?*

Answer: All 3 are homothetic — the slopes (to the extent to which these are defined) of the indifference curves in all three maps are the same along any ray from the origin. The panel (a) perfect complements case is not quasilinear because, for any quantity of right shoes, the "slope" changes from perfectly horizontal to perfectly vertical at some level of left shoes. And for any quantity of left shoes, the "slope" changes from perfectly vertical to perfectly horizontal at some level of right shoes. But the tastes in panels (b) and (c) are quasilinear in both goods — along any horizontal and vertical line, the "slope" remains the same. You can view the latter two as the limit cases of perfect substitutes. For instance, in panel (c) we could add a slight negative slope to the indifference curves, and we would then have indifference curves with the same MRS everywhere. Put differently, we'd have perfect substitutes where we are willing to trade very small numbers of left shoes for many right shoes. Then imagine a sequence of such indifference maps, with each indifference curve in the sequence having a slope that is half the slope of the previous one. Every indifference map in that sequence is similarly one of perfect substitutes with constant MRS, and the limit of that sequence is the indifference map depicted in panel (c).

(e) *In the three different tastes that you graphed, are any of the goods ever "essential"? Are any not essential?*

Answer: A good is essential if there is no way to attain utility greater than what one would attain at the origin without consuming at least some of that good. In panel (a), both goods are therefore essential — because you have to consume the goods in pairs in order to get any utility from consuming either. In panel (b), right shoes are essential but left shoes are not, and in panel (c) left shoes are essential but right shoes are not.

B: *Continue with the description of your tastes given in part A above and let x_1 represent right shoes and let x_2 represent left shoes.*

(a) *Write down a utility function that represents your tastes as illustrated in A(a). Can you think of a second utility function that also represents these tastes?*

Answer: This is just a case of perfect complements — so the utility function $u(x_1, x_2) = \min\{x_1, x_2\}$ would be one that works for representing these tastes. So would a function $v(x_1, x_2) = \alpha \min\{x_1, x_2\}$ for any $\alpha > 0$, or $w(x_1, x_2) = (\min\{x_1, x_2\})^{\beta}$ for any $\beta > 0$, or any number of other transformations that don't alter the ordering of indifference curves.

(b) *Write down a utility function that represents your tastes as graphed in A(b).*

Answer: Since only right shoes (x_1) matter, utility cannot vary with the number of left shoes (x_2). A function like $u(x_1, x_2) = x_1$ would therefore suffice.

(c) *Write down a utility function that represents your tastes as drawn in A(c).*

Answer: Since only left shoes (x_2) matter, utility cannot vary with the number of right shoes (x_1). A function like $u(x_1, x_2) = x_2$ would therefore suffice.

(d) *Can any of the tastes you have graphed in part A be represented by a utility function that is homogeneous of degree 1? If so, can they also be represented by a utility function that is not homogeneous?*

Answer: A function $u(x_1, x_2)$ is homogeneous of degree 1 if $u(tx_1, tx_2) = tu(x_1, x_2)$. The utility function $u(x_1, x_2) = \min\{x_1, x_2\}$ in our answer to part B(a), for instance, is homogeneous of degree 1 because

$$u(tx_1, tx_2) = \min\{tx_1, tx_2\} = t\min\{x_1, x_2\} = tu(x_1, x_2). \quad (5.1.i)$$

Similarly, the functions $u(x_1, x_2) = x_1$ from part B(b) and $u(x_1, x_2) = x_2$ from part B(c) are homogeneous of degree 1. Each of these three functions can be turned into a function that is not homogeneous by simply adding a constant. Adding such a constant does not change the underlying shape of indifference curves — and so it does not alter the kinds of tastes that we are modeling. But, for instance, $f(x_1, x_2) = \alpha + \min\{x_1, x_2\}$ is such that

$$f(tx_1, tx_2) = \alpha + \min\{tx_1, tx_2\} \neq t^k\alpha + t^k\min\{x_1, x_2\} = t^k f(x_1, x_2) \quad (5.1.ii)$$

for any $k > 0$.

(e) *Refer to end-of-chapter exercise 4.13 where the concepts of "strong monotonicity," "weak monotonicity" and "local non-satiation" were defined. Which of these are satisfied by the tastes you have graphed in this exercise?*

Answer: All satisfy local non-satiation because for any bundle, there is always another bundle close by that is more preferred. All satisfy weak monotonicity — because for any bundle, adding more of one of the goods is at least as good as the original bundle. But they don't satisfy strong monotonicity — because in each case there is a way to add more of one good to a bundle without making the individual strictly better off.

(f) *Refer again to end-of-chapter exercise 4.13 where the concepts of "strong convexity" and "weak convexity" were defined. Which of these are satisfied by the tastes you have graphed in this exercise?*

Answer: All satisfy weak convexity because, for any two bundles on a given indifference curve, any weighted average of the bundles (which lies on a line connecting the two bundles) is at least as good as the more extreme bundles. They do not satisfy strong convexity because in each case we can find two bundles that lie on a line segment of the indifference curves — and for those bundles, weighted averages are not strictly better than the extremes.

