CHAPTER

7

Income and Substitution Effects in Consumer Goods Markest

In Chapter 6 we showed how economic circumstances combine with tastes to result in choice or behavior. In Chapter 7 we show how consumer choices (and thus the consumer behavior we observe) change as circumstances change — i.e. as incomes and prices change. Put differently, we will now show how "people respond to incentives" in the consumer goods market.

Chapter Highlights

The main points of the chapter are:

1. There are **two ways in which economic circumstances typically change**: a change in income and a change in opportunity costs.

2. When only **income changes**, we can predict the change in behavior if we know something about **how indifference curves relate to one another** — because we jump from one indifference curve to another. Whether tastes are quasilinear or homothetic, whether goods are normal or inferior — these are statements about that relationship between indifference curves.

3. When only **opportunity costs change** and *real* income remains constant, we don't need to know anything about the relationship of indifference curves to one another — because the change in behavior occurs along a single indifference curve. Thus, **the *shape* of the relevant indifference curve is all that matters** — which is the same as saying that the degree of substitutability of the goods at the margin is all that matters.

4. **Substitution effects** arise as we slide along indifference curves because opportunity costs have changed; **income effects** arise as we jump between indifference curves because real income has changed.

5. **Price changes give rise to both of these effects**. To identify the substitution effect, we only look at the initial indifference curve and thus need to know about the substitutability of goods at the margin; to identify income effects, we have to know how indifference curves relate to one another.

6. In the calculus-based material of Part B of *Microeconomics: An Intuitive Approach with Calculus*, we show how **constrained utility maximization** gives us the choices that people make as incentives change while the **constrained expenditure minimization** problem allows us to disentangle the substitution effect from the income effect.

7A Solutions to Within-Chapter-Exercises for Part A

Exercise 7A.1

Is it also the case that whenever there is a positive income effect on our consumption of one good, there must be a negative income effect on our consumption of a different good?

Answer: No — since it is possible for our consumption of all goods to go up as income increases, the income effect could be positive for all goods.

Exercise 7A.2

Can a good be an inferior good at all income levels? (*Hint*: Consider the bundle (0,0).)

Answer: No. The reason is that, in order for a good to be inferior, it must be that you consume more of it as income falls. But, as income falls toward zero, at some point it will not be possible to consume more as income falls — because there simply won't be enough income to consume more. Thus, around the origin, no good can be inferior.

Exercise 7A.3

Are all inferior goods necessities? Are all necessities inferior goods? (*Hint*: The answer to the first is yes; the answer to the second is no.) Explain.

Answer: If you consume less of a good as income goes up, then it must be true that you spend a smaller fraction of your income on that good as income goes up. Thus, all inferior goods are necessities. At the same time, it may be the case that the fraction of your income spent on a good declines as your income goes up — but you still buy more of the good. (For instance, suppose your income goes up by 10% and you choose to consume 5% more of a good. Then the fraction of income spent on that good is declining even though you are increasing your consumption of the good as your income goes up.) Thus, necessities could be normal goods.

Exercise 7A.4

At a particular consumption bundle, can both goods (in a 2-good model) be luxuries? Can they both be necessities?

Answer: No. In a 2-good model, you will end up spending all your income as income increases. So suppose you are currently spending all your income on the two goods and your income now increases by 10%. If your consumption of both goods increases by more than 10%, then you would now be spending more than your new income. If your consumption of both goods increases by less than 10%, you would be spending less than your new income.

Exercise 7A.5

If you knew only that my brother and I had the same income (but not necessarily the same tastes), could you tell which one of us drove more miles — the one that rented or the one that took taxis?

Answer: Yes. Suppose my brother faces the intersecting budget lines — with the steeper one representing taxis and the shallower one representing rental cars. He chooses the steeper (taxi) budget line. Then we know that he must be consuming a bundle to the left of the intersection point of the two lines — because if he chose to the right of that point, he could have had more of everything on the shallower budget and thus should have chosen the shallower (rental car) budget instead. Thus, by choosing the steeper taxi budget, we know my brother consumes to the left of the intersection point. I, on the other hand, chose the shallower rental car budget. If I were to then choose a bundle to the left of the intersection point, I could have done better choosing the steeper budget because I could get more of both goods. Thus I must be consuming to the right of the intersection point. If my brother and I have the same incomes (and thus face the same taxi and rental car budgets), it therefore must be the case that my brother consumes to the left of the intersection point on the taxi budget and I consume to the right on the rental car budget. We can unambiguously say I consumed more miles driven.

Exercise 7A.6

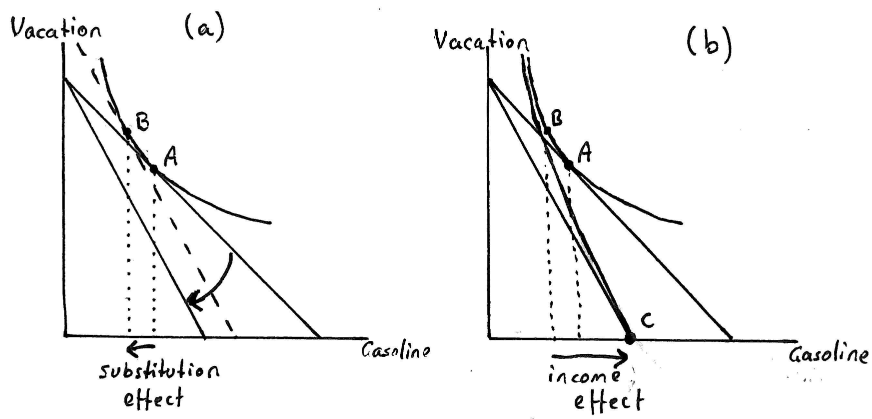
True or False: If you observed my brother and me consuming the same number of miles driven during our vacations, then our tastes must be those of perfect complements between miles driven and other consumption.

Answer: It would at a minimum have to be the case that the indifference curve at the intersection of the two budget lines has a sharp kink at that point. That kink could be such that it forms a right angle — thus creating the typical perfect complements indifference curve. But at a minimum it has to be such that the upper part of the indifference curve is steeper than the taxi budget and the lower part is shallower than the rental car budget — with a kink at the intersection.

Exercise 7A.7

Can you re-tell the Heating Gasoline-in-Midwest story in terms of income and substitution effects in a graph with "yearly gallons of gasoline consumption" on the horizontal axis and "yearly time on vacation in Florida" on the vertical?

Answer: In panel (a) of Exercise Graph 7A.7, bundle A is the original consumption bundle prior to the increase in the price of gasoline. The increase in the price of gasoline then rotates the budget clockwise. Bundle B lies on the compensated budget at the new price of gasoline — and the move from A to B is the substitution effect. As always, the substitution effect causes a decrease in consumption of the good (gasoline) that has become more expensive.



Exercise Graph 7A.7 : Gasoline and Florida Vacation Time

Panel (b) illustrates C — with no consumption of Florida vacation time. This corner solution is rationalized by an indifference curve that crosses the new budget at C — creating an income effect in the opposite direction of the substitution effect. Since the income effect is larger than the substitution effect, the consumer shifts from A before the increase in the price of gasoline to C after the price increase — with an overall increase in gasoline consumption resulting from the price increase.

Exercise 7A.8

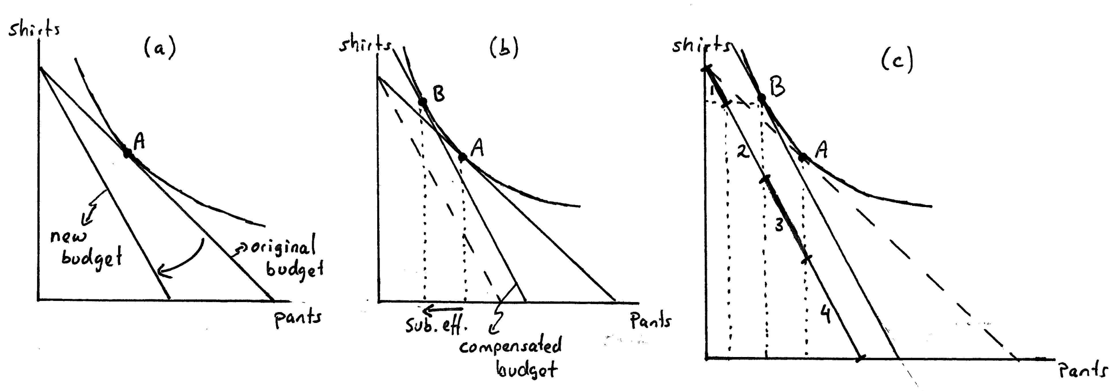
In panel (c) of Graph 7.7, where would the final optimal bundle on the magenta budget lie if tastes were nomothetic? What if they were quasilinear?

Answer: If tastes were nomothetic, the final bundle would lie where a ray from the origin through B intersects the magenta budget. If tastes were quasilinear (in pants), the final bundle would lie where the vertical ray through B intersects the magenta budget (at the borderline between pants being normal and inferior). (If tastes were quasilinear in shirts, the final bundle would lie where the horizontal ray through B intersects the magenta budget.)

Exercise 7A.9

Replicate Graph 7.7 for an increase in the price of pants (rather than a decrease).

Answer: Panel (a) of Exercise Graph 7A.9 illustrates the original consumption bundle A and the change in the budget constraint when the price of pants increases. Panel (b) illustrates the compensated budget and the resulting bundle B — with the substitution effect as the movement from A to B. As always, this effect says the consumer will consume less of what has become more expensive, more of what has become relatively cheaper. Finally, panel (c) identifies four regions (labeled 1, 2, 3 and 4) on the new (uncompensated) budget line. If the consumer ends up optimizing in region 1, her consumption of pants decreases and her consumption of shirts increases with a decline in income (relative to the compensated budget) — which implies that pants are a normal good and shirts are inferior. In region 2, the consumption of both goods declines with income — thus both pants and shirts are normal goods. In regions 3 and 4, consumption of shirts decreases and consumption of pants increases with a drop in income (from the compensated budget) — thus making shirts normal and pants inferior. In region 3, however, the consumer still buys fewer pants as the price increases (i.e. C is to the left of A) — which means pants are regular inferior; in region 4, on the other hand, pants consumption goes up with an increase in price, which makes pants a Giffen good.



