1. (1) $Q_1 = (K^\rho + L^\rho)^{1/\rho}$

$Q_2 = [(2K)^\rho + (2L)^\rho]^{1/\rho} = 2Q_1$

(2) $MP_L = dQ/dL = L^{\rho-1}(K^\rho + L^\rho)^{\frac{1-\rho}{\rho}}$

$MP_K = dQ/dK = K^{\rho-1}(K^\rho + L^\rho)^{\frac{1-\rho}{\rho}}$

(3) $MRTS_{L,K} = MP_L / MP_K = (L/K)^{\rho-1}$

(4) $E_L = \frac{\Delta Q/Q}{\Delta L/L} = L^\rho (K^\rho + L^\rho)^{-1}$

$E_K = \frac{\Delta Q/Q}{\Delta K/K} = K^\rho (K^\rho + L^\rho)^{-1}$

$E_L + E_K = 1$

(5) $L' = L/2 \quad K' = K/2$

$Q' = \frac{1}{2}Q \quad Q_1 = Q_2 = Q'$

$Q_1 + Q_2 = Q$

2. (1) $TC = A + L + 32$

$Q = A^{1/4} L^{1/4} 16^{1/4}$

$$\begin{cases} \text{Min} & TC = A + L + 32 \\ \text{s.t.} & Q = 4A^{1/4}L^{1/4} \end{cases}$$

$\mathcal{L} = A + L + 32 - \lambda(Q - 4A^{1/4}L^{1/4})$

$$\begin{cases} \partial\mathcal{L}/\partial A = 0 \\ \partial\mathcal{L}/\partial L = 0 \\ \partial\mathcal{L}/\partial\lambda = 0 \end{cases} \text{解得} \begin{cases} A = L \\ A = 1/4\, Q^2 \end{cases}$$

$\therefore TC = 1/8\, Q^2 + 32$

$AC = 1/8\, Q + 32/Q$

(2) $TVC = 1/8\, Q^2$

$AVC = 1/8\, Q$

(3) $MC = \frac{\partial TC}{\partial Q} = \frac{1}{4}Q$

3. (1) $LRTC(Q) = L + 4K$

$Q = L^{2/3} K^{1/3}$

$$\begin{cases} \text{Min} & LRTC(Q) = L + 4K \\ \text{s.t.} & Q = L^{2/3} K^{1/3} \end{cases}$$

$\mathcal{L} = L + 4K - \lambda(Q - L^{2/3}K^{1/3})$

$$\begin{cases} \partial\mathcal{L}/\partial K = 0 \\ \partial\mathcal{L}/\partial L = 0 \\ \partial\mathcal{L}/\partial\lambda = 0 \end{cases} \text{解得} \begin{cases} L = 8K \\ L = 2Q \end{cases}$$

$\therefore LRTC(Q) = 2Q + Q = 3Q$

(2) $Q = L^{2/3} K_s^{1/3}$

$L = Q^{3/2} K_s^{-1/2}$

$SRTC(Q) = L + 4K_s = Q^{3/2}K_s^{-1/2} + 4K_s$

(3) 短期内，厂商只能调节劳动投入但不能调节资本投入，因此短期内的成本函数是厂商在既定资本水平下的最低成本；而长期内，厂商既可以调节劳动投入，又可以调节资本投入，因此长期的成本函数是厂商在最优生产规模下的最低成本。从几何上说，厂商的长期成本曲线是短期成本曲线的包络线。因此对于既定的产量，长期成本小于等于短期成本。

4. (1)
$$\begin{cases} \text{Min} & w_1x_1 + w_2x_2 \\ \text{s.t.} & y = x_1^a x_2^b \end{cases}$$

$\mathcal{L} = w_1x_1 + w_2x_2 - \lambda(y - x_1^a x_2^b)$

$$\begin{cases} \partial\mathcal{L}/\partial x_1 = w_1 - \lambda a x_1^{a-1} x_2^b = 0 \\ \partial\mathcal{L}/\partial x_2 = w_2 - \lambda b x_1^a x_2^{b-1} = 0 \\ \partial\mathcal{L}/\partial\lambda = y - x_1^a x_2^b = 0 \end{cases}$$

解得
$$\begin{cases} x_1 = \left(\frac{w_2 a}{w_1 b}\right)^{\frac{b}{a+b}} y^{1/(a+b)} \\ x_2 = \left(\frac{w_1 b}{w_2 a}\right)^{\frac{a}{a+b}} y^{1/(a+b)} \end{cases}$$

(2) $y = x_1^a k^b$

$x_1 = (y/k^b)^{1/a}$

$TC = w_1(y/k^b)^{1/a} + w_2 k$

$AC = w_1 k^{-b/a} y^{\frac{1}{a}-1} + \frac{w_2 k}{y}$

$AVC = w_1 k^{-b/a} y^{\frac{1}{a}-1}$

$AFC = \frac{w_2 k}{y}$

5. (1) $TC_A = 8^2 + 50 = 114$

$TC_B = \frac{1}{2} \times 8^2 + 2 \times 8 + 80 = 128$

$114 < 128 \quad \therefore$ 选方案A

(2) $TC_B \le TC_A$

$\Delta TC = TC_B - TC_A = -\frac{1}{2}Q^2 + 2Q + 30$

$\Delta TC \le 0$ 解得 $Q \ge 10$

(3) $MC_A = MC_B$ 时 TC 最小

$MC_A = \frac{\partial TC_A}{\partial Q_A} = 2Q_A$

$MC_B = \frac{\partial TC_B}{\partial Q_B} = Q_B + 2$

$$\begin{cases} Q_A + Q_B = 22 \\ 2Q_A = Q_B + 2 \end{cases} \text{解得} \begin{cases} Q_A = 8 \\ Q_B = 14 \end{cases}$$

$TC = 114 + (\frac{1}{2} \times 14^2 + 2 \times 14 + 80) = 320$