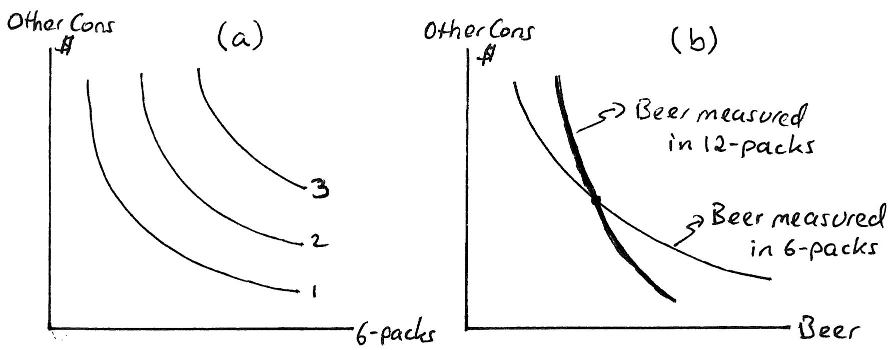
Exercise 5.3

Beer comes in six and twelve-packs. In this exercise we will see how your model of tastes for beer and other consumption might be affected by the units in which we measure beer.

A: *Suppose initially that your favorite beer is only sold in six-packs.*

(a) *On a graph with beer on the horizontal axis and other consumption (in dollars) on the vertical, depict three indifference curves that satisfy our usual five assumptions assuming that the units in which beer is measured is six-packs.*

Answer: An example of 3 such indifference curves is depicted in panel (a) of Exercise Graph 5.3



Exercise Graph 5.3 : Six and 12-packs of Beer

(b) *Now suppose the beer company eliminates six-packs and sells all its beer in twelve-packs instead. What happens to the MRS at each bundle in your graph if 1 unit of beer now represents a twelve-pack instead of a six-pack.*

Answer: At every bundle, you would now be willing to give up twice as many dollars of other consumption for one more unit of beer than you were before — because one more unit of beer is twice as much beer as it was before. Thus, the *MRS* has to be twice as large in absolute value at every consumption bundle.

(c) *In a second graph, illustrate one of the indifference curves you drew in part (a). Pick a bundle on that indifference curve and then draw the indifference curve through that bundle assuming we are measuring beer in twelve-packs instead. Which indifference curve would you rather be on?*

Answer: In panel (b) of Exercise Graph 5.3, this is illustrated — with the indifference curve that measures beer in 12-packs having twice the slope in absolute value as the indifference curve that measures beer in 6-packs. You would of course rather be on the indifference curve with beer measured in 12 packs.

(d) *Does the fact that these indifference curves cross imply that tastes for beer change when the beer company switches from 6-packs to 12-packs?*

Answer: No. The shape of indifference curves on any indifference map is determined in part by the units used to measure quantities of the goods. The two indifference maps from which the indifference curves in panel (b) arise represent the same tastes if they have the same *MRS* adjusted for the units used to measure beer — i.e. if beer is measured in units twice as large, the *MRS* at every bundle has to be twice as large in absolute value.

B: *Let x_1 represent beer and let x_2 represent dollars of other consumption. Suppose that, when x_1 is measured in units of six-packs, your tastes are captured by the utility function $u(x_1, x_2) = x_1 x_2$.*

(a) *What is the MRS of other goods for beer?*

Answer: The *MRS* is

$$MRS = -\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = -\frac{x_2}{x_1} \tag{5.3.i}$$

(b) *What does the MRS have to be if x_1 is measured in units of 12-packs?*

Answer: As we argued already, the *MRS* has to be twice as large in absolute value since now you would be willing to pay twice as much for one more unit of x_1 since it is measured in units twice as large.

(c) *Give a utility function that represents your tastes when x_1 is measured in 12-packs and check to make sure it has the MRS you concluded it must have.*

Answer: The utility function $v(x_1, x_2) = x_1^2 x_2$ would be one function that could represent tastes over 12-packs of beer. The *MRS* of this function is

$$MRS = -\frac{\partial v/\partial x_1}{\partial v/\partial x_2} = -\frac{2x_1 x_2}{x_1^2} = -2\frac{x_2}{x_1}, \tag{5.3.ii}$$

which is twice as large in absolute value as the *MRS* of the original utility function u.

(d) *Can you use this example to explain why it is useful to measure the substitutability between different goods using percentage terms (as in the equation for the elasticity of substitution) rather than basing it simply on the absolute value of slopes at different bundles?*

Answer: The units used to measure goods affect the way that indifference curves look, but they don't affect the underlying tastes represented by those indifference curves. If a measure of substitutability were to use the absolute value of slopes at different bundles, the choice of units would partly determine the value of our measure of substitutability. But by using percentage changes instead of absolute changed in the formula for the elasticity of substitution, the units cancel — and our measure becomes independent of the units. For instance, the utility function we derived in the previous part for the case where we measure beer in 12-packs is Cobb-Douglas just as the utility function we used to measure those same tastes when beer was measured in 6-packs. We know that all Cobb-Douglas utility functions have elasticity of substitution of 1 — and so we know we have not changed the elasticity of substitution when we altered the units used to measure one of the goods. Thus, we have defined in the elasticity of substitution a measure of substitutability that is immune to the units chosen to measure the goods on each axis.