Exercise Graph 7A.9 : Gasoline and Florida Vacation Time

Exercise 7A.10

Can you explain the following Venn Diagram?

Answer: The diagram illustrates that the set of all goods can be divided into two broad subsets — normal goods and inferior goods, with goods at the border between these subsets represented by quasilinear goods. The set of Giffen goods is fully contained in the subset of inferior goods — that is, every Giffen good is an inferior good, but not every inferior good is a Giffen good. (We use the term "regular inferior good" to denote the subset of inferior goods that is not Giffen.) And just as the set of all goods can be subdivided into normal and inferior goods, it can be subdivided into necessities and luxuries, with the borderline between those two subsets representing homothetic goods. The set of luxury goods is then fully contained in the set of normal goods — that is, every luxury good is a normal good but not all normal goods are luxury goods. Necessities, on the other hand, can be normal or inferior (including regular inferior and Giffen).

7B Solutions to Within-Chapter-Exercises for Part B

Exercise 7B.1

Set up my brother's constrained optimization problem and solve it to check that his optimal consumption bundle is indeed equal to this.

Answer: My brother's optimization problem is

$$\max_{x_1,x_2} u(x_1,x_2) = x_1^{0.1}x_2^{0.9} \text{ subject to } x_1 + x_2 = 2000, \tag{7B.1.i}$$

which gives rise to the Lagrange function

$$\mathscr{L}(x_1,x_2,\lambda) = x_1^{0.1}x_2^{0.9} + \lambda(2000 - x_1 - x_2). \tag{7B.1.ii}$$

The first two first order conditions for this problem are then

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial x_1} &= 0.1x_1^{-0.9}x_2^{0.9} - \lambda = 0, \\ \frac{\partial \mathscr{L}}{\partial x_2} &= 0.9x_1^{0.1}x_2^{-0.1} - \lambda = 0. \end{aligned} \tag{7B.1.iii}$$

Moving λ to the other side of each equation, dividing the equations by one another and solving for x_1 gives us $x_1 = x_2/9$. Substituting this into the budget constraint $x_1 + x_2 = 2000$, we get $x_2/9 + x_2 = 2000$ which solves to $x_2 = 1,800$. Plugging this back into $x_1 = x_2/9$ furthermore gives $x_1 = 200$.

Exercise 7B.2

How much did I pay in a fixed rental car fee in order for me to be indifferent in this example to taking taxis? Why is this amount larger than in the Cobb-Douglas case we calculated earlier?

Answer: At B, I am consuming 2,551 miles at a per-mile cost of \$0.2 — for a total of \$510.20. At that bundle, I am also consuming approximately \$918 in other consumption. Thus, I am spending a total of approximately $\$918 + \$510 = \$1,428$ after having paid the fixed fee for the rental car. Since I started with \$2,000, that means the rental car fee must have been $\$2000 - \$1428 = \$572$. This amount is larger than under Cobb-Douglas preferences because the implicit elasticity of substitution is now 2 rather than 1.

Exercise 7B.3

Check to see that this solution is correct.

Answer: The Lagrange function for this optimization problem is

$$\mathcal{L}(x_1, x_2, \lambda) = x_1^{0.5} x_2^{0.5} + \lambda(200 - p_1 x_1 - 10x_2). \tag{7B.3.i}$$

The first two first order conditions for this problem are then

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_1} &= 0.5x_1^{-0.5} x_2^{0.5} - \lambda p_1 = 0, \\ \frac{\partial \mathcal{L}}{\partial x_2} &= 0.5x_1^{0.5} x_2^{-0.5} - 10\lambda = 0. \end{aligned} \tag{7B.3.ii}$$

Moving the λ terms to the other side, dividing the equations by one another and then solving for x_1, we get $x_1 = 10x_2/p_1$. Plugging this into the budget constraint $p_1 x_1 + 10x_2 = 200$ and solving for x_2, we get $x_2 = 10$, and plugging this back into $x_1 = 10x_2/p_1$, we get $x_1 = 100/p_1$.

Exercise 7B.4

Verify the above solutions to the minimization problem.

Answer: The Lagrange function for this optimization problem is

$$\mathcal{L}(x_1, x_2, \lambda) = 10x_1 + 10x_2 + \lambda(u^A - x_1^{0.5} x_2^{0.5}). \tag{7B.4.i}$$

The first two first order conditions for this problem are then

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_1} &= 10 - 0.5\lambda x_1^{-0.5} x_2^{0.5} = 0, \\ \frac{\partial \mathcal{L}}{\partial x_2} &= 10 - 0.5\lambda x_1^{0.5} x_2^{-0.5} = 0. \end{aligned} \tag{7B.4.ii}$$

Solving these for x_1 in the usual way gives us $x_1 = x_2$. Plugging this into the constraint $u^A = x_1^{0.5} x_2^{0.5}$, we then get $x_2 = u^A$, and — given we concluded $x_1 = x_2$, $x_1 = u^A$. Since $u^A \approx 7.071$, this implies $x_1 = x_2 \approx 7.071$.

Exercise 7B.5

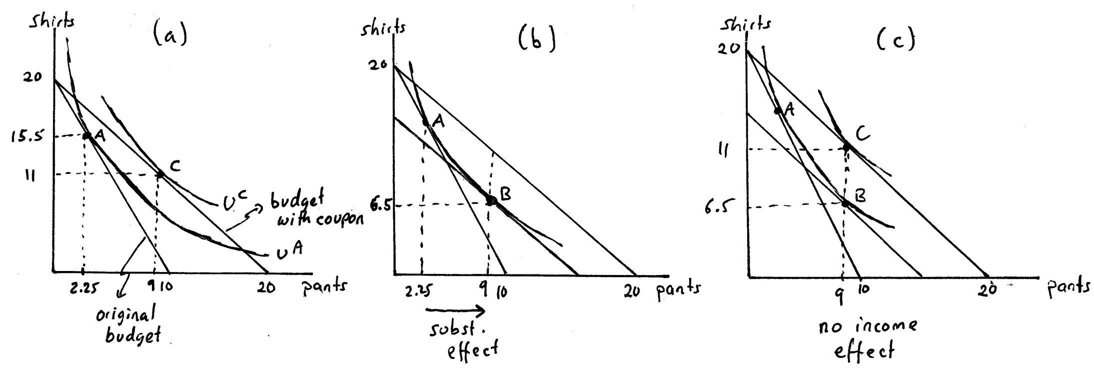
Notice that the ratio of my pants to shirts consumption is the same (= 1) at bundles B and C. What feature of Cobb-Douglas tastes is responsible for this result?

Answer: Cobb-Douglas tastes are homothetic — which implies that optimal consumption bundles lie on the same ray from the origin for all income levels (assuming no price changes).

Exercise 7B.6

Using the previous calculations, plot graphs similar to Graph 7.10 illustrating income and substitution effects when my tastes can be represented by the utility function $u(x_1, x_2) = 6x_1^{0.5} + x_2$.

Answer: This is done in Exercise Graph 7B.6. Notice again that there is no income effect relative to the good x_1 (pants) — which is because of the fact that the utility function represents tastes that are quasilinear in x_1. (Quasilinear goods have no income effects.)



Exercise Graph 7B.6 : Pants and Shirts with Quasilinear Tastes

7C Solutions to Odd Numbered End-of-Chapter Exercises

Exercise 7.1

Here, we consider some logical relationships between preferences and types of goods.

A: *Suppose you consider all the goods that you might potentially want to consume.*

(a) *Is it possible for all these goods to be be luxury goods at every consumption bundle? Is it possible for all of them to be necessities?*

Answer: Neither is possible. If they were all luxuries, then, as income increases by some percentage, consumption of each good would increase by a greater percentage. This is logically impossible. If they were all necessities, then, as income increases by some percentage, consumption of each good would increase by a lesser percentage. This implies that some income would remain unspent, which is inconsistent with optimization.

(b) *Is it possible for all goods to be inferior goods at every consumption bundle? Is it possible for all of them to be normal goods?*

Answer: The first is not possible but the second is. If all goods are inferior, then, as income falls, the consumer would increase her consumption of all goods. But that is logically impossible since income is declining. If all goods are normal goods, than consumption of all increases with increases in income and decreases with decreases in income — which is logically possible.

(c) True or False*: When tastes are homothetic, all goods are normal goods.*

Answer: True. Homothetic tastes are defined by the fact that the *MRS* remains constant along any ray from the origin. Thus, if we find a tangency of an indifference curve with a budget line, we know that, as income changes, indifference curves will always be tangent to the new budget along the ray that connects the original tangency to the origin. Thus, as income increases, consumption of all goods increases, and when income decreases, consumption of all goods decreases.

(d) True or False*: When tastes are homothetic, some goods could be luxuries while others could be necessities.*

Answer: False. We just explained that for homothetic tastes, the optimal bundles (for a given set of prices) lie on rays from the origin as income changes. Thus, as income increases by some percentage, consumption of all goods increases by the same percentage. Thus, all goods are borderline between luxuries and necessities.

(e) True or False*: When tastes are quasilinear, one of the goods is a necessity.*

Answer: True. As income changes, consumption of one of the goods does not change. Thus, as income increases, the percentage of income spent on that good decreases — making that good a necessity.

(f) True or False: *In a two good model, if the two goods are perfect complements, they must both be normal goods.*

Answer: True — since the goods are always consumed as pairs, consumption of both increases as income increases.

(g) True or False: *In a 3-good model, if two of the goods are perfect complements, they must both be normal goods.*

Answer: False. Since there is a third good, it may be that this third good is a normal good while the perfectly complementary goods are (jointly) inferior. Suppose, for instance, that rum and coke are perfect complements for someone, but that the person also has a taste for really good single malt scotch. As income goes up, he increases his consumption of single malt scotch and lowers his consumption of rum and cokes. Rum and coke would be perfect complements, but as income goes up, less of both would be consumed.

B: *In each of the following cases, suppose that a person whose tastes can be characterized by the given utility function has income I and faces prices that are all equal to 1. Illustrate mathematically how his consumption of each good changes with income and use your answer to determine whether the goods are normal or inferior, luxuries or necessities.*

(a) $u(x_1, x_2) = x_1 x_2$

Answer: In each case, we can set up the optimization problem

$$\max_{x_1, x_2} u(x_1, x_2) \text{ subject to } x_1 + x_2 = I \tag{7.1.i}$$

and solve it for x_1 and x_2 as a function of I. For the function $u(x_1, x_2) = x_1 x_2$, this gives us $x_1(I) = x_2(I) = I/2$. Thus, half of all income is spent on x_1 and half on x_2, which implies that, when income doubles, so does consumption of each of the two goods. Thus, the goods are borderline between luxuries and necessities — and they are both normal.

(b) $u(x_1, x_2) = x_1 + \ln x_2$

Answer: Solving this optimization problem again with the new utility function, we get $x_1(I) = I - 1$ and $x_2(I) = 1$. Consumption of x_2 is therefore independent of income — which means the good is borderline between normal and inferior. The fraction of income spent on x_2 declines with income — which means the good is a necessity. Good x_1, on the other hand, is a normal good — and a luxury.

(c) $u(x_1, x_2) = \ln x_1 + \ln x_2$

Answer: For this utility function, we again get $x_1(I) = x_2(I) = I/2$ as in (a). (This makes sense since the utility function here is a monotone transformation of the utility function in (a).) So the same answer as in (a) applies.

(d) $u(x_1, x_2, x_3) = 2\ln x_1 + \ln x_2 + 4\ln x_3$

Answer: We can again solve the same optimization problem, except that we now have 3 choice variables. We would write the Lagrange function as

$$\mathcal{L}(x_1, x_2, x_3, \lambda) = 2\ln x_1 + \ln x_2 + 4\ln x_3 + \lambda(I - x_1 - x_2 - x_3) \quad (7.1.ii)$$

and the first three first order conditions as

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_1} &= \frac{2}{x_1} - \lambda = 0, \\ \frac{\partial \mathcal{L}}{\partial x_2} &= \frac{1}{x_2} - \lambda = 0, \\ \frac{\partial \mathcal{L}}{\partial x_3} &= \frac{4}{x_3} - \lambda = 0. \end{aligned} \quad (7.1.iii)$$

The first and second can be used to write $x_2 = x_1/2$, and the first and third can be combined to give us $x_3 = 2x_1$. Substituting these into the budget constraint $x_1 + x_2 + x_3 = I$ gives us $x_1 + x_1/2 + 2x_1 = I$ which solves to $x_1(I) = 2I/7$. Substituting this back into $x_2 = x_1/2$ and $x_3 = 2x_1$ then gives us $x_2(I) = I/7$ and $x_3(I) = 4I/7$. The consumption of each of the three goods is therefore a constant fraction of income — which implies all three goods are normal and borderline between luxuries and necessities.

(e) $u(x_1, x_2) = 2x_1^{0.5} + \ln x_2$

Answer: Following the same set-up, we get[1]

$$x_1(I) = \left(\frac{-1 + (1+4I)^{1/2}}{2}\right)^2 \text{ and } x_2(I) = \frac{-1 + (1+4I)^{1/2}}{2} \quad (7.1.iv)$$

As income increases, consumption of both goods therefore increases (since I enters positively into both equations). However, it does not increase at a constant rate. Taking the derivative of $x_2(I)$ with respect to I, we get

$$\frac{dx_2(I)}{dI} = \frac{1}{(1+4I)^{1/2}}, \quad (7.1.v)$$

which is a decreasing function of I. Thus, as income increases, the fraction devoted to consumption of x_2 decreases — making x_2 a necessity (and thus x_1 a luxury good).

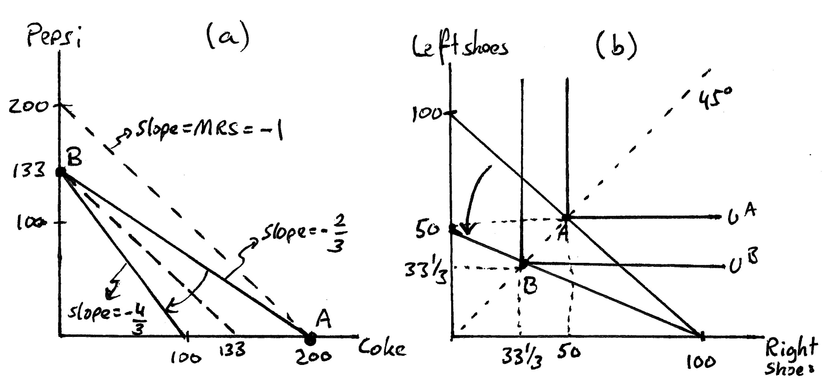
[1] Combining the first 2 first order conditions, we get $x_1 = x_2^2$, and substituting this into the budget constraint, we get $x_2^2 + x_2 - I = 0$. To solve this, we apply the quadratic formula which gives two answers for x_2. However, one of these is clearly negative.

Exercise 7.3

Consider once again my tastes for Coke and Pepsi and my tastes for right and left shoes (as described in end-of-chapter exercise 6.2).

A: *On two separate graphs — one with Coke and Pepsi on the axes, the other with right shoes and left shoes — replicate your answers to end-of-chapter exercise 6.2A(a) and (b). Label the original optimal bundles A and the new optimal bundles C.*

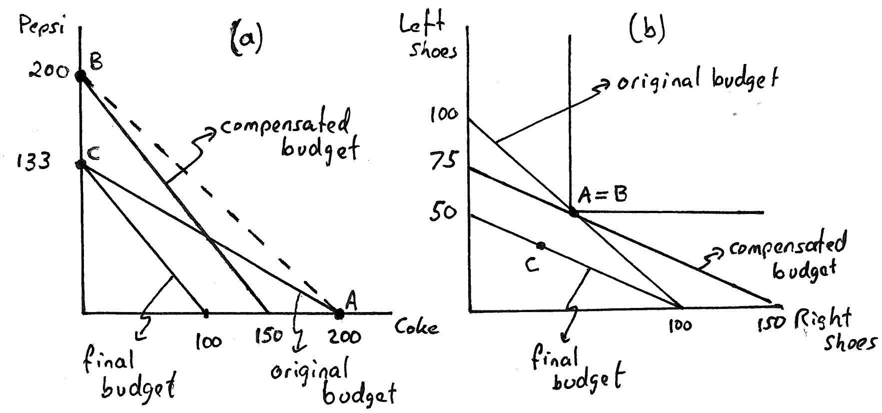
Answer: The graphs from end-of-chapter exercise 6.2A(a) and (b) are replicated in Exercise Graph 7.3(1). Note that indifference curves in panel (a) are dashed while budget lines are solid. Also, note that in this replicated graph, B is the final optimum and should now be labeled C.



Exercise Graph 7.3(1) : Replicated from End-of-Chapter exercise 6.2

(a) *In your Coke/Pepsi graph, decompose the change in behavior into income and substitution effects by drawing the compensated budget and indicating the optimal bundle B on that budget.*

Answer: In panel (a) of Exercise Graph 7.3(2), the original optimum occurs on the dashed indifference curve at bundle A while the final optimum occurs on the final budget at C. (To keep the picture uncluttered, the final indifference curve is left out.) The compensated budget has the same slope as the final budget but sufficient income to reach the original dashed indifference curve — which occurs at B. Thus, the substitution effect takes us from A to B, and the income effect to C. This should make



Exercise Graph 7.3(2) : Inc. and Subst. Effects for Perfect Substitutes and Complements

sense: For the good whose price has changed (coke), the entire change is due to the substitution effect because the goods are perfect substitutes.

(b) *Repeat (a) for your right shoes/left shoes graph.*

Answer: Panel (b) of the graph shows the analogous for perfect complements. The compensated budget has the same slope as the final budget but must be "tangent" to the original indifference curve. This happens at A — which means the usual B that includes the substitution effect lies right on top of A. Thus, there is no substitution effect — which again should make sense since there is no substitutability between the two goods.

B: *Now consider the following utility functions:* $u(x_1, x_2) = \min\{x_1, x_2\}$ *and* $u(x_1, x_2) = x_1 + x_2$.

(a) *Which of these could plausibly represent my tastes for Coke and Pepsi, and which could represent my tastes for right and left shoes?*

Answer: The first could represent tastes for right and left shoes while the second could represent tastes for Coke and Pepsi.

(b) *Use the appropriate function from above to assign utility levels to bundles A, B and C in your graph from 7.3A(a).*

Answer: The appropriate function in this case is $u(x_1, x_2) = x_1 + x_2$. The three bundles are A=(200,0), B=(0,200) and C=(0,133). Thus, the utility levels assigned to each of these bundles is $u(A) = 200 = u(B)$ and $u(C) = 133$.

(c) *Repeat this for bundles A, B and C for your graph in 7.3A(b).*

Answer: The appropriate function now is $u(x_1, x_2) = \min\{x_1, x_2\}$ and the three bundles are A=B=(50,50) and C=(33.33,33.33). The utility values associated with these bundles are $u(A) = u(B) = 50$ and $u(C) = 33.33$.

Exercise 7.5

Return to the analysis of my undying love for my wife expressed through weekly purchases of roses (as introduced in end-of-chapter exercise 6.4).

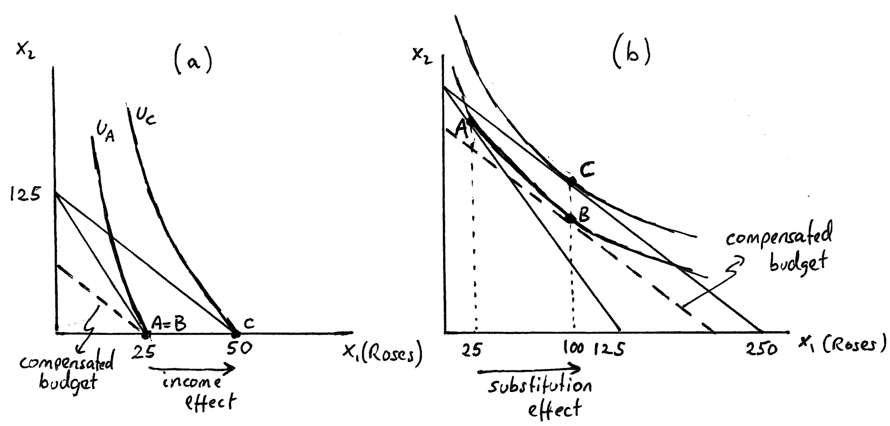
A: *Recall that initially roses cost $5 each and, with an income of $125 per week, I bought 25 roses each week. Then, when my income increased to $500 per week, I continued to buy 25 roses per week (at the same price).*

(a) *From what you observed thus far, are roses a normal or an inferior good for me? Are they a luxury or a necessity?*

Answer: As income went up, my consumption remained unchanged. This would typically indicate that the good in question is borderline normal/inferior — or quasilinear. Since the consumption at the lower income is at a corner solution, however, we cannot be certain that the good is not inferior, with the *MRS* at the original optimum larger in absolute value than the *MRS* at the new (higher income) optimum. Regardless, roses must be a necessity — whether they are borderline inferior/normal or inferior, the percentage of income spent on roses declines as income increases.