Exercise 5.5

Everyday Application: *Personality and Tastes for Current and Future Consumption: Consider two brothers, Eddy and Larry, who, despite growing up in the same household, have grown quite different personalities.*

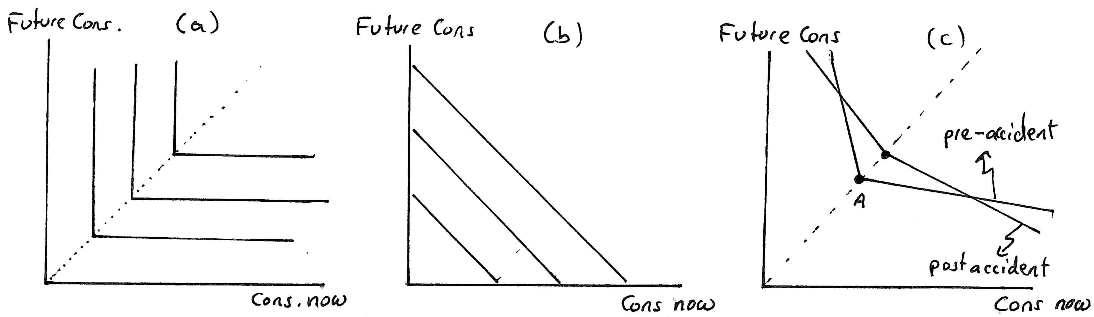
A: *Eddy is known to his friends as "steady Eddy" — he likes predictability and wants to know that he'll have what he has now again in the future. Larry, known to his friends as "crazy Larry", adapts easily to changing circumstances. One year he consumes everything around him like a drunken sailor, the next he retreats to a Buddhist monestary and finds contentment in experiencing poverty.*

(a) *Take the characterization of Eddy and Larry to its extreme (within the assumptions about tastes that we introduced in Chapter 4) and draw two indifference maps with "current consumption" on the horizontal axis and "future consumption" on the vertical — one for steady Eddy and one for crazy Larry.*

Answer: The description indicates that Eddy does not trade off consumption across time very easily while Larry does. In the extreme, that would mean that consumption now and consumption in the future are perfect complements for Eddy and perfect substitutes for Larry. (A less extreme version would have consumption now and consumption in the future be closer to perfect complements for Eddy than for Larry.) The extreme indifference maps for Eddy and Larry are drawn in panels (a) and (b) (respectively) of Exercise Graph 5.5.

(b) *Eddy and Larry have another brother named Daryl who everyone thinks is a weighted average between his brothers' extremes. Suppose he is a lot more like steady Eddy than he is like crazy Larry — i.e. he is a weighted average between the two but with more weight placed on the Eddy part of his personality. Pick a bundle A on the 45 degree line and draw a plausible indifference curve for Daryl through A. (If you take the above literally in a certain way, you would get a kink in Daryl's indifference curve.) Could his tastes be homothetic?*

Answer: His indifference curves would be flatter than Eddy's but not as flat as Larry's, and since he is more like Eddy, they would look more like Eddy's. One plausible such indifference curve — labeled "pre-accident"



Exercise Graph 5.5 : Steady Eddy, Crazy Larry and Unstable Daryl

— through a bundle A on the 45 degree line is drawn in panel (c) of the graph. The indifference curve has a kink at A because at A it is unclear what it would mean to "average" the indifference maps. A less literal interpretation of the problem might not have a kink at that point — but would have a somewhat smoother version of an indifference curve like the one graphed here. Both Eddy's and Larry's indifference maps are homothetic — and an average between their indifference maps should also be homothetic. In panel (c), the other indifference curves would contain parallel line segments emanating from the 45 degree line — and the MRS would therefore be the same along any ray from the origin. The same can easily be true of indifference maps without the sharp kink on the 45 degree line. This illustrates that homotheticity of tastes can allow for many different degrees of substitutability.

(c) *One day Daryl suffers a blow to his head — and suddenly it appears that he is more like crazy Larry than like steady Eddy; i.e. the weights in his weighted average personality have flipped. Can his tastes still be homothetic?*

Answer: Yes, they would simply have indifference curves with line segments flatter than the indifference curve through bundle A — indifference curves like the one labeled "post-accident" in panel (c) of the graph. This would continue to satisfy the homotheticity condition. This would also hold for smoother versions of the indifference curves — i.e. versions that don't have a kink point on the 45 degree line.

(d) *In end-of-chapter exercise 4.9, we defined what it means for two indifference maps to satisfy a "single crossing property". Would you expect that Daryl's pre-accident and post-accident indifference maps satisfy that property?*

Answer: No, they would not. This is easily seen in panel (c) of the graph where the pre- and post-accident indifference curve cross twice. (Note that this conclusion also is not dependent on the kink in the indifference curves.)

(e) *If you were told that either Eddy or Larry saves every month for retirement and the other smokes a lot, which brother is doing what?*

Answer: I would guess that a person who views consumption across time as not very substitutable would make sure to save so that he can consume at the same levels when he stops earning income. At the same time, someone who views consumption now and in the future substitutable might be willing to enjoy a lot of smoking now even if it decreases the quality of life later.

B: *Suppose that one of the brothers' tastes can be captured by the function* $u(x_1, x_2) = \min\{x_1, x_2\}$ *where* x_1 *represents dollars of current consumption and* x_2 *represents dollars of future consumption.*

(a) *Which brother is it?*

Answer: It's steady Eddy — since he is not willing to trade consumption across time periods and thus has indifference curves that treat consumption now and consumption in the future as perfect complements (or something close to it).