(b) *On a graph with weekly roses consumption on the horizontal and "other goods" on the vertical, illustrate my budget constraint when my weekly income is $125. Then illustrate the change in the budget constraint when income remains $125 per week and the price of roses falls to $2.50. Suppose that my optimal consumption of roses after this price change rises to 50 roses per week and illustrate this as bundle C.*

Answer: This is illustrated in panel (a) of Exercise Graph 7.5 where *A* is the original corner solution, *C* is the new corner solution and the dashed line is the compensated budget.



Exercise Graph 7.5 : Love and Roses

(c) *Illustrate the compensated budget line and use it to illustrate the income and substitution effects.*

Answer: This is also illustrated in panel (a) of the graph. In this case, there is no substitution effect (in terms of roses) and only an income effect.

(d) *Now consider the case where my income is $500 and, when the price changes from $5 to $2.50, I end up consuming 100 roses per week (rather than 25). Assuming quasilinearity in roses, illustrate income and substitution effects.*

Answer: This is illustrated in panel (b) of Exercise Graph 7.5 where the dashed line is again the compensated budget line. Unlike in panel (a), the entire change in roses consumption is now due to a substitution effect rather than an income effect.

(e) True or False*: Price changes of goods that are quasilinear give rise to no income effects for the quasilinear good unless corner solutions are involved.*

Answer: This is true. We will often make the statement that income effects disappear if we assume quasilinearity of a good — because then a good is borderline normal/inferior, which implies consumption remains unchanged as income changes. This is true so long as the consumer is at an interior solution. If quasilinear tastes lead to corner solutions, then this may give rise to income effects as we see in panel (a) of the graph.

B: *Suppose again, as in 6.4B, that my tastes for roses (x_1) and other goods (x_2) can be represented by the utility function $u(x_1, x_2) = \beta x_1^{\alpha} + x_2$.*

(a) *If you have not already done so, assume that p_2 is by definition equal to 1, let $\alpha = 0.5$ and $\beta = 50$, and calculate my optimal consumption of roses and other goods as a function of p_1 and I.*

Answer: Solving the optimization problem

$$\max_{x_1,x_2} 50x_1^{0.5} + x_2 \text{ subject to } I = p_1x_1 + x_2, \tag{7.5.i}$$

we get

$$x_1 = \frac{625}{p_1^2} \text{ and } x_2 = I - \frac{625}{p_1}. \tag{7.5.ii}$$

(b) *The original scenario you graphed in 7.5A(b) contains corner solutions when my income is $125 and the price is initially $5 and then $2.50. Does your answer above allow for this?*

Answer: Substituting $I = 125$ and $p_1 = 5$ into our equations (7.5.ii) for x_1 and x_2 from above, we get $x_1 = 625/(5^2) = 25$ and $x_2 = 125 - (625/5) = 0$. This is exactly the original corner solution in the scenario in part A.

Changing the price to $p_1 = 2.5$, we get $x_1 = 625/(2.5^2) = 100$ and $x_2 = 125-(625/2.5) = -125$. Given that the solution from our Lagrange method now gives us a negative consumption level for x_2, we know that the true optimum is the corner solution where all income is spent on x_1 — i.e. the bundle (50,0) just as described in the scenario in A.

At the original price, it turns out that the MRS at the corner solution is exactly equal to the slope of the budget line. At the lower price, the MRS

is large in absolute value than the budget line — which means the indifference curve cuts the budget line at the corner from above. The tangency of an indifference curve with this budget line therefore does not happen until x_2 is negative — which the Lagrange method finds but which is not economically meaningful.

(c) *Verify that the scenario in your answer to 7.5A(d) is also consistent with tastes described by this utility function — i.e. verify that A, B and C are as you described in your answer.*

Answer: Using equations (7.5.ii), we get $x_1 = 625/(5^2) = 25$ and $x_2 = 500 - (625/5) = 375$ when $p_1 = 5$ (and $I = 500$), and we get $x_1 = 625/(2.5^2) = 100$ and $x_2 = 500 - (625/2.5) = 250$ when $p_1 = 2.5$. These correspond to A and C in panel (b) of Exercise Graph 7.5.

To calculate B in the graph, we need to first find the utility level associated with the original bundle A — i.e. $u(25,375) = 50(25^{0.5}) + 375 = 625$. We then need to find what bundle the consumer would buy if she was given enough money to reach that same indifference curve at the new price; i.e. we need to solve the problem

$$\min_{x_1,x_2} 2.5x_1 + x_2 \text{ subject to } 625 = 50x_1^{0.5} + x_2. \tag{7.5.iii}$$

Solving the first order conditions, we then get $x_1 = 100$ and $x_2 = 125$ — consistent with panel (b) of the graph.

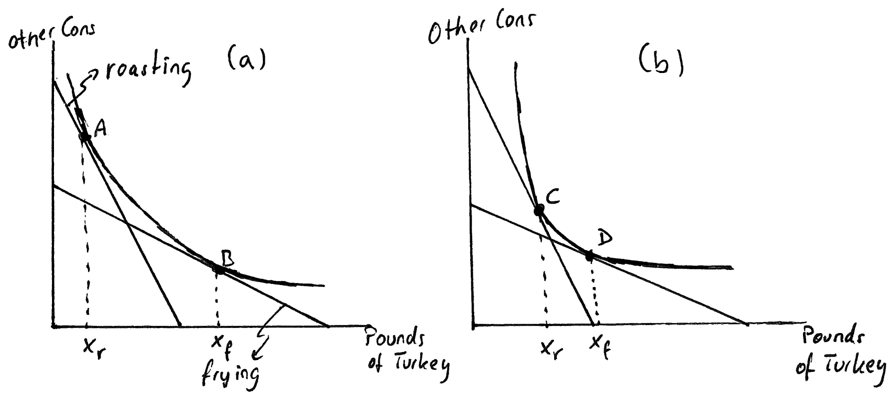
Exercise 7.7

Everyday Application: *Turkey and Thanksgiving: Every Thanksgiving, my wife and I debate about how we should prepare the turkey we will serve (and will then have left over). My wife likes preparing turkeys the conventional way — roasted in the oven where it has to cook at 350 degrees for 4 hours or so. I, on the other hand, like to fry turkeys in a big pot of peanut oil heated over a powerful flame outdoors. The two methods have different costs and benefits. The conventional way of cooking turkeys has very little set-up cost (since the oven is already there and just has to be turned on) but a relatively large time cost from then on. (It takes hours to cook.) The frying method, on the other hand, takes some set-up (dragging out the turkey fryer, pouring gallons of peanut oil, etc. — and then later the cleanup associated with it), but turkeys cook predictably quickly in just 3.5 minutes per pound.*

A: *As a household, we seem to be indifferent between doing it one way or another — sometimes we use the oven, sometimes we use the fryer. But we have noticed that we cook much more turkey — several turkeys, as a matter of fact, when we use the fryer than when we use the oven.*

(a) *Construct a graph with "pounds of cooked turkeys" on the horizontal and "other consumption" on the vertical. ("Other consumption" here is not denominated in dollars as normally but rather in some consumption index that takes into account the time it takes to engage in such consumption.) Think of the set-up cost for frying turkeys and the waiting cost for cooking* *them as the main costs that are relevant. Can you illustrate our family's choice of whether to fry or roast turkeys at Thanksgiving as a choice between two "budget lines"?*

Answer: This is illustrated in panel (a) of Exercise Graph 7.7(1). The set-up cost of the turkey fryer results in a lower intercept for the frying budget on the vertical axis — but the lower cost of cooking turkey results in a shallower slope.



Exercise Graph 7.7(1) : Frying versus Roasting Turkey

(b) *Can you explain the fact that we seem to eat more turkey around Thanksgiving whenever we pull out the turkey fryer as opposed to roasting the turkey in the oven?*

Answer: Since we are indifferent between frying and roasting, our optimal bundle on the two budget lines must lie on the same indifference curve. This is also illustrated in panel (a) of the graph — where it is immediately apparent that we will cook more turkey when frying than when roasting because of the lower opportunity cost.

(c) *We have some friends who also struggle each Thanksgiving with the decision of whether to fry or roast — and they, too, seem to be indifferent between the two options. But we have noticed that they only cook a little more turkey when they fry than when they roast. What is different about them?*

Answer: A possible picture for my friend's family is illustrated in panel (b) of the graph — where the indifference curve is not as flat — making the two goods less substitutable. Since the effect we are demonstrating is a pure substitution effect, it makes sense that with less substitutability between the goods, the difference in behavior is smaller for the two turkey cooking options.

B: *Suppose that, if we did not cook turkeys, we could consume 100 units of "other consumption" — but the time it takes to cook turkeys takes away from that con-*

sumption. Setting up the turkey fryer costs c units of consumption and waiting 3.5 minutes (which is how long it takes to cook 1 pound of turkey) costs 1 unit of consumption. Roasting a turkey involves no set-up cost, but it takes 5 times as long to cook per pound. Suppose that tastes can be characterized by the CES utility function $u(x_1,x_2) = (0.5x_1^{-\rho} + 0.5x_2^{-\rho})^{-1/\rho}$ *where* x_1 *is pounds of turkey and* x_2 *is "other consumption".*

(a) *What are the two budget constraints I am facing?*

Answer: Costs are denominated in "units of consumption" — which implies that p_2, the price of consuming "other goods", is by definition 1. The price of cooking 1 pound of turkey (p_1) is then either 1 if we fry or 5 if we roast. This gives us the budget constraints

$$5x_1 + x_2 = 100 \text{ when roasting, and } x_1 + x_2 = 100 - c \text{ when fying.} \quad (7.7.i)$$

(b) *Can you calculate how much turkey someone with these tastes will roast (as a function of ρ)? How much will the same person fry? (*Hint: *Rather than solving this using the Lagrange method, use the fact that you know the MRS is equal to the slope of the budget line — and recall from chapter 5 that, for a CES utility function of this kind,* $MRS = -(x_2/x_1)^{\rho+1}$*.)*

Answer: At the optimum, we set the MRS equal to the ration $-p_1/p_2$. Setting MRS equal to the ratio of prices then implies

$$\left(\frac{x_2}{x_1}\right)^{\rho+1} = 5 \text{ when roasting, and } \left(\frac{x_2}{x_1}\right)^{\rho+1} = 1 \text{ when fying.} \quad (7.7.ii)$$

Solving for x_2, we get $x_2 = 5^{1/(\rho+1)}x_1$ when roasting and $x_2 = x_1$ when frying. Substituting these into the appropriate budget constraints from equation (7.7.i) and solving for x_1, we get

$$x_1 = \frac{100}{5+5^{1/(\rho+1)}} \text{ when roasting, and } x_1 = \frac{100-c}{2} \text{ when fying.} \quad (7.7.iii)$$

(c) *Suppose my family has tastes with ρ = 0 and my friend's with ρ = 1. If each of us individually roasts turkeys this Thanksgiving, how much will we each roast?*

Answer: My family will roast

$$x_1 = \frac{100}{5+5^1} = 10, \quad (7.7.iv)$$

and my friend's family will roast

$$x_1 = \frac{100}{5+5^{1/2}} = 13.82. \quad (7.7.v)$$

(d) *How much utility will each of us get (as measured by the relevant utility function)? (*Hint*: In the case where* $\rho = 0$*, the exponent* $1/\rho$ *is undefined. Use the fact that you know that when* $\rho = 0$ *the CES utility function is Cobb-Douglas.)*

Answer: To calculate utilities, we first have to calculate how much of x_2 each of us consumes. Just plugging our answers above into the first budget constraint in equation (7.7.i), we get $x_2 = 50$ for my family and $x_2 = 30.9$ for my friends. For my family, $\rho = 0$ — which means we can use the Cobb-Douglas utility function $x_1^{0.5}x_2^{0.5}$ instead of the CES functional form. Plugging $(x_1, x_2) = (10, 50)$ into $x_1^{0.5}x_2^{0.5}$ gives us utility of 22.36. For my friend's family, plugging $(x_1, x_2) = (13.82, 30.90)$ into his utility function (with $\rho = 1$), we get utility of 19.1.

(e) *Which family is happier?*

Answer: We can't know since we generally do not believe that we are measuring utility in units that can be compared across people.

(f) *If we are really indifferent between roasting and frying, what must c be for my family? What must it be for my friend's family? (*Hint*: Rather than setting up the usual minimization problem, use your answer to (b) determine c by setting utility equal to what it was for roasting).*

Answer: We know from our answer in (b) that, when frying, $x_1 = (100 - c)/2$ regardless of ρ. Plugging this into our frying budget constraint $x_1 + x_2 = 100 - c$, this implies that $x_2 = (100 - c)/2$ regardless of ρ. When $\rho = 0$, we can then plug these into the Cobb-Douglas version of the utility function and set it equal to the utility of 22.36 that we determined above my family gets when roasting turkeys; i.e.

$$\left(\frac{100-c}{2}\right)^{0.5}\left(\frac{100-c}{2}\right)^{0.5} = \left(\frac{100-c}{2}\right) = 22.36. \qquad (7.7.\text{vi})$$

Solving for c, we get $c = 55.28$. For my friend's family, we can similarly substitute $x_1 = (100-c)/2$ and $x_2 = (100-c)/2$ into his CES utility function (with $\rho = 1$) and set it equal to the utility he gets from roasting — which we calculated before to be 19.1. Thus,

$$\left[0.5\left(\frac{100-c}{2}\right)^{-1} + 0.5\left(\frac{100-c}{2}\right)^{-1}\right]^{-1} = \frac{100-c}{2} = 19.1. \qquad (7.7.\text{vii})$$

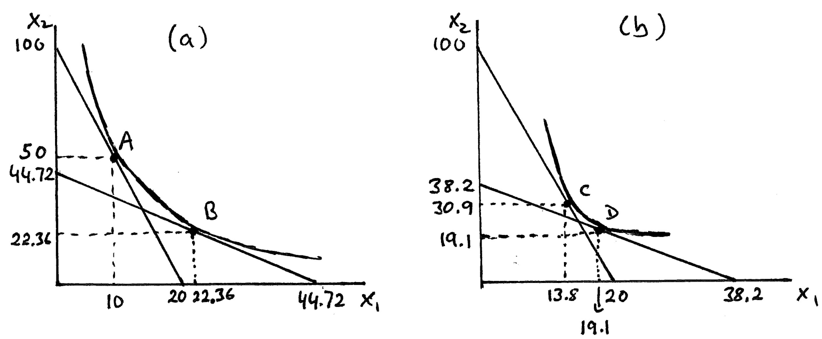
Solving for c, we get $c = 61.8$.

(g) *Given your answers so far, how much would we each have fried had we chosen to fry instead of roast (and we were truly indifferent between the two because of the different values of c we face)?*

Answer: Given that we calculated $c = 55.28$ for my family and $c = 61.8$ for my friend's, we get that $x_1 = (100 - 55.28)/2 = 22.36$ pounds for my family and $x_1 = (100 - 61.8)/2 = 19.1$ pounds for my friend's family.

(h) *Compare the size of the substitution effect you have calculated for my family and that you calculated for my friend's family and illustrate your answer in a graph with pounds of turkey on the horizontal and other consumption on the vertical. Relate the difference in the size of the substitution effect to the elasticity of substitution.*

<u>Answer</u>: My family goes from roasting 10 pounds of turkey to frying 22.23 pounds — a substitution effect of 12.36 pounds. My friend's family goes from roasting 13.82 pounds to frying 19.1 pounds — a substitution effect of 5.28 pounds. The difference, of course, is the greater substitutability that is built into my utility function with $\rho = 0$ as opposed to my friend's with $\rho = 1$. To be precise, my elasticity of substitution is 1 whereas my friend's is 0.5. The results are graphed in Exercise Graph 7.7(2).



Exercise Graph 7.7(2) : Frying versus Roasting Turkey: Part II

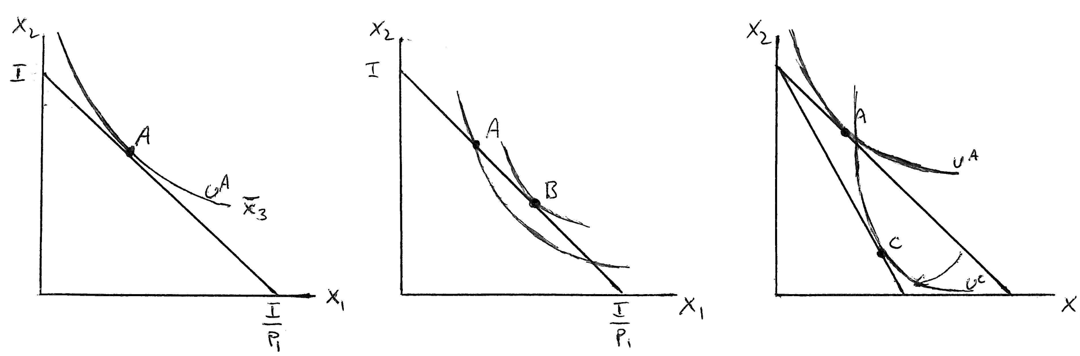
Exercise 7.9

Business Application: *Are Gucci products Giffen Goods? We defined a Giffen good as a good that consumers (with exogenous incomes) buy more of when the price increases. When students first hear about such goods, they often think of luxury goods such as expensive Gucci purses and accessories. If the marketing departments for firms like Gucci are very successful, they may find a way of associating price with "prestige" in the minds of consumers — and this may allow them to raise the price* and *sell more products. But would that make Gucci products Giffen goods? The answer, as you will see in this exercise, is no.*

A: *Suppose we model a consumer who cares about the "practical value and style of Gucci products", dollars of other consumption and the "prestige value" of being seen with Gucci products. Denote these as x_1, x_2 and x_3 respectively.*

(a) *The consumer only has to buy x_1 and x_2 — the prestige value x_3 comes with the Gucci products. Let p_1 denote the price of Gucci products and $p_2 = 1$ be the price of dollars of other consumption. Illustrate the consumer's budget constraint (assuming an exogenous income I).*

Answer: This is just like any typical budget constraint and is illustrated as part of panel (a) of Exercise Graph 7.9.



Exercise Graph 7.9 : Gucci Products and Prestige

(b) *The prestige value of Gucci purchases — x_3 — is something an individual consumer has no control over. If x_3 is fixed at a particular level $\bar{x}_3$, the consumer therefore operates on a 2-dimensional slice of her 3-dimensional indifference map over x_1, x_2 and x_3. Draw such a slice for the indifference curve that contains the consumer's optimal bundle A on the budget from part (a).*

Answer: The 2-dimensional slice of the indifference map will look exactly like our typical indifference maps over 2 goods. The optimal bundle *A* is illustrated as the bundle at the tangency of an indifference curve from this slice with the budget constraint from part (a).

(c) *Now suppose that Gucci manages to raise the prestige value of its products — and thus x_3 that comes with the purchase of Gucci products. For now, suppose they do this without changing p_1. This implies you will shift to a different 2-dimensional slice of your 3-dimensional indifference map. Illustrate the new 2-dimensional indifference curve that contains A. Is the new MRS at A greater or smaller in absolute value than it was before?*

Answer: This is illustrated in panel (b) of the graph. The increase in prestige implies the consumer is willing to pay more for any additional Gucci products — thus the *MRS* increases in absolute value.