(b) *Suppose that when people say that Daryl is the weighted average of his brothers, what they mean is that his elasticity of substitution of current for future consumption lies in between those of his borthers. If Larry and Daryl have tastes that could be characterized by one (or more) of the utility functions from end-of-chapter exercise 4.5, which functions would apply to whom?*

Answer: Crazy Larry's would be perfect substitutes — which are given by utility function (2) in problem 4.5. By looking at MRS's and the ordering of indifference curves, we concluded in the answer to problem 4.5 that the utility functions (1) and (4) represented the same Cobb-Douglas tastes. Cobb-Douglas tastes are members of the family of CES tastes — which have perfect complements and perfect substitutes at the extremes. Thus, one way of thinking about Daryl being the average of his brothers would be to think of his elasticity of substitution being in some sense in between those of his brothers'. In that case, Cobb-Douglas tastes seem plausible tastes for Daryl. Note that is a different notion of what it might mean for Daryl to be the weighted average of his brothers from that which resulted in a kink point in the indifference curves in our graph.

(c) *Which of the functions in end-of-chapter exercise 4.5 are homothetic? Which are quasilinear (and in which good)?*

Answer: When the MRS depends only on the ratio of x_2 to x_1, then this means that it is the same for bundle A as it is for any bundle that multiplies the goods in A by the same constant — i.e. it is the same along any ray from the origin. Utility functions (1), (4) and (5) all have this feature and thus all represent homothetic tastes. Furthermore, utility function (2) gives rise to the same MRS everywhere — so it, too, represents tastes that are homothetic. Utility function (3), on the other hand, has MRS that depends only on x_2 — which means that, if me multiply both goods in a bundle A, we end up getting a different MRS. Thus, utility function (3) is not homothetic.

For a utility function to represent tastes that are quasilinear, the MRS has to be independent of one of the two goods. This is true for utility function (3) where the MRS depends only on x_2. Thus, for any level of x_2, the MRS is unchanged regardless of how much x_1 is in the bundle. Put differently, we can draw a horizontal line in our graph with x_2 on the vertical axis and know that the MRS along that line is constant. Thus, utility function (3) represents tastes quasilinear in x_2. Finally, utility function (2) has the same MRS everywhere and is thus quasilinear in both goods. The other utility functions have MRS varying with both goods and are therefore not quasilinear.

(d) *Despite being so different, is it possible that both steady Eddy and crazy Larry have tastes that can be represented by Cobb Douglas utility functions?*

Answer: No, that does not seem plausible since the description of steady Eddy and crazy Larry clearly indicates very different elasticities of substitution between current and future consumption — and all Cobb-Douglas preferences have elasticity of substitution of 1.

(e) *Is it possible that all their tastes could be represented by CES utility functions? Explain.*

Answer: Yes, this is possible since CES utility functions encompass functions ranging from elasticity of substitution of 0 (perfect complements) to ∞ (perfect substitutes). The essential difference between the three brothers is their elasticity of substitution between current and future consumption — and the CES family of utility functions gives the flexibility to allow that to vary completely across individuals.

Exercise 5.7

Everyday Application: *Tastes for Paperclips*: *Consider my tastes for paperclips and "all other goods" (denominated in dollar units).*

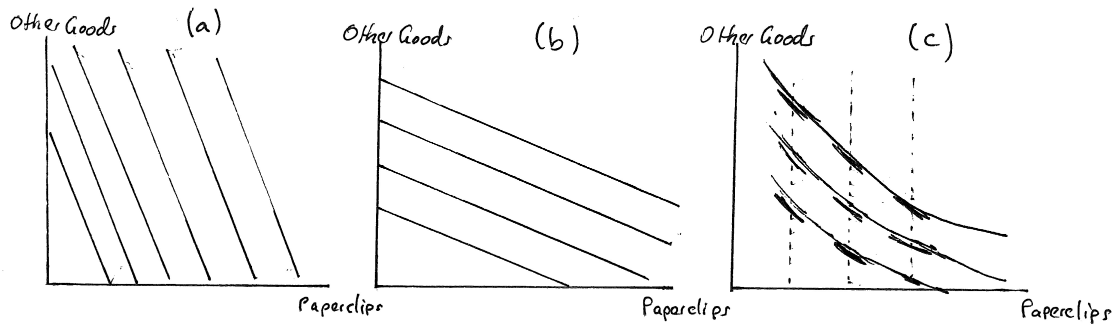
A: *Suppose that my willingness to trade paper clips for other goods does* not *depend on how many other goods I am also currently consuming.*

(a) *Does this imply that "other goods" are "essential" for me?*

Answer: This means that, plotting paperclips on the horizontal axis, the MRS is unchanged along any vertical line we draw — which in turn means that the indifference curves must cross the paperclip axis. When an indifference curve crosses the paperclip axis, it means that I am able to get utility greater than at the origin without consuming any "other good". Thus, other goods are not essential for me.