(d) *Would the consumer consume more or fewer Gucci products after the increase in prestige value?*

Answer: All the bundles that lie above the indifference curve through *A* in panel (b) of the graph contain more Gucci products. The consumer will now optimize at some new bundle such as *B*.

(e) *Now suppose that Gucci manages to convince consumers that Gucci products become more desirable the more expensive they are. Put differently, the prestige value x_3 is linked to p_1, the price of the Gucci products. On a new graph, illustrate the change in the consumer's budget as a result of an increase in p_1.*

Answer: This change in the budget is no different than it would usually be — and is illustrated as part of panel (c) of Exercise Graph 7.9.

(f) *Suppose that our consumer increases her purchases of Gucci products as a result of the increase in the price p_1. Illustrate two indifference curves — one that gives rise to the original optimum A and another that gives rise to the new optimum C. Can these indifference curves cross?*

Answer: This is illustrated in panel (c) of the Graph. Since the indifference curve u^C is drawn from a different 2-dimensional slice of the 3-dimensional indifference curve over x_1, x_2 and x_3 than the indifference curve u^A, the two indifference curves can indeed cross.

(g) *Explain why, even though the behavior is consistent with what we would expect if Gucci products were a Giffen good, Gucci products are not a Giffen good in this case.*

Answer: Gucci products in this example are really bundles of 2 products — the physical product itself, and the prestige value that comes with the product. When price increases, the prestige value increases — which means we are no longer dealing with the same product as before (even though the physical characteristics of the product remain the same). Thus, while the consumer is indeed buying more Gucci products after the price increase, she is also buying more prestige that is bundled with the physical product. In terms of our 3-dimendional indifference curves, she is shifting to a different x_3 level because p_1 is higher. *Holding all else fixed*, she would not buy more Gucci products as price increases — it is only because she is buying more prestige at the higher price that it looks like she is buying more as price increases.

(h) *In a footnote in the chapter we defined the following: A good is a* Veblen good *if* preferences *for the good change as price increases — with this change in preferences possibly leading to an increase in consumption as price increases. Are Gucci products a Veblen good in this exercise?*

Answer: Yes — as price increases, tastes (i.e. the indifference map in 2 dimensions) change in the sense that we are shifting to a different slice of the true 3-D indifference surfaces. The resulting increased consumption of Gucci products as price increases is due to this "change in tastes" — or, to put it more accurately, to the change in the product that looks like a change in tastes when we graph our 2-dimensional indifference curves. This is different from Giffen behavior where the indifference map does not change with an increase in price — but consumption does.

B: *Consider the same definition of x_1, x_2 and x_3 as in part A. Suppose that the tastes for our consumer can be captured by the utility function $u(x_1, x_2, x_3) = \alpha x_3^2 \ln x_1 + x_2$.*

(a) *Set up the consumer's utility maximization problem — keeping in mind that x_3 is not a choice variable.*

Answer: The maximization problem is

$$\max_{x_1,x_2} \alpha x_3^2 \ln x_1 + x_2 \text{ subject to } p_1 x_1 + x_2 = I. \tag{7.9.i}$$

(b) *Solve for the optimal consumption of x_1 (which will be a function of the prestige value x_3).*

Answer: The Lagrange function for this problem is

$$\mathcal{L}(x_1, x_2, \lambda) = \alpha x_3^2 \ln x_1 + x_2 + \lambda(I - p x_1 - x_2). \tag{7.9.ii}$$

Solving this the usual way, we get

$$x_1 = \frac{\alpha x_3^2}{p_1} \text{ and } x_2 = I - \alpha x_3^2. \tag{7.9.iii}$$

(c) *Is x_1 normal or inferior? Is it Giffen?*

Answer: x_1 does not vary with income — thus making it quasilinear. Put differently, x_1 is borderline between normal and inferior. At the same time, x_1 falls with p_1 — implying that consumers will buy less x_1 as p_1 increases all else being equal. Thus, x_1 is not a Giffen good.

(d) *Now suppose that prestige value is a function of p_1. In particular, suppose that $x_3 = p_1$. Substitute this into your solution for x_1. Will consumption increase or decrease as p_1 increases?*

Answer: This implies that

$$x_1 = \frac{\alpha p_1^2}{p_1} = \alpha p_1. \tag{7.9.iv}$$

Thus, consumption of x_1 increases as p_1 increases.

(e) *How would you explain that x_1 is not a Giffen good despite the fact that its consumption increases as p_1 goes up?*

Answer: In order for x_1 to be a Giffen good, consumption of x_1 would have to increase with an increase in p_1 *all else remaining equal.* We showed in (b) that this is not the case — all else (including prestige) remaining constant, an increase in p_1 leads to a decrease in x_1. The only reason that x_1 increases as p_1 increases is that we allow p_1 to change the prestige value of Gucci products — and thus the very nature of those products.

Exercise 7.11

Policy Application: *Substitution Effects and Social Security Cost of Living Adjustments*: In end-of-chapter exercise 6.16, you investigated the government's practice for adjusting social security income for seniors by insuring that the average senior can always afford to buy some average bundle of goods that remains fixed. To simplify the analysis, let us again assume that the average senior consumes only two different goods.

A: *Suppose that last year our average senior optimized at the average bundle A identified by the government, and begin by assuming that we denominate the units of x_1 and x_2 such that last year $p_1 = p_2 = 1$.*

(a) *Suppose that p_1 increases. On a graph with x_1 on the horizontal and x_2 on the vertical axis, illustrate the compensated budget and the bundle B that, given your senior's tastes, would keep the senior just as well off at the new price.*

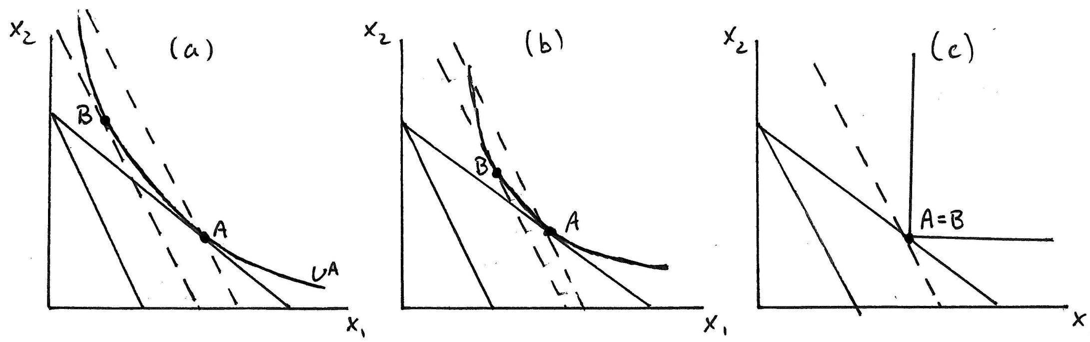
Answer: In panel (a) of Exercise Graph 7.11(1), bundle A lies on the original (solid line) budget. The price increase causes an inward rotation of that budget in the absence of compensation. To compensate the person so that he will be as happy as before, we have to raise income to the lower dashed line in the graph — the line that is tangent to B that lies on the indifference curve u^A.

(b) *In your graph, compare the level of income the senior requires to get to bundle B to the income required to get him back to bundle A.*

Answer: The income required (at the new prices) to get to A is represented by the second dashed line in panel (a) of the graph.

(c) *What determines the size of the difference in the income necessary to keep the senior just as well off when the price of good 1 increases as opposed to the income necessary for the senior to still be able to afford bundle A?*

Answer: The greater the substitutability of the two goods, the greater will be the difference between the two ways of compensating the person. This is illustrated across the three panels in Exercise Graph 7.11(1) where the degree of substitutability falls from left to right.



Exercise Graph 7.11(1) : Hicks and Slutsky Social Security Compensation

(d) *Under what condition will the two forms of compensation be identical to one another?*

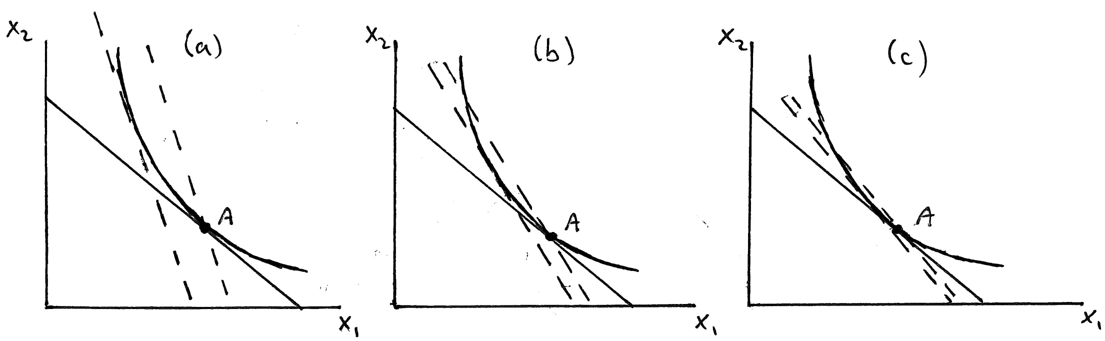
Answer: The difference between the two compensation schemes disappears entirely in panel (c) of the graph when there is no substitutability between the goods (i.e. when they are perfect complements).

(e) *You should recognize the move from A to B as a pure substitution effect as we have defined it in this chapter. Often this substitution effect is referred to as the* Hicksian substitution effect — *defined as the change in behavior when opportunity costs change but the consumer receives sufficient compensation* to remain just as happy. *Let B' be the consumption bundle the average senior would choose when compensated so as to be able to afford the original bundle A. The movement from A to B' is often called the* Slutsky substitution effect — *defined as the change in behavior when opportunity costs change but the consumer receives sufficient compensation* to be able to afford to stay at the original consumption bundle. True or False*: The government could save money by using Hicksian rather than Slutsky substitution principles to determine appropriate cost of living adjustments for social security recipients.*

Answer: The answer is true. The government in fact uses Slutsky compensation as it calculates cost of living adjustments — because it fixes a particular consumption bundle and then adjusts social security checks to make sure that seniors can still afford that bundle. For this reason, you will frequently hear proposals to adjust the way in which cost of living adjustments are calculated — with these proposals attempting to get closer to Hicksian compensation.

(f) True or False*: Hicksian and Slutsky compensation get closer to one another the smaller the price changes.*

Answer: This is true. Larger price changes result in larger substitution effects — and the difference between Hicksian and Slutsky substitution is entirely due to the substitution effect. This is illustrated in the three panels of Exercise Graph 7.11(2) where, going from left to right, the size of the price change (as evidenced in the steepness of the slope of the compensated budget) decreases.



Exercise Graph 7.11(2) : Hicks and Slutsky Social Security Compensation: Part II

B: *Now suppose that the tastes of the average senior can be captured by the Cobb-Douglas utility function $u(x_1, x_2) = x_1x_2$, where x_2 is a composite good (with price by definition equal to $p_2 = 1$). Suppose the average senior currently receives social security income I (and no other income) and with it purchases bundle (x_1^A, x_2^A).*

(a) *Determine (x_1^A, x_2^A) in terms of I and p_1.*

Answer: Solving the usual maximization problem with budget constraint $p_1x_1 + x_2 = I$, we get

$$x_1^A = \frac{I}{2p_1} \text{ and } x_2^B = \frac{I}{2}. \tag{7.11.i}$$

(b) *Suppose that p_1 is currently \$1 and I is currently \$2000. Then p_1 increases to \$2. How much will the government increase the social security check given how it is actually calculating cost of living adjustments? How will this change the senior's behavior?*

Answer: The government compensates so as to make it possible for the senior to keep affording the same bundle as before. With the values $p_1 = 1$ and $I = 2000$, $x_1^A = x_2^A = 1000$. When the price of x_1 goes to \$2, this same bundle costs $2(1000) + 1000 = \$3,000$. Thus, the government is compensating the senior by increasing the social security check by \$1,000.

With an income of \$3,000, equations (7.11.i) then tell us that the senior will consume $x_1 = 3000/(2(2)) = 750$ and $x_2 = 3000/2 = 1,500$. Thus, even though the government makes it possible for the senior to consume bundle A again after the price change, the senior will substitute away from x_1 because its opportunity cost is now higher.

(c) *How much would the government increase the social security check if it used Hicksian rather than Slutsky compensation? How would the senior's behavior change?*

Answer: If the government used Hicksian compensation, it would first need to calculate the bundle B on the original indifference curve that would make the senior just as well off at the higher price as he was at A. At A, the senior gets utility $u^A = x_1^A x_2^A = 1000(1000) = 1,000,000$. The government would then have to solve the problem

$$\min_{x_1,x_2} 2x_1 + x_2 \text{ subject to } x_1 x_2 = 1,000,000. \tag{7.11.ii}$$

Solving the first two first order conditions, we get $x_2 = 2x_1$. Substituting this into the constraint and solving for x_1, we get $x_1 \approx 707.1$, and plugging this back into $x_2 = 2x_1$, we get $x_2 = 1414.2$. This bundle $B = (707.1, 1414.2)$ costs $2(707.1)+1414.2 = 2828.4$. Thus, under Hicksian compensation, the government would increase the senior's social security check by \$828.40 rather than \$1,000.

(d) *Can you demonstrate mathematically that Hicksian and Slutsky compensation converge to one another as the price change gets small — and diverge from each other as the price change gets large?*

Answer: We start with $p_1 = 1$ (and continue to assume $p_2 = 1$).[2] Then suppose p_1 increases to $p_1 > 1$ (or falls to $p_1 < 1$). Slutsky compensation requires that we continue to be able to purchase $A = (1000, 1000)$ — so we have to make sure the senior has income of $1000p_1 + 1000$. Since the senior starts with an income of \$2,000, this implies that Slutsky compensation is $1000p_1 + 1000 - 2000 = 1000p_1 - 1000 = 1000(p_1 - 1)$.

Hicksian compensation, on the other hand, requires we calculate the substitution effect to B as we did in the previous part for $p_1 = 2$. Setting up the same problem but letting the new price of good 1 be denoted p_1 rather than 2, we can calculate $B = (x_1^B, x_2^B) = (1000/p_1^{0.5}, 1000p_1^{0.5})$. This bundle costs

$$p_1 \frac{1000}{p_1^{0.5}} + 1000p_1^{0.5} = 2000p_1^{0.5}. \tag{7.11.iii}$$

Given that the senior starts with \$2000, this means that Hicksian compensation must be equal to $2000p_1^{0.5} - 2000 = 2000(p_1^{0.5} - 1)$.

The difference between Slutsky compensation and Hicksian compensation, which we will call $D(p_1)$ is then

[2] We could start with any other price and change either p_1 or p_2 and the same logic will hold.

$$D(p_1) = 1000(p_1 - 1) - 2000(p_1^{0.5} - 1) = 1000p_1 - 1000 - 2000p_1^{0.5} + 2000$$
$$= 1000 + 1000p_1(1 - 2p_1^{-0.5}). \quad (7.11.iv)$$

As p_1 approaches 1, the second term in the equation goes to -1000 — making the expression go to zero; i.e. the difference between the two types of compensation goes to zero as the price increase (or decrease) gets small. In fact, it is easy to see that this difference reaches it's lowest point when $p_1 = 1$ and increases when p_1 rises above 1 as well as when p_1 falls below 1: Simply take the derivative of $D(p_1)$ which is

$$\frac{dD(p_1)}{dp_1} = 1000\left(1 - 2p_1^{-0.5}\right) + 1000p_1\left(p_1^{-1.5}\right) = 1000\left(1 - p_1^{-0.5}\right). \quad (7.11.v)$$

Then note that $dD/dp_1 < 0$ when $0 < p_1 < 1$, $dD/dp_1 = 0$ when $p_1 = 1$ and $dD/dp_1 > 0$ when $p_1 > 1$. This implies a U-shape for $D(p_1)$, with the U reaching its bottom at $p_1 = 1$ when $D(p_1) = 0$. Put into words, the difference between Slutsky and Hicks compensation is positive for any price not equal to the original price, with the difference increasing the greater the deviation in price from the original price.

(e) *We know that Cobb-Douglas utility functions are part of the CES family of utility functions — with the elasticity of substitution equal to 1. Without doing any math, can you estimate, for an increase in p_1 above 1, the range of how much Slutsky compensation can exceed Hicksian compensation with tastes that lie within the CES family? (*Hint*: Consider the extreme cases of elasticities of subsitution.)*

Answer: We know that if the two goods are perfect complements (with elasticity of substitution equal to 0), then there is no difference between the two compensation mechanisms (because, as we demonstrated in part A of the question, the difference is due entirely to the substitution effect). Thus, one end of the range of how much Slutsky compensation can exceed Hicksian compensation is zero.

The other extreme is the case of perfect substitutes. In that case, it is rational for the consumer to choose bundle A initially since the prices are identical and the indifference curve therefore lies on top of the budget line (making all bundles on the budget line optimal). But any deviation in price will result in a corner solution. Thus, if p_1 increases, the consumer can remain just as well off as she was originally by simply not consuming x_2. Thus, Hicksian compensation is zero while Slutsky compensation still aims to make bundle A affordable — i.e. Slutsky compensation is still $1000(p_1 - 1)$ as we calculated in part (d). So in this extreme case, Slutsky compensation exceeds Hicksian compensation by $1000(p_1 - 1)$.

Depending on the elasticity of substitution, Slutsky compensation may therefore exceed Hicksian compensation by as little as 0 (when the elasticity is 0) to as much as $1000(p_1 - 1)$ (when the elasticity is infinite).

Exercise 7.13

Policy Application: *Public Housing and Housing Subsidies*: *In exercise 2.14, you considered two different public housing programs in parts A(a) and (b) — one where a family is simply offered a particular apartment for a below-market rent and another where the government provides a housing price subsidy that the family can use anywhere in the private rental market.*

A: *Suppose we consider a family that earns \$1500 per month and either pays 50 cents per square foot in monthly rent for an apartment in the private market or accepts a 1500 square foot government public housing unit at the government's price of \$500 per month.*

(a) *On a graph with square feet of housing and "dollars of other consumption", illustrate two cases where the family accepts the public housing unit — one where this leads them to consume less housing than they otherwise would, another where it leads them to consume more housing than they otherwise would.*

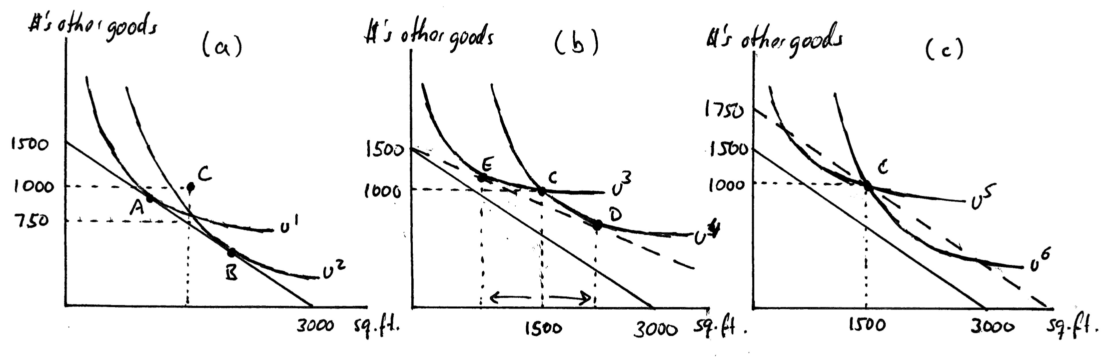
Answer: The budget constraint in the absence of public housing is drawn in panel (a) of Exercise Graph 7.13. Bundle A is optimal under tastes with indifference curve u^1 while bundle B is optimal under tastes with indifference curve u^2. (Since these indifference curves cross, they of course cannot come from the same indifference map — and thus come from different indifference maps representing different tastes.) The public housing unit permits the household to consume C — the 1500 square foot public housing unit costing \$500 (and thus leaving the household with \$1000 of other consumption). Both the household that optimizes at A and the one that optimizes at B in the absence of the public housing option will choose C if it becomes available. For household 1 this implies that public housing increases its housing consumption, but for household 2 it implies that public housing decreases housing consumption.