(b) *Suppose that, in addition, my willingness to trade paperclips for other goods does not depend on how many paperclips I am currently consuming. On two graphs, each with paperclips on the horizontal axis and "dollars of* *other goods" on the vertical, give two examples of what my indifference curves might look like.*

Answer: If my *MRS* does not depend on either my level of paperclip consumption or my level of other good consumption, it means that my *MRS* is the same everywhere. Thus, paperclips and other goods are perfect substitutes — and the only thing we are not sure about is at what rate I am willing to substitute one for the other. Thus, we can get different maps of indifference curves where those maps differ in terms of the *MRS* that holds everywhere within each map. Two such indifference maps are illustrated in panels (a) and (b) of Exercise Graph 5.7.



Exercise Graph 5.7 : Paperclips and other Goods

(c) *How much can the MRS vary* within *an indifference map that satisfies the conditions in part (b)? How much can it vary* between two *indifference maps that both satisfy the conditions in part (b)?*

Answer: As we just concluded, the *MRS* cannot vary *within* an indifference map under these conditions — but it can vary *across* indifference maps that satisfy these conditions.

(d) *Now suppose that the statement in (a) holds for my tastes but the statement in part (b) does not. Illustrate an indifference map that is consistent with this.*

Answer: Tastes are now simply quasilinear in paperclips — and if they are not also quasilinear in other goods, we have ruled out the case of perfect substitutes. An example of such an indifference map is illustrated in panel (c) of Exercise Graph 5.7.

(e) *How much can the MRS vary* within *an indifference map that satisfies the conditions of part (d)?*

Answer: The *MRS* can range from zero to minus infinity within the same map.

(f) *Which condition do you think is more likely to be satisfied in someone's tastes — that the willingness to trade paperclips for other goods is independent of the level of paperclip consumption or that it is independent of the level of other goods consumption?*

Answer: It seems more realistic to assume that our MRS is independent of how much in other goods we are consuming. Paperclips are a small part of our overall consumption bundle — and as we consume more of other goods (because, for instance, our income is increasing), it seems unlikely that we will change how we feel about paperclips on the margin. We have only so much need for paperclips, though — so as we get more paperclips in our consumption bundle, we are probably less willing to pay the same amount for more paperclips as we were willing to pay for the original paperclips that made it into the consumption bundle. So it seems unlikely that our MRS is independent of the level of paperclip consumption.

(g) *Are any of the indifference maps above homothetic? Are any of them quasilinear?*

Answer: The maps in panels (a) and (b) are homothetic and quasilinear, while the map in panel (c) is only quasilinear (in paperclips).

B: *Let paperclips be denoted by x_1 and other goods by x_2.*

(a) *Write down two utility functions, one for each of the indifference maps from which you graphed indifference curves in A(b).*

Answer: Consider the utility function $u(x_1,x_2) = \alpha x_1 + x_2$. When $\alpha = 1$, we have the case of perfect substitutes where the consumer is willing to always trade the goods one for one. More generally, the consumer is willing to trade α of x_2 for one x_1. Thus, when $\alpha < 1$, the consumer is willing to trade less than one unit of x_2 for one unit of x_1 — implying shallower indifference curves as in panel (b); and when $\alpha > 1$, the consumer is willing to trade more than one unit of x_2 for one unit of x_1 — implying steeper indifference curves as in panel (a). You can also see this by simply deriving the MRS as $(-\alpha)$.

(b) *Are the utility functions you wrote down homogeneous? If the answer is no, could you find utility functions that represent those same tastes and are homogeneous? If the answer is yes, could you find utility functions that are not homogeneous but still represent the same tastes?*

Answer: Yes, the utility function $u(x_1,x_2) = \alpha x_1 + x_2$ is homogeneous. In fact, it is homogeneous of degree 1 since

$$u(tx_1,tx_2) = \alpha(tx_1) + tx_2 = t(\alpha x_1 + x_2) = tu(x_1,x_2). \tag{5.7.i}$$

You can transform this into a non-homogeneous utility function by simply adding a constant — say 10 — to get $v(x_1,x_2) = \alpha x_1 + x_2 + 10$. Then

$$v(tx_1,tx_2) = \alpha(tx_1) + tx_2 + 10 \neq t(\alpha x_1 + x_2) + 10t = t^k v(x_1,x_2) \tag{5.7.ii}$$

for all $k > 0$.

(c) *Are the functions you wrote down homogeneous of degree 1? If the answer is no, could you find utility functions that are homogeneous of degree 1 and represent the same tastes? If the answer is yes, could you find utility functions that are not homogeneous of degree k and still represent the same tastes?*

Answer: Equation (5.7.i) demonstrates that the function $u(x_1, x_2) = \alpha x_1 + x_2$ is homogeneous of degree 1. You can turn this into a function that is homogeneous of degree k by taking it to the kth power; i.e. $w(x_1, x_2) = (u(x_1, x_2))^k = (\alpha x_1 + x_2)^k$. Then

$$w(tx_1, tx_2) = (\alpha(tx_1) + tx_2)^k = (t(\alpha x_1) + x_2))^k = t^k(\alpha(x_1) + x_2)^k = t^k w(x_1, x_2). \quad (5.7.iii)$$

(d) *Is there any indifference map you could have drawn when answering A(d) which can be represented by a utility function that is homogeneous? Why or why not?*

Answer: No. Homogeneous functions have the property that they give rise to homothetic indifference maps — with the MRS constant along any ray from the origin. The indifference map in A(d) is quasilinear — and the only way that it can be both quasilinear and homothetic is for the goods to be perfect substitutes. But A(d) specifically ruled out linear indifference curves.