(b) *If we use the household's own judgment about its well-being, is it always the case that the option of public housing makes the households who choose to participate better off?*

Answer: Yes — the household would not choose the option unless it thought it is better off. In panel (a) of the graph, both households end up on higher indifference curves when choosing C.

(c) *If the policy goal behind public housing is to increase the housing consumption of the poor, is it more or less likely to succeed the less substitutable housing and other goods are?*

Answer: The less substitutable housing and other goods are, the sharper the tangency at the optimum on the original budget line. And the sharper the tangency, the less likely it is that a household can consume more than



Exercise Graph 7.13 : Public Housing and Rental Subsidies

1500 square feet of housing in the absence of public housing and still become better off at C in our graph. For instance, in panel (a) of the graph it is possible for A to be optimal and C to be better even if housing and other goods are perfect complements — but this is not true for B.

(d) *What is the government's opportunity cost of owning a public housing unit of 1500 square feet? How much does it therefore cost the government to provide the public housing unit to this family?*

Answer: The government could charge the market price of \$0.50 per square foot for the 1500 square foot public housing unit. It is therefore giving up \$750 in rent by not renting it on the open market — and it is collecting only \$500 from the public housing participant. Thus, the cost the government incurs is \$250 per month. You can also see this in panel (a) of our graph — as the vertical difference between C and the budget line.

(e) *Now consider instead a housing price subsidy under which the government tells qualified families that it will pay some fraction of their rental bills in the private housing market. If this rental subsidy is set so as to make the household just as well off as it was under public housing, will it lead to more or less consumption of housing than if the household chooses public housing?*

Answer: Panel (b) of Exercise Graph 7.13 illustrates that such a subsidy could lead to more or less consumption of housing.

(f) *Will giving such a rental subsidy cost more or less than providing the public housing unit? What does your answer depend on?*

Answer: It may cost more or less. If the household consumes less housing under the rental subsidy (as with indifference curve u^3), it will definitely cost less. (In the graph, the cost is the vertical difference between E and the original budget constraint — which must be smaller than the \$250 difference between C and the original constraint.) But if the rental subsidy results in more housing consumption than public housing (as with indifference curve u^D), it may cost the government more or less depending on just how much more housing is consumed.

(g) *Suppose instead that the government simply gave cash to the household. If it gave sufficient cash to make the household as well off as it is under the public housing program, would it cost the government more or less than \$250? Can you tell whether under such a cash subsidy the household consumes more or less housing than under public housing?*

Answer: It will definitely cost the government less (or at least no more) but we can't tell whether it will result in greater or lesser housing consumption. This is illustrated in panel (c) of Exercise Graph 7.13 where the dashed budget line results from the government giving \$250 in cash — the same as it spends under the public housing program. Unless the slope of the indifference curve at C just happens to be the same as the slope of the budget line, the new budget line will cut the indifference curve that contains C either from above (as in u^5) or from below (as in u^6). Either way, the household would be able to make itself better off by reaching a higher indifference curve. Thus, except for the special case where the budget line has the same slope as the indifference curve at C, it will cost the government less than \$250 to make the household as well off as it is under public housing. Put differently, there are smaller budgets with the same slope that are tangent to u^5 and u^6. But at those tangencies, housing consumption will fall below 1500 square feet in the case of u^5 and rise above 1500 in the case of u^6.

B: *Suppose that household tastes over square feet of housing (x_1) and dollars of other consumption (x_2) can be represented by* $u(x_1, x_2) = \alpha \ln x_1 + (1-\alpha)\ln x_2$.

(a) *Suppose that empirical studies show that we spend about a quarter of our income on housing. What does that imply about α?*

Answer: These are Cobb-Douglas tastes (equivalent to $u(x_1, x_2) = x_1^{\alpha} x_2^{(1-\alpha)}$ which, when transformed by the natural log, turns into the one given in the problem). When the exponents of a Cobb-Douglas utility function sum to 1, the exponents denote the fraction of income spent on each good. Thus, if households spend a quarter of their income on housing, then $\alpha = 0.25$.

(b) *Consider a family with income of \$1,500 per month facing a per square foot price of $p_1 = 0.50$. For what value of α would the family not change its housing consumption when offered the 1500 square foot public housing apartment for \$500?*

Answer: 1500 square feet cost \$750 — which is half of the household's income of \$1,500. Given what we said about exponents in Cobb-Douglas utility functions representing budget shares, α would have to be 0.5 in order for the household to spend half its income on housing.

(c) *Suppose that this family has α as derived in B(a). How much of a rental price subsidy would the government have to give to this family in order to make it as well off as the family is with the public housing unit?*

Answer: With the public housing unit, the family consumes the bundle (1500, 1000) — which gives utility

$$u(1500,1000) = 0.25\ln 1500 + 0.75\ln 1000 \approx 7.009. \tag{7.13.i}$$

If you solve the maximization problem

$$\max_{x_1,x_2}\ 0.25\ln x_1 + 0.75\ln x_2 \ \text{ subject to } \ p_1x_1 + x_2 = 1500, \tag{7.13.ii}$$

you get

$$x_1 = \frac{0.25(1500)}{p_1} = \frac{375}{p_1} \ \text{ and } \ x_2 = 0.75(1500) = 1125. \tag{7.13.iii}$$

Plugging these back into the utility function, we get

$$\begin{aligned} u\left(\frac{375}{p_1}, 1125\right) &= 0.25\ln\frac{375}{p_1} + 0.75\ln 1125 \\ &= 0.25\ln 375 - 0.25\ln p_1 + 0.75\ln 1125 \approx 6.751 - 0.25\ln p_1. \end{aligned} \tag{7.13.iv}$$

In order for p_1 to be subsidized to a point where it makes the household indifferent between getting the subsidy and participating in the public housing program, this utility has to be equal to the utility of public housing (which is 7.009); i.e.

$$6.751 - 0.25\ln p_1 = 7.009. \tag{7.13.v}$$

Solving this for p_1, we get $p_1 \approx 0.356$. Thus, the subsidy that would make the household indifferent requires that the government pay a fraction of about 0.288 of rental housing (which reduces the price from 0.5 to 0.356).

(d) *How much housing will the family rent under this subsidy? How much will it cost the government to provide this subsidy?*

Answer: The household would rent

$$x_1 = \frac{0.25(1500)}{0.356} \approx 1053, \tag{7.13.vi}$$

which is less than it consumes under public housing. A house with 1053 square feet costs 1053(0.5) = 526.50 to rent — and the government under this subsidy pays 28.8% of this cost — i.e. the program costs 0.288(526.5) ≈ 151.63.

(e) *Suppose the government instead gave the family cash (without changing the price of housing). How much cash would it have to give the family in order to make it as happy?*

Answer: We already determined that the utility of participating in the public housing program is 7.009. You can find the amount of income necessary to get to that utility level in different ways. One way is to solve the minimization problem

$$\min_{x_1,x_2} \; 0.5x_1 + x_2 \; \text{ subject to } \; 0.25\ln x_1 + 0.75\ln x_2 = 7.009. \qquad (7.13.vii)$$

The first two first order conditions give us $x_2 = 1.5x_1$. Substituting into the constraint, we get

$$7.009 = 0.25\ln x_1 + 0.75\ln(1.5x_1) = \ln x_1 + 0.75\ln(1.5) \approx \ln x_1 + 0.304. \qquad (7.13.viii)$$

Solving for x_1, we get $x_1 \approx 816.5$, and substituting back into $x_2 = 1.5x_1$, $x_2 \approx 1224.75$. This bundle costs $0.5(816.5) + 1224.75 = 1633$. Since the household starts with \$1,500, this implies that a monthly cash payment of \$133 would make the household as well off as the public housing program (that costs \$250 per month).

(f) *If you are a policy maker whose aim is to make this household happier at the least cost to the taxpayer, how would you rank the three policies? What if your goal was to increase housing consumption by the household?*

Answer: We have calculated that the public housing policy costs \$250 per month, the rent subsidy costs approximately \$156 per month and the cash subsidy costs \$133 per month. All three policies result in the same level of household utility. So if increasing happiness at the least cost is the goal, the cash subsidy would be best, followed by the rental subsidy and then the public housing program.

We also calculated that housing consumption will be 1500 square feet under public housing, 1053 square feet under the rental subsidy and 816.5 square feet under the cash subsidy. If the goal is to increase housing consumption, the public housing program dominates the rental subsidy which dominates the cash subsidy.

Conclusion: Potentially Helpful Reminders

1. *Important Graphing Hint*: When graphing income and substitution effects, it is very helpful to draw the original indifference curve with lots of substitutability — i.e. with relatively little curvature — unless specifically told to do otherwise. If you do this, it becomes much harder to trick yourself into thinking that something which is logically impossible is actually happening in your graphs.

2. Keep in mind the following: Substitution effects always occur *along a single indifference curve* and income effects always involve *jumping from one indifference curve to another across two parallel budgets.*

3. Since concepts like homotheticity, quasilinearity, normal and inferior goods, and luxuries and necessities are definitions about how indifference curves within an indifference map relate to one another, they are relevant only for determining income effects. In fact, we can get both large and small substitution effects for any of these types of tastes and goods — with the size of the substitution effect depending on the curvature of the original indifference curve (which has no relation to whether goods are normal or inferior or homothetic, etc.).

4. In the text, we emphasize the more common of the two types of substitution effects that economists talk about — the effect that holds "real welfare" fixed and thus occurs along an indifference curve. This effect is also called the *Hicks substitution effect* and it differs from a second type of substitution effect (called the *Slutsky substitution effect*) that assumes a consumer is compensated enough to afford the original bundle (rather than to reach the original indifference curve). This second type of substitution effect is almost identical to the first, particularly for small changes in prices — and it appears in end-of-chapter exercises 7.6 and 7.11 for you to explore.

5. Often students confuse Giffen goods with a certain type of "prestige good" that people value more as it gets more expensive. That is definitely not what a Giffen good is — and you can do end-of-chapter exercise 7.9 to work through the difference between these two types of goods.

6. The math (in part B of *Microeconomics: An Intuitive Approach with Calculus*) follows straightforwardly from the graphical intuitions: Maximize utility subject to budget constraints to get what people do at bundles A and C (when income and substitution effects are combined) — but minimize the expenditure it takes to get to the original utility level at the new prices to find B (and thus the substitution effect).