Exercise 5.9

Everday Application: *Syllabi-Induced Tastes over Exam Grades*: *Suppose you are taking two classes, economics and physics. In both classes, only two exams are given during the semester.*

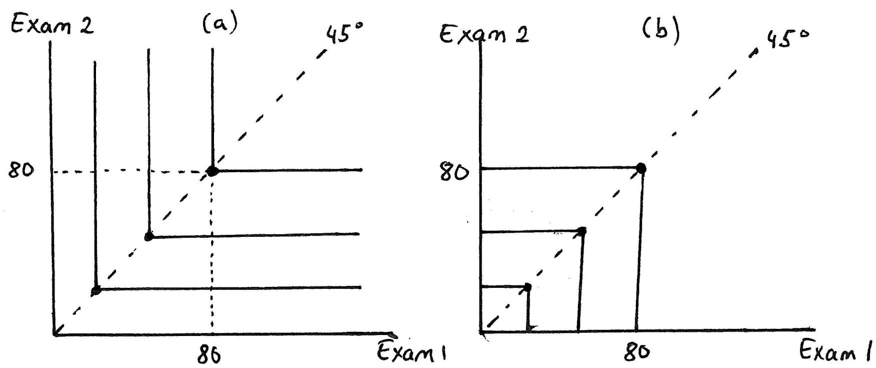
A: *Since economists are nice people, your economics professor drops the lower exam grade and bases your entire grade on the higher of the two grades. Physicists are another story. Your physics professor will do the opposite — he will drop your highest grade and base your entire class grade on your lower score.*

(a) *With the first exam grade (ranging from 0 to 100) on the horizontal axis and the second exam grade (also ranging from 0 to 100) on the vertical, illustrate your indifference curves for your physics class.*

Answer: These are illustrated in panel (a) of Exercise Graph 5.9. Since the physics professor will only count the lower score, only the lower score matters for generating utility. Thus, the two exam scores are perfect complements — the only way to get more utility is for the minimum score to increase.

(b) *Repeat this for your economics class.*

Answer: Since the economics professor is only counting the higher score, only the higher score matters for utility. Thus, if you have an 80, for instance, on your first exam, your utility will remain the same if your second exam is an 80 or less. Similarly, if your second exam is an 80, your



Exercise Graph 5.9 : Physics and Economics Exams

first exam does not matter if it is less than or equal to 80. One of your indifference curves therefore is the point (80,80) as well as the bundles (x,80) and (80,x) for all $x \le 80$. This forms one of the indifference curves in panel (b) of Exercise Graph 5.9, with the remaining indifference curves in the graph similarly derived.

(c) *Suppose all you care about is your final grade in a class and you otherwise value all classes equally. Consider a pair of exam scores* (x_1, x_2) *and suppose you knew before registering for a class what that pair will be — and that it will be the same for the economics and the physics class. What must be true about this pair in order for you to be indifferent between registering for economics and registering for physics?*

Answer: The fact that you care only about your final grade and you value all classes equally means that getting an 80 in your economics class means exactly as much to you as getting an 80 in your physics class. The highest indifference curve in panel (a) of the graph illustrates all the different ways of getting an 80 in your physics class. Similarly, the highest indifference curve in panel (b) of the graph illustrates all the different ways of getting an 80 in the economics class. Thus, any of the pairs of grades on the relevant indifference curve in panel (a) is just as good as any of the pairs of grades on the relevant indifference curve in panel (b). But those indifference curves only share one pair of grades in common — the pair (80,80). The same reasoning holds for any other pair of exam grades that might make you indifferent between the two classes. Thus, it must be that $x_1 = x_2$.

B: *Consider the same scenario as the one described in part A.*

(a) *Give a utility function that could be used to represent your tastes as you described them with the indifference curves you plotted in A(a)?*

Answer: The simples example of such a utility function is $u(x_1, x_2) = \min\{x_1, x_2\}$.

(b) *Repeat for the tastes as you described them with the indifference curves you plotted in A(b).*

Answer: Now it is the maximum, not the minimum, grade that matters. So the corresponding utility function could take the form $u(x_1, x_2) = \max\{x_1, x_2\}$.

Exercise 5.11

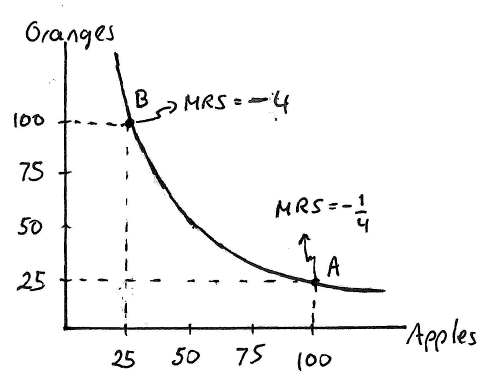
In this exercise, we are working with the concept of an elasticity of substitution. This concept was introduced in part B of the Chapter. Thus, this entire question relates to material from part B, but the A-part of the question can be done simply by knowing the formula for an elasticity of substitution while the B-part of the question requires further material from part B of the Chapter. In Section 5B.1, we defined the elasticity of substitution as

$$\sigma = \left| \frac{\%\Delta(x_2/x_1)}{\%\Delta MRS} \right|. \tag{5.11}$$

A: *Suppose you consume only apples and oranges. Last month, you consumed bundle A=(100,25) — 100 apples and 25 oranges, and you were willing to trade at most 4 apples for every orange. Two months ago, oranges were in season and you consumed B=(25,100) and were willing to trade at most 4 oranges for 1 apple. Suppose your happiness was unchanged over the past two months.*

(a) *On a graph with apples on the horizontal axis and oranges on the vertical, illustrate the indifference curve on which you have been operating these past two months and label the MRS where you know it.*

Answer: This is illustrated in Exercise Graph 5.11.



Exercise Graph 5.11 : Elasticity of Subsitution

(b) *Using the formula for elasticity of substitution, estimate your elasticity of substitution of apples for oranges.*

Answer: This is

$$\sigma = \left| \frac{((100/25) - (25/100))/(100/25)}{(-4 - (-1/4))/(-4)} \right| = \left| \frac{(15/4)/4}{(15/4)/4} \right| = 1. \tag{5.11.i}$$

(c) *Suppose we know that the elasticity of substitution is in fact the same at every bundle for you and is equal to what you calculated in (b). Suppose the bundle C=(50,50) is another bundle that makes you just as happy as bundles A and B. What is the MRS at bundle C?*

Answer: Using B and C in the elasticity of substitution formula, setting σ equal to 1 and letting the MRS at C be denoted by x , we get

$$\left|\frac{((100/25)-(50/50))/(100/25)}{(-4-x)/(-4)}\right| = \left|\frac{3/4}{(4+x)/4}\right| = 1, \qquad (5.11.ii)$$

and solving this for x, we get $x = -1$ — i.e. the MRS at C is equal to -1.

(d) *Consider a bundle $D = (25,25)$. If your tastes are homothetic, what is the MRS at bundle D?*

Answer: Since it, like bundle C, lies on the 45 degree line, homotheticity implies the MRS is again -1.

(e) *Suppose you are consuming 50 apples, you are willing to trade 4 apples for one orange and you are just as happy as you were when you consumed at bundle D. How many oranges are you consuming (assuming the same elasticity of substitution)?*

Answer: Let the number of oranges be denoted y. Using the bundle $(50, y)$ and $D = (25,25)$ in the elasticity formula and setting it to 1, we get

$$\left|\frac{((50/y)-(25/25))/(50/y)}{(-4-(-1))/(-4)}\right| = \left|\frac{((50/y)-1)/(50/y)}{(3/4)}\right| = 1. \qquad (5.11.iii)$$

Solving this for y, we get $y = 12.5$.

(f) *Call the bundle you derived in part (e) E. If the elasticity is as it was before, at what bundle would you be just as happy as at E but would be willing to trade 4 oranges for 1 apple?*

Answer: If the elasticity is 1 from D to E and is again supposed to be 1 from D to this new bundle, there must be symmetry around the 45 degree line (as there was between A and B). At $E = (50,12.5)$, the MRS is $-1/4$, and the necessary symmetry then means that $MRS = -4$ at (12.5,50).

B: *Suppose your tastes can be summarized by the utility function $u(x_1, x_2) = \left(\alpha x_1^{-\rho} + (1-\alpha)x_2^{-\rho}\right)^{-1/\rho}$.*

(a) *In order for these tastes to contain an indifference curve such as the one containing bundle A that you graphed in A(a), what must be the value of ρ? What about α?*

Answer: The elasticity of substitution for the CES utility function can be written as $\sigma = 1/(1+\rho)$. We already determined that the elasticity of substitution in this problem is 1. Thus, $1 = 1/(1+\rho)$ which implies $\rho = 0$. Since our graph is symmetric around the 45 degree line, it must furthermore be true that $\alpha = 0.5$ — i.e. x_1 and x_2 enter symmetrically into the utility function.

(b) *Suppose you were told that the same tastes can be represented by* $u(x_1, x_2) = x_1^\gamma x_2^\delta$. *In light of your answer above, is this possible? If so, what has to be true about* γ *and* δ *given the symmetry of the indifference curves on the two sides of the 45 degree line?*

Answer: Yes — it is possible because we determined that the elasticity of substitution is 1 everywhere, which is true for Cobb-Douglas utility functions of the form $u(x_1, x_2) = x_1^\gamma x_2^\delta$. The symmetry implies $\gamma = \delta$.

(c) *What exact value(s) do the exponents* γ *and* δ *take if the label on the indifference curve containing bundle A is 50? What if that label is 2,500? What if the label is 6,250,000?*

Answer: If the utility at A is 50, it means $50^\gamma 50^\delta = 50$. Since we just concluded in (a) that $\gamma = \delta$, this implies that $\gamma = \delta = 0.5$. If the utility is 2,500, then $\gamma = \delta = 1$, and if the utility is 6,250,000, $\gamma = \delta = 2$.

(d) *Verify that bundles A, B and C (as defined in part A) indeed lie on the same indifference curve when tastes are represented by the three different utility functions you implicitly derived in B(c). Which of these utility functions is homogeneous of degree 1? Which is homogeneous of degree 2? Is the third utility function also homogeneous?*

Answer: The bundles are A=(100,25), B=(25,100) and C=(50,50). The following equations hold, verifying that these must be on the same indifference curve for each of the three utility functions: $u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$, $v(x_1, x_2) = x_1 x_2$ and $w(x_1, x_2) = x_1^2 x_2^2$:

$$\begin{aligned} u(100,25) &= u(25,100) = u(50,50) = 50 \\ v(100,25) &= v(25,100) = v(50,50) = 2,500 \\ w(100,25) &= w(25,100) = w(50,50) = 6,250,000. \end{aligned} \tag{5.11.iv}$$

The following illustrate the homogeneity properties of the three functions:

$$\begin{aligned} u(tx_1, tx_2) &= (tx_1)^{0.5}(tx_2)^{0.5} = tx_1^{0.5} x_2^{0.5} = tu(x_1, x_2) \\ v(tx_1, tx_2) &= (tx_1)(tx_2) = t^2 x_1 x_2 = t^2 v(x_1, x_2) \\ w(tx_1, tx_2) &= (tx_1)^2 (tx_2)^2 = t^4 x_1^2 x_2^2 = t^4 w(x_1, x_2). \end{aligned} \tag{5.11.v}$$

Thus, u is homogeneous of degree 1, v is homogeneous of degree 2 and w is homogeneous of degree 4.

(e) *What values do each of these utility functions assign to the indifference curve that contains bundle D?*

Answer: Recall that $D = (25, 25)$. Thus, the three utility functions assign values of $u(25,25) = 25^{0.5} 25^{0.5} = 25$; $v(25,25) = 25(25) = 625$; and $w(25,25) = 25^2(25^2) = 390,625$.

(f) True or False: *Homogeneity of degree 1 implies that a doubling of goods in a consumption basket leads to "twice" the utility as measured by the homogeneous function, whereas homogeneity greater than 1 implies that a* *doubling of goods in a consumption bundle leads to more than "twice" the utility.*

Answer: This is true. We already showed an example of this. More generally, you can see this from the definition of a function that is homogeneous of degree k; i.e. $u(tx_1, tx_2) = t^k u(x_1, x_2)$. Substituting $k = 2$, $u(2x_1, 2x_2) = 2^k u(x_1, x_2)$. When $k = 1$ — i.e. when the utility function is homogeneous of degree 1, this implies $u(2x_1, 2x_2) = 2u(x_1, x_2)$ — a doubling of goods leads to a doubling of utility assigned to the bundle. More generally, a doubling of goods leads to 2^k times as much utility assigned to the new bundle — and 2^k is greater than 2 when $k > 1$ (and less than 2 when $k < 1$.)

(g) *Demonstrate that the MRS is unchanged regardless of which of the three utility functions derived in B(c) is used.*

Answer: The MRS of a Cobb-Douglas utility function $u(x_1, x_2) = x_1^{\gamma} x_2^{\delta}$ is $MRS = -(\gamma x_2)/(\delta x_1)$ which reduces to $-x_2/x_1$ when $\gamma = \delta$ which is the case for all three of the utility functions. Thus, the MRS is the same for the three functions.

(h) *Can you think of representing these tastes with a utiliy function that assigns the value of 100 to the indifference curve containing bundle A and 75 to the indifference curve containing bundle D? Is the utility function you derived homogeneous?*

Answer: The function $u(x_1, x_2) = x_1^{0.5} x_2^{0.5} + 50$ would work. This function is not homogeneous (but it is homothetic).

(i) True or False: *Homothetic tastes can always be represented by functions that are homogeneous of degree k (where k is greater than zero), but even functions that are not homogeneous can represent tastes that are homothetic.*

Answer: This is true. We showed in the text that $MRS(tx_1, tx_2) = MRS(x_1, x_2)$ for homogeneous functions — thus, for homogeneous functions, the MRS is constant along any ray from the origin, the definition of homothetic tastes. At the same time, we just saw in the answer to the previous part an example of a non-homogeneous function that still represents homothetic tastes.

(j) True or False*: The marginal rate of substitution is homogeneous of degree 0 if and only if the underlying tastes are homothetic.*

Answer: For any set of homothetic tastes, the MRS is constant along rays from the origin; i.e. $MRS(tx_1, tx_2) = MRS(x_1, x_2)$. Thus, for homothetic tastes, the MRS is indeed homogeneous of degree 0. But $MRS(tx_1, tx_2) = MRS(x_1, x_2)$ defines homotheticity — so non-homothetic tastes will not have this property, which implies their MRS is not homogeneous of degree zero. The statement is therefore true.

Conclusion: Potentially Helpful Reminders

1. Keep in mind the distinction between how the *MRS* changes along an indifference curve (which tells us about substitutability) and how the *MRS* changes across the indifference map (which leads to ideas like homotheticity and quasilinearity).

2. The idea of substitutability will become critical in Chapter 7 when we introduce substitution effects (which will depend only on the shape of an indifference curve). The ideas of homotheticity and quasiliearity become important as we introduce income effects (in Chapter 7) — which will be measured across an indifference map (rather than along an indifference curve).

3. Extremes like perfect substitutes and perfect complements are useful to keep in mind because they make it easy to remember which way an indifference map looks if the goods are relatively more substitutable as opposed to relatively more complementary and vice versa.

4. Special cases like homothetic and quasilinear tastes will become useful borderline cases in Chapter 7 — with homothetic tastes being the borderline case between luxury goods and necessities, and with quasilinear tastes being the borderline case between normal and inferior goods. (These terms are defined in Chapter 7.)